Relative density concept is not a reliable criterion

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Many years ago, a new concept called relative density was developed with the intention of appropriately defining the looseness and denseness of sand or sand–gravel soils in a meaningful way. Soon after, relative density found its way into ground improvement as an acceptance criterion by engineers who were more familiar with the construction of engineered backfilling rather than thick mass treatment. There are considerable amounts of research and publications that are able to well demonstrate the unreliability of relative density as an acceptance criterion. Relative density has no real influence on the soil's performance, its range of application does not span across all soil types, and it is subject to large inherent errors that make its use a technical risk. Here, the reasons why the concept of relative density is unreliable and should not be used for a ground improvement acceptance criterion are presented and discussed.

Notation

$C_{\gamma d}$	error propagation factor for γ_d
$C_{\gamma d\max}$	error propagation factor for $\gamma_{d\max}$
$C_{\gamma d\min}$	error propagation factor for $\gamma_{d\min}$
D_d	relative density
D_{50}	mean particle diameter
e	the in situ or stated void ratio of a soil deposit or fill
$e_{\max}$	maximum index void ratio or the reference void ratio of a soil at the minimum index density/unit weight
$e_{\min}$	minimum index void ratio or the reference void ratio of a soil at the maximum index density/unit weight
m	number of measurements
S_{Dd}	standard deviation for D_d
$S_{\gamma d}$	standard deviation for γ_d
$S_{\gamma d\max}$	standard deviation for $\gamma_{d\max}$
$S_{\gamma d\min}$	standard deviation for $\gamma_{d\min}$
ΔD_d	deviation in relative density
$\Delta \gamma_d$	deviation in γ_d
$\Delta \gamma_{d\max}$	deviation in $\gamma_{d\max}$
$\Delta \gamma_{d\min}$	deviation in $\gamma_{d\min}$
ρ_d or γ_d	dry density/unit weight of a soil deposit or fill at the given void ratio
$\rho_{d\max}$ or $\gamma_{d\max}$	the reference dry density/unit weight of a soil in the densest state of compactness that can be attained using a standard laboratory compaction procedure that minimises particle segregation and breakdown
$\rho_{d\min}$ or $\gamma_{d\min}$	the reference dry density/unit weight of a soil in a standard state of compactness at which it can be placed using a standard laboratory procedure that prevents bulking and minimises particle segregation

1.   Introduction: history

The concept of relative density (D_d) was first introduced by Terzaghi (1925) to bring the behaviour characteristics of soils together on a common basis in consistent and practically useful relations and to provide a tool for communications between engineers (Burmister, 1948). It was suggested that this parameter would be an appropriate means to define the looseness and denseness of sand or sand–gravel soils in a meaningful way because important properties were assumed to correlate quite well by this means.

Relative density is a definition rather than an inherent property of the soil. Thus, by itself, it has no significance and influence on performance and it is possible to satisfy design criteria with a higher safety factor without even complying with the relative density criterion (Hamidi *et al.*, 2010a, 2010b).

In fact, confusion in the use of relative density began as soon as engineers started utilising it as a soil parameter because there was no common definition or set of standards to work from (Holtz, 1973). Recognising that some kind of standard would be necessary, Section D, Subcommittee 3 of Committee D-18 of ASTM was established in 1954 with the aim of determining the minimum and maximum densities of sand and gravel soils. The work of the subcommittee resulted in ASTM D2049-69 (ASTM, 1969) which was approved by Committee D-18 in 1964.

It was the opinion of the majority of subcommittee members that the benefits of a reasonably good standard method far outweighted any disadvantages of the particular test that was then proposed (Holtz, 1973). In addition, it was felt that by getting a tentative standard published, many people and organisations would work with it, evaluate its suitability, and suggest beneficial modifications and improvements.

The introduction of relative density as a criterion for acceptance, in which it is compared with a critical value to give a yes or no type of answer has led to a considerable amount of research, most of which points toward the insufficiency of its accuracy and reliability, and it is the opinion of the authors that although still popular and systematically used by those engineers who have never looked deeply into this concept, the truth is that after years of having had its suitability evaluated, relative density has not fulfilled expectations and should be abandoned as a criterion altogether. That seems to be already under way. There was a time when liquefaction analysis was founded on the concept of relative density (Seed and Idriss, 1967, 1971); however, today, although the same methodology has been retained, relative density has lost credence, and is no longer part of the liquefaction evaluation procedure (Youd *et al.*, 2001). In the process of re-evaluating the concept of relative density, as proposed by Selig and Ladd (1973), ASTM withdrew its D2049-69 standard for measuring relative density as early as 1984 (Hamidi *et al.*, 2011), and has replaced it with the more meaningful standards for measuring maximum and minimum index densities (ASTM, 2006a, 2006b). In its new standards, ASTM (2006a) cautiously states that it is generally recognised that either relative density or percentage compaction is a good indicator of the state of compactness of a given soil mass. However, the engineering properties, such as strength, compressibility and permeability of a given soil, compacted by various methods to a given state of compactness can vary considerably. Therefore, considerable engineering judgment must be used in relating the engineering properties of soil to the state of compactness. Mayne (2006) cautiously notes that in his opinion, relative density is a rather weak parameter. Likewise, Bowles (1996) states that in his opinion, the relative density test is not of much value because it is difficult to obtain maximum and minimum unit weight values within a range of about ± 0.5 kN/m^3.

Herein, the authors will review the limitations and the unreliability of the concept of relative density; thus revealing the potential risks that a project is unnecessarily exposed to simply due to a poor choice of criteria.

2. Relative density and points of concern

2.1 The definition of relative density and its intended range of application

ASTM (2006a, 2006b) defines relative density as a ratio, expressed as a percentage, of the difference between the maximum index void ratio and any given void ratio of a cohesionless, free-draining soil to the difference between its maximum and minimum index void ratios, or

1.
$$D_d = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100$$

where e_{max} is the maximum index void ratio or the reference void ratio of a soil at the minimum index density/unit weight; e_{min} is the minimum index void ratio or the reference void ratio of a soil at the maximum index density/unit weight; and e is the in situ or stated void ratio of a soil deposit or fill.

Equation 1 can also be expressed in terms of maximum and minimum indexes and dry densities or dry unit weights (ASTM, 2006a, 2006b) as formulated in Equations 2 and 3

2.
$$D_d = \frac{\rho_{dmax}(\rho_d - \rho_{dmin})}{\rho_d(\rho_{dmax} - \rho_{dmin})} \times 100$$

3.
$$D_d = \frac{\gamma_{dmax}(\gamma_d - \gamma_{dmin})}{\gamma_d(\gamma_{dmax} - \gamma_{dmin})} \times 100$$

where ρ_{dmax} or γ_{dmax} are the reference dry density/unit weight of a soil in the densest state of compactness that can be attained using a standard laboratory compaction procedure that minimises particle segregation and breakdown; ρ_{dmin} or γ_{dmin} are the reference dry density/unit weight of a soil in a standard state of compactness at which it can be placed using a standard laboratory procedure that prevents bulking and minimises particle segregation; and ρ_d or γ_d are the dry density/unit weight of a soil deposit or fill at the given void ratio

ASTM makes a number of statements that deserve review and consideration. First, the applications of ASTM testing methods are conditioned to a range of selected soils, or in other words, the concept of relative density is not applicable to all other soils. According to ASTM (2006a, 2006b), for its testing methods to be applicable, the soil can contain up to 15%, by dry mass, of soil particles passing a 75 μm sieve, provided they still have cohesionless, free-draining characteristics. Furthermore, for determination of minimum index density and unit weight of soils, the three

accepted methods are applicable to soil in which 100%, by dry mass, of soil particles pass, respectively, 75, 19 and 9·5 mm sieves. In the first method, the soil may contain up to 30%, by dry mass, of soil particles retained on a 37·5 mm sieve, and the third method is applicable only to fine and medium sands that may contain up to 10%, by dry mass, of soil particles retained on a 2 mm sieve (ASTM, 2006b). To summarise, as shown in Figure 1, at best, relative density is applicable only to soils with less than 15% of the material being silt or clay and can have gravel up to a diameter of 75 mm, provided that 70% of the grain size is less than 37·5 mm and the soil is still cohesionless and free draining.

ASTM has rightfully introduced the term 'free draining' even though the applicable zone of relative density is for sands and gravels. As much as sands and gravels are assumed to be free draining, the authors have come across non-free-draining sands with less than 15% fines content, but containing high amounts of clay. For example, in the dynamic compaction project of Salam Resort, on the seashore of Bahrain, the sieve analyses showed that the fines content of the soil was less than 20% and the soil was identified as sand; however, clay content was relatively higher than normally encountered and was approximately 10%. When ground improvement was carried out on the site, it was observed that the soil was not draining as would be expected in sand, and reduction of pore water pressure to initial values required a period of more than one week.

Engineers who have been involved in reclamation projects know very well that even if specifications limit the soil's grain sizes to the limits defined by ASTM, there is still a good chance that due to segregation of material or change of material source, layers or bands of soil with higher fines content will be realised here and there. At the other extreme, there are many situations in which the soil contains larger cobbles and boulders.

When developing a specification, it would seem rational to stipulate acceptance criteria that will most likely not require
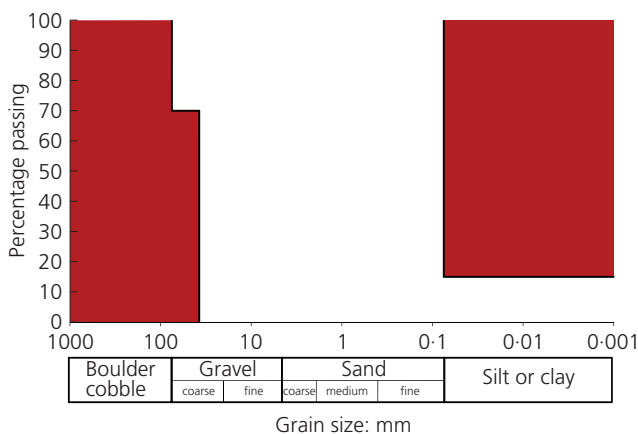

Figure 1. Applicable range of relative density (non-shaded areas)

reassessment and revision during the project. In the author's opinion relative density does not comply with this statement, as in the event that further supplementary investigations prove that the soil is non-free draining, then the specifications will not apply to the soil any longer, and alternative criteria must be sought. It may be simply a more reasonable approach to stipulate the alternative specification in the first place.

If the oversize and percentage of undersize materials are identified in advance, the oversize may be separated or crushed or the material may be mixed to reduce the fines content, but there are techniques such as dynamic compaction that can treat soils containing more fines and oversized cobbles than allowed by ASTM, and the extra activities and costs could have been easily avoided by stipulation of an appropriate acceptance criterion.

If surpassing the material size limits are detected after reclamation, one ill-advised solution may be to remove the soil and to replace it with soil that meets the required grading; however, this is more easily said than done. Alternatively, there are others who may re-define relative density standards to fit their needs. Of course, these scenarios could be totally avoided simply by defining a reliable criterion.

For many types of free-draining, cohesionless soils, ASTM-specified test methods cause a moderate amount of degradation (particle breakdown) of the soil (ASTM, 2006a). When degradation occurs, typically, there is an increase in the maximum index density/unit weight obtained, and comparable test results may not be obtained when different size moulds are used to test a given soil. This raises the question of comparability of test results, and as will be discussed later, any small changes to the limit indices will result in dramatic changes in relative density.

Referring to Holtz (1973), ASTM (2006a, 2006b) also acknowledges that there are published data to indicate that these test methods have a high degree of variability, but proposes that the variability can be greatly reduced by careful calibration of equipment. Of course, calibration of equipment can only reduce systematic errors caused by equipment conditions, but it will have no effect on random errors. The sources of errors and variations of relative density testing require further review.

2.2 Errors in relative density testing
Relative density testing is subject to a combination of systematic errors, random errors and mistakes. Systematic error is a measure of accuracy and the difference between the correct value and the measured average of a set of repeated tests. Random error is the precision of a quantity, and is measured by the scatter in the results of a group of repeated tests (Selig and Ladd, 1973).

The reliability of relative density measurements has, for the most part, been taken for granted by most engineers, but not by all. Yoshimi and Tohno (1973) have assumed a statistical approach, and note that since relative density is proportional to

$(\gamma_d - \gamma_{d\min})/\gamma_d$, even a small variation in γ_d or $\gamma_{d\min}$ can cause a considerable variation in relative density when $\gamma_d - \gamma_{d\min}$ is small; that is, when relative density is low. For example, when $\gamma_{d\min} = 13 \cdot 5 \text{ kN/m}^3$, $\gamma_{d\max} = 16 \cdot 37 \text{ kN/m}^3$ and $\gamma_d = 14 \cdot 25 \text{ kN/m}^3$, relative density will be 30%. If $\gamma_{d\min}$ is increased by 1% to $13 \cdot 635 \text{ kN/m}^3$, D_d reduces to 25·8%, which is 14% less than the initial value. In other words, the relative deviation in relative density is 14 times that of $\gamma_{d\min}$.

Yoshimi and Tohno (1973) express the influence of dry unit weight random errors on relative density using Equation 4

$$4. \quad \left(\frac{S_{D_d}}{D_d}\right)^2 = C^2_{\gamma_{d\max}}\left(\frac{S_{\gamma_{d\max}}}{\gamma_{d\max}}\right)^2 + C^2_{\gamma_{d\min}}\left(\frac{S_{\gamma_{d\min}}}{\gamma_{d\min}}\right)^2 + C^2_{\gamma_d}\left(\frac{S_{\gamma_d}}{\gamma_d}\right)^2$$

The terms in the parentheses are coefficients of variation. Parameters S_{Dd}, $S_{\gamma d\max}$, $S_{\gamma d\min}$ and $S_{\gamma d}$ are, respectively, the standard deviations for D_d, $\gamma_{d\max}$, $\gamma_{d\min}$ and γ_d, and $C_{\gamma d\max}$, $C_{\gamma d\min}$ and $C_{\gamma d}$ are error propagation factors that are expressed in the forms of Equations 5 to 7

$$5. \quad C_{\gamma_{d\max}} = \frac{\gamma_{d\min}}{(\gamma_{d\max} - \gamma_{d\min})}$$

$$6. \quad C_{\gamma_{d\min}} = \frac{\gamma_{d\max}(1 - D_d)}{(\gamma_{d\max} - \gamma_{d\min})D_d} = \frac{(C_{\gamma_{d\max}} + 1)(1 - D_d)}{D_d}$$

$$7. \quad C_{\gamma_d} = \frac{\gamma_{d\max}}{(\gamma_{d\max} - \gamma_{d\min})D_d} - 1 = \frac{(C_{\gamma_{d\max}} + 1)}{D_d} - 1$$

As illustrated in Figure 2 for the case of $C_{\gamma d\max} = 4\cdot 7$, which is a representative value for clean sand with low uniformity, these equations show that while $C_{\gamma d\max}$ is independent of relative density, $C_{\gamma d\min}$ and $C_{\gamma d}$ increase as relative density decreases until they reach infinity when relative density reduces to zero.

Random errors can be reduced to any desired degree by repeating the test and averaging the results (Selig and Ladd, 1973). For the case when the coefficients of variation are equal, for a defined coefficient of variation, the number of measurements, m, for each dry unit weight was calculated by Yoshimi and Tohno (1973) to be as expressed in Equation 8
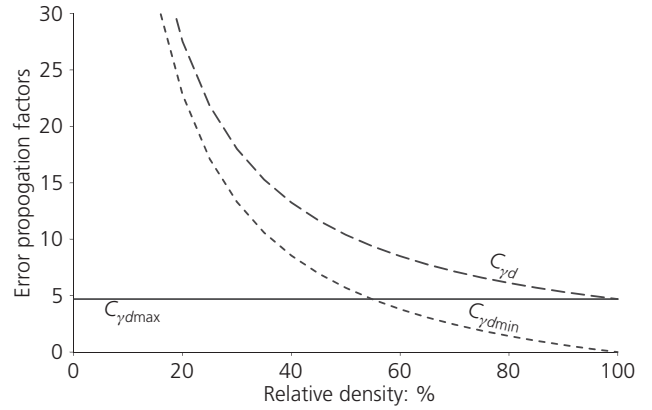

Figure 2. Error propagation factors for relative density, redrawn from Yoshimi and Tohno (1973)

$$8. \quad m = \frac{C^2_{\gamma_{d\max}} + C^2_{\gamma_{d\min}} + C^2_{\gamma_d}}{(S_{D_d}/D_d)^2}$$

For example, when $C_{\gamma d\max} = 4\cdot 7$, if $D_d = 50\%$, $(S_{D_d}/D_d) = 5\%$ and the coefficient of variation equals 1%, then seven tests must be made for each of the maximum index, minimum index and specimen unit weights. For $D_d = 22\%$, with the previous other parameters, 40 tests for each unit weight will be required. Unfortunately, and even with the acknowledgement of ASTM (2006a, 2006b), it seems that, in practice, this recommendation of Yoshimi and Tohno (1973) has gone unheard, and most commonly, regardless of the value of relative density, only one test for each parameter is carried out.

When studying the influence of dry unit weight systematic errors on relative density, Yoshimi and Tohno (1973) formulate relative deviation in relative density, $\Delta D_d/D_d$, due to a deviation in any one of the dry unit weights, namely $\Delta\gamma_{d\max}$, $\Delta\gamma_{d\min}$ and $\Delta\gamma_d$, and express them in the forms of Equations 9 to 11

$$9. \quad \left.\frac{\Delta D_d}{D_d}\right]_{\Delta\gamma_{d\min}=\Delta\gamma=0} = \frac{\gamma_{d\min}}{\gamma_{d\max} - \gamma_{d\min} + \Delta\gamma_{d\max}}\frac{\Delta\gamma_{d\max}}{\gamma_{d\max}}$$

$$10. \quad \left.\frac{\Delta D_d}{D_d}\right]_{\Delta\gamma_{d\max}=\Delta\gamma=0} = \frac{(1 - D_d)\gamma_{d\max}}{D_d(\gamma_{d\max} - \gamma_{d\min} - \Delta\gamma_{d\min})}\frac{\Delta\gamma_{d\min}}{\gamma_{d\min}}$$

$$11. \quad \left.\frac{\Delta D_d}{D_d}\right]_{\Delta\gamma_{d\max}=\Delta\gamma_{d\min}=0} = \frac{\gamma_d\gamma_{d\min}}{(\gamma_d - \gamma_{d\min})(\gamma_d + \Delta\gamma_d)}\frac{\Delta\gamma_d}{\gamma_d}$$

For simultaneous errors in $\gamma_{d\max}$, $\gamma_{d\min}$ and γ_d, the resulting values of $\Delta D_d/D_d$ may be added, keeping in mind the correct

signs. In view of relatively large values of $\Delta\gamma_{d\max}$ or $\Delta\gamma_{d\min}$ due to the systematic errors of the unit weight measurements, it can be understood that $\Delta D_d/D_d$ may reach tens of per cent.

Referring to Equations 1 to 3, there are three parameters that must be determined for calculation of relative density, namely $e_{\max}$, $\rho_{\min}$ or $\gamma_{\min}$, which describe the most loose state of the soil, $e_{\min}$, $\rho_{\max}$ or $\gamma_{\max}$, which describe the most dense state of the soil and e, ρ_d or γ_d, which describe the in situ state of the soil.

Due to the relative magnitude of the maximum index, minimum index and in-situ dry unit weights, relative density is computed from the ratio of small differences between large numbers (Tavenas *et al.*, 1973). This implies that small variations of the large numbers will be magnified to produce a great variability in the computed result. The simple application of the theory of errors led Tavenas and La Rochelle (1970) to conclude that any laboratory determination of relative density would be affected by a large variability, even if the ASTM standard method was used.

As part of the study of Tavenas *et al.* (1973), 87 laboratories in the United States and Canada participated in grading tests, minimum and maximum index density tests. Some very important, if not dramatic, conclusions that they were able to draw from their findings included the following properties.

(*a*) Even though ASTM standard tests for determining the minimum and maximum (index) density of cohesionless materials were considered as 'normally accurate' soil mechanics tests with an observed coefficient of variability of the order of 2·5% and a coefficient of reproducibility of the order of 0·8%, the use of these parameters in the relative density formula leads to a result of poor quality, since it is characterised by coefficients of variability of the order of 15 to 40% and by coefficients of reproducibility of the order of 3 to 15% in most of the usual cases. Thus, and simply due to the formulation of relative density, the variability was multiplied by a factor of 10.

(*b*) The variability of the relative density increases as the maximum grain size of the tested materials increases. Standard deviations were found to be 60 to 100% larger for gravelly sand than for fine sand. As the fine sand tested was close to the ideal material with a small maximum grain size and a coefficient of uniformity of the order of 3, and no particles passed sieve 0·075 mm, the variability and reproducibility observed on this material were the best possible with the testing technique. Thus, it cannot be expected to determine any relative density with a width of the 95% interval, less than 10% if the results obtained by one technician only are considered, and less than 40% if the results obtained by different laboratories are analysed. The results obtained by different operators in the same laboratory fell in between.

(*c*) A third basic parameter influencing the determination of the relative density is the actual dry unit weight itself. This parameter is also affected by a certain error. In the most important case of the measurement of the in situ unit weight, the methods were such that any value of dry unit weight could not have been defined with an error less than $\pm\,30\ \text{N/m}^3$. The error on the in situ dry unit weight had to be combined with the previously discussed variability to give the final variability of the relative density. As shown in Figure 3, Tavenas *et al.* (1973) demonstrated that the width of the 95% interval for the correlation between the in situ dry density and relative density of fine sand and gravelly sand is, respectively, 64 and 94%, which is close to the full range of possible values for the relative density, and they assessed that under such circumstances, the probability of evaluating the correct relative density by a wild guess is at least equal to that of measuring it by the standard method.

(*d*) Due to the very large variability of the relative density between laboratories, the comparison of relative densities measured by different laboratories was totally non-significant. There were important practical implications of this fact: all established correlations between relative density and various properties of cohesionless soils such as standard penetration index, point resistance in a static penetration test, friction angle, modulus of compressibility, shear wave velocity, etc. are useless to anyone but the operator who has established them, since that person is the only one who can reproduce the relative density of the considered soil with sufficient accuracy.

(*e*) It appeared that due not so much to the variability of the minimum and maximum unit weights, but essentially to the formulation of relative density itself, the resulting accuracy of this parameter was so poor that its use was related to major uncertainties (the best case was the ideal material such as the tested fine sand, and was deemed to be practically meaningless in most of the other cases).

Similarly, Tiedemann (1973) has conducted research on the results of minimum and maximum index unit weight tests performed by 14 US Bureau of Reclamation soil laboratories, and has concluded that based on results obtained from the cooperative test programme and other studies, it appeared that the variations associated with the minimum and maximum unit weight tests investigated were approximately the same as, or less than, those associated with the impact-type compaction test. However, when the results are used to compute relative density, large variations can occur. For the extreme variations in both the minimum and maximum density, a dry unit weight of 17·3 kN/m³ could be reported as a relative density varying from 40 to 76% on a between-laboratory basis or from 52 to 66% if the limiting unit weights were determined by a single operator.

One may argue that in reality, the chances of the occurrence of the worst-case scenarios as envisaged by Tavenas *et al.* (1973) and Tiedemann (1973) are quite low. As true as that may be, these worst-case scenarios are just an indication that as unlikely as may be, relative density criteria could seriously misrepresent
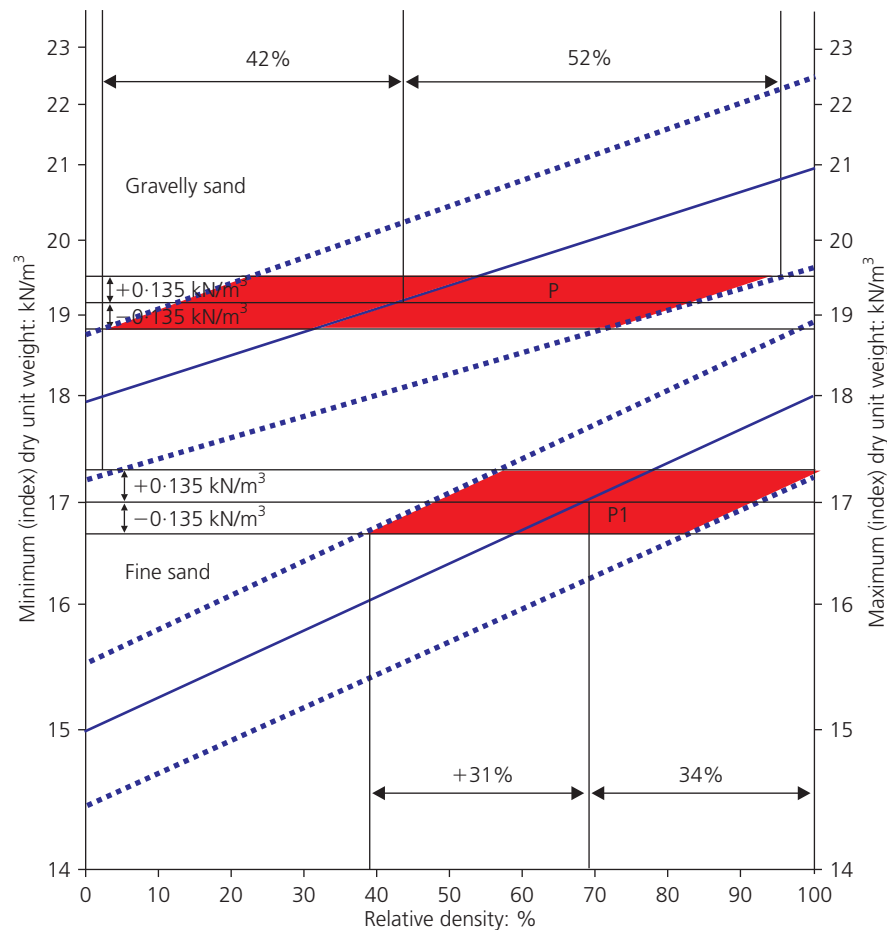
Figure 3. Error on the in situ relative density, redrawn in SI units
from Tavenas *et al.* (1973)

the soil conditions to the point of rejecting a well-compacted soil or lead to a total disaster by accepting a loose soil. Of course, there are all the other non-worst-case scenarios that nevertheless also lead to totally misrepresented ground conditions. It would indeed be much more appropriate not to tie a project's destiny to a parameter that is referred to as being as accurate as a guess.

Tiedemann (1973) also reports that variations in duplicate tests were less than for duplicate specimens, indicating that there was considerable variation in the specimen. According to Holtz (1973), it is usually more difficult to produce uniform samples and uniform specimens of the coarse sand and gravel soils than fine silty and clayey soils. The difficulty usually becomes greater when the soils are cohesionless and as the range of particle sizes increase. This is primarily due to segregation of the material. Segregation can even affect the uniformity of sampling when closely controlled quartering and splitting methods are used. The scooping and placing of cohesionless, coarse-grained soils in moulds for the standard minimum and maximum index density tests can cause segregation, which can produce inhomogeneity of

gradation within a test specimen and gradation variations between the so-called duplicate specimens.

Similar findings have been also reported by others, such as Gupta and McKeown (1973) who encountered problems for applying the relative density criterion for Kettle Generating Station in Canada. In the end, they concluded that the effects of variations in minimum unit weight on relative density were startling. They noted that although the variation decreased with an increase of relative density, nevertheless, it still created a dilemma for effective quality control in the field in terms of enforcing the requirements of 75% relative density as stipulated in the contract.

Gupta and McKeown (1973) used four technicians for each test to carry out 10 tests on each soil sample, thus resulting in 40 tests per sample. For the analysis of the tests, they only used those tests that fell within the middle 90% spread.

Among the numerous batches of 40 tests, in the best case, relative density at 50 and 75%, respectively, had errors of ± 5% and

$\pm 1.6\%$. In the worst case, a 50 and 75% relative density, respectively, had errors of $\pm 26\%$ and $\pm 6.6\%$. This means that a sample with relative density of 50% could have been reported as 64% and a sample with relative density equal to 75% could have been reported as 69%. Although the true condition of one sample is loose and the other dense, errors may lead to a condition in which the material may be practically assumed to have similar test values. Obviously, this may lead to a non-compliance with project specifications solely due to the inherent defaults of the choice of specification. Had Gupta and McKeown (1973) not limited the tests to the middle 90% spread, the variations would have been even more concerning.

Reviewing the work of others during the ASTM symposium for *Evaluation of Relative Density and its Role in Geotechnical Projects involving Cohesionless Soils* at the 75th Annual Meeting, Selig and Ladd (1973) assess that the error in $\gamma_{d\max}$ has the largest effect for high D_d, while the error in $\gamma_{d\min}$ has the greatest effect for low D_d and add that, in general, relative density has associated with it a random error of about ± 10 to 15 standard deviations and a systematic error of range 25 to 30. They conclude that in their opinion, relative density has value, but that it has frequently been overextended with a false sense of reliability, or improperly used. Selig and Ladd (1973) were probably using a mild tone not to upset any supporters of relative density. If there is any value in relative density, with all that has been discussed, the value is not in being an acceptance criterion of ground improvement, and it should be sought elsewhere.

3. A second glance at the standards

After this discussion, it may be interesting to take a second look at ASTM standards (ASTM, 2006a, 2006b). ASTM allows four alternative methods for determination of maximum index density/ unit weight, of which two methods are on oven-dried soil and two are on wet soil. The wet method may be conducted on either oven-dried soil to which sufficient water is added or, if preferred, on wet soil from the field.

Although it may be expected that the application of any of the methods should yield the same result, ASTM notes that the wet method can yield significantly higher values of maximum index density/unit weight for some soils, and adds that such a higher maximum index density, when considered along with the minimum index density/unit weight, will be found to significantly affect the value of relative density.

ASTM does not define wetness, and it is not clear what would be considered as the minimum water content to be considered as wet.

ASTM notes that although the dry method of testing is often preferred because results can usually be obtained more quickly, as a general rule, the wet method should be used if it is established that it produces maximum index densities/unit weights that would significantly affect the use of the value of

relative density. However, ASTM does not define what it considers as being significant. Noting that any small changes in limit indexes are magnified several times in changes of relative density, any difference may in fact be significant.

Likewise, ASTM (2006b) specifies three testing methods for determination of minimum index density/unit weight, but allows the individual assigning the test to specify the method to be used. If no method is specified, the provisions of the first method shall govern. The two other methods are provided for guidance of testing used in conjunction with special studies, especially where there is not enough material. Once again, surely the application of these three testing methods may yield different results that could result in values of relative density with significant differences.

ASTM (2006a, 2006b) is well aware of relative density deviations and has performed a series of three replicate tests per type and single test per type for poorly graded sand and specifies acceptable ranges of results for it. For single tests, the acceptable range between two results is 1.15 kN/m^3. It has already been discussed that for a much smaller value of 0.135 kN/m^3, Yoshimi and Tohno (1973) calculated 14% change in relative density; hence, it could be expected that the acceptable range of ASTM could result in larger errors.

4. The reliable criteria

It would be very incomplete to have demonstrated how unreliable relative density is as an acceptance criteria without proposing a reliable way forward. Hamidi *et al.* (2011) have studied different methods to stipulate acceptance criteria and have concluded that it is much more meaningful and affordable to base acceptance criteria on design criteria themselves rather than on defining a designed work procedure, or stipulating minimum test values. Truly, what can be a better way to ensure design criteria have been satisfied than to evaluate the ground condition for each and every criterion independently and to verify that they are all satisfied.

5. Conclusion

Due to its formulation, relative density is prone to errors with magnitudes of tens of per cent. Putting it in one simple sentence, this amount of inaccuracy is unacceptable for a parameter that itself is to be the criterion of acceptance, and it is high time that the engineering community become aware of this fundamental drawback of relative density and abandon it as a criterion once and for all.

Regardless of unavoidable errors, standards have imposed numerous limitations, conditions and alternatives that further complicate the application of relative density, and can result in a condition where there is a void in contractual specifications.

Finally, relative density is not a soil property, it is only a definition. It has no real influence on performance, and there is no benefit in stipulating it as a criterion. It would be more

rational to ensure that design criteria have been satisfied by directly specifying them as the acceptance criteria (Hamidi *et al.*, 2011). It is possible to satisfy design criteria with higher safety factors without complying to relative density requirements (Hamidi *et al.*, 2010a, 2010b).

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