Relationship of light scattering at an angle in the backward direction to the backscattering coefficient

Emmanuel Boss and W. Scott Pegau

We revisit the problem of computing the backscattering coefficient based on the measurement of scattering at one angle in the back direction. Our approach uses theory and new observations of the volume scattering function (VSF) to evaluate the choice of angle used to estimate b_b. We add to previous studies by explicitly treating the molecular backscattering of water (b_{bw}) and its contribution to the VSF shape and to b_b. We find that there are two reasons for the tight correlation between observed scattering near 120° and the backscattering coefficient reported by Oishi [Appl. Opt. **29,** 4658, (1990)], namely, that (1) the shape of the VSF of particles (normalized to the backscattering) does not vary much near that angle for particle assemblages of differing optical properties and size, and (2) the ratio of the VSF to the backscattering is not sensitive to the contribution by water near this angle. We provide a method to correct for the water contribution to backscattering when single-angle measurements are used in the back direction (for angles spanning from near 90° to 160°) that should provide improved estimates of the backscattering coefficient. © 2001 Optical Society of America

OCIS codes: 010.3920, 290.1350, 010.4450.

1. Introduction

The backscattering coefficient, the integral of the volume scattering function (VSF) in the back direction, is an important inherent optical property in the ocean. Both the irradiance reflectance and the remote sensing reflectance are proportional to it.[1] In addition, the ratio of backscattering to total scattering has been found to be an indicator of the bulk index of refraction.[2] Measuring backscattering is complicated, and different approaches have been used to obtain the backscattering coefficient. A common approach is to measure scattering at one angle in the back direction and multiply that measurement by a constant to estimate the backscattering coefficient.

Oishi[3] showed, based on Mie calculations of the particle's VSF [$\beta(\theta)$] and historical measurements of the VSF, that measurements of $\beta(120°)$ provide a good proxy for the backscattering coefficient (b_b). Maffione and Dana[4] argued that measuring $\beta(140°)$ provides a good proxy b_b as well.

Using theory and new observations of the VSF, we examine the application of measurements of scattering at a single angle to estimate b_b and reevaluate the choice of angle used to estimate b_b. We add to the previous studies in that we explicitly treat the molecular backscattering of water (b_{bw}) and its contribution to the VSF shape and to b_b. Although water is often negligible in the total scattering coefficient, it is an important contributor to b_b. Water can account for approximately 80% of b_b in the blue part of the spectrum in the clearest waters.[5,6] The contribution of water to b_b varies spectrally, decreasing with approximately the forth power of wavelength.[7] In Fig. 1 we illustrate how different relative amounts of a particulate and water can affect the shape of the VSF.

We find that there are two reasons for the tight correlation between observed scattering near 120° and the backscattering coefficient, namely, that (1) the shape of the VSF of particles near 120° (normalized by the backscattering coefficient) does not vary much between particle assemblages of differing optical properties and size distributions (as shown by Oishi[3]), and (2) the ratio of $\beta(\theta)$ to b_b is not sensitive to the contribution by water near this angle (e.g., Fig. 1). These conclusions change little when we vary the type of particulate phase function used. In addition, we provide a method to correct for the water contribution to backscattering by using single-angle measurements in the back direction (for angles spanning from near 90° to 160°) that should provide improved estimates of the backscattering coefficient.

E. Boss (boss@oce.orst.edu) and W. S. Pegau are with the College of Ocean and Atmospheric Sciences, Oregon State University, 104 Ocean Administration Building, Corvallis, Oregon 97331.

Received 16 March 2001; revised manuscript received 2 July 2001.

0003-6935/01/305503-05$15.00/0

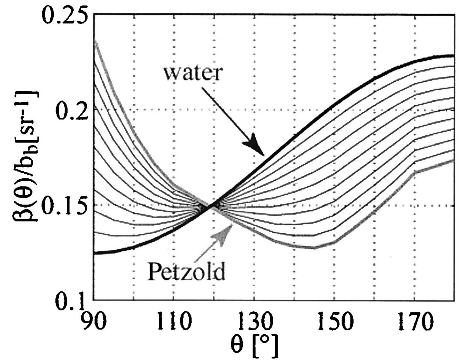

Fig. 1. VSF normalized by the backscattering coefficient $[\beta(\theta)/b_b]$ for cases in which water contributes 0% (gray curve) to 100% (bold black curve) of the backscattering coefficient in increments of 10%. For this illustrative example, we chose the average particulate Petzold VSF.[8,9]

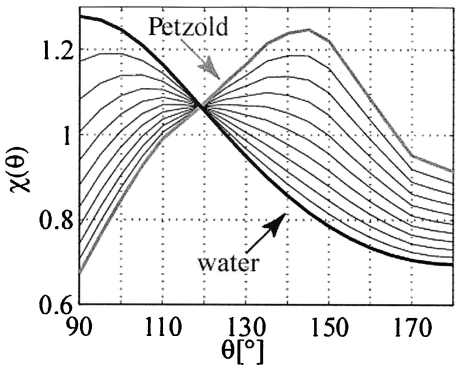

Fig. 2. $\chi(\theta) = b_b/[2\pi\,\beta(\theta)]$ for cases in which water contributes 0% (gray curve) to 100% (bold black curve) of the backscattering coefficient in increments of 10%. For this illustrative example, we chose the average particulate Petzold VSF.[9] Note that, for these curves, all the curves intersect near 118°.

2. Relationship between $\beta(\theta)$ and b_b

Current instrumentation measures scattering at best a few discrete angles in the backward direction, and these measurements are used to estimate the backscattering coefficient. This measurement limitation requires us to assume that we have an accurate VSF measurement $\beta(\theta)$ in a given direction θ, from which we need to compute the value of the backscattering coefficient b_b. We evaluate two approaches to this problem: one in which water is included in the measurement throughout the analysis (the total approach) and another in which water is removed from the measurement prior to conversion to backscattering (the water removal approach).

$\beta(\theta)$ contains scattering by both water and particles (denoted by subscript w and p, respectively):

$$\beta(\theta) = \beta_w(\theta) + \beta_p(\theta). \tag{1}$$

Similarly,

$$b_b = b_{\text{bw}} + b_{\text{bp}}, \tag{2}$$

because, by definition,

$$b_b = 2\pi \int_{\pi/2}^{\pi} \beta(\theta)\sin\theta d\theta. \tag{3}$$

Morel[7] provided a formula for determining $\beta_w(\theta)$:

$$\beta_w(\theta) = A(\lambda, S)^*[1 + \cos^2\theta(1 - \delta)/(1 + \delta)], \tag{4}$$

where the amplitude $A(\lambda, S)$ depends primarily on wavelength λ and salinity S. The depolarization ratio δ varies between 0.07 and 0.11. Morel[7] suggests using $\delta = 0.09$. Using this depolarization ratio, his Table 4, and assuming a linear relationship with salinity, we can determine $A(\lambda, S)$:

$$\begin{aligned} A(\lambda, S) = {} & 1.38(\lambda/500\text{ nm})^{-4.32} \\ & \times (1 + 0.3S/37\text{ psu}) \\ & \times 10^{-4}\text{ m}^{-1}\text{ sr}^{-1}, \end{aligned} \tag{5}$$

where psu is the practical salinity unit.

The uncertainty in $A(\lambda, S)$ is ±15%, based on a comparison of measurements and theory.[7] Assuming we know $\beta_w(\theta)$ sufficiently well, we can compute $\beta_p(\theta_0)$ from measurements of the VSF using Eq. (1).

Following Maffione and Dana,[4] we introduce a nondimensional variable $[\chi(\theta)]$ to relate the $\beta(\theta)$ to b_b. For each constituent, we define

$$2\pi\beta_w(\theta)\chi_w(\theta) = b_{\text{bw}},\ 2\pi\beta_p(\theta)\chi_p(\theta) = b_{\text{bp}}. \tag{6}$$

Similarly,

$$2\pi\beta(\theta)\chi(\theta) = b_b. \tag{7}$$

From Eqs. (1), (2), (6), and (7) we find

$$\begin{aligned} \chi(\theta) &= \chi_w(\theta)\beta_w(\theta)/\beta(\theta) + \chi_p(\theta)\beta_p(\theta)/\beta(\theta) \\ &= \chi_w(\theta)(y) + \chi_p(\theta)(1 - y), \end{aligned} \tag{8}$$

where $y \in [0, 1]$. In practice $y \le 0.8$.[5]

From Eq. (8) it follows that χ falls between χ_w and χ_p. Equality occurs only when $\chi_p = \chi_w$. From Eq. (4) we find

$$\chi_w(\theta) = \left(1 + \frac{1}{3}\frac{1 - \delta}{1 + \delta}\right) \Big/ \left(1 + \frac{1 - \delta}{1 + \delta}\cos^2\theta\right). \tag{9}$$

Note that χ_w does not depend on $A(\lambda, S)$.

3. Total Approach

It follows from the above that the best angle to measure scattering to predict backscattering directly is the angle where $\chi(\theta) = \chi_p(\theta) = \chi_w(\theta)$. At that angle there is no need to know the contribution of $\beta_w(\theta)$ to $\beta(\theta)$ exactly (although a specific VSF shape has to be assumed). For water and the average particulate Petzold function,[8,9] we find that it occurs near $\theta = 118°$ (in Fig. 2. note that Petzold's measurements[8] had an angular resolution of 10° and that no measurements were done beyond 170°).

A. Mie Theory

We would like to know the range of angles at which $\chi(\theta) = \chi_p(\theta) = \chi_w(\theta)$ for the likely range of water and

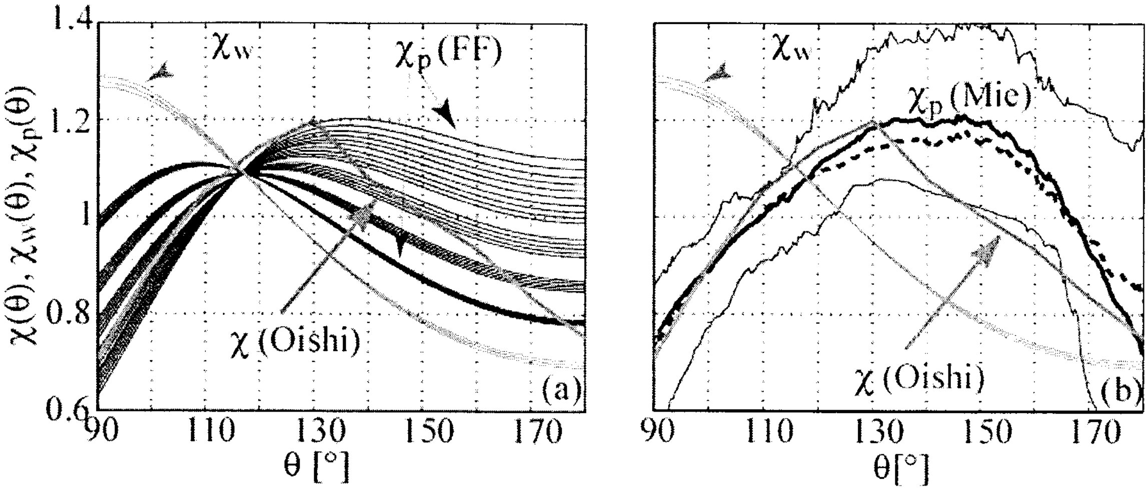

Fig. 3. χ based on Oishi (bold gray curve) and χ_w ($\delta = 0.07, 0.09, 0.11$, thin gray curves). (a) χ_p based on the Fournier and Forand (FF)[10] and the Fournier and Jonasz[11] approximations for $n = 1.05$–1.17 and Junge slope 3.3–4.5 (black curves denoted by FF). (b) χ_p based on Mie calculations (bold solid black curve, dashed black curve; 10th and 90th percentiles, thin black curves).

particulate VSFs. One approach to this problem is to use Mie theory to calculate the $\beta_p(\theta)$ associated with a range of particle types and size distributions. In Fig. 3(a) the curves are based on a two-parameters approximation to Mie theory[10,11] (designated as the Fournier–Forand particulate VSF). This approximation is based on the anomalous diffraction approximation,[12] which is generally assumed to be a good approximation for absorption, scattering, and attenuation by marine particles.[13] Although this approximation may be in error for calculation of backscattering when particles smaller than the wavelengths contribute significantly,[14] it has been found to successfully simulate observed VSF.[15] In Fig. 3(b) we present the results of 150 Mie calculations for populations of particles of sizes varying from 0.01 to 300 μm with a distribution represented by a hyperbolic function with differential slopes varying between 3 and 4.5, a wavelength $\lambda = 530$ nm, real indices of refraction varying from 1.02 to 1.2, and the imaginary part varying from 0 to 0.01. We also superimpose the values of χ_w based on Eq. (8) using $\delta = 0.07, 0.09, 0.11$ (thin gray curves).

Based on Fig. 3, the χ_w and the χ_p curves cross each other between 115° and 123° (mean near 118°) near the value of 120° suggested by Oishi.[3] Based on the Mie calculations, $\chi = 1.07 \pm 10\%$ at these angles. For the Fournier–Forand particulate VSF,[10,11] the χ_p curves cross each other close to where they cross the χ_w curve. This occurs near 117° with a value of $\chi = 1.1 \pm 0.01$.

From Mie theory we find that changes in particulate size distribution and indices of refraction have the smallest standard deviation in χ at angles between 105° and 120°. We also note that χ_w changes little for the ranges of δ based on theory and measurements.[7]

We superimposed on Fig. 3 an estimate of χ reported by Oishi.[3] Oishi linearly regressed many historical measurements of b_b and β (his Table 4) at intervals of 10°, from 90° to 180°. We use his slope estimates (divided by 2π) to estimate χ, although the intercept of Oishi's regression was not zero (except near 116°, based on interpolation of his data). Within the error bars of Oishi,[3] we find that χ falls between χ_w and χ_p as expected. Also note that Oishi's χ is closer to χ_p at small angles and is more influenced by χ_w beyond 130°. This is because β_w monotonously increases with angle [Eq. (3)] whereas β_p generally monotonously decreases with angle down to ~150°.

B. Observations

In Fig. 4 we present χ based on 44 measured $\beta(\theta)$

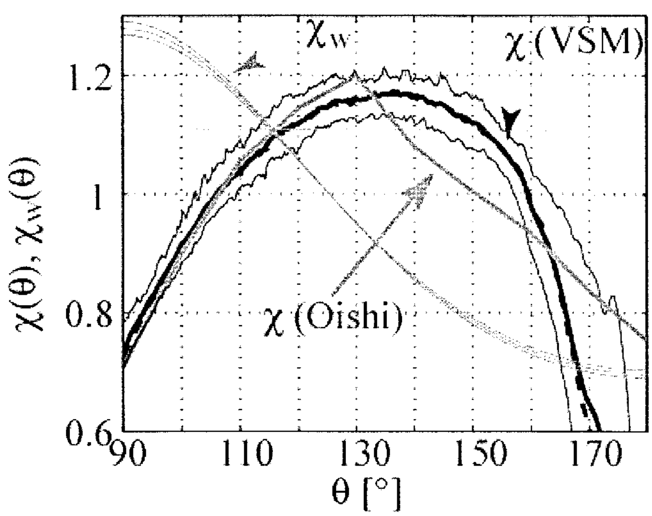

Fig. 4. χ based on 44 VSF measurements by use of the VSM instrument (bold solid black curve, dashed black curve; 10th and 90th percentiles, thin black curves). χ based on Oishi (bold gray curve) and χ_w ($\delta = 0.07, 0.09, 0.11$) (thin gray curves).

Table 1. χ_p Based on 41 VSF Measurements[a]

	Angle (deg)								
	90	100	110	120	130	140	150	160	170
χ_p	0.71	0.9	1.03	1.12	1.17	1.18	1.13	1	0.62
Percent error	4.3	2.6	3.1	4.2	3.3	3.5	4.2	6.4	34.8

[a] b_{bw} is less than 6% of b_b, and the estimated percent error is based on half of the difference between the 10th and 90th percentile.

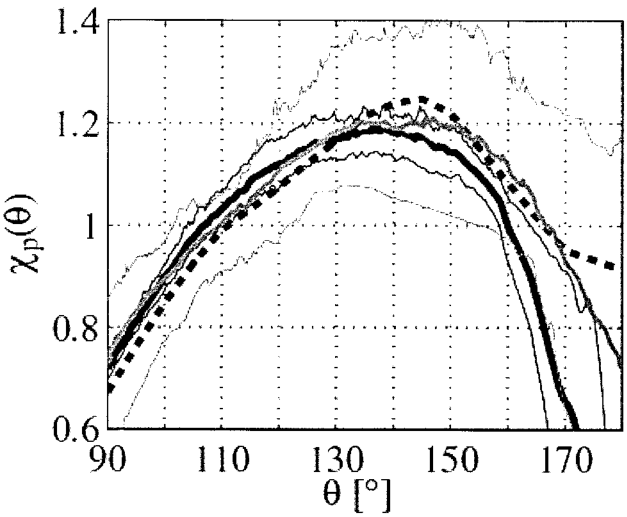

Fig. 5. χ_p based on 41 VSM measurements in turbid waters (black curves) and based on 150 Mie theory calculations (gray curves). The thin curves denote the 10th and 90th percentiles in each angle. The dashed black curve denotes χ_p based on the average Petzold particulate phase function.[8,9]

recently collected in a coastal shelf off New Jersey with a prototype volume scattering meter (VSM) developed by a group of scientists from the Marine Hydrophysical Institute, Sebastopol, Ukraine.[16] The angular resolution of this instrument is 0.3° and its wavelength is 532 nm. We use $\beta(\theta)$ measurements from 90° to 177.3°. The measurements encompassed many different water types; the water varied from being phytoplankton dominated to being dominated by inorganic sediment particles (particulate backscattering ratios varied from 0.005 to 0.033).

In 80% of the cases, $\chi = \chi_w$ for $114° < \theta < 119°$. Based on these observations, $\chi(117°) = 1.1 \pm 4\%$. Oishi[3] found that, for 94% of the observations he analyzed, $\chi(120°) = 1.14 \pm 10\%$, which is consistent with the data in Fig. 4.

We find the total approach to work best for the angles of 117° ± 3°. At other angles the variable contribution of water can cause large biases in the estimate of b_b from β at one angle (e.g., Fig. 3b).

4. Water Removal Approach

When a scattering measurement is done at an angle outside the range recommended above, removing the water prior to the calculation of the backscattering coefficient can minimize the error that is due to the variable water contribution:

$$b_b = \chi_p(\theta)[\beta(\theta) - \beta_w(\theta)] + b_{\mathrm{bw}}. \tag{10}$$

This procedure is simple but requires having an estimate of $\chi_p(\theta)$ and knowledge of the water VSF (β_w) (These are provided in Table 1 and Eqs. (4) and (5), respectively).

Examples of estimates of χ_p are presented in Fig. 5. They are based on 41 VSF observations to which water contributed less than 6% of the b_b (black curves) and based on the same Mie computation as in Fig. 3 (gray curves). The water VSF subtracted from the VSM measurements was modeled based on Eqs. (4) and (5) with $\delta = 0.09$. For contrast we also add χ_p based on the average Petzold particulate phase function.[8,9]

It can be seen that the recent observations and Mie calculations are in agreement on the value of χ_p up to approximately 145°. Differences at angles close to 180° are expected because of the nonsphericity of particles in natural samples.[17] Mie theory assumes the particles are spherical and homogeneous. The VSM measurements suggest that the potential error in χ_p for angles between 90° and 145° is less than 10%, with higher possible errors predicted from Mie theory, especially at angles greater than 120°. The average particulate phase function based on Petzold's measurement[8] is within 5% of the VSM data for the angles of 90°–140° (Fig. 5). In Table 1 we tabulated χ_p based on the VSM measurements.

5. Discussion and Summary

We found the angle where χ_p and χ_w intersect to be similar for VSFs from an approximation to Mie theory, calculations using Mie theory, as well as new and historic measurements. We find that, in the vicinity of 117°, $\beta_w(\theta)/b_{\mathrm{bw}}$, $\beta_p(\theta)/b_{\mathrm{bp}}$, and $\beta(\theta)/b_b$ are equal. From our analysis, we suggest using a value $\chi(117°) = 1.1$ with a likely error of less than 4%. These findings are consistent with Oishi's[3] conclusion that a measurement of total scattering near 120° provides a good estimate for backscattering.

It is advised that, for measurements at other angles than near 117°, the scattering by water should be removed prior to calculation of the backscattering coefficient of the particulate component [see Eq. (10) in Section 4]. The backscattering coefficient of water can then be added to the backscattering coefficient of particles to determine the total backscattering coefficient. This approach would minimize errors caused when a χ value is used that has an assumed proportion of scattering by water.

Mie calculations (e.g., Fig. 3) suggest that there will likely be a higher uncertainty in χ_p because of variability in size distribution and index of refraction in measurements at other angles than near 117°. Recent VSF observations, however, suggest that there is little increase in the possible range of χ_p for a given angle up to 160° (Table 1). One should take into account, however, the possible errors that are due to uncertainties in the VSF of water, which increase with the relative contribution of water to backscattering. Another caveat is the limited measurements to date of the VSF in natural waters; more measurements are needed to provide estimates on the error of the method presented here.

We do not wish to suggest that estimation of the backscattering coefficient from scattering measurements at a single angle or a few angles is preferable to measurement of the full VSF. Given the currently available instrumentation, it is important to understand how to interpret and process data collected at a single angle to estimate backscattering. In conclusion, we encourage that single-angle backscattering measurements be conducted at an angle near 117° where the processing is simplest. When it is not measured at this angle, we recommend that the scattering by water be removed before it is multiplied by χ_p and then the backscattering by water be added back to provide the best estimate of the total backscattering coefficient.

This research was supported by the National Oceanographic Partnership Program under National Science Foundation award OCE 9911037 and by the Environmental Optics Program of the U.S. Office of Naval Research as part of the Hyperspectral Coastal Ocean Dynamics Experiment initiative. We thank M. Lee, M. Shibanov, and G. Korotaev of the Marine Hydrophysical Institute, Sebastopol, Ukraine, for providing the VSM data. We thank J. R. V. Zaneveld for valuable discussions.

References

1. H. R. Gordon, O. B. Brown, R. E. Evans, J. W. Brown, R. C. Smith, K. C. Baker, and D. C. Clark, "A semianalytic model of ocean color," J. Geophys. Res. D **96,** 10909–10924 (1988).
2. M. S. Twardowski, E. Boss, J. B. Macdonald, W. S. Pegau, A. H. Barnard, and J. R. V. Zaneveld, "A model for retrieving oceanic particle composition and size distribution from measurements of the backscattering ratio and spectral attenuation," J. Geophys. Res. **106**, 14129–14142 (2001).
3. T. Oishi, "Significant relationship between the backward scattering coefficient of sea water and the scatterance at 120°," Appl. Opt. **29,** 4658–4665 (1990).
4. R. A. Maffione and D. R. Dana, "Instruments and methods for measuring the backward-scattering coefficient of ocean waters," Appl. Opt. **36,** 6057–6067 (1997).
5. A. Morel and B. Gentili, "Diffuse reflectance of oceanic waters: its dependence on Sun angle as influenced by the molecular scattering contribution," Appl. Opt. **30,** 4427–4438 (1991).
6. A. Morel and B. Gentili, "Diffuse reflectance of oceanic waters. II. Bidirectional aspects," Appl. Opt. **32,** 6864–6878 (1993).
7. A. Morel, "Optical properties of pure water and pure sea water," in *Optical Aspects of Oceanography*, N. G. Jerlov and E. S. Neilsen, eds. (Academic, New York, 1974), pp. 1–24.
8. T. J. Petzold, "Volume scattering functions for selected ocean waters," Rep. 72–78 (Scripps Institution of Oceanography, La Jolla, Calif., 1972).
9. C. D. Mobley, B. Gentili, H. R. Gordon, Z. Jin, G. W. Kattawar, A. Morel, P. Reinersman, K. Stamnes, and R. H. Stavn, "Comparison of numerical models for computing underwater light fields," Appl. Opt. **32,** 7484–7505 (1993).
10. G. R. Fournier and J. L. Forand, "Analytic phase function for ocean water," in *Ocean Optics XII*, J. S. Jaffe, ed., Proc. SPIE **2258,** 194–201 (1994).
11. G. P. Fournier and M. Jonasz, "Computer-based underwater imaging analysis," in *Airborne and In-Water Underwater Imaging*, G. D. Gilbert, ed., Proc. SPIE **3761,** 62–70 (1999).
12. H. C. van de Hulst, *Light Scattering by Small Particles* (Wiley, New York, 1957).
13. A. Morel and A. Bricaud, "Light attenuation and scattering by planktonic cells: a theoretical modeling," Appl. Opt. **25,** 571–580 (1986).
14. D. Stramski and D. A. Kiefer, "Light scattering by microorganisms in the open ocean," Prog. Oceanogr. **28,** 343–383 (1991).
15. C. D. Mobley, Sequoia Scientific, Inc., 15317 NE 90th St., Redmond, Wash. 98052 (personal communication, 2001).
16. V. I. Haltrin, M. E. Lee, and O. V. Martynov, "Polar nephlometer for sea truth measurements," in *Proceedings of the Second International Airborne Remote Sensing Conference and Exhibition* (Environmental Research Institute of Michigan, Ann Arbor, Mich., 1996), pp. 444–450.
17. C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles* (Wiley, New York, 1983).