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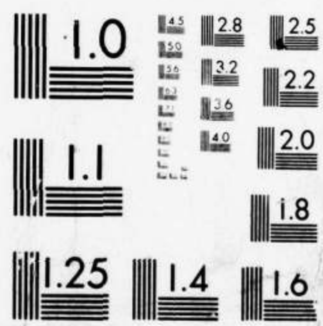

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(10) W. Symes

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RELATIONS AMONG GENERALIZED KORTEWEG-deVRIES SYSTEMS

W. Symes

Technical Summary Report #1823
January 1978

ABSTRACT

This report presents certain relations among the completely integrable Hamiltonian systems introduced by Gel'fand and Dikii. These relations generalize a formula of A. Lenard linking the higher-order Korteweg-deVries equations, of which the Gel'fand-Dikii Systems are a generalization. The general form of the relations, which connect the various isospectral deformations of linear differerential operators, is described, and two examples are given explicitly.

AMS(MOS) Subject Classification: 35A25

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Calculus of Symbols

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SIGNIFICANCE AND EXPLANATION

In recent years several nonlinear partial differential equations of applied mathematics have been discovered to have the extraordinary property known as complete integrability: that is, roughly speaking, they possess the maximum number of constants of motion possible for the type of system considered. These equations arise in the study of shallow water waves, accoustic waves in plasmas (Korteweg-deVries equation) nonlinear optics, Josephson junction theory (sine-Gordon equation), other plasma phenomena (nonlinear Schrödinger equation), and other areas. The complete integrability property - again, roughly speaking - allows unusually explicit solution of these equations.

These equations are undoubtedly very special. It seems important to understand the place occupied by these special systems in the general aggregate of partial differential equations. Recently, I. M. Gel'fand and L. A. Dikii have succeeded in isolating several key facets of the structure of these systems, simultaneously constructing families of hitherto unknown systems with the complete integrability property. This construction reveals slightly more clearly the nature of these systems.

Each of the aforementioned partial differential equations - Korteweg-deVries, sine-Gordon, nonlinear Schrödinger - is embedded in a heirachy of equations, each having the complete integrability property, and each related to the others by a common rule, discovered in the case of the Korteweg-deVries heirarchy by A. Lenard. This relation allows a transparent derivation of some of the extraordinary structure of the Korteweg-deVries equation. This report presents a similar rule relating the various hierarchies of systems introduced by Gel'fand and Dikii.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

RELATIONS AMONG GENERALIZED KORTEWEG-deVRIES SYSTEMS

W. Symes

§0. Introduction

In this short note we prove the existence of certain relationships amongst the Hamiltonian systems constructed by Gel'fand and Dikii in [1]. These relations generalize a formula of A. Lenard linking the higher-order Korteweg-deVries equations, of which the systems in [1] are a generalization. We refer the reader to [2], [3] for a description of Lenard's result, which is also reproduced as an example at the end of this paper, and for an explanation of its importance in the theory of the Korteweg-deVries equation and its higher-order relatives. We anticipate similar applications to the Gel'fand-Dikii systems.

Mark Adler has derived the generalized Lenard relations of this paper in somewhat different ways, first for hierarchies of systems related to the Boussinesq equation and to a certain fourth-order eigenvalue problem, [3], then in general for the Gel'fand Dikii systems [4]. His work will appear elsewhere.

I would like to thank Mark Adler, Charles Conley, and Neil Fenichel for a number of illuminating conversations on this subject.

§1. Formal Isospectral Deformations.

Let L be a linear ordinary differential expression of order n:

$$L = D^n + \sum_{k=0}^{n-2} q_k D^k , \qquad D = -i\frac{d}{dx} . \tag{1}$$

The domain of definition of the coefficients q_k, which are supposed infinitely differentiable, is some open interval, U, finite or not, in the real line.

A <u>formal isospectral deformation</u> of L is a specification

$$\dot{L} = \sum_{k=0}^{n-2} \dot{q}_k D^k = [P,L] \tag{2}$$

where P is a differential expression whose coefficients depend polynomially on $q_0,\ldots,q_{n-2}$ and their derivatives, having the property that the commutator appearing on the right-hand side of (2) is of order $n-2$ or less. Thus (2) may be regarded as a collection of $(n-1)$ partial differential equations for the coefficients q_k, where the dot is interpreted to mean differentiation with respect to a ("time") parameter. Peter Lax has shown that, if L is provided with a suitable function space domain, becoming an operator and attaining a spectrum,

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then a solution of such a system of partial differential equations represents a deformation of L preserving its spectrum (See [5]).

For example, let $n = 2$. Then

$$L = D^2 + q_0 , \quad P = D^3 + \frac{3}{2} qD + \frac{3}{4} (Dq)$$

and the equation (2) is equivalent to the Korteweg-deVries equation for the coefficient q_0:

$$-i \dot{q}_0 = \frac{1}{4} D^3 q_0 + \frac{3}{4} q Dq_0 .$$

Of course, the existence of a suitable expression P is clearly a local matter, having nothing whatever to do with selection of a domain for L. Gel'fand and Dikii present a construction in [1] for all such P, based on the local algebra of symbols, which is the formal side of the calculus of pseudodifferential operators. We give a very brief review of this construction in §§2,3, and 4. All of the statements in §§2,3 are formal counterparts of results of Seeley, [6], and proofs are also sketched in [1]. We therefore omit the proofs.

§2. Symbol Algebra

By intentional abuse of notation, denote also by L the symbol of L,

$$L[q,\xi] = \xi^n + \sum_{k=0}^{n-2} q_k \xi^k \tag{3}$$

where we have emphasized the dependence of the *symbol* L on the coefficient vector $q = (q_0,\ldots,q_{n-2})$.

More generally, a symbol is a formal sum

$$A(x,\xi) = \sum_{\ell=0}^{\infty} A_\ell(x,\xi) \tag{4}$$

where A_ℓ is smooth and complex-valued in $U \times \mathbb{R} \setminus \{0\}$, and homogeneous of degree d_ℓ in ξ for large $|\xi|$, with

$$d_0 > d_1 > \ldots > d_\ell > \ldots \to -\infty$$

a sequence of real numbers tending to $-\infty$.

Symbols are considered the same if they agree, term-by-term, with each other for ξ outside some sompact neighborhood of $0 \in \mathbb{R}$. In that case we will write the sign of equality, remembering that it means "for large $|\xi|$".

The symbols form a module over $C^\infty(U)$, with the module operations defined pointwise. They form an algebra over $\mathbb{C}$, with the multiplication operation $\circ$ defined by

$$(A,B) \mapsto A \circ B = \sum_{\nu \geq 0} \frac{1}{\nu!} \partial^{\nu} A \, D^{\nu} B \tag{5}$$

where $\partial = \frac{\partial}{\partial \xi}$ and $D = -i \frac{\partial}{\partial x}$ are applied term-by-term. Note that the sum on the right of (5) is not in the canonical form (4); however, only finitely many products of the same homogeneous degree appear in (5), and rearrangement into the form (4) is easy.

Note that the class of symbols all of whose homogeneous pieces have integral degree, and the class of symbols which for large $|\xi|$ can be written

$$A = \sum_{\ell \geq 0} A_{\ell} [q,\xi]$$

with $A_{\ell}[q,\xi] = a_{\ell}[q] \, \xi^{d_{\ell}}$ and $a_{\ell}[q]$ a polynomial in q and its x-derivatives (that is, a polynomial in $q_0,\ldots,q_{n-2}$ and their x-derivatives), both form subalgebras. The latter subalgebra will be called the class of <u>q-symbols</u>.

The order ord A of a symbol A is the homogeneous degree of the term of highest homogeneous degree appearing in an expansion (4) - that is, ord $A = d_0$ in the notation of (4).

Note that

$$\operatorname{ord}(A \circ B) \leq \operatorname{ord} A + \operatorname{ord} B$$

and

$$\operatorname{ord}\,[A,B] \leq \operatorname{ord} A + \operatorname{ord} B - 1$$

where

$$[A,B] = A \circ B - B \circ A .$$

Note that differential expressions such as (1), and more generally differential expressions whose coefficients depend polynomially on q and its x-derivatives, correspond under the replacement $D \mapsto \xi$ to polynomial (in ξ) q-symbols. Moreover, this correspondence commutes with the formation of products and sums. Therefore, Lax equations (2) are in 1-1 correspondence with polynomial q-symbols P for which ord $[L,P] \leq n-2$.

§3. Complex Powers

Define the resolvent symbol $R(\lambda)$ for L by the equation

$$R(\lambda) \circ (L-\lambda) = 1 .$$

According to the definition of the product (5), we can re-write this equation in the form

$$(\xi^n-\lambda)\ R(\lambda) = 1 - R(\lambda) \circ \left(\sum_{k=0}^{n-2} q_k\ \xi^k \right) .$$

From this formula it is easy to see that $R(\lambda)$ admits an expansion in the form

$$R(\lambda) = \sum_{\ell=n}^{\infty} R_\ell\ (\lambda,\xi) \tag{6}$$

where R_ℓ is homogeneous of degree $-\ell$ in $\xi, \sqrt[n]{\lambda}$, and is defined recursively by the formulae

$$R_n(\lambda) = (\xi^n-\lambda)^{-1}, \quad R_{n+1}(\lambda) \equiv 0 ,$$

$$R_\ell(\lambda) = - \sum_{j=0}^{\ell-n-2} \ \sum_{k=\max(0,2n+j-1)}^{n-2} \frac{\xi^k(\xi^n-\lambda)^{-1}}{(-2n-j+\ell+k)!}\ \partial^{-2n-j+\ell+k}\ R_{n+j}\ D^{-2n-j+\ell+k}\ q_k .$$

In particular,

$$R_\ell(\lambda) = \sum_{m=2}^{\ell-n} B_{\ell m}\ (\xi^n + \lambda)^{-m} , \qquad \ell \geq n+2 \tag{7}$$

where $B_{\ell m}$ is a homogeneous polynomial in ξ of degree $nm-\ell$ whose coefficients are polynomials in q and its x-derivatives.

Observe that, by virtue of (7), the expansion (6) can be rearranged for large $|\xi|$ into the form (4), showing that $R(\lambda)$ is a q-symbol. $R(\lambda)$ should be regarded as a local version of the resolvent of L. Indeed, R obeys the resolvent equation

$$R(\lambda) - R(\mu) = (\lambda - \mu)\ R(\lambda) \circ R(\mu).$$

In further analogy to the usual theory of the resolvent, we use $R(\lambda)$ to define complex powers of the symbol L.

For the remainder of this paper, define

$$\lambda^s = \exp(s \log \lambda), \quad \lambda, s \in \mathbb{C}$$

where the principal branch of log is selected, with the branch cut down the negative imaginary axis.

Define the symbol L^s, for complex s, by

$$L^s = \frac{1}{2\pi i} \oint d\lambda\ \ \lambda^s\ R(\lambda)\ \chi(\xi) \tag{8}$$

The contour goes up the ray $\operatorname{Re} \lambda = 1/2$, around the semicircle $|\lambda| = 1/2$ counter-clockwise, and back down the ray $\operatorname{Re}\lambda = -1/2$. The function $\chi(\xi)$ is smooth and satisfies

$$\chi(\xi) = \begin{cases} 1 \ , & |\xi| \geq 1 \\ 0 \ , & |\xi| \leq 1/2 \end{cases} .$$

The integral (8) is evaluated by inserting the expansion (6), (7) for $R(\lambda)$ in (8) and integrating term-by-term, using the Residue Theorem. Each integral converges and admits evaluation by residues for $\operatorname{Re} s < -1$: We obtain

$$L^s = \sum_{p=0}^{\infty} A_p(s) \tag{9}$$

where $\operatorname{ord} A_p = n \operatorname{Re} s - p$,

$$A_p(s) \equiv (\xi^n)^s \quad (\xi^n)^{1-m} \sum_{m=2}^{p} B_{p+n,m} \quad (-1)^{m-1} \binom{s}{m-1} \tag{10}$$

with

$$\binom{s}{m-1} \equiv \frac{1}{(m-1)!} \quad \sum_{j=0}^{m-2} (s-j)$$

According to the convention, that symbols are identified when they agree for large $|\xi|$, the above results (9), (10) are independent of the choice of χ , which appears in (8) merely to rule out poles on the integration contour. All mention of χ in the following will therefore be suppressed.

Thus L^s is a q-symbol, of order $n \operatorname{Re} s$, for $\operatorname{Re} s < -1$. Formulae (9), (10) allow continuation of L^s as an entire function of s . Mimicing arguments in [6], one easily shows that the Resolvent Formula implies the semigroup properties

$$L^0 = 1 \ , \ L^1 = L \ , \ L^s \, L^t = L^{s+t} ,$$

In particular, for any complex s,t,

$$[L^s \ , \ L^t] = 0 \ .$$

§4. Lax Pairs

Let m be any positive integer. Then $L^{m/n}$ is a q-symbol of order m, whose homogeneous pieces have integral degree. Set

$$L^{m/n} = P^m + N^m$$

where

$$P^m = \sum_{p=0}^{m} A_p \left(\frac{m}{n} \right)$$

$$N^m = \sum_{p=m+1}^{\infty} A_p \left(\frac{m}{n} \right).$$

Thus P^m is a polynomial q-symbol, and ord $N^m \leq -1$. If m is not divisible by n, it is easy to check that $A_p\left(\frac{m}{n} \right) \not\equiv 0$ for $p \geq m+1$.

Because $[L, L^{m/n}] = 0$,

$$[P^m, L] = [L, N^m] \quad .$$

Since the left-hand side of this equation is a polynomial q-symbol, so is the right. On the other hand, since the right-hand side has order $\leq n-2$, so does the left-hand side. Thus

$$\dot{L} = [P^m, L]$$

is a Lax equation for each positive integer m. This is Gel'fand and Dikii's construction of Lax pairs.

A simple degree-counting argument shows that P^m is essentially unique polynomial q-symbol of order m, whose commutator with L is of order $n-2$.

§5. Generalized Lenard Relations

Denote by X^m the polynomial q-symbol

$$X^m \equiv [P^m, L] = [L, N^m] \ .$$

In view of equation (2), it is natural to call X^m the m^{th} Lax vector field for L. The objects of this section is to relate X^m to X^{m+n}. Since $L^{m/n + 1} = L \circ L^{m/n}$, we have

$$\begin{aligned} P^{m+n} &= (L \circ L^{m/n})_+ \\ &= L \circ P^m + (L \circ N^m)_+ \end{aligned}$$

where the subscript "+" signifies the sum of parts of positive homogeneous degree, i.e. the polynomial part. Thus

$$X^{m+n} = L \circ X^m - [L, (L \circ N^m)_+] \ . \tag{11}$$

The equation

$$X^m_\ell = \sum_{k=\ell+2}^{n} \sum_{p=m+1}^{m+k-\ell-1} [A_p(\frac{m}{n}), q_k \xi^k]_\ell \tag{12}$$

(where the subscript ℓ denotes the part of homogeneous degree ℓ) shows that X^m depends only on $A_p(\frac{m}{n})$, $m+1 \leq p \leq m+n-1$. We shall show that Equation (11) can also be rewritten to express X^{m+n} in terms of $A_p(\frac{m}{n})$, $m+1 \leq p \leq m+n-1$. This will be our generalized Lenard relation.

In fact,

$$(L \circ N^m)_+ = \xi^n A_{m+n}(\frac{m}{n}) + \ldots$$

where the terms denoted by dots involve only $A_p(\frac{m}{n})$, $m+1 \leq p \leq m+n-1$. On the other hand, according to the result of §4,

$$\begin{aligned} 0 &= [L, N^m]_{-1} \\ &= \sum_{p=1}^{n} \binom{n}{n+1-p} D^{n+1-p} a_{m+p}(\frac{m}{n}) \\ &+ \sum_{k=0}^{n-2} \sum_{p=1}^{k} \binom{k}{k+1-p} \{q_k D^{k+1-p} a_{m+p}(\frac{m}{n}) - a_{m+p}(\frac{m}{n})(-D)^{k+1-p} q_k\} \end{aligned}$$

where we have written $A_p(\frac{m}{n}) = a_p(\frac{m}{n})\xi^{m-p}$. This can be re-written

$$\begin{aligned} -n D a_{m+n}(\frac{m}{n}) &= \sum_{p=1}^{n-1} \binom{n}{n+1-p} D^{n+1-p} a_{m+p}(\frac{m}{n}) \\ &+ \sum_{k=0}^{n-2} \sum_{p=1}^{k} \binom{k}{k+1-p} \{q_k D^{k+1-p} a_{m+p}(\frac{m}{n}) - a_{m+p}(\frac{m}{n})(-D)^{k+1-p} q_k\} \end{aligned} \tag{13}$$

The r.h.s of (13) is an exact derivative, so we can express $a_{m+n}(\frac{m}{n})$ in terms of $a_{m+p}(\frac{m}{n})$, $1 \leq p \leq n-1$. On the other hand, we don't even need to do that, since only $DA_{m+n}(\frac{m}{n})$ appears in $[L, (L \circ N^n)_+]$. In any case, we have expressed X^{m+n} in terms of $A_{m+p}(\frac{m}{n})$, $1 \leq p \leq n-1$, as was desired.

§5. The Hamiltonian Formalism

In order to write the relations derived in the last section in the form in which the Lenard relations are usually presented, we introduce the Hamiltonian formalism of [1].

Suppose that F is a polynomial in q and its derivatives. Define the formal variational derivative of F by

$$\frac{\partial F}{\partial q_k} = \sum_{j=0}^{\infty} (-D)^j \frac{\partial F}{\partial (D^j q_k)}$$

$$\frac{\partial F}{\partial q} = \left(\frac{\partial F}{\partial q_0}, \ldots, \frac{\partial F}{\partial q_{n-2}} \right)^t$$

In [1] it is proven (Equation (21)) that

$$a_p\left(\frac{m}{n}\right) = \frac{n}{m+n} \sum_{\nu=0}^{p-m-1} \binom{p-m-1}{\nu} (-D)^{\nu} \frac{\delta}{\delta q_{p-m-\nu-1}} a_{m+n+1}\left(\frac{m+n}{n}\right) \tag{14}$$

for $p = m+1,\ldots,m+n-1$.

Using (12) and (14), it is easy to express the Lax vector field X^m in the form

$$(X_0^m,\ldots,X_{n-2}^m)^t = \frac{n}{m+n} J \frac{\delta}{\delta q} H_m \tag{15}$$

where $H_m \equiv a_{m+n+1}\left(\frac{m+n}{n}\right)$, and J is a certain matrix of differential expressions, whose coefficients depend polynomially on q and its derivatives.

In [1] it is shown that J defines a symplectic structure on the space of coefficient vectors q, in a certain local sense, and that the Lax vector field is a Hamiltonian vector field, by virtue of (15).

Now Equations (11), (12), (13), (14) and (15), allow us to write the relations derived in the last section in the succinct form

$$\frac{n}{m+2n} J \frac{\delta}{\delta q} H_{m+n} = \frac{n}{n+m} K \frac{\delta}{\delta q} H_m \tag{16}$$

where K is another $(n-1)\times(n-1)$ matrix of differential expressions whose coefficients depend polynomially on q and its derivatives. Equation (16) is, for $n=2$, the form in which the Lenard relations are usually stated. The general form of J is very complicated, and that of K even more so. Rather than exhibit these general forms, we compute J and K explicitly for $n=2,3$ in the next section.

§7. Examples

I. $n = 2 : L = \xi^2 + q_0$, and

$$X_0^m = 2\, D\, a_{m+1}\left(\frac{m}{2}\right) = \frac{4}{m+2}\, D\, \frac{\delta}{\delta q_0}\, a_{m+3}\left(\frac{m}{2}+1\right)$$

so in this case J is the 1×1 matrix $2D$.

Equation (11) yields

$$X_0^{m+2} = D^2 X_0^m + q X_0^m - 2D^3 a_{m+1}\left(\frac{m}{2}\right) - D^2 a_{m+2}\left(\frac{m}{2}\right) + a_{m+1}\left(\frac{m}{2}\right) Dq \; .$$

Equation (13) yields

$$D\, a_{m+2} = -1/2\, D^2\, a_{m+1} \; .$$

Hence

$$X_0^{m+2} = \left(\frac{1}{2} D^3 + 2\, qD + (Dq)\right) a_{m+1}\left(\frac{m}{2}\right) .$$

So K is the 1×1 matrix

$$K = \frac{1}{2} D^3 + q\, D + D_{0q} \; .$$

This is the expression discovered by Lenard.

II. $n = 3$. Set $H_m = \frac{3}{m+3}\, a_{m+4}\left(\frac{m}{3}+1\right)$.

Then $a_{m+1} = \frac{\delta H_m}{\delta q_0}$

$$a_{m+2} = \frac{\delta H_m}{\delta q_1} - D\,\frac{\delta H_m}{\delta q_0}$$

according to formula (14) .

The Lax vector field is

$$X^m = 3\xi D a_{m+1}\left(\frac{m}{3}\right) + 3\left(D^2\, a_{m+1}\left(\frac{m}{3}\right) + D\, a_2\left(\frac{m}{3}\right)\right)$$

Thus

$$J = \begin{pmatrix} 0 & 3D \\ 3D & 0 \end{pmatrix} .$$

From (13) we obtain

$$D\, a_{m+3}\left(\frac{m}{3}\right) = -\frac{1}{3}\left(3D^2\, a_{m+2}\left(\frac{m}{3}\right) + D^3\, a_{m+1}\left(\frac{m}{3}\right) + (D\circ q_0)\, a_{m+1}\left(\frac{m}{3}\right)\right)$$

and, after tedious computation

$$K = \begin{pmatrix} K_{00} & K_{01} \\ K_{10} & K_{11} \end{pmatrix}$$

where

$$K_{00} = -\frac{2}{3} D^5 + \frac{1}{3} D^3 \circ q_1 - \frac{2}{3} q_1 D^3 - \frac{1}{3} q_1 (D \circ q_1) + D \circ (Dq_0) + (Dq_0)D$$

$$K_{01} = D^4 - 2q_1 D^2 + 3q_0 D - Dq_0$$

$$K_{10} = -D^4 + 2 D^2 \circ q_1 - 2(Dq_0) + 3q_0 D$$

$$K_{11} = 2D^3 + q_1 D + D \circ q_1 .$$

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This report presents certain relations among the completely integrable Hamiltonian systems introduced by Gel'fand and Dikii. These relations generalize a formula of A. Lenard linking the higher-order Korteweg-deVries equations, of which the Gel'fand-Dikii Systems are a generalization. The general form of the relations, which connect the various isospectral deformations of linear differential operators, is described, and two examples are given explicitly.

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