Bulletin of the Psychonomic Society
1975, Vol. 6 (2), 161-164

Relations, operators, predicates, and the syntax of (verbal) propositional and (spatial) operational memory

WAYNE A. WICKELGREN
University of Oregon, Eugene, Oregon 97403

Relational, operator, and predicate systems are distinguished on the basis that they correspond to the three possible pair-wise bracketings into two constituents of the three parts of a proposition: relation, subject, and object. It is asserted that the verbal propositional modality (left hemisphere) uses a predicate grammar, while the spatial-image operational modality (right hemisphere) uses an operator grammar. Verbal propositional memory has the capacity for extensive propositional embedding while spatial operational memory does not.

Consider a proposition expressed in functional or operator form as $f(x) = y$ or $f(x) \rightarrow y$. The meaning of such an expression is that the result of applying operator f to the argument x yields the result y. It is immediately obvious once it is pointed out that such a proposition in the preceding operator syntax can be expressed completely equivalently using a relational syntax of the form F(x,y) equals unity when the relation F holds (is true) for the pair of arguments x and y and zero when it does not hold (is false) for the pair of arguments x and y. The two modes represent precisely the same information whenever F(x,y) has the value 1, if and only if $f(x) = y$. This equivalence has fascinated me since it was pointed out to me some 15 years ago, because, while the operator and relational notations are mathematically equivalent, they seem to be, by no means, psychologically equivalent. I have always found it very difficult to think in relational notation, and so far as I know, everyone else does too. Operator syntax seems natural for the mind, while relational syntax does not.

This is a specific example of the very general problem concerned with the relation between syntax and thought. Presumably, of all the possible types of syntactic organization of concepts into phrases and propositions, the mind uses certain types and not others. It may be, as Anderson and Bower (1973) appear to assert, that at the highest level of cognitive processing, the mind uses a single type of syntax for organizing semantic memory, regardless of the verbal or spatial character of the information.

However, there is now abundant evidence supporting the functional differentation of the left and right hemispheres, with the left hemisphere being specialized for verbal representation and the right hemisphere for a variety of nonverbal (primarily, but not exclusively, spatial) functions (e.g., Gazzaniga, 1970; Milner, 1966; Milner & Taylor, 1972; Sperry, 1968). In light of the anatomical separation of these modalities, it is attractive to consider possible functional differences in both the structure of representation and the cognitive processes that operate in these modalities. The present paper presents a theory of the difference in the syntax used for representation in (verbal) propositional memory and (spatial) operational memory.

This work was supported by Grant 3-0097 from the National Institute of Education and by contract F 44620-73-C-0056 from the Advanced Research Projects Agency, D. O. D. to Ray Hyman.

RELATIONS, OPERATORS, AND PREDICATES

Whatever other differences there may be among them, a principal difference between relational, operator, and predicate grammars is the difference in grouping (chunking, parenthesization, or bracketing) of the three fundamental constituents of most propositions: subject, relation, and object. According to this notion, relations, operators (or functions), and verbs all belong to the same class of terms that might be broadly termed "relations." Subjects and objects are often the same kind of thing, differing psychologically in that the subject is the starting point for thinking. The subject represents the topic under discussion. By contrast, the object(s) are the new elements (in the output state) that are added by applying the operator to the subject (input state) in an operator grammar.

If one accepts this analogy between the subject of a sentence and the input argument for an operator, then it is clear that in a relational grammar, the subject and object are first bracketed together to form the argument set before being combined with the relational term to form the entire proposition. In a predicate grammar, the relational term (verb) and the object are first conjoined to form the predicate, and thereafter the predicate is conjoined with the subject to form a proposition. Finally, in an operator grammar, the subject is first conjoined with the relation (operator), and thereafter

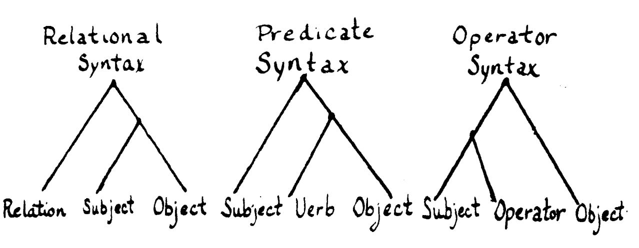

Figure 1. The three possible pairwise (nonoverlapping) bracketing systems.

this pair is conjoined with the object (result) to form the entire proposition. In general, the subject and object refer to sets of elements, rather than individual elements, but the sets are psychologically distinguished into these two classes. Viewed in this way, there are only three possible pairwise bracketings of the three fundamental constituents of a proposition, and this bracketing difference is the primary distinguishing characteristic of relational, operator, and predicate systems. The three systems are illustrated in Figure 1.

VERBAL AND OPERATIONAL MEMORY

With the exception of the case grammar approach to representation in verbal semantic memory (e.g., Fillmore, 1968; Rumelhart, Lindsay, & Norman, 1972), most theories of representation in verbal memory have assumed some type of predicate grammar (e.g., Anderson & Bower, 1973; Chomsky, 1957). And, of course, the attempt to mathematize thought in symbolic logic made use of a predicate grammar.

By contrast, a case grammar can be considered to be a relation system, with the verb taking cases in the same way that a relation takes arguments. This interpretation of case grammar conflicts with linguistic intuition since it asserts that the subject and object together form a constituent and have a closer relation to each other than either has to the verb. Another reasonable interpretation of case grammar is outside the present three-way classification of bracketing systems. According to this classification, the verb has relations to the subject and to the object, but the subject and object do not form a constituent and have no direct relation to each other. This is the interpretation adopted by Rumelhart, Lindsay, and Norman (1972). For a verb and two cases, it can be considered to be an overlapping bracketing system, for example: $(_a\text{Subject}(_b\text{Verb})_a\text{Object})_b$. Discussion of this sort of representation system is outside the scope of the present paper.

My current preference is for a predicate system of representation in verbal propositional memory. In any event, no one seems to think an argument can be made for an operator system of representation in verbal propositional memory.

By contrast, I think it is very attractive to consider the possibility that modalities in the right hemisphere of the brain, primarily the (visual) spatial modality, use an operator grammar.

The ability of human beings to mentally rotate images of objects to judge the congruence of two objects (Cooper & Shepard, 1973; Metzler & Shepard, 1974) seems quite easy to account for in terms of an operator system. Assume that there are operators for a 20-deg rotation in each of a number of different directions (around a number of different axes). One of these operators applied to the constituents of a (visual) spatial image yields a new image corresponding to the object after a 20-deg rotation around the specified axis. Rotation through an angle of 40 deg requires two applications of the operator and takes twice as much time as a single application. In principle, there could be operators for every angle of rotation. However, the findings of Cooper and Shepard (1973) and Metzler and Shepard (1974) suggest that there is a maximum step size for image rotation on the order of 20 deg, since, as the angle of rotation increases, there is a linear increase in the reaction time to judge the congruence of two views of the same object. Between 20-deg steps, there may be smaller-step rotational operators, or the congruence judgment mechanism may operate successfully with minor angular disparities.

Mental rotation has been extensively investigated of late, but it hardly begins to exhaust all of the possible operators we can apply to spatial images. Besides rotation, there would appear to be deletion, addition, substitution, reflection, translation, size changes, and various types of distortions, etc. Many pattern recognition schemes have used an operator system including centering (translation), filling in gaps (addition), deleting stray marks (deletion), size changes, rotation (to some extent), etc. An illustration of a series of operators applied to an image is shown in Figure 2.

More generally, our spatial operational memory presumably contains our cognitive maps for all of the locations of things we know about in the world and for what the front, back, top, bottom, and sides of various objects look like. In general, the image of various views of complex objects may not be derivable from mental rotation of some three-dimensional image of that object. To give an example of the latter, consider applying the operator "back of" to the image of the front of my television set to obtain an image of the back of my television set.

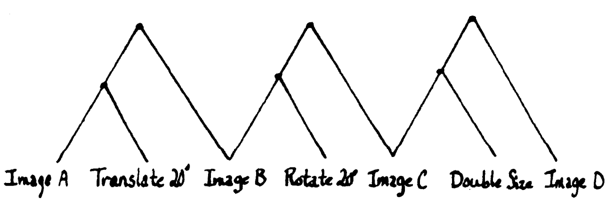

Figure 2. A succession of three operators applied to Image A resulting in Image D in operational memory, using vertical associations to and from propositional nodes.

Memory for what is inside of my house or my living room might consist of a set of images linked by operators such as "left of," "right of," "above," "below," "in front of," "in back of," "lower left corner of," etc. Memory for locality maps too complex for incorporation into a single simultaneously available image could consist of a set of images linked by various spatial operators, for example, the downtown area, the university area, your neighborhood, etc.

It is interesting to conjecture that animal cognitive maps make exclusive use of an operator grammar, with little or no reliance on propositional thought, which uses a predicate grammar.

PROPOSITIONAL EMBEDDING

Another bold conjecture regarding the differences between the presumed modalities of propositional and operational memory is that propositional memory involves extensive embedding of one proposition in another while operational memory does not (Wickelgren, 1975). If operational memory consists of nodes representing operators applied to input images which are associated to resulting output image, then it is not clear to me what meaning would attach to the embedding of such a "proposition" into any higher order construction, let alone the need for it. If this conjecture is true, there would appear to be no need to have any superordinate propositional nodes in operational memory. A node representing an operator applied to an image could be directly (horizontally) associated to an output image, without the need to go indirectly by means of "vertical" associations to and from a propositional node representing the entire triple. Such a system is illustrated in Figure 3.

By contrast, in verbal propositional memory, there is a clear need to assume the capacity for propositional embedding. For example, we store such embedded propositions as "John thinks that it is false that nothing travels faster than light." To represent such embedded propositions, it is very useful to have unitary nodal representation of entire propositions for embedding in higher order propositions.

The representation of kinship relations (father, mother, son, daughter, aunt, uncle, cousin, etc.) appears to use an operator grammar. Relying primarily upon the ease of defining rules representing the inferences that humans make within the kinship system, Winkelman (1975) has shown that an operator grammar provides a far more elegant and adequate representation than a predicate grammar or a semantic feature system.

Of course, it is not clear whether the encoding of kinship relationships occurs primarily in the right hemisphere along with spatial images, chess, music, etc., but there does appear to be a considerable degree of spatial imagery in the representation of kinship relations. Furthermore, the argument is not that kinship relations are necessarily spatial, but that they use an operator grammar as opposed to a predicate grammar. Finally, there is nothing in the abstract distinction between modalities using an operator grammar as opposed to modalities using a predicate grammar that requires complete anatomical localization with respect to cerebral hemispheres, and indeed, some individuals exhibit little or no anatomical lateralization of verbal vs. nonverbal capacities.

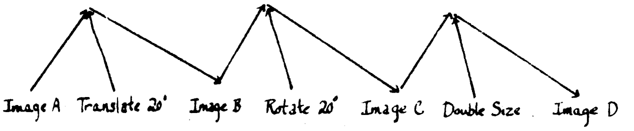

Figure 3. A succession of three operators applied to Image A resulting in Image D, using "horizontal" unidirectional associations with no use of a propositional node.

An operator system without propositional embedding seems quite adequate, indeed, perhaps ideally suited to the translation of cognitive maps into goal-directed behavior. Following a line of reasoning similar to that of Deutsch (1960), one assumes that the image of the current situation partially activates all of the operator + images nodes to which it is connected which, in turn, activate their consequence image nodes, and so on. At the same time, the image of the goal activates antecedent operator + image nodes which activate their antecedents until this source of spreading activation intersects with the spreading activation from the starting point. When they meet, a particular chain of operator + image nodes and their image consequence nodes will have been activated, leading from the current location to the goal (provided the animal has learned the cognitive map leading from the current location to the goal.)

No propositional embedding appears necessary for the translation of such elementary cognitive maps into goal-directed behavior, and the mental (spatial) operators would appear to be exactly the type of relations most easily translated into overt responses. Of course, people can also translate their verbal propositional memory into speech and writing, but the process appears to be considerably more complicated. Viewed in this way, one might go so far as to suggest that verbal propositional memory is the type of thinking specialized for abstract thought, only distantly related to actions in the real world. By contrast, operational memory is specialized for concrete thought, which is more directly related to input from and output to the external world. Thus, in righthanders, the right hemisphere is the concrete hemisphere, and the left hemisphere is the abstract hemisphere.

REFERENCES

Anderson, J. H., & Bower, G. *Human associative memory*. New York: Wiley, 1973.
Chomsky, N. *Syntactic structures*. Netherlands: Mouton, 1957.

Cooper, L. A., & Shepard, R. N. Chronometric studies of the rotation of mental images. In W. G. Chase (Ed.), *Visual information processing*. New York: Academic Press, 1973, 75-176.

Deutsch, J. A. *The structural basis of behavior*. Chicago: University of Chicago Press, 1960.

Fillmore, C. J. The case for case. In E. Bach and R. T. Harms, *Universals in linguistic theory*. New York: Holt, Rinehart, & Winston, Inc., 1968, 1-88.

Gazzaniga, M. S. *The bisected brain*. New York: Appleton-Century-Crofts, 1970.

Metzler, J., & Shepard, R. N. Transformational studies of the internal representation of three-dimensional objects. In R. L. Solso (Ed.), *Theories in cognitive psychology: The Loyola Symposium*, 1974. Pp. 147-202.

Milner, B., & Taylor, L. Right-hemisphere superiority in tactile pattern-recognition after cerebral commissurotomy: Evidence for nonverbal memory. Neuropsychologia, 1972, 10, 1-15.

Milner, B. Amnesia following operation on the temporal lobes. In C. W. M. Whitty & O. L. Zangwill (Eds.), *Amnesia*. London: Butterworths, 1966. Pp. 109-133.

Rumelhart, D. E., Lindsay, P. H., & Norman, D. A. A process model for long-term memory. In E. Tulving and W. Donaldson (Eds.), *Organization of memory*. New York: Academic Press, 1972, 198-245.

Sperry, R. W. Hemisphere deconnection and unity in conscious awareness. American Psychologist, 1968, 23, 723-733.

Tulving, E. Episodic and semantic memory. In E. Tulving and W. Donaldson (Eds.), *Organization of memory*. New York: Academic Press, 1972, 328-402.

Wickelgren, W. A. Subproblems of semantic memory: A review of "Human associative memory" by J. R. Anderson and G. H. Bower. Journal of Mathematical Psychology, in press, 1975.

Winkelman, J. H. Semantic representation of kinship systems. Journal of Psycholinguistic Research, 1975, 4. Pp. 133-140.

(Received for publication April 28, 1975.)