# Relationship Banking, Fragility, and the Asset-Liability Matching Problem 

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#### Abstract

We address a fundamental question in relationship banking: why do banks that make relationship loans finance themselves primarily with core deposits and when would it be optimal to finance such loans with purchased money? We show that not only are relationship loans informationally opaque and illiquid, but they also require the relationship between the bank and the borrower to endure in order for the bank to add value. However, the informational opacity of relationship loans gives rise to endogenous withdrawal risk that makes the bank fragile. Core deposits are an attractive funding source for such loans because the bank provides liquidity services to core depositors and this diminishes the likelihood of premature deposit withdrawal, thereby facilitating the continuity of relationship loans. That is, we show that banks will wish to match the highest value-added liabilities with the highest valueadded loans and that doing so simultaneously minimizes the bank's fragility owing to withdrawal risk and maximizes the value the bank adds in relationship lending. We also examine the impact of interbank competition on the bank's asset-liability matching and extract numerous testable predictions. (JEL G21, G28, D82, D86)


"We have entirely lost the idea that any undertaking likely to pay, and seen to be likely, can perish for want of money; yet no idea was more familiar to our ancestors, or is more common now in most countries."

-Walter Bagehot, Lombard Street, Richard D. Irwin, Inc., Homewood, Ill, 1962, p. 119.

Banks are inherently fragile [Allen and Gale (2001), Diamond and Rajan (2001), and Freixas and Rochet (1999)]. This fragility arises because banks provide liquidity by financing themselves with demand deposits which are part of "core deposits." These deposits create risk for the bank owing to unanticipated withdrawals that may be precipitated by adverse perceptions of depositors about the bank's payoffs [Chari and Jagannathan (1988)] due to macroeconomic shocks

[^0][Gorton (1988)], or perceived excessive bank portfolio risk [Calomiris and Kahn (1991)]. Hence, banking fragility may be diminished by reducing banks' risk-taking incentives on the asset side. It has been suggested that this can be achieved by increasing banks' profits and charter values [Keeley (1990)], possibly by insulating them against excessive competition [Bhattacharya and Thakor (1993)]. Boot and Thakor (2000) show that banks could achieve this insulation by engaging in information-intensive relationship lending [see also Petersen and Rajan (1994)].

It appears then that banking profitability and fragility are linked to the interaction between the asset and liability activities of the bank, and that fragility can be addressed by examining how banks fund themselves and where they invest. For instance, if the essence of banking lies in the liquidity services provided to core depositors, then why not reduce fragility by having core-deposit-funded banks invest in informationally transparent assets like transaction loans and marketable securities that are less likely to induce unanticipated deposit withdrawals? ${ }^{1}$ Alternatively, if relationship lending is a high value-added activity, why not have these informationally opaque loans provided by institutions that avoid demand deposits and hence are not inherently fragile? But in practice, banks finance with core deposits and make relationship loans. Why?

We address these issues by taking a fresh approach to modeling core deposits and relationship lending. We model a bank funded exclusively by short-maturity liabilities that can be withdrawn at a moment's notice, and later establish the optimality of such liabilities. The bank can choose between two types of short-maturity liabilities: "core deposits"-passbook savings accounts, checking accounts, and the like-on which it provides the depositors transaction and advisory services, and "purchased money"-brokered CDs, large time deposits, etc.-on which no such services are provided. For convenience, we call both of them "deposits," and refer to transaction and advisory services as "liquidity services." On the asset side, the bank can choose between transaction loans on which it provides no valued-added services and relationship loans on which it does [Boot and Thakor (2000)]. The transaction loans are informationally transparent to depositors, whereas relationship loans are informationally opaque. The question we address is one of the oldest in banking: how should the bank match its assets and liabilities? However, the matching here is not the usual one with respect to maturity; rather, it is with respect to matching assets and liabilities based on how much value the bank is adding.

[^1]Our main result is that it is efficient for a bank to finance sufficiently informationally opaque relationship loans with core deposits; less informationally opaque relationship loans as well as transaction loans are financed with purchased money. Thus, the loans where the bank adds the most value are funded by deposits where it adds the most value, and the loans where it adds the least value are funded by deposits where it adds the least value. The intuition is as follows. High informational opaqueness of relationship loans means a high probability that the bank and the depositors will disagree on the value of the loan portfolio. This disagreement arises because of potentially different but correlated prior beliefs about the value of the relationship loan portfolio. Consequently, depositors will prematurely withdraw their deposits when they believe that the bank's loan portfolio has low value and should not continue to be funded, but the bank assesses a high value and wishes to continue to fund it. The illiquidity of relationship loans means that they cannot be sold to help the bank cope with unanticipated deposit withdrawals. This forces the bank to prematurely liquidate its relationship loans, and to minimize the likelihood of this happening the bank finds it efficient to fund relationship loans with deposits subject to the least withdrawal risk among demand deposits. We show that these are core deposits whose withdrawal sluggishness arises endogenously owing to the bank's liquidity services. ${ }^{2}$ The diminished withdrawal risk associated with core deposits increases the likelihood of continuation of the relationship loan and hence increases the value the bank adds to the relationship loan. For less informationally opaque relationship loans, the disagreement probability is relatively low, and the informational transparency of transaction loans means there is no possibility of disagreement between the bank and depositors. Consequently, the bank making such loans finds it efficient to fund them with purchased money which has a lower average cost than core deposits due to the fixed cost the bank has to incur with core deposits in providing liquidity services to depositors. ${ }^{3}$

In our model, banking fragility is caused not by depositor coordination failures that lead to panic runs on banks, but by premature deposit withdrawals triggered by disagreement between the bank and depositors regarding the value of informationally opaque loans. These loans, however, are precisely where the bank adds value on the asset side. Thus, our analysis

[^2]identifies an important link between the two sides of the bank's balance sheet: the value added by the bank on its asset side has withdrawal repercussions on the liability side that make the bank fragile. The bank's deposit choice attempts to diminish this fragility and in doing so the bank is simultaneously able to enhance the value it adds on the asset side. In other words, by matching the highest value-added liabilities with the highest value-added loans, the bank minimizes its withdrawal-risk-induced fragility and maximizes the value it adds in relationship lending.

In addition to this main result, our analysis generates testable predictions related to the impact of interbank competition for relationship loans. We find that increased interbank competition reduces banks' reliance on core deposits and also decreases the total surplus enjoyed by banks as well as depositors; in some cases, it may also reduce borrowers' surplus. Moreover, greater interbank competition also increases the withdrawal risk faced by each bank and thereby adds to banking fragility. These results are based on specific aspects of relationship lending in our analysis as well as the result that relationship loans are financed in part by core deposits. While others have noted that higher competition can lower banks' margins and diminish bank stability by inducing banks to take greater asset portfolio risk [e.g., Gorton and Rosen (1995), and Keeley (1990)], we show that the source of increased fragility due to higher interbank competition can also be a change in the bank's liability mix.

Our analysis departs from the standard asymmetric information approach used to characterize the informational opaqueness of relationship loans [e.g., Rajan (1992), and Sharpe (1990)]. We agree that informational asymmetries generated by the incumbent lender's access to proprietary information are an important aspect of relationship banking. However, in relationship lending models, asymmetric information serves to increase the relationship bank's profit because it permits greater rent extraction from the borrower; this should increase depositors' confidence in the bank and make it less susceptible to deposit runoffs. That is, asymmetric information, as introduced in existing relationship lending models, cannot be the reason for bank fragility. We believe, however, that an important factor in banking fragility is that banks invest in assets that are often difficult to value because of "soft" payoff-relevant information [Stein (2002)] that is amenable to multiple interpretations, some of which may be at odds with each other. We model this through the device of rational but heterogenous prior beliefs [Kurz (1994a,b)] about
the precision of an interim signal about the value of the bank's relationship loan portfolio. ${ }^{4}$

Apart from the relationship banking literature [e.g., Berger and Udell (1995), Boot (2000), Degryse and Van Cayseele (2000), Ongena and Smith (2000), and Slovin et al. (1993)], the two articles most complementary to ours are Berlin and Mester (1999) and Kashyap et al. (2002). Berlin and Mester (1999) hypothesize that banks can intertemporally smooth loan prices because they have access to core deposits, whose interestinelasticity insulates them against exogenous economic shocks. They provide supporting evidence that banks with greater access to core deposits provide borrowers more insurance against credit shocks. However, their largely empirical analysis does not address the portfolio-matching problem we focus on since it takes as a given that relationship loans are financed with core deposits, nor does it deal with the impact of core deposits and relationship lending on the bank's withdrawal risk. Kashyap et al. (2002) suggest that a bank's deposit taking and loan commitment activities are "two manifestations of one primitive function: the provision of liquidity on demand," and banks engage in both activities in order to share the same costs of liquid asset holdings. ${ }^{5}$ Their analysis implies that a bank with a high ratio of demand deposits to total deposits will also have a high ratio of loan commitments to total loans. By contrast, our analysis suggests that banks will tend to fund high value-added assets with high value-added liabilities. That is, rather than focusing on liquidity on demand, we analyze why the bank funds a highly illiquid loan with a deposit liability that is liquid but has a low likelihood of premature withdrawal.

The rest is structured as follows. Section 1 describes the basic model. Section 2 contains the equilibrium analysis. Section 3 examines the impact of interbank competition on the equilibrium. Section 4 concludes with empirical implications. Proofs are in the Appendix.

[^3]
## 1. The Model

In this section, we describe the model, including the agents, their preferences, the economic environment, and potential disagreement among agents regarding project profitability. All the variables introduced in the model are summarized in the Appendix.

### 1.1 The agents and economic environment

We consider an overlapping generation (OLG) economy with universal risk neutrality. At every date $t$ a new generation of agents is born that lives for two periods. Each generation has three agent types: borrowers, banks, and depositors. We capture a snapshot of this economic environment by describing a subset of it in a three-date time frame: $t=0,1$, and 2. At $t=0$, a new generation of agents is born; these agents live until $t=2$.

### 1.1.1 The borrowers.

1.1.1.1 Investment Opportunities of Borrowers. Each borrower has the potential to invest in a two-period project. The project needs a $\$ 1$ initial investment at $t=0$, with its payoff realized at $t=2$. Each project is defined by its payoff attributes and its informational transparency. On the basis of its payoff attributes, the project can be either good $(G)$ or bad $(B)$. If the project is good, its payoff at $t=2$ is $H>1$ for sure. A bad project always pays off zero at $t=2$. The project's informational transparency has to do with how much is known at the outset $(t=0)$ about its payoff attributes. The project (regardless of whether it is $G$ or $B$ ) is either informationally transparent or informationally opaque. If it is informationally transparent, the project's payoff distribution is common knowledge at $t=0$, that is, everybody knows whether it is $G$ or $B$. A bad informationally transparent project will never be funded. If it is informationally opaque, there is uncertainty at $t=0$ about whether the project is $G$ or $B$. The common prior belief at $t=0$ about the project's quality is that with probability $\theta \in(0,1)$ the project is good, and with probability $1-\theta$ it is bad. We assume $\theta H<1$, that is, a priori the informationally opaque project has negative NPV even ignoring the cost of deposits to the bank. Deposit funding for the project is raised at $t=0$ and depositors intend to keep their funds with the bank until $t=2$, unless they receive adverse information at the interim date $t=1$.

One could think of the informationally transparent project as one that employs a well-established technology and involves a well-known entrepreneur/firm. Such a project would be operated at its peak payoff potential, with its payoff distribution known to all. An example would be a loan to a Fortune 500 company in connection with a routine financing need like working capital financing. By contrast, the informationally opaque project is one that employs a relatively undeveloped or new technology
and involves a less well-known entrepreneur/firm, say a small or mid-sized firm. An example may be financing for a new project like the commercial development of a biotech engineering project or entry into a new market for a small firm.

### 1.1.1.2 Financing Possibilities for Borrowers. The borrower can finance

 his project by taking a $\$ 1$ bank loan at $t=0$. This can be either a relationship loan or a transaction loan. Following Boot and Thakor (2000), we assume that a relationship loan allows the lender to become deeply involved with the project and enhance the project payoff at $t=2$ if it is an informationally opaque good project; the payoff enhancement, which may be due to the bank's "sector specialization," is $e$, with a cost to the bank of $\kappa e^{2} / 2$. There is no payoff enhancement possible for an informationally opaque bad project or for any informationally transparent project. ${ }^{6}$ Thus, the good project yields a payoff of $H+e$ if it is informationally opaque and the relationship bank exerts effort $e$ to enhance the project payoff, while it pays off $H$ regardless of the bank's effort if it is informationally transparent. The bad project always pays off zero. The bank's investment in project payoff enhancement is specific to the bank and nontransferable, that is, the payoff is enhanced by $e$ only if the loan is extended by the bank that exerted $e$. We assume that no project payoff enhancement is possible for a transaction loan.
### 1.1.2 The banks.

1.1.2.1 Bank Assets and Liabilities. On the asset side, although many banks have a mix of relationship and transaction loans, there are also banks that are specialized relationship lenders [see Ergungor (2005)] and others that engage primarily in transaction lending (e.g., mortgage lenders). For simplicity, we assume that a bank is either an exclusive relationship lender or an exclusive transaction lender.

On the liability side, the bank can choose at $t=0$ to be funded by either core deposits or purchased money. Core deposits, which include retail demand and savings deposits, transaction accounts (e.g., checking accounts), money market deposit accounts, and small time deposits (usually with face value below $\$ 100,000$ ), come from depositors who value the bank's transaction and advisory services. We label these "liquidity

[^4]services" and interpret them rather broadly as including both routine transactional liquidity services like check-writing and overdraft privileges, as well as more relationship-oriented liquidity services like access to live bank tellers, cash management advice, etc. ${ }^{7}$ The bank must make a fixed investment of $F$ at $t=0$ in order to provide liquidity services at future dates. This can be viewed as the bank's investment in the physical infrastructure of branches, tellers, employees who can provide depositors cash management advice and other services, and related expenses. We assume that for each $\$ 1$ core deposit that is deposited at $t=0$, the utility gained by the depositor from the bank's liquidity services is $\tau_{1}$ at $t=1$ for sure, whereas at $t=2$ that utility depends on the bank's financial solvency (Section 1.1.2.B discusses the bank's financial solvency). If the bank fails at $t=2$, which occurs when it cannot fully repay depositors, the value of the bank's liquidity services to the depositor at $t=2$ is zero. If the bank is solvent at $t=2$, the value of the bank's liquidity services to the depositor at $t=2$ is $\tau_{2}>\tau_{1}$. We assume that the utility gain of $\tau_{2}$ accrues to the depositor only if deposits are kept in the bank until $t=2$ and the bank stays solvent. ${ }^{8}$

Depositors and banks have a temporal association due to the OLG structure of the economy. There are one-period and two-period depositors among the core depositors of the bank at $t=0$. One-period depositors are those born at $t=-1$ who deposited with banks that also came into existence at $t=-1$, but withdrew their deposits from those banks and switched to the current bank (which came into existence at $t=0$ ). These depositors live until $t=1$ and hence will only keep their deposits in the current bank for one period until $t=1$. Two-period depositors are those born at $t=0$ who intend to keep their deposits with the current bank until $t=2$, unless they receive adverse information at $t=1$ and decide to switch for the second period to other banks that will be born at $t=1$. Thus, one-period depositors will only gain $\tau_{1}$ from the current bank's liquidity services, whereas two-period depositors will gain $\tau_{1}$ at $t=1$ and $\tau_{2}$ at $t=2$ if they do not switch to another bank at $t=1$ and the current bank is solvent at $t=2 .{ }^{9}$ If the depositors switch, they will receive $\tau_{1}$ from

[^5]the current bank's liquidity services at $t=1$ and another $\tau_{1}$ from the new (born at $t=1$ ) bank's liquidity services at $t=2$.
Since $\tau_{2}>\tau_{1}$, switching banks at $t=1$ is costly to the depositor if the current bank remains solvent at $t=2$, because the provision of liquidity services is a bank (or branch)-specific activity. The idea is that liquidity services include both transaction-oriented and relationship-oriented services. While the transaction services can be conducted electronically and through other impersonal means, relationship-oriented liquidity services involve some face-to-face interaction between the depositor and the bank, so that the physical location of the bank and the bank employees the depositor interacts with become important. Repeated interactions with the bank can increase the value of relationship-oriented liquidity services to the depositor, in either a real or a perceived sense, and switching from the bank can thus engender a cost. At the very least, this will be an incremental transportation cost as in a spatial model, but it may include other costs as well, such as a perceived loss in utility for the depositor in dealing with less familiar personnel at the new bank and related psychological costs [Kim et al. (2003)]. There is empirical evidence that bank customers experience switching costs, and when a bank closes a branch it typically loses some of its core depositors even though there may be another branch some distance away, which indicates that location convenience matters. For example, Sharpe (1997) finds that retail deposit rates are positively affected by switching costs. ${ }^{10}$

Purchased money includes large time deposits, brokered negotiable CDs with short remaining maturities, overnight funds purchases, advances, and other short-term borrowings whose price and supply fluctuate with credit market conditions [e.g., Berlin and Mester (1999), and Feldman and Schmidt (2001)]. Moreover, the providers of purchased money do not value the bank's liquidity services, and they typically deal with the bank in faceless transactions, so the physical location of the bank and the characteristics of its employees are irrelevant. Hence, these depositors face no switching costs in moving to another bank. While not all of the components of purchased money are strictly deposits, they share the common feature that they either reprice in a very short time or can be withdrawn at a moment's notice, so that they will need to be replaced. For simplicity, we will refer to these as deposits throughout.

[^6]1.1.2.2 Bank Fragility and Failure. We distinguish between bank fragility and failure. Fragility arises from withdrawal risk. A bank's fragility is monotonically increasing in the probability of unanticipated deposit withdrawal at $t=1$, which is when depositors withdraw and the bank cannot replace the deposits even though it wishes to continue funding the project. ${ }^{11}$ We assume that withdrawal at $t=1$ forces the bank to call back the loan immediately, compelling the borrower to liquidate the project prematurely. In this case, the project pays off $\delta \leqslant 1$, and the bank returns the proceeds to depositors. For the informationally transparent bad project, we have $\delta=0$, but such a project would never be funded at $t=0$. We assume, without loss of generality, that $\delta=1$ for other projects, so that depositors withdrawing at $t=1$ receive $\$ 1$. The bank shuts down in this event, the bank's payoff enhancement effort is wasted, and the depositors receive only first-period liquidity service from the initial bank; the depositors may switch to another bank for second-period liquidity service, but that may entail a switching cost. That is, premature deposit withdrawal may have deadweight costs. However, since the payment to depositors in the event of withdrawal at $t=1$ is predictable, depositors are only promised $\$ 1$ if they withdraw at $t=1$, so there is no randomness in their payoff at $t=1$.

As mentioned before, bank failure occurs when the bank cannot fully repay depositors what it promised them. Failure occurs at $t=2$ if the borrower's project fails and the loan defaults, imposing a failure cost of $\xi>0$ on the bank, which includes, among other things, the cost of losing its charter. We assume that $\xi$ is sufficiently large that the bank will never deliberately pursue projects that it believes do not have positive NPV. We will make this precise later.
1.1.3 The depositors. There are two types of depositors, and each requires an expected two-period payoff of $r_{d}>1$ for depositing $\$ 1$ at $t=0$ with the bank until $t=2$, and an expected one-period payoff of 1 for depositing $\$ 1$ at $t=0$ with the bank until $t=1$. The first type is motivated by liquidity needs to invest in bank deposits and hence values the bank's liquidity services with core deposits. The second type is motivated solely by investment return in choosing bank deposits and hence does not value the bank's liquidity services. Thus, the first type prefers investing in core deposits and the second type prefers investing in purchased money. Among the core depositors, there are one-period and two-period depositors at $t=0$. However, since it is incentive-compatible for these depositors to reveal their types at $t=0$, a bank financing with core deposits will raise all

[^7]the money it needs for funding loans from the two-period core depositors. Money deposited by one-period core depositors is kept in liquid assets and returned to the depositors in its entirety at $t=1$. Thus, one-period core deposits do not affect the subsequent analysis.

### 1.2 The information and beliefs structure

We now describe an information structure that results in banks and depositors having heterogenous beliefs about project quality. Each borrower seeks a bank loan to finance the project. If the project is informationally transparent, its type is known at $t=0$, and financing is sought only if the project is $G$. If the project is informationally opaque, its type is unknown at $t=0$, and a public signal regarding its type is observed at $t=1$; prior to observing the signal, everybody agrees the project has negative expected NPV. The signal is $s \in\left\{s_{G}, s_{B}\right\}$, where $s_{G}$ is a good signal and $s_{B}$ is a bad signal. Everybody sees the same signal, and all agree on whether it is good or bad. Moreover, the prior probabilities are $\operatorname{Pr}\left(s=s_{G}\right)=\theta \in(0,1)$ and $\operatorname{Pr}\left(s=s_{B}\right)=1-\theta$.

Although all agents see the same signal and have the same prior beliefs about the values $\left(s_{G}\right.$ or $\left.s_{B}\right)$ the signal will take, they have different priors about the precision, $p$, of the signal; $p$ can be either precise (I), not-precise $(N)$ or uninformative $(U)$. The probabilities of drawing $I, N$ and $U$ are $q_{I}, q_{N}$ and $q_{U}$, respectively, with $q_{j} \in(0,1) \forall j \in\{I, N, U\}$ and $\sum_{j} q_{j}=1$. A precise signal is viewed as perfect and results in a posterior belief $\operatorname{Pr}\left(G \mid s=s_{G}, p=I\right)=1$. A not-precise signal is viewed as noisy but informative and causes the posterior belief about project NPV to be a weighted average of the prior belief and the signal, for example, $\operatorname{Pr}\left(G \mid s=s_{G}, p=N\right)=\hat{\theta} \in(\theta, 1)$. And an uninformative signal is disregarded so that the posterior belief about project quality stays at the prior belief, for example, $\operatorname{Pr}\left(G \mid s=s_{G}, p=U\right)=\theta$. Moreover, we assume that $\hat{\theta} H=1$, that is, the NPV of the project, ignoring the cost of deposits to the bank, $\hat{\theta} H-1$, is zero when the signal is $s_{G}$ and the prior belief about the signal precision is that it is not-precise.

If the signal is $s_{B}$, the posterior probabilities are as follows: $\operatorname{Pr}(G \mid s=$ $\left.s_{B}, p=I\right)=0, \operatorname{Pr}\left(G \mid s=s_{B}, p=N\right)=\check{\theta} \in(0, \theta)$, and $\operatorname{Pr}\left(G \mid s=s_{B}, p=\right.$ $U)=\theta$. In all three cases, the project has negative NPV, so the project is rejected regardless of prior beliefs about signal precision. To ensure that the bank will wish to terminate a project that it views as not having positive NPV, we assume that the failure penalty $\xi \geq H /[H-1]$, where $H$ is the payoff of the good project. ${ }^{12}$

[^8]We model heterogeneity of prior beliefs between the bank and the depositors regarding the signal precision as follows. ${ }^{13}$ They draw their prior beliefs randomly, with $p_{d} \in\{I, N, U\}$ being the depositors' prior belief and $p_{b} \in\{I, N, U\}$ being the bank's prior belief. Thus, the bank's beliefs may differ from the depositors' and we assume the correlation structure to be

$$
\begin{align*}
& \operatorname{Pr}\left(p_{d}=I \mid p_{b}=I\right)=\rho \in[0,1],  \tag{1}\\
& \operatorname{Pr}\left(p_{d}=N \mid p_{b}=I\right)=[1-\rho]\left[\frac{q_{N}}{q_{N}+q_{U}}\right] \equiv[1-\rho] \beta,  \tag{2}\\
& \operatorname{Pr}\left(p_{d}=U \mid p_{b}=I\right)=[1-\rho]\left[\frac{q_{U}}{q_{N}+q_{U}}\right] \equiv[1-\rho][1-\beta], \tag{3}
\end{align*}
$$

where $\beta \equiv q_{N} /\left[q_{N}+q_{U}\right]$ is the relative likelihood that the signal is not precise, conditional on the signal being either not precise or uninformative. The precision drawn by the bank is privately observed by the bank and not verifiable by others. Similarly, the precision drawn by the depositors is privately observed by the depositors and not verifiable by others.

The value of $\rho$ measures the "degree of agreement" between the bank and depositors, with a higher $\rho$ representing greater agreement in the sense of a higher probability that their prior beliefs about signal precision will coincide; $\rho=1$ indicates perfect agreement and $\rho=0$ indicates perfect disagreement. The agreement parameter $\rho$ is affected by the attributes of the borrower's project and/or business characteristics that affect the project's informational opaqueness. That is, even among the class of informationally opaque projects, there are some that may be more opaque than others. If a project involves a radically new product or business design, there may be little hard historical data to gauge the future prospects of the project. Project evaluation may thus have to be based largely on soft information that is inherently subjective in nature [Stein (2002)], possibly causing $\rho$ to be low. By contrast, for a project that is somewhat more familiar in the sense that similar projects have been tried in the past, there may be a more balanced mix of hard historical data and soft information, so the value of $\rho$ may be relatively high. Even though there are possibly multiple depositors, we will assume that their prior beliefs about signal precision are perfectly correlated, so that depositors act as a monolithic group. ${ }^{14}$

[^9]All agents have "rational beliefs" as defined by $\operatorname{Kurz}(1994 a, b)$, who provides a theoretical foundation for heterogenous priors. Although Kurz's theory of rational beliefs has many aspects, the two aspects most relevant for our analysis are that agents have different priors and that all these priors are consistent with the data in the sense that none can be precluded by historical data. That is, what we are modeling is a setting in which the economic observables based on which agents form beliefs about informationally opaque projects are "stable" but not "stationary," whereas transparent projects are associated with "stationary" beliefs. Thus, with informationally-opaque projects, agents will not be able to uniquely derive the precision of the signal from historical data, and many different distributions may be consistent with the data. ${ }^{15}$

### 1.3 Competitive structure of the deposit and loan markets, and time line

We now describe competition in the loan and deposit markets and the sequence of events.
1.3.1 The deposit market. A depositor's reservation two-period expected payoff from a $\$ 1$ deposit, $r_{d}>1$, can be thought of as the riskless interest factor. The expected payoff is composed of the interest paid on deposits, the repayment of principal, and the value of the bank's liquidity services. Note that the interest rate promised to depositors, which is constrained to be nonnegative, can be conditioned only on events that can be contracted upon. The only such event here is the timing of deposit withdrawal. If depositors withdraw at $t=1$, the bank must call back the loan, and collect $\$ 1$; hence it promises $\$ 1$ to depositors. It is this possibility that makes the bank fragile. If deposits are withdrawn at $t=2$, depositors are given their entire promised repayment to the extent permitted by the bank's payoff. Since this payoff is observable to all at $t=2$, there will be no disagreement over repayment to depositors. ${ }^{16}$ In the following analysis, we solve for the equilibrium promised repayment. The deposit market is perfectly competitive, so each depositor receives exactly $r_{d}$ in expectation at $t=2$. There is no deposit insurance for either core deposits or purchased money in this base model. We introduce deposit insurance in Section 2.7.

[^10]

Figure 1
Sequence of events
1.3.2 The loan market. We assume the loan market is imperfectly competitive, so the project surplus is shared between the bank and the borrower. The participation constraints for the bank and the borrower are that their expected payoffs from project investment should be nonnegative. We assume that the bank gets $\alpha \in(0,1)$ share of the surplus and the borrower gets the remaining $1-\alpha$. We show later that such a specification is isomorphic to a debt contract between the bank and the borrower. We therefore refer to the financing extended by the bank as a loan. Initially $\alpha$ is taken as given; we endogenize it in Section 3 via interbank competition.

Figure 1 shows the sequence of events.

## 2. The Analysis

We now analyze the basic model. We begin by collecting our assumptions about the exogenous parameters. The Appendix [(A1)-(A4)] contains the precise restrictions corresponding to these assumptions. We then examine project continuation/termination at $t=1$ contingent upon the signal precision beliefs of the bank and the depositors, and the payoffs to borrowers, banks, and depositors based on that. We subsequently
study self-selection by banks, and self-selection among the depositors and borrowers. Interbank competition is examined in Section 3.

Assumption 1. The additional value of liquidity services for two-period core deposits versus one-period core deposits from a solvent bank, $\tau_{2}-\tau_{1}$, is neither too high nor too low.

This assumption guarantees that the additional value of liquidity services from keeping core deposits for two periods (from $t=0$ to $t=2$ ) in a solvent bank should be high enough to ensure that core deposits are stable in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=N\right\}$, but should not be so high so that core deposits will be withdrawn in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=U\right\}$.

Assumption 2. The upfront investment for liquidity services $(F)$ is neither too high nor too low.

As we will see in Section 2.3, this assumption ensures that the fixed investment in liquidity services for core deposits should be high enough to induce the transaction bank to finance with purchased money even though the marginal cost of core deposits is lower than that of purchased money. However, this assumption also guarantees that $F$ is not so high that it deters the relationship bank from financing with core deposits.

Assumption 3. The payoff of the good project (H) is high enough.
This assumption makes the good project sufficiently attractive so that a higher agreement parameter $\rho$ always leads to a higher profit for a relationship bank financed with core deposits.

Assumption 4. The expected return demanded by depositors ( $r_{d}$ ) is sufficiently high.

This assumption is sufficient to ensure that issuing demand deposits dominates the issuance of two-period deposits for a bank (see Section 2.6).

### 2.1 Signal-contingent project continuation/termination and payoffs

The project will be prematurely terminated at $t=1$ either because the bank wishes to call back the loan or because depositors withdraw prematurely even though the bank wishes to continue. At $t=1$, the bank will call back the loan and return $\$ 1$ to the depositors if the signal on the informationally opaque project is either $s_{B}$ or is $s_{G}$ coupled with the bank's prior belief about signal precision, $p_{b} \in\{N, U\}$. To see this, note that if the bank continues to invest in the project when $s=s_{G}$ and $p_{b}=N$, then the bank assesses the probability of failure at $t=2$ as $1-\hat{\theta}$ and its expected payoff (before repaying depositors) as $\hat{\theta} H-[1-\hat{\theta}] \xi \leq 0$, since $\hat{\theta} H=1$ and $\xi \geq H /[H-1]$. Thus, the bank will terminate the project. Since $\hat{\theta}>\theta$,
the bank will also terminate the project when $s=s_{G}$ and $p_{b}=U$. Project termination when $s=s_{B}$ is obvious. ${ }^{17}$

If $s=s_{G}$ and $p_{b}=I$, the bank will wish to continue with the project. However, whether depositors want to continue to fund the project depends on their signal precision and the type of deposit (core deposits or purchased money). The bank's fragility refers to the state in which $s=s_{G}$ and $p_{b}=I$ so that the bank wishes to continue, and yet the depositors withdraw their deposits and receive $\$ 1$. The reason for premature project termination is that the relationship loan cannot be sold to another bank because of its illiquidity that arises from the relationship-specific nature of the bank's payoff enhancement effort. We say more on this later.

An obvious question this raises is: why can the bank not cope with its fragility by issuing two-period deposits? While this appears to be an alternative to demand deposits, we show in Section 2.6 that the two-period deposit contract is dominated by demand deposits.

Clearly, the informationally transparent project would never have its funding cut off if it were known at $t=0$ that the project was good. The informationally opaque project is a different matter, however. Whether the loan is funded with core deposits or purchased money will make a difference in whether the depositors withdraw their deposits at $t=1$ when they do not wish to continue to fund the project. ${ }^{18}$ As shown later (see Proposition 1), this causes the project enhancements to be different for different types of deposit financing for a relationship bank. So we now use $e_{p u r}$ to denote the enhancement with purchased money and $e_{\text {core }}$ to denote the enhancement with core deposits. Let $r_{j}^{i}$ represent the repayment obligation (to depositors) of the $i$ bank funded by $j$, with $i \in\{R, T\}$ and $j \in\{$ pur, core $\}$. Here, " $R$ " represents "relationship," " $T$ " represents "transaction," "pur" represents "purchased money" and "core" represents "core deposits." In each case below, we will focus on $s=s_{G}$ and $p_{b}=I$, which is the combination of interest for banking fragility, and compare the depositors' payoff from withdrawing deposits to that from not withdrawing, so as to determine whether deposits will be withdrawn. We consider three main cases corresponding to the informationally opaque project being funded by a relationship bank with purchased money and with core deposits, and the informationally transparent project being funded by a transaction bank with either purchased money or core deposits. In each of these three cases, we

[^11]examine subcases defined by different realizations of $p_{d}$ combined with $\left\{s=s_{G}, p_{b}=I\right\}$. We focus primarily on depositors' payoffs; payoffs to other agents are specified in Figure 2.

## Possibility 1. The informationally opaque project is funded by a relationship bank with purchased money:

(i) $\mathbf{s}=\mathbf{s}_{\mathbf{G}}$, and $\mathbf{p}_{\mathbf{b}}=\mathbf{p}_{\mathbf{d}}=\mathbf{I}$. Deposits are not withdrawn and the project is continued. Both the bank and depositors believe that the project will yield a payoff of $H+e_{p u r}$ for sure at $t=2$ and the total surplus is $H+e_{p u r}-r_{p u r}^{R}-\kappa e_{p u r}^{2} / 2$, with the depositors receiving $r_{p u r}^{R}$ if they do not withdraw, which exceeds their payoff from withdrawal.
(ii) $\mathbf{s}=\mathbf{s}_{\mathbf{G}}, \mathbf{p}_{\mathbf{b}}=\mathbf{I}$, and $\mathbf{p}_{\mathbf{d}}=\mathbf{N}$. Deposits are withdrawn and the project is terminated. If deposits are withdrawn at $t=1$, the project is terminated and the bank's project enhancement effort is wasted. The total (negative) surplus, $-\kappa e_{p u r}^{2} / 2$, is shared between the borrower and the bank, and depositors are paid off $\$ 1$. However, if deposits are not withdrawn, the project is continued since both the bank and the borrower believe that the project will pay off $H+e_{p u r}$ and they will pay the depositors $r_{p u r}^{R}$ for sure at $t=2$. The depositors, however, believe that the promised repayment will be made at $t=2$ only with probability $\hat{\theta}$, and hence the expected payoff perceived by the depositors themselves is $\hat{\theta} r_{p u r}^{R}$, where $\hat{\theta} r_{\text {pur }}^{R}<1$ since $r_{p u r}^{R}<H$ and $\hat{\theta} H=1$. Thus, deposits will be withdrawn in this case, since depositors' payoff from withdrawal is higher.
(iii) $\mathbf{s}=\mathbf{s}_{\mathbf{G}}, \mathbf{p}_{\mathbf{b}}=\mathbf{I}$, and $\mathbf{p}_{\mathrm{d}}=\mathbf{U}$. Deposits are withdrawn and the project is terminated. If deposits are withdrawn, the project is terminated and the depositors receive $\$ 1$ at $t=2$. If deposits are not withdrawn, depositors perceive their expected payoff to be $\theta r_{\text {pur }}^{R}$. Since $\theta r_{p u r}^{R}<\hat{\theta} r_{p u r}^{R}<1$ (as explained above), deposits are withdrawn.

## Possibility 2. The informationally opaque project is funded by a relationship bank with core deposits:

(i) $\mathbf{s}=\mathbf{s}_{\mathbf{G}}$, and $\mathbf{p}_{\mathbf{b}}=\mathbf{p}_{\mathbf{d}}=\mathbf{I}$. Deposits are not withdrawn and the project is continued. Absent a withdrawal at $t=1$, the total payoff to the depositors is $r_{\text {core }}^{R}+\tau_{1}+\tau_{2}$, which exceeds their payoff from withdrawal. Hence, core deposits will not be withdrawn.
(ii) $\mathbf{s}=\mathbf{s}_{\mathbf{G}}, \mathbf{p}_{\mathrm{b}}=\mathbf{I}$, and $\mathbf{p}_{\mathrm{d}}=\mathbf{N}$. Deposits are not withdrawn and the project is continued. If deposits are withdrawn, depositors switch their $\$ 1$ deposit to a new bank born at $t=1$, earning a financial payoff of $\$ 1$ and a utility of $\tau_{1}$ from the new bank's liquidity services, plus a first-period utility of $\tau_{1}$ from the original bank's liquidity services. Thus, the payoff to the depositors is $1+\tau_{1}+\tau_{1}$.
A. Informationally Opaque Project Funded by a Relationship Bank with Purchased Money

B. Informationally Opaque Project Funded by a Relationship Bank with Core Deposits


Figure 2
A schematic of the investment payoffs for informationally opaque projects
Note: The first item in each bracket is the payoff to the borrower, the second is the payoff to the bank and the third is the payoff to the depositors. The payoff to an agent is from that agent's own perspective.

If deposits are not withdrawn, the project is continued and depositors perceive their expected payoff to be $\hat{\theta} r_{\text {core }}^{R}+\tau_{1}+\hat{\theta} \tau_{2}$. Assumption 1 implies $\hat{\theta} r_{\text {core }}^{R}+\tau_{1}+\hat{\theta} \tau_{2}>1+\tau_{1}+\tau_{1}$, and hence core deposits will not be withdrawn.
(iii) $\mathbf{s}=\mathbf{s}_{\mathbf{G}}, \mathbf{p}_{\mathbf{b}}=\mathbf{I}$, and $\mathbf{p}_{\mathbf{d}}=\mathbf{U}$. Deposits are withdrawn and the project is terminated. If deposits are withdrawn, depositors' payoff at $t=2$ is $1+\tau_{1}+\tau_{1}$. If deposits are not withdrawn, depositors perceive their expected payoff is $\theta r_{\text {core }}^{R}+\tau_{1}+\theta \tau_{2}$. Assumption 1 ensures that $\theta r_{\text {core }}^{R}+\tau_{1}+\theta \tau_{2}<1+\tau_{1}+\tau_{1}$, and hence core deposits will be withdrawn.

This analysis indicates that core deposits are more stable than purchased money in that core deposits are less likely to be withdrawn when the bank and depositors disagree about the precision of the project value signal. That is, with $s=s_{G}$, purchased money is withdrawn whenever depositors' prior belief is $p_{d} \in\{U, N\}$, but core deposits are withdrawn only if $p_{d}=U$ and not if $p_{d}=N$. The greater stability of core deposits arises because purchased money financiers care only about their financial return and thus withdraw whenever their disagreement with the bank causes the expected value of their return to fall below that from withdrawing at $t=1$, whereas core depositors care about financial return and the value of liquidity services. The source of the relative stability of core deposits in the $\left\{s=s_{G}, p_{b}=I, p_{d}=N\right\}$ state is thus the additional liquidity value of core deposits in a solvent bank, $\tau_{2}-\tau_{1}$, if these deposits are kept in the bank until $t=2$. Consequently, core depositors choose not to withdraw when the additional financial return from withdrawal at $t=1$ is exceeded by the liquidity benefit from keeping deposits with the bank until $t=2$. A measure of core deposit stability is $\beta$, which is the likelihood of getting a not-precise $(N)$ signal relative to the likelihood of getting an uninformative $(U)$ signal. Figure 2 sketches the payoffs for an informationally opaque project funded by a relationship bank with purchased money and core deposits.

## Possibility 3. The informationally transparent project is funded by a transaction bank with purchased money or core deposits:

Deposits are not withdrawn and the project is continued. Informationally transparent loans involve no uncertainty or disagreement. The payoffs to the borrower, the bank, and the depositors at $t=2$ are $\alpha[H-$ $\left.r_{p u r}^{T}\right],[1-\alpha]\left[H-r_{p u r}^{T}\right]$, and $r_{p u r}^{T}$, respectively, with purchased money, and $\alpha\left[H-r_{\text {core }}^{T}\right],[1-\alpha]\left[H-r_{\text {core }}^{T}\right]$, and $r_{\text {core }}^{T}$, respectively, with core deposits. With either purchased money or core deposits, depositors are better off not withdrawing.

### 2.2 Self-selection in the loan market: the link between the informational transparency of the loan and the type of bank funding the loan

For simplicity we have assumed that banks specialize as either relationship or transaction lenders. The following result is straightforward given the setup of the model.

Lemma 1. Informationally opaque projects will always be funded by the relationship bank. Informationally transparent projects will always be funded by the transaction bank.

The intuition is as follows. Because a relationship loan adds value to an informationally opaque borrower but a transaction loan does not,
it is optimal to have all informationally opaque borrowers funded by a relationship loan. An informationally transparent borrower would be indifferent to the distinction between a relationship and a transaction loan, if the deposit funding costs (for the same kind of deposit) were equal for relationship and transaction banks and the project-surplus sharing rules were also identical (which we have assumed they are). However, the deposit funding costs will be ceteris paribus higher for a relationship bank, reflecting its funding of informationally opaque projects. Thus, the transaction bank will be at an advantage relative to the relationship bank in funding a transaction loan.

### 2.3 Self-selection in the deposit market: the link between the type of the bank and the type of deposit funding source it chooses

We show that self-selection in the deposit market results in the relationship bank choosing to finance with core deposits and the transaction bank choosing to finance with purchased money.

To do this, we first determine the repayments at $t=2$ that must be promised to depositors per dollar of deposits, $r_{j}^{i}, i \in\{R, T\}$ and $j \in\{p u r$, core $\}$. These repayments can be made only when the project is good and continued at $t=1$. The promised repayments are determined so that the expected two-period returns to core deposits and purchased money are both $r_{d}$, as they should be in equilibrium. Explicit expressions for these promised repayments appear below.

Lemma 2. The repayments promised to depositors at $t=2$ per dollar of deposits when the project is good and continued, are

$$
\begin{align*}
r_{\text {core }}^{R}= & \frac{r_{d}-\left\{1-\theta q_{I}[\rho+(1-\rho) \beta]\right\}-2 \tau_{1}}{\theta q_{I}[\rho+(1-\rho) \beta \hat{\theta}]} \\
& -\left\{\tau_{2}-\left[\frac{\rho+(1-\rho) \beta}{\rho+(1-\rho) \beta \hat{\theta}}\right] \tau_{1}\right\},  \tag{4}\\
r_{\text {pur }}^{R}= & \frac{r_{d}-\left(1-\theta q_{I} \rho\right)}{\theta q_{I} \rho},  \tag{5}\\
r_{\text {core }}^{T}= & r_{d}-\tau_{1}-\tau_{2},  \tag{6}\\
r_{\text {pur }}^{T}= & r_{d} . \tag{7}
\end{align*}
$$

To understand the intuition, consider the differences between core deposits and purchased money and what they entail for the relationship and transaction banks. Core deposits have two features that affect the repayment promised to depositors, $r_{\text {core }}^{i}$ (for $i \in\{R, T\}$ ): (i) the value of the bank's liquidity services, and (ii) the relative stability of these deposits in the case of disagreement. For a transaction bank, core deposits stability is irrelevant since this bank funds only informationally-transparent loans
for which disagreement is absent. Consequently, only the relative liquidity of core deposits affects the deposit pricing for a transaction bank, and this bank pays $\tau_{1}+\tau_{2}$ less on its core deposits than on its purchased money.

Now consider the relationship bank's repayment obligation on purchased money. First note that $r_{p u r}^{R}>r_{p u r}^{T}$, since $\left[r_{d}-\left(1-\theta q_{I} \rho\right)\right]\left(\theta q_{I} \rho\right)^{-1}>r_{d}$. The relationship bank pays more than the transaction bank for purchased money because there is a possibility of disagreement between the relationship bank and the purchased money providers that does not exist for a transaction bank; note that a decrease in disagreement reduces the spread $r_{p u r}^{R}-r_{p u r}^{T}$, since $\partial r_{p u r}^{R} / \partial \rho<0$. As expected, the pricing of the relationship bank's purchased money is affected by disagreement but not by the value of liquidity services.

We now turn to the relationship bank's repayment obligation on core deposits, $r_{\text {core }}^{R}$. In addition to the liquidity value of core deposits, $r_{\text {core }}^{R}$ is affected both by the agreement parameter $(\rho)$ and the relative conditional likelihood of the signal's being not precise $(\beta)$. Note that $\rho$ affects the stability of all deposits and $\beta$ determines the stability of core deposits relative to purchased money. Thus, $\partial r_{\text {core }}^{R} / \partial \rho<0$ and $\partial r_{\text {core }}^{R} / \partial \beta<0$. Further, $r_{\text {core }}^{R}>r_{\text {core }}^{T}$, for reasons similar to those underlying $r_{p u r}^{R}>r_{p u r}^{T}$. And, $r_{\text {core }}^{R}<r_{p u r}^{R}$ due to the liquidity value of core deposits.

To summarize, the marginal cost of core deposits is always less than the marginal cost of purchased money for either a relationship bank or a transaction bank, that is, $r_{\text {core }}^{j}<r_{\text {pur }}^{j}$ for $j \in\{R, T\}$. However, the fixed cost of core deposits $(F>0)$ is higher than the fixed cost of purchased money (0) for either a relationship or a transaction bank. Moreover, we have $r_{\text {pur }}^{T}<r_{\text {pur }}^{R}$ and $r_{\text {core }}^{T}<r_{\text {core }}^{R}$, which means that the marginal deposit funding cost is always higher for the relationship bank than for the transaction bank.
2.3.1 The transaction bank's choice of deposit funding source. A transaction bank financing with core deposits generates a total net surplus of $H-r_{\text {core }}^{T}-F$, because it only finances informationally transparent good projects. Since the transaction bank receives a fraction $\alpha$ of the surplus, ${ }^{19}$ the transaction bank's expected profit is given by $\pi_{\text {core }}^{T}=\alpha\left[H-r_{\text {core }}^{T}-F\right]$. If the transaction bank finances with purchased money, it generates a total net surplus of $H-r_{p u r}^{T}$. Since the transaction

[^12]bank receives a fraction $\alpha$ of the surplus, ${ }^{20}$ the transaction bank's expected profit is given by $\pi_{p u r}^{T}=\alpha\left[H-r_{p u r}^{T}\right]$. Note that
\[

$$
\begin{equation*}
\pi_{p u r}^{T}-\pi_{\text {core }}^{T}=\alpha\left[F-\tau_{1}-\tau_{2}\right] . \tag{8}
\end{equation*}
$$

\]

Assumption 2 guarantees that the bank's fixed investment for liquidity services is large enough to satisfy $F>\tau_{1}+\tau_{2}$, making $\pi_{p u r}^{T}-\pi_{\text {core }}^{T}>0$ and implying that the transaction bank prefers to fund with purchased money. Given the equivalence between our specification and a loan contract between the bank and the borrower, henceforth we will not explicitly show this equivalence.
2.3.2 The relationship bank's choice of deposit funding source. If the relationship bank finances with core deposits, it invests $F$ to provide liquidity services to core depositors and exerts an effort $e_{\text {core }}$ with a cost of $\kappa e_{\text {core }}^{2} / 2$. Thus, the total net surplus is $\theta q_{I}[\rho+(1-$ $\rho) \beta]\left[H+e_{\text {core }}-r_{\text {core }}^{R}\right]-\kappa e_{\text {core }}^{2} / 2-F$. The relationship bank's expected profit is determined by the solution to the following optimization problem at $t=0$ :
$\pi_{\text {core }}^{R} \equiv \max _{\{\text {ccore }\}} \alpha\left\{\theta q_{I}[\rho+(1-\rho) \beta]\left[H+e_{\text {core }}-r_{\text {core }}^{R}\right]-\frac{\kappa e_{\text {core }}^{2}}{2}-F\right\}$.
If the relationship bank finances with purchased money, it exerts an effort $e_{p u r}$ with a cost of $\kappa e_{p u r}^{2} / 2$. There is no fixed investment for liquidity services and hence the total net surplus is $\theta q_{I} \rho\left[H+e_{p u r}-r_{p u r}^{R}\right]-\kappa e_{p u r}^{2} / 2$. The relationship bank's expected profit is given by the solution to the following optimization problem at $t=0$ :

$$
\begin{equation*}
\pi_{p u r}^{R} \equiv \max _{\left\{e_{p u r}\right\}} \alpha\left\{\theta q_{I} \rho\left[H+e_{p u r}-r_{p u r}^{R}\right]-\frac{\kappa e_{p u r}^{2}}{2}\right\} \tag{10}
\end{equation*}
$$

We now have the following proposition.

Proposition 1. The transaction bank always finances with purchased money. For values of the agreement parameter $\rho$ lower than a cutoff $\rho^{*}$, the relationship bank finances with core deposits. For $\rho \geq \rho^{*}$, the relationship bank finances with purchased money. Moreover, for any fixed value of $\rho$, the relationship bank exerts more effort in enhancing the value of the

[^13]borrower's project when it finances with core deposits than when it finances with purchased money, that is, $e_{\text {core }} \geq e_{p u r}$.

The proposition can be understood as follows. For the transaction bank, deposit stability adds no value since there is never disagreement between the bank and the depositors. The bank's liquidity services result in a decrease in the marginal cost of core deposit financing, and this increases the bank's expected profit. But this benefit comes at the cost of a fixed investment, $F$. Because $F$ outweighs the benefit of the lower marginal cost due to the "liquidity discount" $\left(\tau_{1}+\tau_{2}\right)$, the average cost of core deposit financing is higher than that of purchased money for the transaction bank, and the bank prefers purchased money despite its higher marginal cost.

For the relationship bank, core deposits produce an added benefit that is absent for the transaction bank. Because of their greater relative stability, core deposits yield a lower probability that a profitable, informationally opaque project will be terminated owing to premature deposit withdrawal caused by disagreement between the bank and the depositors. The borrower who takes a relationship loan is thus willing to pay the bank more when it funds with core deposits than when it funds with purchased money. This additional benefit of core deposits for the relationship borrower makes core deposits more valuable for the relationship bank than for the transaction bank, despite the fact that the average cost of core deposits exceeds that of purchased money. And when the agreement parameter $\rho$ is small enough, this additional benefit of core deposits makes the total benefits of core deposits outweigh the relative cost in the case of relationship lending. The reason the superiority of core deposits over purchased money for the relationship bank depends on $\rho$ being sufficiently small is that the relative stability value of core deposits diminishes as $\rho$ increases; for example, the stability of core deposits relative to purchased money has no value when $\rho=1$ (no disagreement). Moreover, for any fixed value of $\rho$, core deposits are less likely to be withdrawn in the event of disagreement and hence less likely than purchased money to cause premature termination of the relationship loan. Thus, for any fixed value of $\rho$, the bank invests more in project enhancement when it is financed with core deposits ( $e_{\text {core }}$ ) than when it is financed with purchased money ( $e_{\text {pur }}$ ).

We now present a result about the cutoff $\rho^{*}$, which is the value of the agreement parameter $\rho$ below which the relationship bank prefers to finance with core deposits.

## Corollary 1.

$\frac{d \rho^{*}}{d\left[\tau_{2}-\tau_{1}\right]}>0$ and $\frac{d \rho^{*}}{d F}<0$.
A bigger $\tau_{2}-\tau_{1}$ means that the depositors attach a higher value to the liquidity benefit of core deposits if they are kept in the original bank for


Figure 3
Relation between expected profits and agreement parameter for relationship lenders
Note: Here $\pi_{\text {core }}^{R}$ is the expected profit of the relationship bank with core deposit financing and $\pi_{p u r}^{R}$ is the expected profit of the relationship bank with purchased money financing.
two periods than if they are switched to another bank at $t=1$. So the cutoff $\rho^{*}$ increases with $\tau_{2}-\tau_{1}$ and core deposits are preferred for a larger set of $\rho$ values. A bigger $F$ has the opposite effect.

Numerical Example: Now we provide a numerical example to illustrate the bank's choice of funding source. In Figure 3, we allow $\rho$ to vary and hold fixed $r_{d}=1.091, q_{I}=3 / 8, q_{N}=4 / 8, q_{U}=1 / 8, \beta=$ $0.8, \tau_{1}=0.0157, \tau_{2}=0.26, \kappa=0.028, \theta=0.444, H=1.35, \hat{\theta}=0.7407$, $F=0.2758$ and $\alpha=0.5$. This figure depicts the relationship bank's expected profits with core deposit financing ( $\pi_{\text {core }}^{R}$ ) and purchased money financing $\left(\pi_{p u r}^{R}\right)$ for different values of $\rho$. With these parameter values, we have $\rho^{*}=0.67$, and as predicted by Proposition 1, the bank enjoys a higher expected profit with core deposit financing when $\rho<\rho^{*}=0.67$, and a higher expected profit with purchased money financing when $\rho>\rho^{*}=0.67$. Although $\pi_{\text {core }}^{R}$ and $\pi_{p u r}^{R}$ are both increasing in $\rho, \pi_{p u r}^{R}$ is more sensitive than $\pi_{\text {core }}^{R}$ to changes in $\rho$.

### 2.4 Self-selection among depositors

Suppose there are two groups of depositors: group $\Psi_{l}$ that values the bank's liquidity services, and group $\Psi_{n}$ that does not value the bank's liquidity
services. Because the transaction bank always finances with purchased money, and the relationship bank finances with either core deposits or purchased money depending on the value of $\rho$, core deposits are provided only to a relationship bank, with expected payoff:

$$
\begin{align*}
r_{d}= & \left\{\left(1-\theta q_{I}\right)+\theta q_{I}(1-\rho)(1-\beta)+\theta q_{I}[\rho+(1-\rho) \beta \hat{\theta}] r_{\text {core }}^{R}\right\} \\
& +\left\{\theta q_{I} \rho\left(\tau_{1}+\tau_{2}\right)+\theta q_{I}(1-\rho) \beta\left(\tau_{1}+\hat{\theta} \tau_{2}\right)\right. \\
& \left.+\left[1-\theta q_{I}[\rho+(1-\rho) \beta]\right]\left(2 \tau_{1}\right)\right\}, \tag{11}
\end{align*}
$$

where the term $\theta q_{I} \rho\left(\tau_{1}+\tau_{2}\right)+\theta q_{I}(1-\rho) \beta\left(\tau_{1}+\hat{\theta} \tau_{2}\right)+\left[1-\theta q_{I}[\rho+\right.$ $(1-\rho) \beta]]\left(2 \tau_{1}\right)$ represents the value of the bank's liquidity services. This leads to self-selection among depositors:

Proposition 2. Depositors who value the bank's liquidity services (group $\left.\Psi_{l}\right)$ prefer core deposits and depositors who do not value these liquidity services (group $\Psi_{n}$ ) prefer purchased money.

This is clearly a partial equilibrium result since the depositors' reservation return $r_{d}$ is taken as exogenous. In a general equilibrium, $r_{d}$ will be endogenously determined by the value of liquidity services and other factors. But as long as all depositors have the same reservation payoffs across core deposits and purchased money, regardless how $r_{d}$ is determined, a depositor from group $\Psi_{l}$ will prefer core deposits since he values the liquidity service that is embedded in the expected payoff of core deposits. Similarly, a depositor from group $\Psi_{n}$ prefers purchased money since he does not value the liquidity service provided by core deposits.

### 2.5 How informationally-opaque borrowers choose banks based on their funding source

Suppose that within the set of borrowers with informationally-opaque projects, there are two groups: group $\Phi_{l}$ with agreement parameter $\rho<\rho^{*}$ and group $\Phi_{h}$ with $\rho>\rho^{*}$. We interpret group $\Phi_{l}$ as having relatively highly informationally opaque projects subject to greater potential disagreement, and group $\Phi_{h}$ as having projects with relatively low informational opaqueness.

Proposition 3. The borrowers in group $\Phi_{l}$, whose projects have relatively high informational opaqueness ( $\rho<\rho^{*}$ ), borrow from relationship banks that finance with core deposits, whereas borrowers in group $\Phi_{h}$, whose projects have relatively low informational opaqueness ( $\rho>\rho^{*}$ ), borrow from relationship banks that finance with purchased money.

This proposition reveals a novel aspect of asset-liability matching: the assets where the bank adds the most value are economically efficiently
matched with the liabilities where it adds the most value. On the asset side, the bank enhances the payoffs of relationship loans through effort expended in loan advisory services, and the magnitude of this payoff enhancement is increasing in the probability that the loan will not be prematurely called back due to early deposit withdrawal. Thus, the nature of the deposits used to finance the loan affects the value of that loan as perceived by the borrower. The disagreement-induced likelihood of premature termination of a relationship loan financed with purchased money diminishes the perceived value the bank adds to a relationship loan. But this diminution in value is relatively small if the likelihood of disagreement between the bank and depositors is relatively low (high $\rho$ ). So in this case the bank prefers to fund with purchased money due to its lower average cost, even at the expense of the (relatively low) probability of premature withdrawal. The liquidity services associated with core deposits act as a buffer against the depositors' propensity to withdraw in the case of disagreement, which increases the perceived value of the relationship loan. This protection against premature deposit withdrawal is sufficiently valuable when the probability of disagreement is high (low $\rho$ ), so the bank prefers core deposits despite their higher average cost.

In practice, banks often specialize in certain types of loans, and sometimes choose a predominance of either relationship or transaction lending. Proposition 3 then predicts that banks that specialize in relationship lending will rely more on core deposit funding than banks that specialize in transaction loans. Alternatively, if the bank's funding mix is relatively rigid, then banks that rely more on core deposits will make more relationship loans and those that rely more on purchased money will make transaction loans. Since large banks rely more on purchased money than small banks, ${ }^{21}$ Proposition 3 implies that small banks will tend to specialize in informationally opaque relationship loans and large bank will specialize in informationally-transparent loans. This prediction seems to be consistent with the recent evidence provided by Berger et al. (2005). ${ }^{22}$

Since banking fragility here is caused by premature deposit withdrawal, one might ask why the bank cannot simply go out and raise deposits to replace the withdrawn deposits or sell its relationship loans. Raising replacement deposits is not feasible since the new depositors would not have a different assessment of the value of the bank's loan portfolio from that held by the old depositors. So even if new core deposits were purchased, the bank is likely to suffer rapid attrition of these deposits. Likewise, selling relationship loans is unlikely to be feasible because these loans

[^14]are highly illiquid. This illiquidity is linked to the source of value in the relationship loan which derives from the bank's investing effort in project payoff enhancement. Because this effort is inherently nontransferable if the loan is sold, the loan is illiquid.

### 2.6 The optimality of deposits withdrawable on demand

We have assumed that the bank finances with deposits that are withdrawable on demand, whether these are core deposits or purchased money with very short maturity. This seems like a strong assumption in a model in which the bank could eliminate its withdrawal risk by simply matching its loan and deposit maturities. That is, if the deposit contract were to make it impossible for the depositors to withdraw until $t=2$, how would the analysis change? We show below that such a deposit contract is generally not optimal.

Proposition 4. Either a core deposit or a purchased money contract that makes it impossible for the depositors to withdraw until $t=2$ is dominated by a similar contract that permits the depositors to withdraw at $t=1$.

The intuition is that the interim liquidity of deposits, as manifested in the depositors' ability to withdraw at $t=1$, is valuable to depositors because it protects them against the bank's continuing to invest in a relationship loan that gives depositors a lower expected payoff at $t=2$ than they would get from withdrawing at $t=1$. If the bank denies depositors this protection, the bank enjoys the benefit of continuing with the project when it wishes to do so, but depositors demand a higher promised repayment from the bank, and the bigger the $r_{d}$, the bigger the increase in the promised repayment due to the inability to withdraw at $t=1$. When $r_{d}$ is high enough, the cost outweighs the benefit and it is efficient for the bank to permit interim deposit withdrawal.

### 2.7 The analysis with deposit insurance

There is no deposit insurance associated with either core deposits or purchased money in the above analysis. Our motivation for this feature is that deposit insurance in the United States is incomplete-it protects only the first $\$ 100,000$ of deposits - and banks often operate with fairly significant portions of their total funding being uninsured; this is over $40 \%$ for large banks [Feldman and Schmidt (2001)]. Nonetheless, it is useful to examine the impact of deposit insurance since banking fragility arises in our model from premature withdrawal of core deposits and even partial deposit insurance can add to the sluggishness of core deposit withdrawals.

We begin by noting that the impact of deposit insurance on premature core depositor withdrawals may be limited in our model since a bank failure disrupts the provision of liquidity services to depositors even though the deposit insurer guarantees their financial claims; a failed bank
provides no second-period liquidity services in our model. Depositors will thus associate a liquidity-services-related cost with bank failure and may transfer their deposits to another bank at $t=1$ despite deposit insurance if they disagree with the initial bank's assessment of the value of its relationship loans. More specifically, when the depositors' prior belief about signal precision is uninformative, that is, $\left\{s=s_{G}, p_{b}=I, p_{d}=U\right\}$, the value of the second-period liquidity services that they expect to receive from their original bank is $\theta \tau_{2}$ if they do not switch to another bank at $t=1$. If $\theta$ is sufficiently low such that $\theta \tau_{2}<\tau_{1}$, and the depositors demand at least $2 \tau_{1}$ of liquidity services from core deposits, then they will switch to another bank at $t=1$ even though their financial claims are protected by deposit insurance. That is, core deposits may not be immune to withdrawal risk despite complete deposit insurance if the depositors' demand for liquidity services is sufficiently high. Moreover, as is well recognized now, using deposit insurance to reduce banking fragility has the flavor of a Faustian bargain due to the accompanying loss of market discipline [e.g., Calomiris and Kahn (1991), and Nier and Baumann (2006)].

We examine the impact of deposit insurance both when depositors are concerned solely with its effect on their expected repayment and when the depositors' value of liquidity services is sufficiently high that they care about the effect of deposit insurance on their payoffs as well as the impact of bank failure on liquidity services. For this, it is useful to note that purchased money consists largely of uninsured funding in the form of jumbo negotiable CDs, federal funds, etc. ${ }^{23}$ Of course, some purchased funds may also be insured (e.g., purchased retail deposits). Thus, we now analyze two cases with deposit insurance: (i) core deposits are fully insured while purchased money is not insured, and (ii) both core deposits and purchased money are partially insured but the insurance protection is higher for core deposits than for purchased money.
2.7.1 Core deposits are fully insured while purchased money is not insured. If core deposits are fully insured and $\theta$ is sufficiently high, then the depositors will never withdraw their deposits at $t=1$ even in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=U\right\}$. That is, core deposits are more stable in this case than when there is no deposit insurance. If $\theta$ is sufficiently low and the depositors' demand for liquidity services is sufficiently high, then as discussed above, core deposits will be withdrawn in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=U\right\}$ even if they are fully insured. However, uninsured purchased money will be invariably withdrawn whenever there is disagreement between the bank and depositors, as in our previous

[^15]analysis. Thus, core deposits are more stable than purchased money with complete deposit insurance.
2.7.2 Both core deposits and purchased money are partially insured with higher coverage for core deposits. Now suppose both core deposits and purchased money are partially insured, with coverage $D_{\text {core }}$ (core deposits) and $D_{p u r}$ (purchased money) satisfying $D_{\text {core }}>D_{p u r}$. We first analyze the case in which the informationally opaque project is funded with purchased money. Recall that in the absence of deposit insurance, the depositors will withdraw the deposits at $t=1$ when their prior belief about signal precision is $N$, that is, $\left\{s=s_{G}, p_{b}=I, p_{d}=N\right\}$. One should expect that if $D_{p u r}$ is sufficiently high, then deposits will not be withdrawn in this disagreement state. The cutoff level of deposit insurance can be determined as follows. Note that in order for the depositors to withdraw their deposits in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=U\right\}$ but not in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=N\right\}$, it must be true that
\[

$$
\begin{equation*}
1 \in\left(\theta r_{p u r}^{R}+(1-\theta) D_{p u r}, \hat{\theta} r_{p u r}^{R}+(1-\hat{\theta}) D_{p u r}\right), \tag{12}
\end{equation*}
$$

\]

where the explicit mathematical expression for the repayment obligation $r_{p u r}^{R}$ is given in the Appendix. This condition implies that

$$
\begin{equation*}
D_{p u r} \in\left(D_{p u r}^{*}, D_{p u r}^{* *}\right), \tag{13}
\end{equation*}
$$

where $D_{p u r}^{*}$ is defined such that $\hat{\theta} r_{p u r}^{R}+(1-\hat{\theta}) D_{\text {pur }}^{*}=1$, and $D_{p u r}^{* *}$ is defined such that $\theta r_{p u r}^{R}+(1-\theta) D_{p u r}^{* *}=1$. The explicit mathematical expressions for $D_{p u r}^{*}$ and $D_{p u r}^{* *}$ are given in the Appendix. Thus, as long as $D_{p u r}<D_{p u r}^{*}$, we have $\hat{\theta} r_{p u r}^{R}+(1-\hat{\theta}) D_{p u r}<1$, and depositors will still withdraw their purchased money deposits in the state $\left\{s=s_{G}, p_{b}=\right.$ $\left.I, p_{d}=N\right\}$, as they do in the case without deposit insurance. Given our earlier result that core deposits will not be withdrawn in this state even in the absence of deposit insurance, core deposits are more stable than purchased money regardless of the value of $D_{\text {core }}$. All our previous results hold.
However, if $D_{p u r} \in\left(D_{\text {pur }}^{*}, D_{p u r}^{* *}\right)$, purchased money deposits will become stable in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=N\right\}$. In order for core deposits to continue to be more stable than purchased money, $D_{\text {core }}$ will need to be sufficiently high so that core deposits will not be withdrawn even in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=U\right\}$. That is,

$$
\begin{equation*}
\theta r_{\text {core }}^{R}+(1-\theta) D_{\text {core }}+\tau_{1}+\theta \tau_{2}>1+\tau_{1}+\tau_{1}, \tag{14}
\end{equation*}
$$

where the explicit mathematical expression for the repayment obligation $r_{\text {core }}^{R}$ is given in the Appendix. This condition implies that

$$
\begin{equation*}
D_{\text {core }}>D_{\text {core }}^{*}, \tag{15}
\end{equation*}
$$

where $D_{\text {core }}^{*}$ is defined such that $\theta r_{\text {core }}^{R}+(1-\theta) D_{\text {core }}^{*}=1-\left(\theta \tau_{2}-\tau_{1}\right)$; see the Appendix. For $D_{\text {core }}>D_{\text {core }}^{*}$, core deposits will be stable despite disagreement between the bank and the depositors, and hence will continue to be more stable than purchased money even when $D_{p u r}$ is so high that purchased money is stable in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=N\right\}$. This leads to:

Proposition 5. Even with complete deposit insurance, core deposits are not immune to withdrawal risk. If core deposits are fully insured but purchased money is not, core deposits are always more stable than purchased money, even if core depositors do not face bank-switching costs. If both core deposits and purchased money are partially insured, but core deposits have higher coverage, then core deposits are more stable than purchased money if the insurance coverage for purchased money is below a critical fraction of the insurance coverage for core deposits.

Given the fact that core deposits are largely covered by deposit insurance, whereas much of purchased money is uninsured, all our main results in the base model without deposit insurance are qualitatively unaffected by deposit insurance, and are even strengthened in some cases as deposit insurance can make core deposits even stickier.
2.7.3 Discussion of the stability of core deposits relative to purchased money. We have seen that either liquidity services cum switching costs or deposit insurance can contribute to the greater withdrawal sluggishness of core deposits relative to purchased money. Which factor is more important in determining this relative withdrawal sluggishness is ultimately an empirical issue. Fortunately, it is unimportant for our results whether the primary cause of core deposit stickiness is the combination of liquidity services and switching costs associated with core deposits or the greater deposit insurance coverage for core deposits compared to purchased money or both. ${ }^{24}$ All that matters is that the net effect of all the factors that affect deposit withdrawal is to make core deposits more sticky than purchased money.

We have argued that, by its very nature, purchased money will not be provided by investors who care about the bank's liquidity services, so these services will always be valued more by core depositors. So the question is: do core deposits always have higher deposit insurance coverage than

[^16]purchased money, as we have assumed? We believe the answer is yes, primarily because core deposits are defined this way in the empirical literature. For example, Feldman and Schmidt (2001) write: ". . . we rely on information from the regulatory reports that banks file each quarter in which deposits are divided into five categories: transaction accounts (for example, checking accounts), savings accounts, money market deposit accounts, small time deposits (certificates of deposits with a face value less than $\$ 100,000$ ) and large time deposits. The first four categories represent the primary funding sources of most banks and are collectively known as core deposits. Although it is possible for some of the funds held in these core deposits to be uninsured (an example would be a checking account with a balance greater than $\$ 100,000$ ), the vast majority will be covered by the insurance fund. As such, core deposits can serve as a proxy for the amount of insured deposits in the banking system." Similarly, Berlin and Mester (1999) state that their measure of the core deposit ratio is deposits with denominations less than $\$ 100,000$ as a fraction of the bank's total liabilities. These studies and others clearly indicate that core deposits will be associated with higher insurance coverage than noncore deposits. Further, consistent with our analysis, core deposits are considered more stable than other types of deposits. See, for example, Berlin and Mester (1999) and Feldman and Schmidt (2001).

## 3. Extension of the Analysis: The Role of Ex Ante and Ex Post Competition

In this section, we extend the analysis to examine ex post interbank competition by asking: how does greater competition in relationship lending affects the liability side of the bank's balance sheet, namely its reliance on core deposits? We consider the case in which the likelihood of disagreement between the bank and the depositors is sufficiently high so that the relationship bank finances with core deposits (Proposition 1). We begin with an analysis of ex post competition that unfolds after banks have decided to enter relationship banking. Although a fixed investment $F$ is needed to provide liquidity services with core deposits, this investment is sunk and plays no role in the ex post competition among core-depositfunded relationship lenders.

We model loan market competition using a spatial model [Salop (1979)], and continue to assume perfect competition in the deposit market. At $t=0$, there are $M$ identical borrowers with agreement parameter $\rho$, distributed uniformly along the circumference of a unit circle, with $c>0$ as the borrower's unit transportation cost. There are $\tilde{N}$ identical relationship banks uniformly spaced along the same circle. Viewed at $t=0, N$ is a random variable specified by the commonly known continuously differentiable probability density function $f(N)=A N$, where $A>0$ is a constant. Thus, the support of $f(N)$ is $[0, \sqrt{2 / A}]$. The realized value of $\tilde{N}$
will be known to all at $t=1$. Denote by $\mathbf{E}(N) \equiv \bar{N}$ the expected value of $\tilde{N}$. Ex ante at $t=0$, each bank $i$ invests $F$ to provide liquidity services. $E x$ post when the extent of competition within relationship banking becomes known at $t=1$, each bank determines the effort, $e$, it exerts at $t=1$ to enhance the borrower's project payoff and also the rent it extracts via its share $\alpha$ of the project surplus. Using backward induction, we first analyze each bank's ex post choices of $e$ and $\alpha$ at $t=1$, and then we examine each bank's entry strategy ex ante at $t=0$.

### 3.1 Ex post choice of effort exertion and rent extraction of relationship bank financed with core deposits

Suppose the realized value of $\tilde{N}$ is $N$ and that a relationship bank $j$ 's nearest competitor exerts a project enhancement effort of $\hat{e}_{\text {core }}$ and takes $\hat{\alpha}_{\text {core }}$ fraction of the project payoff from each borrower. If the relationship bank $j$ chooses effort $e_{\text {core }}$ and project payoff share $\alpha_{\text {core }}$, it captures all the borrowers lying within a distance $d_{\text {core }}$, where $d_{\text {core }}$ is determined by: ${ }^{25}$

$$
\begin{aligned}
& \left(1-\alpha_{\text {core }}\right)\left\{\theta q_{I}[\rho+(1-\rho) \beta]\left[H+e_{\text {core }}-r_{\text {core }}^{R}\right]\right. \\
& \left.-\frac{\kappa e_{\text {core }}^{2}}{2}-\frac{F}{M / N}\right\}-c d_{\text {core }} \\
& =\left(1-\hat{\alpha}_{\text {core }}\right)\left\{\theta q_{I}[\rho+(1-\rho) \beta]\left[H+\hat{e}_{\text {core }}-r_{\text {core }}^{R}\right]\right. \\
& \left.-\frac{\kappa \hat{e}_{\text {core }}^{2}}{2}-\frac{F}{M / N}\right\}-c\left(1 / N-d_{\text {core }}\right),
\end{aligned}
$$

where $r_{\text {core }}^{R}$ is given by Equation (4). For a borrower that is located a distance $d_{\text {core }}$ away from bank $j$ and a distance $1 / N-d_{\text {core }}$ away from bank $j$ 's nearest competitor, the left-hand-side (LHS) and right-hand-side (RHS) are the borrower's expected payoffs if he borrows from bank $j$ and bank $j$ 's nearest competitor, respectively. Hence, the number of borrowers that the relationship bank $j$ captures (denoted as $\Omega_{\text {core }}$ ) is given by: ${ }^{26}$

$$
\begin{aligned}
\Omega_{\text {core }} & =2 M d_{\text {core }} \\
& =\frac{M}{N}+\frac{M}{c}\left\{\begin{array}{c}
\left(1-\alpha_{\text {core }}\right)\left[\theta q_{I}[\rho+(1-\rho) \beta]\right. \\
\left.\left[H+e_{\text {core }}-r_{\text {core }}^{R}\right]-\frac{\kappa e_{\text {core }}^{2}}{2}-\frac{F}{M / N}\right] \\
-\left(1-\hat{\alpha}_{\text {core }}\right)\left[\theta q_{I}[\rho+(1-\rho) \beta]\right. \\
\left.\left[H+\hat{e}_{\text {core }}-r_{\text {core }}^{R}\right]-\frac{\kappa \hat{e}_{\text {core }}^{2}}{2}-\frac{F}{M / N}\right]
\end{array}\right\} .
\end{aligned}
$$

[^17]Bank $j$ 's ex post problem is to choose $e_{\text {core }}$ and $\alpha_{\text {core }}$ to maximize its total expected profit $\left(\pi_{\text {core }}^{E P}(N)\right)$, which is the expected profit per borrower times the number of captured borrowers:

$$
\begin{align*}
& \pi_{\text {core }}^{E P}(N) \equiv \max _{\left\{e_{\text {core },}, \alpha_{\text {core } e}\right.} \alpha_{\text {core }} \\
& \times\left\{\theta q_{I}[\rho+(1-\rho) \beta]\left[H+e_{\text {core }}-r_{\text {core }}^{R}\right]-\frac{\kappa e_{\text {core }}^{2}}{2}\right\} \times \Omega_{\text {core }} \tag{16}
\end{align*}
$$

We focus on a symmetric equilibrium in which $e_{\text {core }}=\hat{e}_{\text {core }}$ and $\alpha_{\text {core }}=\hat{\alpha}_{\text {core }}$. The next lemma characterizes the bank's ex post profitability:

Lemma 3. There exists a cutoff $N^{*}$ such that the bank which finances relationship loans with core deposits is unprofitable ex post $\left(\pi_{\text {core }}^{E P}(N)<0\right)$ if the actual number of competing banks $N>N^{*}$, and the bank is profitable ex post $\left(\pi_{\text {core }}^{E P}(N) \geq 0\right)$ if $N \leq N^{*}$.
3.2 Ex ante entry strategy of relationship bank financed with core deposits

We now turn to the bank's ex ante entry strategy at $t=0$, which accounts for the fixed investment, $F$. The bank's ex ante expected profit (denoted as $\pi_{\text {core }}^{E A}$ ) is given by:

$$
\begin{equation*}
\pi_{\text {core }}^{E A}=\mathbf{E}\left(\pi_{\text {core }}^{E P}(N)-\alpha_{\text {core }} F\right) . \tag{17}
\end{equation*}
$$

The bank enters the relationship loan market as long as $\pi_{\text {core }}^{E A} \geq 0$, and its entry decision will depend on its expectation about $\tilde{N}$. The bank's entry strategy is characterized below:

Proposition 6. For each relationship bank financed with core deposits, there exists a cutoff $\bar{N}^{*}$ such that it enters the relationship loan market if $\mathbf{E}(N)=\bar{N} \leq \bar{N}^{*}$; and does not enter the relationship loan market if $\mathbf{E}(N)=\bar{N}>\bar{N}^{*}$.

What is at play here is the usual intuition that an increase in ex post competition increases the numbers of states in which the bank's ex ante expectation is that it will make a loss ex post. The bank's ex ante expected profit is thus decreasing in $\mathbf{E}(N)$, and at some point it falls below $F$ for $\mathbf{E}(N)$ large enough, inducing core-deposit-financed banks to avoid relationship lending.

### 3.3 Impact of competition on the total surplus of relationship banking industry

The above analysis focuses on an individual relationship bank. We now examine the impact of interbank competition on the total surplus of the relationship banking industry, focusing on the case in which $\bar{N} \leq \bar{N}^{*}$, which occurs when core-deposit-financed banks enter the market
(Proposition 6). Conditional on entry, each bank's ex ante expected profit is given by Equation (17), which is decreasing in $\bar{N}$. The total surplus of all relationship banks, denoted as $\Pi_{\text {core }}^{E A}$, is

$$
\begin{equation*}
\Pi_{\text {core }}^{E A}=N\left[\pi_{\text {core }}^{E A}\right] . \tag{18}
\end{equation*}
$$

Proposition 7. The larger the expected number of competitors $(\bar{N})$ in the loan market, the lower the total surplus for each relationship bank financed with core deposits as well as for the entire relationship banking industry.

The intuition behind why the total surplus of the entire relationship banking industry falls owing to higher competition is that competition reduces each bank's share of the borrower's project payoff and hence the marginal return to effort for each bank.

### 3.4 The effect of loan market competition on the relationship bank's choice of funding source

We now analyze how competition affects the relationship bank's ex ante choice of funding source for informationally-opaque projects. The analysis above assumes that the relationship banks finance with core deposits, with the only decision made at $t=0$ being whether to enter the relationship loan market, and if so, whether to invest $F$ to provide liquidity services for core deposits. Instead of fixing the bank's funding source, we assume now that each bank makes two choices at $t=0$. First, it decides whether to finance the informationally opaque projects with core deposits or with purchased money. Second, if it decides to finance with core deposits, it must also decide whether to enter the loan market; by contrast, if it decides to finance with purchased money, it always enters the loan market. ${ }^{27}$

To focus on the more interesting case, we consider the situation in which $\bar{N}<\bar{N}^{*}$. We consider a symmetric equilibrium in which either all the banks finance with core deposits or all the banks finance with purchased money. If the banks finance with purchased money at $t=0$, then following analysis similar to that of core deposit financing, we can show that ex post the bank's project enhancement effort ( $e_{p u r}$ ) and rent extraction $\left(\alpha_{p u r}\right)$ are given by $e_{p u r}=\frac{\theta q_{I} \rho}{\kappa}$, and $\alpha_{p u r}=\frac{c / N}{\theta q_{I} \rho\left[H+e_{p u r}-r_{p u r}^{R}\right]-\frac{\kappa e_{p u r}^{2}}{2}}$, where $r_{p u r}^{R}$ is given by Equation (5). In a symmetric equilibrium, the number of borrowers captured by each bank is $\Omega_{p u r}=M / N$. The ex post expected profit for a bank financed with purchased money (denoted as $\pi_{p u r}^{E P}(N)$ ) is

[^18]thus:
\[

$$
\begin{equation*}
\pi_{p u r}^{E P}(N)=\alpha_{p u r}\left\{\theta q_{I} \rho\left[H+e_{p u r}-r_{p u r}^{R}\right]-\frac{\kappa e_{p u r}^{2}}{2}\right\} \times \Omega_{p u r}=\frac{M c}{N^{2}} \tag{19}
\end{equation*}
$$

\]

Hence, the bank's ex ante expected profit with purchased money (denoted as $\left.\pi_{p u r}^{E A}\right)$ is:

$$
\begin{equation*}
\pi_{p u r}^{E A}=\mathbf{E}\left(\pi_{p u r}^{E P}(N)\right)=M c \mathbf{E}\left(1 / N^{2}\right) \tag{20}
\end{equation*}
$$

Recall that the bank's ex ante expected profit with core deposit financing is given by Equation (17):

$$
\begin{align*}
\pi_{\text {core }}^{E A} & =\mathbf{E}\left(\pi_{\text {core }}^{E P}(N)-\alpha_{\text {core }} F\right) \\
& =M c \mathbf{E}\left(\frac{1}{N^{2}\left(1-\frac{F}{y M / N}\right)}-\frac{F /(M y)}{N\left(1-\frac{F}{y M / N}\right)}\right), \tag{21}
\end{align*}
$$

where $y \equiv \theta q_{I}[\rho+(1-\rho) \beta]\left[H+e_{\text {core }}-r_{\text {core }}^{R}\right]-\kappa e_{\text {core }}^{2} / 2$. The effect of ex post loan market competition on the bank's ex ante funding source is described in the next proposition:

Proposition 8. For a fixed $\bar{N}$, there exists a cutoff $\rho^{* *}$ such that for $\rho>\rho^{* *}$ the relationship bank chooses to finance with purchased money, and for $\rho<\rho^{* *}$ the relationship bank chooses to finance with core deposits. Moreover, the value of the cutoff $\rho^{* *}$ is decreasing in the ex ante expected number of competitors, i.e., $d \rho^{* *} / d \bar{N}<0$. An increase in $\bar{N}$ has the following effects: ( $i$ ) it makes the relationship bank and depositors worse off; (ii) it increases the expected payoffs of borrowers with very high values of $\rho$ (i.e., the borrowers who are always funded with purchased money) and with very low values of $\rho$ (i.e., the borrowers who are always funded with core deposits); and (iii) it decreases the expected payoffs of borrowers with intermediate values of $\rho$ (i.e., an increase of $\bar{N}$ will result in those borrowers being funded with purchased money rather than with core deposits), if $\bar{N}$ increases beyond a threshold value $\bar{N}^{* *}$.

This proposition exposes a new link between interbank competition in relationship lending and each bank's choice of deposit funding source. In particular, banks that make relationship loans rely less on core deposit financing as competition for relationship loans increases. This implies that interbank competition in relationship lending will elevate withdrawal risk
and bank fragility as banks rely more on purchased money that has greater withdrawal risk. ${ }^{28}$

The intuition for the decline of core deposit financing due to greater competition is as follows. As competition in relationship lending increases, the measure of the set of ex post states (the range of realized values of $\tilde{N}$ ) in which the bank loses money after accounting for the fixed investment $F$ increases. In these states, therefore, core-deposit-financed relationship lending is unprofitable ex ante. However, relationship lending financed with purchased money can still avoid negative profits in these states because there is no upfront fixed investment. Hence, heightened competition for relationship lending pushes banks away from core deposits. ${ }^{29}$

This proposition also asserts that greater relationship lending competition may even reduce the expected payoffs of the borrowers. This happens when the borrowers' projects have intermediate informational opacity (intermediate value of $\rho$ ) and loan market competition is already sufficiently high (high $\bar{N}$ ). We saw in Proposition 7 that all banks are collectively worse off due to greater competition. What Proposition 8 adds to this is that borrowers and depositors may also be worse off. The reason is that greater competition leads banks to rely less on core deposits to finance relationship loans, which causes those borrowers with informationally opaque projects to bear more deposit withdrawal risk. The negative effect of this on borrowers can outweigh the benefit for borrowers from an increase in $\bar{N}$ that leads the bank to extract less rents. Moreover, the lower reliance on core deposits means less liquidity services for depositors.

### 3.5 Impact of competition on transaction bank financed with purchased money

We now discuss how competition affects the transaction bank financed with purchased money.

Proposition 9. The borrower's expected payoff is increasing and the transaction bank's expected payoff is decreasing in the degree of competition in the transaction loan market. The depositors' expected payoff is unaffected.

Since transaction loans are always financed with purchased money, which does not require any upfront investment, a transaction bank always enters the market. Stronger loan-market competition lowers the rents the

[^19]bank can extract from the borrower, and hence increasing the borrower's expected payoff and decreasing the transaction bank's expected payoff.

## 4. Conclusion

We have explored the complementarity between the liquidity services provided by the bank in connection with core deposits and the valueadded services by the bank in connection with its relationship lending. We have shown that self-selection occurs among depositors, banks and borrowers. Depositors who value the bank's liquidity services choose core deposits, whereas those who do not value these services invest in purchased money deposits. Each bank chooses to be either a transaction bank or a relationship bank. Transaction banks finance with purchased money, whereas relationship banks finance with either purchased money or core deposits depending on the informational opaqueness of the loans they make. We also show that interbank competition for relationship loans can have significant effects on the volume and nature of relationship lending, on the level of core deposit financing by banks, and on welfare.

Our analysis produces numerous testable predictions. First, there will be a positive correlation between the level of a bank's relationship lending and its core deposit financing. We do not know of any direct empirical evidence of this prediction. However, an implication of this is that small banks will engage more in informationally opaque lending and large banks will engage more in informationally transparent lending, consistent with the evidence in Berger et al. (2005). ${ }^{30}$ Second, the correlation between relationship lending and core deposits will decline as banking becomes more competitive and more relationship loans are financed with purchased money. ${ }^{31}$ This prediction appears to have empirical support. Recent studies [DeYoung et al. (2004), and Genay (2000)] show that core deposit financing is declining, and deposits are increasingly composed of interest-ratesensitive instruments. Finally, for banks already involved in relationship lending, greater interbank competition may cause banks to lose money on average. Since our analysis indicates that the fate of core deposits is intertwined with that of relationship lending, this decline in relationship lending profitability should predictably lead to a decline in the premia paid in interbank sales of branches-the dominant points of receipt of core deposits-and/or a decline in core deposit funding.

In exploring the complementarity between the bank's deposit-taking and lending activities, we have highlighted a new aspect of relationship lending,

[^20]namely, that to the extent that such loans are informationally opaque, there is no a priori reason why the bank will find that its depositors agree with its assessment of the payoff prospects of a relationship loan. This creates endogenous deposit withdrawal risk with relationship lending even when the bank's own managers are convinced that the bank is in sound financial health. This risk can arise even with deposit insurance and our results are likely to be strengthened by deposit insurance. While we did not include bank capital in our analysis, it is clear that, with partial deposit insurance, the introduction of bank capital will provide additional payoff protection for depositors. This will increase the stability of deposits, facilitating relationship lending. In other words, banks that engage in more relationship lending may find it optimal to keep more capital. It is interesting that the potential divergence of beliefs about project values can lead not only to the raison d'être for the emergence of banks [Coval and Thakor (2005)] and predictions about the borrower's choice of bank versus capital market financing [Allen and Gale (1999)], but also to the endogenous withdrawal risk that defines the inherent fragility of banking.

Future research can be directed at examining why some banks specialize in relationship lending and core deposits, whereas others specialize in transaction lending and purchased money. It would also be interesting to examine the role of the regulator and the impact of a potential divergence of beliefs between the bank and the regulator. Moreover, there may be possible applications in the (short-term) financing and capital budgeting decisions of nonfinancial corporations. Payables could be interpreted as analogous to core deposits and commercial paper could be viewed as purchased money. One could then examine how the relative mix of these two financing sources affects the firm's capital budgeting.

## Appendix

Variables Used in the Model: A summary of our model's notation is given below for easy reference.

| $\theta$ | Prior, and posterior (receiving an uninformative good signal) probabilities of <br> project being good |
| ---: | :--- |
| $\hat{\theta}$ | Posterior probability of project being good after receiving a not-precise good <br> signal |
| $\check{\theta}$ | Posterior probability of project being good after receiving a not-precise bad <br> signal |
| $H$ | Payoff of a good project |
| $F$ | Bank's fixed investment for liquidity services |
| $\tau_{1}$ | Value of the bank's first-period liquidity services |
| $\tau_{2}$ | Value of the bank's second-period liquidity services if the bank doesn't fail at |
| $\rho$ | $t=2$ |
| $s_{G}$ | Degree of agreement between the bank and depositors |
| $s_{B}$ | Bood signal |


| $p_{b}$ | Bank's signal precision |
| :---: | :---: |
| $p_{d}$ | Depositors' signal precision |
| $q_{I}$ | Probability of drawing a precise signal (I) |
| $q_{N}$ | Probability of drawing a not-precise signal ( $N$ ) |
| $q_{U}$ | Probability of drawing an uninformative signal ( $U$ ) |
| $\beta$ | $\beta \equiv q_{N} /\left[q_{N}+q_{U}\right]:$ the relative likelihood that the signal is not precise, conditional on the signal being either not-precise or uninformative |
| $r_{d}$ | Depositor's reservation two-period expected payoff from a $\$ 1$ deposit |
| $r_{\text {core }}^{R}$ | Promised repayment to depositors per dollar of deposits for a relationship bank financed with core deposits |
| $r_{p u r}^{R}$ | Promised repayment to depositors per dollar of deposits for a relationship bank financed with purchased money |
| $r_{\text {core }}^{T}$ | Promised repayment to depositors per dollar of deposits for a transaction bank financed with core deposits |
| $r_{p u r}^{T}$ | Promised repayment to depositors per dollar of deposits for a transaction bank financed with purchased money |
| $e_{\text {core }}$ | Project-enhancement effort exerted by a relationship bank financed with core deposits |
| $e_{\text {pur }}$ | Project-enhancement effort exerted by a relationship bank financed with purchased money |
| $\kappa$ | Bank's project-enhancement-effort cost parameter |
| $\alpha$ | Bank's share of project investment surplus |
| $\delta$ | Premature liquidation value of the project at $t=1$ |
| $\xi$ | Penalty of bank failure |
| $\rho^{*}$ | Cutoff value of the agreement parameter such that for projects with $\rho<\rho^{*}$ a relationship bank prefers to finance with core deposits, and for projects with $\rho>\rho^{*}$ a relationship bank prefers to finance with purchased money |
| $\Psi_{l}$ | Group of depositors who attach value to the bank's liquidity services |
| $\Psi_{n}$ | Group of depositors who do not attach value to the bank's liquidity services |
| $\Phi_{l}$ | Group of borrowers with agreement parameter $\rho<\rho^{*}$ |
| $\Phi_{h}$ | Group of borrowers with agreement parameter $\rho>\rho^{*}$ |

## Parametric Restrictions Corresponding to Assumptions 1 to 4:

## Assumption 1

$$
\begin{equation*}
\frac{\left(r_{d}-1\right)-q_{I} \beta(1 / H-\theta)}{2+q_{I} \beta(1 / H-\theta)}<\tau_{1}<\max \left\{\frac{\left(r_{d}-1\right)-\theta q_{I}(H-1)}{2+\theta q_{I}(H-1)}, \tau_{2}\right\} . \tag{A1}
\end{equation*}
$$

## Assumption 2

$$
\begin{equation*}
\tau_{1}+\tau_{2}<F \leq \theta q_{I} \beta\left(\tau_{2}-H \tau_{1}\right)+\frac{\left(\theta q_{I} \beta\right)^{2}}{2 \kappa}-H\left(r_{d}-1-2 \tau_{1}\right) \tag{A2}
\end{equation*}
$$

## Assumption 3

$$
\begin{align*}
& \frac{\left(\theta q_{I}\right)^{2} \beta(1-\beta)}{\kappa}+\theta q_{I}(1-\beta) H+\beta(1-1 / H)\left(r_{d}-1-2 \tau_{1}\right) \\
& \geq \theta q_{I}\left[-(1-\beta) \tau_{2}+(1-2 \beta+\beta / H)\left(1+\tau_{1}\right)\right] \\
& \text { and } \\
& H>F+r_{d}-\tau_{1}-\tau_{2} \tag{A3}
\end{align*}
$$

## Assumption 4

$$
\begin{equation*}
r_{d}>\max \left\{r_{d 1}, r_{d 2}\right\}, \tag{A4}
\end{equation*}
$$

where

$$
\begin{aligned}
& r_{d 1} \equiv 1+\theta q_{I}\left\{\begin{array}{l}
\left.\frac{H\left[H+\theta q_{I} / \kappa-2\right]+[\beta+(1-\beta) \theta H]}{H-[\beta+(1-\beta) \theta H]}\right\}, \\
r_{d 2} \equiv 1+\theta q_{I}\left\{\begin{array}{l}
(1-\beta) H+\frac{\theta q_{I}(1-\beta)}{\kappa}+\left(\tau_{2}-\tau_{1}-1+\frac{2 \tau_{1}}{\theta q_{I}}\right)\left(\frac{1-x}{x^{2}}\right) \\
+\left(\tau_{2}-\tau_{1}-1\right)\left(\frac{1-\beta}{1-\beta / H}\right)^{2}-\frac{\beta(1-1 / H)\left(2 \tau_{1}\right)}{(\beta / H)^{2}}
\end{array} 1\right.
\end{array}\right\},
\end{aligned}
$$

and $x \equiv \beta / H+(1-\beta) \theta$.
Proof of Lemma 1: First, it is always optimal for an informationally opaque project to be funded by a relationship bank instead of a transaction bank, since the payoff of the informationally opaque project can be enhanced (if it turns out to be good) by the relationship bank, while such payoff enhancement is impossible via the transaction bank. Second, it would be indifferent for an informationally transparent project to be funded by either a transaction bank or a relationship bank, if the deposit funding costs are not considered. However, since the transaction bank will choose the deposit source that is most efficient for funding informationally transparent projects, while the relationship bank's choice of deposit source has to take into account those informationally opaque projects, and the deposit funding cost for informationally opaque projects is higher than that for informationally transparent projects, it is optimal for an informationally transparent project to be funded by a transaction bank.

Proof of Lemma 2: The deposit repayments are determined so that the expected payoff to the depositor is $r_{d}$ in equilibrium. Thus, they are determined by the following equations:

$$
\begin{align*}
r_{d}= & \left\{(1-\theta)+\theta\left(1-q_{I}\right)+\left[\theta q_{I}(1-\rho)(1-\beta)\right]\right\}\left(1+\tau_{1}+\tau_{1}\right) \\
& +\left[\theta q_{I} \rho\right]\left(r_{\text {core }}^{R}+\tau_{1}+\tau_{2}\right)+\left[\theta q_{I}(1-\rho) \beta\right]\left(\hat{\theta} r_{\text {core }}^{R}+\tau_{1}+\hat{\theta} \tau_{2}\right),  \tag{A5}\\
r_{d}= & (1-\theta)+\theta\left(1-q_{I}\right)+\theta q_{I}(1-\rho)+\left(\theta q_{I} \rho\right) r_{\text {pur }}^{R},  \tag{A6}\\
r_{d}= & r_{\text {core }}^{T}+\tau_{1}+\tau_{2},  \tag{A7}\\
r_{d}= & r_{\text {pur }}^{T} \tag{A8}
\end{align*}
$$

Solving the equations yields the four promised repayments.

Proof of Proposition 1: The transaction bank's preference for purchased money financing is guaranteed by the parametric assumption in (A2), that is, $F>\tau_{1}+\tau_{2}$. Solving the relationship bank's optimization problem in Equation (9), we have

$$
\begin{equation*}
e_{\text {core }}=\frac{\theta q_{I}[\rho+(1-\rho) \beta]}{\kappa} \tag{A9}
\end{equation*}
$$

Thus, the expected profit for a relationship bank financed with core deposits is given by:

$$
\pi_{\text {core }}^{R}=\alpha\left\{\begin{array}{l}
\theta q_{I}[\rho+(1-\rho) \beta] H+\left[\frac{\theta q_{I}[\rho+(1-\rho) \beta]^{2}}{\rho+(1-\rho) \beta \hat{\theta}}\right]\left\{\left[\frac{\rho+(1-\rho) \beta \hat{\theta}}{\rho+(1-\rho) \beta}\right] \tau_{2}-\tau_{1}-1\right\}  \tag{A10}\\
+\frac{\left[\theta q_{I}[\rho+(1-\rho) \beta]\right]^{2}}{2 \kappa}-\left[\frac{\rho+(1-\rho) \beta}{\rho+(1-\rho) \beta \hat{\theta}}\right]\left(r_{d}-1-2 \tau_{1}\right)-F
\end{array}\right\}
$$

Solving the relationship bank's optimization problem in Equation (10), we have

$$
\begin{equation*}
e_{p u r}=\frac{\theta q_{I} \rho}{\kappa} . \tag{A11}
\end{equation*}
$$

It is obvious that $e_{\text {core }} \geq e_{\text {pur }}$ for each fixed value of $\rho$, and the inequality is strict for $\rho<1$. The expected profit for a relationship bank financed with purchased money is given by:

$$
\begin{equation*}
\pi_{p u r}^{R}=\alpha\left\{\left[\frac{\left(\theta q_{I}\right)^{2}}{2 \kappa}\right] \rho^{2}+\theta q_{I}(H-1) \rho-\left(r_{d}-1\right)\right\} . \tag{A12}
\end{equation*}
$$

Note that $\pi_{\text {pur }}^{R}$ is a quadratic function of $\rho$. Its positive root is:

$$
\begin{equation*}
\rho_{l}=\frac{-(H-1)+\sqrt{(H-1)^{2}+2\left(r_{d}-1\right) / \kappa}}{\theta q_{I} / \kappa} . \tag{A13}
\end{equation*}
$$

For $\rho<\rho_{l}$, we have $\pi_{p u r}^{R}<0$, and the comparison between core deposit financing and purchased money financing is meaningless. Thus, for the rest of analysis, we focus on $\rho \in\left[\rho_{l}, 1\right]$ for which $\pi_{p u r}^{R} \geq 0$.

Note two facts: (i) $d \pi_{p u r}^{R} / d \rho>0$, and (ii) $\pi_{\text {core }}^{R}<\pi_{p u r}^{R}$ when $\rho=1$. Thus, the sufficient conditions for the existence of a cutoff $\rho^{*}$ are: (i) $\pi_{\text {core }}^{R} \rho_{\rho=0} \geq 0$, and (ii) $d \pi_{\text {core }}^{R} / d \rho \geq 0$.

Note that

$$
\begin{equation*}
\left.\pi_{\text {core }}^{R}\right|_{\rho=0}=\alpha\left[\theta q_{I} \beta\left(\tau_{2}-H \tau_{1}\right)+\frac{\left(\theta q_{I} \beta\right)^{2}}{2 \kappa}-H\left(r_{d}-2 \tau_{1}-1\right)-F\right] . \tag{A14}
\end{equation*}
$$

Thus, the first condition $\left.\pi_{\text {core }}^{R}\right|_{\rho=0} \geq 0$ is equivalent to

$$
F \leq \theta q_{I} \beta\left(\tau_{2}-H \tau_{1}\right)+\frac{\left(\theta q_{I} \beta\right)^{2}}{2 \kappa}-H\left(r_{d}-2 \tau_{1}-1\right),
$$

which is guaranteed by (A2). The second condition $d \pi_{\text {core }}^{R} / d \rho \geq 0$ is guaranteed by the sufficient condition in (A3). Finally, we characterize the explicit parametric conditions for the stability of core deposits, that is, $1+\tau_{1}+\tau_{1} \in\left(\theta r_{\text {core }}^{R}+\tau_{1}+\theta \tau_{2}, \hat{\theta} r_{\text {core }}^{R}+\tau_{1}+\hat{\theta} \tau_{2}\right)$. Combining this with (4) yields Equation (A1).

Proof of Corollary 1: Define $h(\rho) \equiv \pi_{\text {core }}^{R}-\pi_{\text {pur }}^{R}$. Note that $h\left(\rho^{*}\right)=0$. Under our parametric assumptions, it is easy to see that $d h(\rho) / d \rho<0$. Thus,

$$
\begin{align*}
\frac{d \rho^{*}}{d\left[\tau_{2}-\tau_{1}\right]} & =-\frac{d h / d\left[\tau_{2}-\tau_{1}\right]}{\left.(d h / d \rho)\right|_{\rho=\rho^{*}}} \propto d h / d\left[\tau_{2}-\tau_{1}\right]>0,  \tag{A15}\\
\frac{d \rho^{*}}{d F} & =-\frac{d h / d F}{\left.(d h / d \rho)\right|_{\rho=\rho^{*}}} \propto d h / d F<0 . \tag{A16}
\end{align*}
$$

Proof of Proposition 2: We see from Equation (11) that a depositor's expected payoff for core deposit investment consists of two parts: one from the pure monetary investment return, and the other from the liquidity gain. Since the depositors from group $\Psi_{n}$ do not attach any value to the bank's liquidity services, only the monetary investment return is relevant to them and hence the expected payoff of core deposit investment is smaller than $r_{d}$. Thus, depositors from group $\Psi_{n}$ prefer purchased money. The depositors from group $\Psi_{l}$ obviously prefer core deposit investment since banks do not provide liquidity through purchased money.

Proof of Proposition 3: This result is straightforward in our model because the profits for the borrower and the bank are completely aligned as $\frac{\text { Profit for the borrower }}{\text { Profit for the bank }}=\frac{1-\alpha}{\alpha}$. Thus, the borrower's choice is the same as the bank's choice, that is, those borrowers in group $\Phi_{l}$ with $\rho<\rho^{*}$ find it optimal to borrow from relationship banks that finance with core deposits; and those borrowers in group $\Phi_{h}$ with $\rho>\rho^{*}$ find it optimal to borrow from relationship banks that finance with purchased money.

Proof of Proposition 4: We consider two cases: relationship bank financed with purchased money and relationship bank financed with core deposits. ${ }^{32}$

First, for a relationship bank financed with purchased money, if the bank allows the depositors to withdraw at $t=1$, the bank's expected profit is given by (A12). If the bank does not allow interim withdrawal at $t=1$, it needs to promise the depositors a higher repayment (denoted as $\tilde{r}_{p u r}^{R}$ ), which is determined as follows:

$$
\begin{equation*}
r_{d}=(1-\theta)+\theta\left(1-q_{I}\right)+\left(\theta q_{I} \rho\right) \tilde{r}_{p u r}^{R}+\theta q_{I}(1-\rho) \beta\left(\hat{\theta} \tilde{r}_{p u r}^{R}\right)+\theta q_{I}(1-\rho)(1-\beta)\left(\theta \tilde{r}_{p u r}^{R}\right) \tag{A17}
\end{equation*}
$$

Thus, we have

$$
\begin{equation*}
\tilde{r}_{p u r}^{R}=\frac{r_{d}-\left(1-\theta q_{I}\right)}{\theta q_{I}[\rho+(1-\rho)[\beta \hat{\theta}+(1-\beta) \theta]]} . \tag{A18}
\end{equation*}
$$

The bank's expected profit is determined by the solution to the following optimization problem:

$$
\begin{equation*}
\tilde{\pi}_{p u r}^{R} \equiv \max _{\{\tilde{e} p u r\}} \alpha\left\{\theta q_{I}\left[H+\tilde{e}_{p u r}-\tilde{r}_{p u r}^{R}\right]-\frac{\kappa \tilde{e}_{p u r}^{2}}{2}\right\} \tag{A19}
\end{equation*}
$$

where $\tilde{e}_{p u r}$ is the bank's project-enhancement effort exertion in this case in which there is no interim deposit withdrawal possibility. Solving the problem (A19), we have $\tilde{e}_{p u r}=\frac{\theta q_{I}}{\kappa}$. Thus, the bank's expected profit is given by:

$$
\begin{equation*}
\tilde{\pi}_{p u r}^{R}=\alpha\left\{\frac{\left(\theta q_{I}\right)^{2}}{2 \kappa}+\theta q_{I}\left[H-\frac{1}{\rho+(1-\rho)[\beta \hat{\theta}+(1-\beta) \theta]}\right]-\frac{r_{d}-1}{\rho+(1-\rho)[\beta \hat{\theta}+(1-\beta) \theta]}\right\} . \tag{A20}
\end{equation*}
$$

In order to show the optimality of demand deposit, we only need to show $\pi_{p u r}^{R}>\tilde{\pi}_{p u r}^{R}$ for $\forall \rho \in[0,1]$. Note $d \pi_{p u r}^{R} / d \rho>0, d \tilde{\pi}_{p u r}^{R} / d \rho>0$ and $\left.\pi_{p u r}^{R}\right|_{\rho=1}=\left.\tilde{\pi}_{p u r}^{R}\right|_{\rho=1}$. Thus, it is sufficient to require: (i) $\left.\pi_{p u r}^{R}\right|_{\rho=0}>\left.\tilde{\pi}_{p u r}^{R}\right|_{\rho=0}$ and (ii) $d \pi_{p u r}^{R} / d \rho<d \tilde{\pi}_{p u r}^{R} / d \rho$. The two sufficiency conditions can be expressed by the exogenous parameters as follows:

$$
r_{d}>1+\theta q_{I}\left\{\frac{H\left[H+\theta q_{I} / \kappa-2\right]+[\beta+(1-\beta) \theta H]}{H-[\beta+(1-\beta) \theta H]}\right\}
$$

which is ensured by (A4).
Second, for a relationship bank financed with core deposits, if the bank allows interim deposit withdrawal, its expected profit is given by (A10). If the bank does not allow interim deposit withdrawal, it needs to promise the depositors a higher repayment (denoted as $\tilde{r}_{\text {core }}^{R}$ ),

[^21]which is determined as follows:
\[

$$
\begin{align*}
r_{d}= & \left(1-\theta q_{I}\right)\left(1+2 \tau_{1}\right)+\left(\theta q_{I} \rho\right)\left(\tilde{r}_{\text {core }}^{R}+\tau_{1}+\tau_{2}\right) \\
& +\theta q_{I}(1-\rho) \beta\left(\hat{\theta} \tilde{r}_{\text {core }}^{R}+\tau_{1}+\hat{\theta} \tau_{2}\right)+\theta q_{I}(1-\rho)(1-\beta)\left(\theta \tilde{r}_{\text {core }}^{R}+\tau_{1}+\theta \tau_{2}\right) . \tag{A21}
\end{align*}
$$
\]

Thus, we have:

$$
\begin{equation*}
\tilde{r}_{\text {core }}^{R}=\frac{r_{d}-\left(1-\theta q_{I}\right)}{\theta q_{I}[\rho+(1-\rho)[\beta \hat{\theta}+(1-\beta) \theta]]}-\tau_{2}-\frac{\left(2-\theta q_{I}\right) \tau_{1}}{\theta q_{I}[\rho+(1-\rho)[\beta \hat{\theta}+(1-\beta) \theta]]} \tag{A22}
\end{equation*}
$$

The bank's expected profit is determined by the following optimization problem:

$$
\begin{equation*}
\tilde{\pi}_{\text {core }}^{R} \equiv \max _{\left\{\tilde{e}_{\text {core }}\right\}} \alpha\left\{\theta q_{I}\left[H+\tilde{e}_{\text {core }}-\tilde{r}_{\text {core }}^{R}\right]-\frac{\kappa \tilde{e}_{\text {core }}^{2}}{2}-F\right\} \tag{A23}
\end{equation*}
$$

where $\tilde{e}_{\text {core }}$ is the bank's project-enhancement effort exertion in the case in which there is no interim deposit withdrawal possibility. Solving the problem (A23), we have $\tilde{e}_{\text {core }}=\frac{\theta q_{I}}{\kappa}$. Thus, the bank's expected profit is given by:

$$
\tilde{\pi}_{\text {core }}^{R}=\alpha\left\{\begin{array}{c}
\theta q_{I} H+\left[\frac{\left(2-\theta q_{I}\right) \tau_{1}}{\rho+(1-\rho)[\beta \hat{\theta}+(1-\beta) \theta]}+\theta q_{I} \tau_{2}\right]  \tag{A24}\\
+\frac{\left(\theta q_{I}\right)^{2}}{2 \kappa}-\left[\frac{r_{d}-1+\theta q_{I}}{\rho+(1-\rho)[\beta \hat{\theta}+(1-\beta) \theta]}\right]-F
\end{array}\right\} .
$$

We need to show $\pi_{\text {core }}^{R}>\tilde{\pi}_{\text {core }}^{R}$, for $\forall \rho \in[0,1]$. Similar to the analysis for purchased money, it is sufficient to require: (i) $\left.\pi_{\text {core }}^{R}\right|_{\rho=0}>\left.\tilde{\pi}_{\text {core }}^{R}\right|_{\rho=0}$ and (ii) $d \pi_{\text {core }}^{R} / d \rho<d \tilde{\pi}_{\text {core }}^{R} / d \rho$. The two sufficiency conditions can be expressed by the exogenous parameters as follows:

$$
r_{d}>1+\theta q_{I}\left\{\frac{\begin{array}{l}
(1-\beta) H+\frac{\theta q_{I}(1-\beta)}{\kappa}+\left(\tau_{2}-\tau_{1}-1+\frac{2 \tau_{1}}{\theta q_{I}}\right)\left(\frac{1-x}{x^{2}}\right) \\
+\left(\tau_{2}-\tau_{1}-1\right)\left(\frac{1-\beta}{1-\beta / H}\right)^{2}-\frac{\beta(1-1 / H)\left(2 \tau_{1}\right)}{(\beta / H)^{2}}
\end{array} 1-x-\beta(1-1 / H)}{1-x, ~}\right.
$$

where $x \equiv \beta / H+(1-\beta) \theta$. This condition is ensured by (A4).

Proof of Proposition 5: Consider the case with complete insurance for core deposits. Suppose $\theta$ is sufficiently low such that $\theta \tau_{2}<\tau_{1}$, and the depositors demand at least $\tau^{\prime}$ of liquidity services, where $\tau^{\prime} \in\left(\theta \tau_{2}+\tau_{1}, 2 \tau_{1}\right]$. If the depositors do not switch to another bank at $t=1$ in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=U\right\}$, then the total amount of liquidity services that they expect to receive from the original bank is $\tau_{1}+\theta \tau_{2}<\tau^{\prime}$ since they perceive that the original bank fails at $t=2$ with probability $1-\theta$, generating no liquidity services despite complete deposit insurance. Switching to another bank gives the depositors $\tau_{1}$ of liquidity services at $t=2$ from the new bank. Thus, switching gives them $2 \tau_{1} \geq \tau^{\prime}$ of liquidity services in total, and hence in this case they will switch at $t=1$ in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=U\right\}$.

Suppose that purchased money is stable in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=N\right\}$ but not in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=U\right\}$. The repayment obligation $r_{p u r}^{R}$ is determined by:

$$
r_{d}=(1-\theta)+\theta\left(1-q_{I}\right)+\theta q_{I}(1-\rho)(1-\beta)+\theta q_{I}(1-\rho) \beta\left[\hat{\theta} r_{p u r}^{R}+(1-\hat{\theta}) D_{p u r}\right]+\theta q_{I} \rho r_{p u r}^{R}
$$

which yields

$$
\begin{equation*}
r_{p u r}^{R}=\frac{r_{d}-\left\{1-\theta q_{I}[\rho+(1-\rho) \beta]\right\}}{\theta q_{I}[\rho+(1-\rho) \beta \hat{\theta}]}-\left[\frac{(1-\rho) \beta(1-\hat{\theta})}{\rho+(1-\rho) \beta \hat{\theta}}\right] D_{p u r} . \tag{A25}
\end{equation*}
$$

For the depositors to withdraw their deposits in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=U\right\}$ but not in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=N\right\}$, it must be true that

$$
1 \in\left(\theta r_{p u r}^{R}+(1-\theta) D_{p u r}, \hat{\theta} r_{p u r}^{R}+(1-\hat{\theta}) D_{p u r}\right),
$$

that is,

$$
\begin{equation*}
D_{p u r} \in\left(D_{p u r}^{*}, D_{p u r}^{* *}\right), \tag{A26}
\end{equation*}
$$

where

$$
\begin{align*}
D_{p u r}^{*} & \equiv 1-\frac{r_{d}-1}{\theta q_{I} \rho(H-1)}  \tag{A27}\\
D_{p u r}^{* *} & \equiv 1-\frac{r_{d}-1}{(\hat{\theta}-\theta) q_{I}(1-\rho) \beta+(1-\theta) q_{I} \rho} \tag{A28}
\end{align*}
$$

Suppose that core deposits are stable in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=U\right\}$. The repayment obligation $r_{\text {core }}^{R}$ is determined by:

$$
\begin{aligned}
r_{d}= & \left(1-\theta q_{I}\right)\left(1+2 \tau_{1}\right)+\theta q_{I} \rho\left(r_{\text {core }}^{R}+\tau_{1}+\tau_{2}\right)+\theta q_{I}(1-\rho) \beta \\
& \times\left[\hat{\theta} r_{\text {core }}^{R}+(1-\hat{\theta}) D_{\text {core }}+\tau_{1}+\hat{\theta} \tau_{2}\right] \\
& +\theta q_{I}(1-\rho)(1-\beta)\left[\theta r_{\text {core }}^{R}+(1-\theta) D_{\text {core }}+\tau_{1}+\theta \tau_{2}\right]
\end{aligned}
$$

which yields

$$
\begin{align*}
r_{\text {core }}^{R}= & \frac{r_{d}-\left(1-\theta q_{I}\right)}{\theta q_{I}[\rho+(1-\rho)[\beta \hat{\theta}+(1-\beta) \theta]]}-\tau_{2}-\frac{\left(2-\theta q_{I}\right) \tau_{1}}{\theta q_{I}[\rho+(1-\rho)[\beta \hat{\theta}+(1-\beta) \theta]]} \\
& -\left[\frac{(1-\rho)[\beta(1-\hat{\theta})+(1-\beta)(1-\theta)]}{\rho+(1-\rho)[\beta \hat{\theta}+(1-\beta) \theta]}\right] D_{\text {core }} . \tag{A29}
\end{align*}
$$

In order to have core deposits be more stable than purchased money in the state $\left\{s=s_{G}, p_{b}=I, p_{d}=U\right\}$, we need to have

$$
\theta r_{\text {core }}^{R}+(1-\theta) D_{\text {core }}+\tau_{1}+\theta \tau_{2}>1+2 \tau_{1},
$$

that is,

$$
\begin{equation*}
D_{\text {core }}>D_{\text {core }}^{*} \equiv 1+\tau_{1}-\frac{r_{d}-1-2 \tau_{1}}{(\hat{\theta}-\theta) q_{I}(1-\rho) \beta+(1-\theta) q_{I} \rho} . \tag{A30}
\end{equation*}
$$

Proof of Lemma 3: Solving the optimization problem in (16), we have the following results:

$$
\begin{aligned}
e_{\text {core }} & =\frac{\theta q_{I}[\rho+(1-\rho) \beta]}{\kappa}, \\
\alpha_{\text {core }} & =\frac{c / N}{\theta q_{I}[\rho+(1-\rho) \beta]\left[H+e_{\text {core }}-r_{\text {core }}^{R}\right]-\frac{\kappa e_{\text {core }}^{2}}{2}-\frac{F}{M / N}} .
\end{aligned}
$$

In the symmetric equilibrium, $\Omega_{\text {core }}=M / N$. Thus, the bank's ex post profit (excluding the fixed investment $F$ ) is

$$
\begin{equation*}
\pi_{\text {core }}^{E P}(N)=\left[\frac{M c}{N^{2}}\right] \times\left[\frac{\theta q_{I}[\rho+(1-\rho) \beta]\left[H+e_{\text {core }}-r_{\text {core }}^{R}\right]-\frac{\kappa e_{\text {core }}^{2}}{2}}{\theta q_{I}[\rho+(1-\rho) \beta]\left[H+e_{\text {core }}-r_{\text {core }}^{R}\right]-\frac{\kappa e_{\text {core }}^{2}}{2}-\frac{F}{M / N}}\right] . \tag{A31}
\end{equation*}
$$

Note that the bank's ex post profit is given by:

$$
\begin{equation*}
\alpha_{\text {core }} f \propto f \tag{A32}
\end{equation*}
$$

where $f \equiv\left\{\left[\theta q_{I}[\rho+(1-\rho) \beta]\left[H+e_{\text {core }}-r_{\text {core }}^{R}\right]-\frac{\kappa e_{\text {core }}^{2}}{2}\right]\left[\frac{M}{N}\right]-F\right\}$. It is easy to see that there exists a cutoff $N^{*}$ such that for $N>N^{*}, f<0$ and for $N \leq N^{*}, f \geq 0$.

Proof of Proposition 6: The bank's ex ante profit is give by:

$$
\begin{align*}
\pi_{\text {core }}^{E A} & =\mathbf{E}\left(\pi_{\text {core }}^{E P}(N)-\alpha_{\text {core }} F\right) \\
& \propto M\left\{\theta q_{I}[\rho+(1-\rho) \beta]\left[H+e_{\text {core }}-r_{\text {core }}^{R}\right]-\frac{\kappa e_{\text {core }}^{2}}{2}\right\} \mathbf{E}(1 / N)-F \tag{A33}
\end{align*}
$$

Thus, the bank enters the market if

$$
\mathbf{E}(1 / N) \geq \frac{F}{M\left\{\theta q_{I}[\rho+(1-\rho) \beta]\left[H+e_{\text {core }}-r_{\text {core }}^{R}\right]-\frac{\kappa e_{\text {core }}^{2}}{2}\right\}} \equiv z .
$$

Note that (since $f(N)=A N) \mathbf{E}(1 / N)=\sqrt{2 A}$, and $\mathbf{E}(N)=\sqrt{8 /(9 A)}$. Thus, the bank-entry condition $\mathbf{E}(1 / N) \geq z$ is equivalent to $\mathbf{E}(N) \leq 4 /(3 z) \equiv \bar{N}^{*}$.

Proof of Proposition 7: The claim that the larger the $\bar{N}$ the smaller is $\pi_{\text {core }}^{E A}$ has been demonstrated in the proof of Proposition 6, that is,

$$
\begin{equation*}
\frac{d \pi_{\text {core }}^{E A}}{d \bar{N}}<0 \tag{A34}
\end{equation*}
$$

For the total surplus of the relationship industry, $\Pi_{\text {core }}^{E A}$, note that

$$
\begin{equation*}
\frac{d \Pi_{\text {core }}^{E A}}{d \bar{N}}=N\left[\frac{d \pi_{\text {core }}^{E A}}{d \bar{N}}\right]<0 . \tag{A35}
\end{equation*}
$$

Proof of Proposition 8: Define

$$
g \equiv \frac{\mathbf{E}\left(1 / N^{2}\right)}{\mathbf{E}\left(\frac{1}{N^{2}\left(1-\frac{F}{y M / N}\right)}-\frac{F /(M y)}{N\left(1-\frac{F}{y M / N}\right)}\right)} .
$$

Note that if $g>1$, banks choose to finance with purchased money, and if $g<1$, banks choose to finance with core deposits. Given the facts that $\mathbf{E}\left(1 / N^{2}\right)<\mathbf{E}\left(\frac{1}{N^{2}\left(1-\frac{F}{y M / N}\right)}\right)$ and $\mathbf{E}\left(\frac{F /(M y)}{N\left(1-\frac{F}{y M / N}\right)}\right)>0$, the existence of a $\rho^{* *}$ such that $g\left(\rho^{* *}\right)=1$ is guaranteed. Note that $d y / d \rho>0$ and $d g / d y>0$. Hence, $d g / d \rho>0$. Thus, it is easy to see that if $\rho>\rho^{* *}, g>1$ and banks choose purchased money financing, and if $\rho<\rho^{* *}, g<1$ and banks choose core deposit financing. Meanwhile, note that $d g / d \bar{N}>0$. Thus, $d \rho^{* *} / d \bar{N}=-\frac{d g / d \bar{N}}{\left.(d g / d \rho)\right|_{\rho=\rho^{* *}}}<0$.

To see the effect of an increase of $\bar{N}$ on the depositors' welfare, note: (i) for a depositor who does not attach value to the bank's liquidity services (group $\Psi_{n}$ ), his expected payoff (denoted as $\pi_{d n}$ ) is: $\pi_{d n}=[\operatorname{Pr}($ entry $)]\left[r_{d}\right]+[1-\operatorname{Pr}($ entry $)][1]$, where $\operatorname{Pr}$ (entry) is the bank's entry probability; and (ii) for a depositor who attaches value to the bank's liquidity services (group $\Psi_{l}$ ), his expected payoff (denoted as $\pi_{d l}$ ) is: $\pi_{d l}=\left[\operatorname{Pr}(\right.$ entry, core) $]\left[r_{d}\right]+$ $[1-\operatorname{Pr}($ entry, core $)][1]$, where $\operatorname{Pr}$ (entry, core) is the probability that the bank enters the relationship lending market and at the same time finances with core deposits. Note that $d[\operatorname{Pr}($ entry $)] / d \bar{N}<0$, and $d[\operatorname{Pr}($ entry, core $)] / d \bar{N}=d[\operatorname{Pr}$ (entry) $\operatorname{Pr}$ (core|entry) $] / d \bar{N}<0$ (since $d[\operatorname{Pr}($ entry $)] / d \bar{N}<0$ and $d[\operatorname{Pr}$ (core|entry) $] / d \bar{N}<0$, where the second inequality comes from the first part of this proposition proved above). Thus, we have $d \pi_{d n} / d \bar{N}<0$ and $d \pi_{d l} / d \bar{N}<0$.

To see the effect of an increase of $\bar{N}$ on the borrowers' welfare, note:
(i) If a borrower has a high agreement parameter such that he is always funded with purchased money, his ex post payoff (denoted as $\hat{\pi}_{p u r}^{E P}(N)$ ) is given by:

$$
\begin{align*}
\hat{\pi}_{p u r}^{E P}(N) & =\left[1-\alpha_{p u r}\right]\left\{\theta q_{I} \rho\left[H+e_{p u r}-r_{p u r}^{R}\right]-\frac{\kappa e_{p u r}^{2}}{2}\right\} \\
& =\theta q_{I} \rho\left[H+e_{p u r}-r_{p u r}^{R}\right]-\frac{\kappa e_{p u r}^{2}}{2}-\frac{c}{N} \tag{A36}
\end{align*}
$$

Thus, his ex ante expected payoff, given by $\hat{\pi}_{p u r}^{E A}=\mathbf{E}\left(\hat{\pi}_{p u r}^{E P}(N)\right)$, is increasing in $\bar{N}$.
(ii) By similar argument, we can show that an increase in $\bar{N}$ increases the ex ante expected payoff for a borrower with a low agreement parameter such that he is always funded with core deposits.
(iii) For a borrower with an intermediate agreement parameter that is initially funded with core deposits, an increase in $\bar{N}$ will lead the bank to finance him with purchased money. Suppose that is the case. Before the increase of $\bar{N}$, the borrower is funded with core deposits and his ex ante expected payoff is given by:

$$
\begin{align*}
\hat{\pi}_{\text {core }}^{E A}= & \mathbf{E}\left(\hat{\pi}_{\text {core }}^{E P}(N)\right)=\mathbf{E}\left(\theta q_{I}[\rho+(1-\rho) \beta]\left[H+e_{\text {core }}-r_{\text {core }}^{R}\right]\right. \\
& \left.-\frac{\kappa e_{\text {core }}^{2}}{2}-\frac{F}{M / N}-\frac{c}{N}\right) \tag{A37}
\end{align*}
$$

After the increase of $\bar{N}$, the borrower is funded with purchased money and his ex ante expected payoff is given by:

$$
\begin{equation*}
\hat{\pi}_{p u r}^{E A}=\mathbf{E}\left(\hat{\pi}_{p u r}^{E P}(N)\right)=\mathbf{E}\left(\theta q_{I} \rho\left[H+e_{p u r}-r_{p u r}^{R}\right]-\frac{\kappa e_{p u r}^{2}}{2}-\frac{c}{N}\right) \tag{A38}
\end{equation*}
$$

Note that for those values of $\rho$ such that

$$
\begin{aligned}
& \theta q_{I}[\rho+(1-\rho) \beta]\left[H+e_{\text {core }}-r_{\text {core }}^{R}\right] \\
& \quad-\frac{\kappa e_{\text {core }}^{2}}{2}-\frac{F}{M / N}>\theta q_{I} \rho\left[H+e_{p u r}-r_{\text {pur }}^{R}\right]-\frac{\kappa e_{\text {pur }}^{2}}{2}
\end{aligned}
$$

switching from core deposit financing to purchase money financing causes a loss of welfare to the borrower. Although $d \hat{\pi}_{\text {pur }}^{E A} / d \bar{N}>0$, we have $d^{2} \hat{\pi}_{\text {pur }}^{E A} / d \bar{N}^{2}<0$. Thus, beyond a certain cutoff $\bar{N}^{* *}$, although a further increase in $\bar{N}$ increases $\hat{\pi}_{p u r}^{E A}$, such increase is outweighed by the loss from the financing source switching.

Proof of Proposition 9: In the symmetric equilibrium, the share of the surplus extracted by the transaction bank is given by (the analysis is similar to the analysis for the case of relationship bank):

$$
\begin{equation*}
\alpha_{p u r}^{T}=\frac{c / N}{H-r_{p u r}^{T}}=\frac{c / N}{H-r_{d}} \tag{A39}
\end{equation*}
$$

Note $\alpha_{p u r}^{T}$ is decreasing in $N$. By similar analysis to the case of relationship bank, we can show that the transaction bank's welfare decreases, while the borrower's welfare increases as $N$ increases. The depositor's expected payoff is always $r_{d}$ and hence unaffected by the degree of competition.

## References

Allen, F., and D. Gale. 1999. Diversity of Opinion and Financing of New Technologies. Journal of Financial Intermediation 8:68-89.

Allen, F., and D. Gale. 2001. Comparing Financial Systems. The MIT Press.
Berger, A. N., and G. F., Udell. 1995. Relationship Lending and Lines of Credit in Small Firm Finance. Journal of Business 68:351-81.

Berger, A. N., N. H. Miller, M. A. Petersen, R. G. Rajan, and J. C. Stein. 2005. Does Function Follow Organizational Form? Evidence from the Lending Practices of Large and Small Banks. Journal of Financial Economics 76:237-69.

Berlin, M., and L. J. Mester. 1999. Deposits and Relationship Lending. Review of Financial Studies 12:579-607.

Bhattacharya, S., and A. V. Thakor. 1993. Contemporary Banking Theory. Journal of Financial Intermediation 3:2-50.

Boot, A. W. A. 2000. Relationship Banking: What Do We Know? Journal of Financial Intermediation 9:7-25.

Boot, A. W. A., and A. V. Thakor. 2000. Can Relationship Banking Survive Competition? Journal of Finance 55:679-713.

Boot, A. W. A., R. Gopalan, and A. V. Thakor. 2006. The Entrepreneur's Choice Between Private and Public Ownership. Journal of Finance 61:803-36.

Calomiris, C. W., and C. M. Kahn. 1991. The Role of Demandable Debt in Structuring Optimal Banking Arrangements. American Economic Review 81:497-513.

Chari, V. V., and R. Jagannathan. 1988. Banking Panics, Information, and Rational Expectations Equilibrium. Journal of Finance 43:749-61.

Coval, J. D., and A. V. Thakor. 2005. Financial Intermediation as a Beliefs-Bridge Between Optimists and Pessimists. Journal of Financial Economics 75:535-69.

Degryse, H., and P. Van Cayseele. 2000. Relationship Lending within a Bank-Based System: Evidence from European Small Business Data. Journal of Financial Intermediation 9:90-109.

DeYoung, R., W. C. Hunter, and G. F. Udell. 2004. The Past, Present, and Probable Future for Community Banks. Journal of Financial Services Research 25:85-133.

Diamond, D. W., and R. G. Rajan. 2001. Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking. Journal of Political Economy 109:287-327.

Ergungor, O. E. 2005. The Profitability of Bank-Borrower Relationships. Journal of Financial Intermediation 14:485-512.

Feldman, R., and J. Schmidt. 2001. Increased Use of Uninsured Deposits: Implications for Market Discipline. Federal Reserve Bank of Minneapolis-fedgazette March:18-9.

Flannery, M. J. 1994. Debt Maturity and the Deadweight Cost of Leverage: Optimally Financing Banking Firms. American Economic Review 84:320-31.

Freixas, X., and J. C. Rochet. 1999. Microeconomics of Banking. The MIT Press.
Garmaise, M. 2001. Rational Beliefs and Security Design. Review of Financial Studies 14:1183-213.
Genay, H. 2000. Recent Trends in Deposit and Loan Growth: Implications for Small and Large Banks. Chicago Fed Letter 160(December):.

Gorton, G. 1988. Banking Panics and Business Cycles. Oxford Economic Papers 40:751-81.
Gorton, G., and R. Rosen. 1995. Corporate Control, Portfolio Choice, and the Decline of Banking. Journal of Finance 50:1377-420.

Kandel, E., and N. D. Pearson. 1995. Differential Interpretation of Public Signals and Trade in Speculative Markets. Journal of Political Economy 103:831-72.

Kashyap, A. K., R. G. Rajan, and J. C. Stein. 2002. Banks as Liquidity Providers: An Explanation for the Coexistence of Lending and Deposit-Taking. Journal of Finance 57:33-73.

Keeley, M. C. 1990. Deposit Insurance, Risk, and Market Power in Banking. American Economic Review 80:1183-200.

Kim, M., D. Kliger, and B. Vale. 2003. Estimating Switching Costs: The Case of Banking. Journal of Financial Intermediation 12:25-56.

Kreps, D. M. 1990. A Course in Microeconomic Theory. Princeton University Press.
Kurz, M. 1994a. On the Structure and Diversity of Rational Beliefs. Economic Theory 4:877-900.
Kurz, M. 1994b. On Rational Belief Equilibria. Economic Theory 4:859-76.
Kurz, M., and M. Motolese. 2001. Endogenous Uncertainty and Market Volatility. Economic Theory 17:497-544.

Manove, M., and A. J. Padilla. 1999. Banking (Conservatively) with Optimists. RAND Journal of Economics 30:324-50.

Morris, S. 1995. The Common Prior Assumption in Economic Theory. Economics and Philosophy 11:227-53.

Nier, E., and U. Baumann. 2006. Market Discipline, Disclosure and Moral Hazard in Banking. Journal of Financial Intermediation 15:332-61.

Ongena, S., and D. C. Smith. 2000. What Determines the Number of Bank Relationships? Cross-Country Evidence. Journal of Financial Intermediation 9:26-56.

Petersen, M. A., and R. G. Rajan. 1994. The Benefits of Lending Relationships: Evidence from Small Business Data. Journal of Finance 49:3-37.

Qi, J. 1998. Deposit Liquidity and Bank Monitoring. Journal of Financial Intermediation 7:198-218.

Rajan, R. G. 1992. Insiders and Outsiders: The Choice between Informed and Arm's-Length Debt. Journal of Finance 47:1367-400.

Salop, S. C. 1979. Monopolistic Competition with Outside Goods. Bell Journal of Economics 10:141-56.
Samuelson, L. 2004. Modeling Knowledge in Economic Analysis. Journal of Economic Literature 42:367-403.

Sharpe, S. A. 1990. Asymmetric Information, Bank Lending and Implicit Contracts: A Stylized Model of Customer Relationships. Journal of Finance 55:1069-987.

Sharpe, S. A. 1997. The Effect of Consumer Switching Costs on Prices: A Theory and its Application to the Bank Deposit Market. Review of Industrial Organization 12:79-94.

Slovin, M. B., M. E. Sushka, and J. A. Polonchek. 1993. The Value of Bank Durability: Borrowers as Bank Stakeholders. Journal of Finance 48:247-66.

Stein, J. C. 2002. Information Production and Capital Allocation: Decentralized versus Hierarchical Firms. Journal of Finance 57:1891-921.

Tagaki, S. 1991. Exchange Rate Expectations: A Survey of Survey Studies. International Monetary Fund Staff Papers 38:156-83.

Van den Steen, E. 2004. Rational Overoptimism (and Other Biases). American Economic Review 94:1141-51.
von Thadden, E. L. 1998. Intermediated versus Direct Investment: Optimal Liquidity Provision and Dynamic Incentive Compatibility. Journal of Financial Intermediation 7:177-97.


[^0]:    The helpful comments of Viral Acharya, Arnoud Boot, Chris James, Rafael Repullo, Elu von Thadden, participants at the Banco de Portugal Conference on Financial Fragility and Bank Regulation in Lisbon (June 2005), Robert McDonald (the editor), and an anonymous referee are gratefully acknowledged. Address correspondence to Anjan V. Thakor, Olin School of Business, Washington University in St. Louis, Campus Box 1133, One Brookings Drive, St. Louis, MO 63130, or e-mail: thakor@wustl.edu.

[^1]:    ${ }^{1}$ And even if there are withdrawals, the liquidity of these assets means that the bank will be able to cope with these withdrawals efficiently by selling off these assets or borrowing using them as collateral.

[^2]:    ${ }^{2}$ Additional factors that may contribute to the sluggishness of core deposits are switching costs for depositors and deposit insurance. We analyze these as well and they do not qualitatively affect our analysis. Regardless of the source of the relatively higher sluggishness of core deposits, the key is that core deposits are more sluggish than purchased money, which is a well-established stylized fact that we discuss later.
    ${ }^{3}$ In our analysis, even though purchased money has a lower average cost, it has a higher marginal cost than core deposits, consistent with the empirical fact that the marginal cost of purchased money is typically higher for a bank than the marginal cost of core deposits.

[^3]:    ${ }^{4}$ The Harsanyi doctrine asserts uniform prior beliefs, and Samuelson (2004) justifies these on the grounds that it is just "a welcome source of modeling discipline." However, economic theory stipulates rationality as dealing with the revision of prior beliefs and has little to say about how these priors themselves are arrived at. These are viewed as part of the primitives, along with preferences and endowments. Kreps (1990) argues that heterogenous priors represent a more general specification than uniform priors, and Morris (1995) explains that heterogenous priors are consistent with Bayesian rationality. Numerous papers have employed heterogenous prior beliefs, including Allen and Gale (1999), Manove and Padilla (1999), Garmaise (2001), Kurz and Motolese (2001), Coval and Thakor (2005), Boot et al. (2006), and Van den Steen (2004); see Kandel and Pearson (1995) and Tagaki (1991) for empirical evidence.
    ${ }^{5}$ Other explanations for the maturity mismatching across the bank's balance sheet include, for example, Flannery (1994) and Qi (1998). However, our analysis differs significantly from theirs. For example, Flannery (1994) explains why long-maturity bank assets are financed with short-maturity liabilities, thereby focusing on the classic maturity mismatching problem. Qi (1998) rationalizes maturity mismatching on the grounds that it provides incentives for banks to monitor their borrowers. That is, these papers focus on explaining why demand deposits fund long-maturity assets.

[^4]:    ${ }^{6}$ Our assumption that no payoff enhancement is possible even with relationship loans for the informationally-transparent good projects is motivated by the observation that these projects involve well-established technologies and relatively large firms, so there is little the bank can do to enhance project payoffs beyond what the borrowers have already come up with on their own. Allowing payoff enhancement for an informationally-opaque bad project financed with a relationship loan is easy to do but adds little except a bit of additional algebra as long as the project remains uncreditworthy even after the payoff enhancement.

[^5]:    ${ }^{7}$ Our view of liquidity services is therefore distinct from the standard view in the banking literature that the liquidity value of a bank deposit to the depositor is the ability to withdraw at a moment's notice [von Thadden (1998)]. In our model, this liquidity value exists both with core deposits and purchased money and hence can be normalized to zero without loss of generality.
    ${ }^{8}$ The assumption that the value of the bank's liquidity services falls to zero when it fails at $t=2$ is made to capture the intuition that a failed bank may be unable to provide the same quality of liquidity services as a solvent bank, due to the disruptive nature of bank failure even in circumstances where deposits' financial claims are settled by federal deposit insurance. The value is assumed to be zero solely for algebraic simplicity. Our results would be unchanged if we were to assume that the value of the liquidity services to the depositors of a failed bank at $t=2$ is $\tau_{2}^{\prime} \in\left(0, \tau_{2}\right)$.
    ${ }^{9}$ If the current bank fails at $t=2$ and they don't switch at $t=1$, then they gain nothing from the current bank's liquidity services at $t=2$.

[^6]:    ${ }^{10}$ One can also endogenize these switching costs in a "matching" model in which different depositors have different preferences for various combinations of transaction-oriented liquidity services and relationshiporiented liquidity services, and these preferences are unknown at the outset to the bank as well as the depositors. It takes the bank a period to learn the depositor's preferences and provide the appropriate matching. This learning is lost if the depositor switches banks. We have examined this formally and found that this specification is consistent with the rest of our model; details are available upon request.

[^7]:    ${ }^{11}$ As Section 2.1 explains, such withdrawal happens because the bank and depositors have different valuations for the loan. As we discuss there, once deposits are withdrawn by one group, the bank will be unable to convince any other group to invest in the bank.

[^8]:    ${ }^{12}$ This restriction simply ensures that there is no divergence between what maximizes the value of the bank's equity and what maximizes the value of the bank. This well-known incentive problem would merely complicate the analysis without adding new insights.

[^9]:    ${ }^{13}$ We assume that there is no disagreement between the bank and the borrowers. Adding this layer of disagreement will not qualitatively change the analysis as long as we continue to assume that the opinions of the bank can differ from those of borrowers, and that depositors may disagree with the bank and borrowers.
    ${ }^{14}$ Since the prior beliefs of depositors will not generally be perfectly correlated in practice, we should expect banks to be less fragile in practice than suggested by our analysis.

[^10]:    ${ }^{15}$ If diverse beliefs are based on nonstationary variables, they will not converge even with countably infinite observations. However, informationally opaque projects in our model will additionally be characterized by a paucity of historical data, further impeding convergence. In this case, the rational expectations hypothesis requires agents to have information about underlying processes that cannot be derived from historical data, whereas the rational beliefs hypothesis requires only that their beliefs be consistent with the data.
    ${ }^{16}$ The bank thus effectively contracts with depositors over the project payoff at $t=2$, since repayment is constrained by the size of this payoff. However, what is ruled out is readjusting the promised repayment at $t=1$ based on the observed signal, since it is not possible to contract on the privately-observed precisions of this signal.

[^11]:    ${ }^{17}$ Our assumption is that the bank cannot be forced to invest in what it believes is a bad loan, which rules out cases in which depositors believe the loan is good but the bank does not. The justification is that the depositors can never evaluate loan applications the bank does not give them an opportunity to.
    ${ }^{18}$ We focus on informationally-opaque projects financed by the relationship bank. We show later in Lemma 1 that the self-selection in the loan market results in the informationally-opaque projects being financed by the relationship bank and the informationally-transparent projects being financed by the transaction bank.

[^12]:    ${ }^{19}$ Note that the transaction bank receiving a fraction $\alpha$ of the surplus is isomorphic to the bank charging an interest factor of $T R_{\text {core }}=\alpha\left[H-r_{\text {core }}^{T}\right]+[1-\alpha] F$. The expected payoffs to the borrower and the bank are $H-r_{\text {core }}^{T}-T R_{\text {core }}$ and $T R_{\text {core }}-F$, respectively. In order to satisfy both the borrower's and the bank's participation constraints, we need $T R_{\text {core }} \in\left[F, H-r_{\text {core }}^{T}\right]$. This requires that $H>F+r_{\text {core }}^{T}=F+r_{d}-\tau_{1}-\tau_{2}$, which is guaranteed by Assumption 3.

[^13]:    ${ }^{20}$ Equivalently, the bank charges a loan interest factor of $T R_{p u r}=\alpha\left[H-r_{p u r}^{T}\right]$. The expected payoffs to the borrower and the bank are $H-r_{p u r}^{T}-T R_{p u r}$ and $T R_{p u r}$, respectively. In order to satisfy both the borrower's and the bank's participation constraints, we need $T R_{p u r} \in\left[0, H-r_{p u r}^{T}\right]$. This holds because $\alpha \in(0,1)$ and $H>r_{\text {pur }}^{T}=r_{d}$, which is guaranteed by $H>F+r_{d}-\tau_{1}-\tau_{2}$ (Assumption 3) and $F>\tau_{1}+\tau_{2}$ (Assumption 2).

[^14]:    ${ }^{21}$ See Feldman and Schmidt (2001) for data indicating that in 2000, large banks (assets over $\$ 1$ billion) had roughly $60 \%$ insured (core) deposits and $40 \%$ uninsured deposits (purchased money in our model), whereas small banks (those with assets below $\$ 1$ billion) were funded with roughly $80 \%$ insured deposits and $20 \%$ uninsured deposits.
    ${ }^{22}$ Berger et al. (2005) explain their finding based on organization structure grounds.

[^15]:    ${ }^{23}$ For example, the typical lot size for negotiable CDs is $\$ 1$ million and multiples thereof, which is well above the deposit insurance limit.

[^16]:    ${ }^{24}$ Since deposit insurance is in fact incomplete, in reality liquidity services, switching costs and deposit insurance probably work in concert to make core deposits display greater withdrawal sluggishness than purchased money.

[^17]:    ${ }^{25}$ We assume that each borrower that borrows from relationship bank $j$ shares $\left(1-\alpha_{\text {core }}\right) /(M / N)$ portion of the bank's fixed investment cost $F$. In the symmetric equilibrium demonstrated later, each relationship bank captures $M / N$ borrowers. Thus, the relationship bank $j$ bears the remaining ( $1-\alpha_{\text {core }}$ ) portion of the cost.
    ${ }^{26}$ On each side, bank $j$ captures $M d_{\text {core }}$ borrowers. Thus, bank $j$ captures $2 M d_{\text {core }}$ borrowers in total.

[^18]:    ${ }^{27}$ This is because there is no fixed investment for purchased money financing and hence the bank's ex ante and ex post choice problems coincide.

[^19]:    ${ }^{28}$ Moreover, although our model does not deal with intertemporal volatility in loan interest rates, combining our finding with Berlin and Mester (1999) about the link between banks' reliance on core deposits and their ability to smooth loan interest rates suggests that interbank competition may induce higher loan interest rate volatility.
    ${ }^{29}$ Note that this is an ex ante choice or a choice related to the bank acquiring incremental core deposit gathering capability. In an ex post sense, once a bank has invested $F$, it always pays to finance to the maximum extent possible with core deposits since these have a lower marginal cost.

[^20]:    ${ }^{30}$ As indicated earlier, the Berger et al. (2005) evidence may also be due to factors outside of our model.
    ${ }^{31}$ Again, as mentioned earlier, this refers to new banks entering the relationship lending market that decide ex ante to rely on purchased funds to finance relationship loans, or to existing banks making expansion decisions.

[^21]:    ${ }^{32}$ The depositors will never withdraw their deposits from a transaction bank, since a transaction bank only funds informationally-transparent good projects.

