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Relativistic beaming and quasar statistics

M. J. L. Orr and I. W. A. Browne *The University of Manchester, Nuffield Radio Astronomy Laboratories, Jodrell Bank, Macclesfield, Cheshire*

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Summary. The statistical predictions of a unified scheme for the radio emission from quasars are explored. This scheme attributes the observed differences between flat- and steep-spectrum quasars to projection and the effects of relativistic beaming of the emission from the nuclear components. We use a simple quasar model consisting of a compact relativistically beamed core with spectral index zero and unbeamed lobes, spectral index -1, to predict the proportion of flat-spectrum sources in flux-limited samples selected at different frequencies. In our model this fraction depends on the core Lorentz factor, γ, and we find that a value of ~ 5 gives satisfactory agreement with observation. In a similar way the model is used to construct the expected number/flux density counts for flat-spectrum quasars from the observed steep-spectrum counts. Again, good agreement with the observations is obtained if the average core Lorentz factor is about 5. Independent estimates of γ from observations of superluminal motion in quasars are of the same order of magnitude. We conclude that the statistical properties of quasars are entirely consistent with the predictions of simple relativistic-beam models.

1 Introduction

In Paper I (Browne *et al.* 1982) we presented evidence for the existence of extended steep-spectrum regions of emission (haloes) surrounding the bright compact cores of some flat-spectrum quasars mapped with the MTRLI. We argued that this diffuse emission could be identified with the superposed lobes of normal double sources and that the high brightness of the cores could be explained by relativistic beaming. We were thus led to consider the possibility that all core-dominated quasars are normal doubles whose axes are pointing close to the line-of-sight and, additionally, all normal double quasars are just unaligned core-dominated ones. Henceforth we will refer to this as the 'unified scheme'.

The rudiments of such a unifying scheme, involving Doppler boosting, have been discussed by Scheuer & Readhead (1979). In their scheme, flat-spectrum radio quasars and radio-quiet QSOs are one and the same thing. They propose that in the cores of QSOs there are radio-emitting regions moving at speeds approaching that of light. These, when travelling close to the line-of-sight, would show all the properties of flat-spectrum quasars. For other alignments the core radio emission would be very weak and the QSO classified as radio quiet. However, the common occurrence of low-brightness emission surrounding flat-spectrum quasars (Paper I; Perley, Fomalont & Johnston 1980) indicates that the simple Scheuer & Readhead picture is not viable. The unaligned QSOs would not be radio quiet; they should have more than enough flux density to be detected in searches for radio emission from optically selected QSOs whatever their spectral indices.

The scheme we consider attempts to unify flat-spectrum with steep-spectrum quasars and not flat-spectrum quasars with radio-quiet ones. In this respect we disagree with Scheuer & Readhead (1979) who deduced, mistakenly we believe, that the core Lorentz factors in steep-spectrum quasars are low ($\gamma < 2$) and thus cannot be unaligned core-dominated objects. As we discuss in more detail in Section 2.1, their estimate was based on a sample of quasars which was incomplete in a way that would lead them to underestimate the value of γ. In fact we shall show that high values of γ (~ 5) are needed for consistency with observation. We make use of the demand of the unified scheme that the statistical properties of samples containing quasars of predominantly the flat-spectrum type or the steep-spectrum type are not independent, because both types share the same intrinsic (i.e. beaming independent) properties. In Section 2 a model of an ideal quasar consistent with the scheme is developed. In Section 3 we use the model to predict the fraction of flat-spectrum objects in samples of quasars selected at different frequencies and with different flux density cut-offs, and in Section 4 to predict the high-frequency number counts of flat-spectrum quasars. We deduce the value of γ which gives the best agreement between the predictions and the observations.

2 The canonical quasar

In this section a canonical model of a quasar, based on the unified scheme, is developed. We restrict attention just to quasars in order to simplify the model, as observational evidence suggests that quasars form a statistically distinct group separate from the rest of the radio source population, certainly so far as their core strengths are concerned. For example, most flat-spectrum sources are quasars while most steep-spectrum sources are galaxies or are unidentified, and in Cambridge 5-km maps of 3C sources (Jenkins, Pooley & Riley 1977, and references therein) the proportion of sources with a detected compact core (CC) is higher for quasars than for others.

2.1 THE PARAMETER *R*

Following Hine & Scheuer (1980) we define the parameters

$$R(\theta) = \frac{\text{flux density of beamed CC}}{\text{flux density of unbeamed components}}$$

$$R_{\mathrm{T}} = R(90^\circ),$$

where θ is the angle between the line-of-sight and the direction of motion of the approaching side of the CC. The CC is assumed to be produced by emission from the unresolved bases of two oppositely directed jets, each of which is composed of a quasi-continuous stream of material which has a finite emitting lifetime and which moves with speed βc (Lorentz factor γ) relative to the central optical object. The unbeamed components are the lobes of normal doubles, or, when these are superposed, the diffuse emission in core-dominated sources. Under these conditions, and on the assumption that the jet spectra are flat ($S \propto \nu^{\alpha}$, $\alpha = 0$), the relation between R and its value at transverse alignment, R_T, is given by

$$R = \tfrac{1}{2}R_T(1-\beta\cos\theta)^{-2} + \tfrac{1}{2}R_T(1+\beta\cos\theta)^{-2}. \qquad (1)$$

This is a straightforward application of equation (1) of Scheuer & Readhead (1979) except that the contribution of the receding side of the CC (the second term on the right in equation 1 above) is also included.

In a randomly orientated sample of quasars all with the same values of R_T and γ, the distribution of R is given by

$$P(R)\,dR = d(\cos\theta) \qquad (2)$$

where, from equation (1),

$$\cos\theta = \frac{1}{\beta}\left\langle\frac{1}{2R}\left\{2R + R_T - [R_T(8R+R_T)]^{1/2}\right\}\right\rangle^{1/2} \qquad (3)$$

and the minimum and maximum values of R are R_T and $R_T\gamma^2(2\gamma^2-1)$ respectively. In order to assign a numerical value to R_T we have taken the 38 quasars from the 166-source sample of Jenkins *et al.* (1977) and compiled the distribution of R at 5 GHz. We compare the observed distribution with that expected from equations (2) and (3). The sources in the Jenkins *et al.* sample were selected at 178 MHz, and therefore almost entirely by the strength of their steep-spectrum emission, so the 38 quasars should be close to randomly orientated. Values of R were taken mostly from Cambridge 5-km maps (Jenkins *et al.* 1977, and references therein; Laing 1981) and K-corrected assuming $\alpha = 0$ for the CCs and taking steep-spectrum indices from Laing & Peacock (1980). For the quasars 3C 68.1, 196, 288.1 and 432 we can find only upper limits to the strengths of their central components. Six more compact objects: 3C 48, 138, 186, 286, 287 and 343 have not been mapped with sufficient resolution to be able to distinguish a central component unambiguously. These last six have been excluded from the distribution, which is shown in Fig. 1(a). We suspect that most of them have high values of R since objects similar in compactness and spectrum, like 3C 147, 309.1 and 380 do, and because Gopal-Krishna, Preuss & Schilizzi (1980) find that, on average, small-diameter sources have much more prominent cores than large-diameter sources.

The expected distribution for $\gamma = 5$ and $R_T = 0.024$ is shown in Fig. 1(b). We will justify the choice of $\gamma = 5$ later, though for our present purpose, to estimate the value of R_T, the choice is not important. Fig. 1(c) shows the Fig. 1(b) distribution smoothed to simulate the effect of an order of magnitude dispersion in R_T (the convolution function is also shown). The observed distribution is clearly very similar to the convolved one. The lower cut-off in R is determined by the value of R_T only (not γ) and thus R_T is relatively well constrained. This should be true even though six sources have been excluded from the observed distribution, provided that, as we suspect, these all have $R \gtrsim 0.1$. As the dispersion of our fitted value of R_T is much smaller than the total range of R, we shall for simplicity take R_T as a constant (= 0.024 at 5 GHz emitted).

The R-distribution is not a good way to determine γ. This is because the differences between the predicted distributions with the same R_T but different γ only become large for high R values where the number of expected objects is low. Also the sample we have used is incomplete for medium to large values of R because of the exclusion of the six compact objects. In fact, Scheuer & Readhead (1979) do attempt to use the spread in the R-distribution of 3CR quasars to put constraints on γ and conclude that $\gamma \lesssim 2$. We believe this result to be incorrect because they exclude strong core sources from the distribution, and it is just these which provide the useful constraints on γ. (A similar criticism applies to Hine &

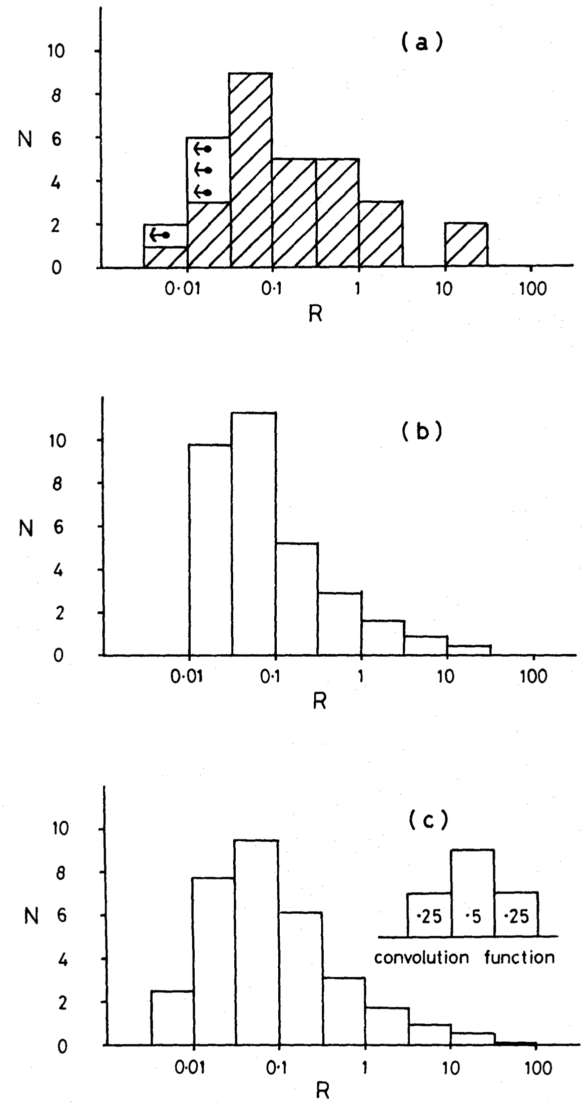

Figure 1. (a) The observed R distribution amongst 32 3CR quasars. (b) The predicted R distribution for $\gamma = 5$, $R_T = 0.024$ and $N = 32$. (c) The predicted distribution as in (b) but convolved with the function shown.

Scheuer 1980.) In Scheuer & Readhead's sample the spread in R is 64 : 1, whereas we know from the existence of sources like 3C 345 ($R \sim 20$) and 3C 454.3 ($R \sim 20$) and other similar core-dominated sources (Paper I) that the true spread in R is $\gtrsim 1000:1$. Such a large range of R requires $\gamma \gtrsim 5$ (the spread is $\sim 2\gamma^4 : 1$).

We next consider the variation of R and R_T with observing frequency ν MHz and redshift z. We shall assume that $\alpha = 0$ for the compact core over the range 178 MHz–5 GHz. For the steep-spectrum emission, the mean spectral index from 750 MHz–5 GHz of 3CR quasars (Laing & Peacock 1980) is $\alpha = -1.0 \pm 0.1$ (37 sources). Then, provided that $750 < \nu(\text{MHz}) < 5000$,

$$R_T(\nu, z) = 0.024 \frac{\nu}{5000} (1 + z). \tag{4a}$$

At frequencies much lower than 750 MHz a correction must be made to allow for spectral curvature of the steep-spectrum emission. This correction is about 15 per cent at 178 MHz and 6 per cent at 408 MHz. Thus, to the required accuracy, equation 4(a) remains valid at

408 MHz while at 178 MHz we have

$$R_T(178, z) = 0.0010(1+z). \tag{4b}$$

Hereafter we denote the observed value of R at ν MHz by R_ν. This is then given by equation (1) by replacing R by R_ν and R_T by $R_T(\nu, z)$. $R_\nu \propto \nu$ over the range 408 MHz–5 GHz, while $R_\nu = R_{178}(\nu/209)$.

2.2 THE RELATION BETWEEN OVERALL SPECTRUM AND R_ν

The overall spectrum of our canonical quasar is assumed to be the sum of contributions from the CC and unbeamed components only. Although it is true that some core-dominated quasars have large-scale (a few arcsec) jets, their contribution to the spectrum can usually be ignored. We can justify doing this by considering two possibilities. The first is that the large-scale jet is not highly relativistic and that any Doppler boosting is small compared to that of the core. In this case the observation that in quasars with normal double structure (face-on quasars) the jets are usually weaker than the cores means that, when the cores are seen end-on, the jet contribution will be negligible. The second possibility is that the jets are relativistic, having speeds as large as, but not greater than, the core material. This we regard as by far the most likely situation, as jets are in fact observed in core-dominated sources and, if the unified scheme is correct, these must be the Doppler-boosted counterparts of those weak jets seen in some face-on sources. However, in the core-dominated sources the jets are weaker than the cores even at 408 MHz (Paper I). So at higher frequencies and all orientations it is clear that the jet flux will always be dominated by that from the core, thus making a negligible contribution to the overall spectrum.

We classify the overall spectrum as either flat or steep depending on whether the spectral index, $\alpha_{\nu_1}^{\nu_2}$, is > or < -0.5. The condition on $\alpha_{\nu_1}^{\nu_2}$ can be re-expressed as a condition on R, since if S_{ν_1} and S_{ν_2} are the total fluxes at frequencies ν_1 and ν_2 respectively, then

$$\frac{S_{\nu_2}}{S_{\nu_1}} = \frac{(\nu_1/\nu_2) + R_{\nu_1}}{1 + R_{\nu_1}}.$$

Substitution of this equation into the definition of $\alpha_{\nu_1}^{\nu_2}$ leads to

$$R_{\nu_1} > (\nu_1/\nu_2)^{1/2}$$

as the condition for a flat overall spectrum. (If one of the frequencies is 178 MHz then a slight adjustment is required to account for the low-frequency curvature of the steep-spectrum emission.)

This completes the model building necessary for the statistical predictions we will make in the next two sections. Our canonical quasar can be described by the following information:

(a) the intrinsic properties represented by the values of R_T and the spectral indices of the CC and the unbeamed components,

(b) the extrinsic (i.e. beaming-dependent) properties as expressed by equation (1), and

(c) an equation connecting the model parameter R and a relatively easily obtained observable, the spectral index α.

The only unknown parameters are γ and θ which both appear in equation (1); since the statistical behaviour of θ is known ($\cos\theta$ is intrinsically uniformly distributed), this leaves only the value of γ to determine the predictions.

3 Statistical properties of flux limited samples of quasars

3.1 THE DISTRIBUTION OF R AND THE FRACTION OF QUASARS WITH FLAT SPECTRA

A sample of quasars selected at frequency ν above a flux density limit S_0 will favour sources in which Doppler boosting of the core flux occurs and so include an excess of sources with small values of θ. This bias will increase with $R_T(\nu, z)$, i.e. with increasing frequency ν, and redshift z. Below we use the canonical quasar model to predict the distribution of R and the fraction F of flat-spectrum quasars in several samples. By comparing the predicted and observed values of F we derive estimates of γ.

If a quasar with total flux $> S_0$ has some specified value of R_ν then the unbeamed flux (the steep-spectrum components alone) is given by

$$S_{\text{steep}} > \frac{S_0}{1+R_\nu}. \tag{5}$$

Therefore the number of quasars with R_ν in the range R_ν to $R_\nu + dR_\nu$ is proportional to the product of the probability of the appropriate beam-orientation (to give R_ν) with the number of quasars with unbeamed flux greater than $S_0/(1+R_\nu)$. The latter is given by the number counts for 'unbeamed quasars' which we take to be

$$N(>S_{\text{steep}}) \propto S_{\text{steep}}^{-\delta}. \tag{6}$$

Combining equations (5) and (6) and using (2), (3) and (4) for the orientation probability P, the distribution of R is given by

$$N(R_\nu) \propto \left(\frac{S_0}{1+R_\nu}\right)^{-\delta} \int_0^{z^{\max}} \rho(z) P(R_\nu) dz \tag{7}$$

where $\rho(z)$, the redshift distribution of unbeamed quasars, has been assumed to be independent of flux density and vanishes for $z > z^{\max}$. The observed redshift histogram in a sample of quasars selected at low frequency should be a good approximation to $\rho(z)$. R_ν ranges from $R_T(\nu, 0)$ up to $R_T(\nu, z^{\max})\gamma^2(2\gamma^2-1) = R_\nu^{\max}$. The distribution given by equation (7), when normalized by dividing by the integral over all R, gives the fraction of quasars in the sample with flat spectra as

$$F = \int_{R_\nu^{\min}}^{R_\nu^{\max}} N(R_\nu) dR_\nu \bigg/ \int_{R_T(\nu,0)}^{R_\nu^{\max}} N(R_\nu) dR_\nu, \tag{8}$$

where $R_\nu^{\min}$ is the minimum value of R_ν needed for a flat overall spectrum, and depends on the definition of 'flat' (see Section 2.2).

We estimate δ by the slope of the quasar counts at the lowest available frequency (i.e. 408 MHz) and at a flux density corresponding to S_0 (i.e. $(\nu/408)S_0$). The 408-MHz quasar counts, which we discuss further in Section 4.1, are a good approximation to the counts for unbeamed quasars, since the CCs are still intrinsically weak at this low frequency. We take $\rho(z)$ from Fig. 11 of Wills & Lynds (1978) for their sample of 4C quasars (selected at 178 MHz).

The only unknown parameter in equation (8), determining F, is γ. In Table 1 we list the value of γ which gives the best fit between the predicted and observed values of F in seven different samples of quasars. The columns in the table are:

1. The parent survey name and reference number
2. The selection frequency ν MHz

Table 1. The percentages of quasars with flat spectra in surveys made at different frequencies. γ is the CC Lorentz factor required to predict the observed percentages.

(1) Sample	(2) ν	(3) S_0	(4) δ	(5) R_T	(6) Flat spectrum condition	(7) R_ν^{min}	(8) R_{5000}^{min}	(9) F	(10) γ
3C (1)	178	10	1.6	0.0010	$\alpha_{750}^{5000} > -0.5$	0.09	2.58	8 ± 4	3.2 ± 0.9
Wills & Lynds (2)	178	2/3	1.5	0.0010	$\alpha_{178}^{2695} > -0.5$	0.21	5.76	17 ± 6	5.8 ± 0.8
B2 (3)	408	0.9	1.4	0.0020	$\alpha_{408}^{5000} > -0.5$	0.29	3.50	18 ± 5	4.9 ± 0.7
Jodrell Bank (4)	996	1.0	1.4	0.0046	$\alpha_{966}^{2695} > -0.5$	0.60	3.10	21 ± 4	3.7 ± 0.3
Parkes (5)	2695	0.35	1.4	0.013	$\alpha(1\,\mathrm{GHz}) > -0.5$	2.70	5.00	60 ± 8	5.2 ± 0.5
Kühr *et al.* (6)	5000	1.0	1.4	0.024	$\alpha_{2695}^{5000} > -0.5$	1.36	1.36	82 ± 6	5.3 ± 0.3
NRAO Deep Survey (7)	5000	0.1	1.1	0.024	$\alpha_{2695}^{8085} > -0.5$	1.07	1.07	56 ± 18	4.5 ± 1.1

References for samples

1. Hewitt & Burbidge 1980; Laing & Peacock 1980; Kellermann, Pauliny-Toth & Williams 1969.
2. Wills & Lynds 1978, combination of their subsamples 1–4.
3. Grueff & Vigotti 1973.
4. Cohen *et al.* 1977; Porcas *et al.* 1980; Moore *et al.* 1981.
5. Bentley *et al.* 1976.
6. Kühr *et al.* 1981.
7. Condon, Balonek & Jauncey 1975.

3. The flux density limit
4. The estimated value of δ
5. The value of $R_T(\nu, 0)$
6. The definition of flat spectrum
7. The corresponding definition in terms of R_ν
8. The corresponding definition in terms of R_{5000}
9. The percentage of quasars in the parent sample which have flat spectra, errors proportional to the square root of the actual number found
10. The best-fitting value of γ with errors corresponding to the errors in (9).

It is clear from Table 1 that the best-fitting values of γ for the various surveys are similar. We find the weighted mean Lorentz factor to be 4.7, though it should be noted that not all the surveys are statistically independent.

Fig. 2 displays the expected distribution of R_{5000} for five of the samples in Table 1 using $\gamma = 4.7$. This diagram illustrates the way in which most flat-spectrum quasars selected at high frequencies have values of R well in excess of that required for a flat spectrum ($R_{5000} \sim 3$) and in this sense are truly core-dominated ($R_{5000} \sim 40$). This is in agreement with the observations of Paper I where only one out of nine flat-spectrum sources (4C 39.25) had a core flux which was less than that of the diffuse emission at 408 MHz.

3.2 THE DISTRIBUTION OF REDSHIFTS

The differential form of equation (7) is

$$N(R_\nu, z) = \frac{S_0}{(1 + R_\nu)} P(R_\nu)\rho(z), \tag{7a}$$

so that the fraction of flat-spectrum quasars as a function of redshift, $F(z)$, is given by

$$F(z) = \int_{R_\nu^{min}}^{R_\nu^{max}} N(R_\nu, z)\, dR_\nu \Bigg/ \int_{R_T(\nu,0)}^{R_\nu^{max}} N(R_\nu, z)\, dR_\nu . \tag{8a}$$

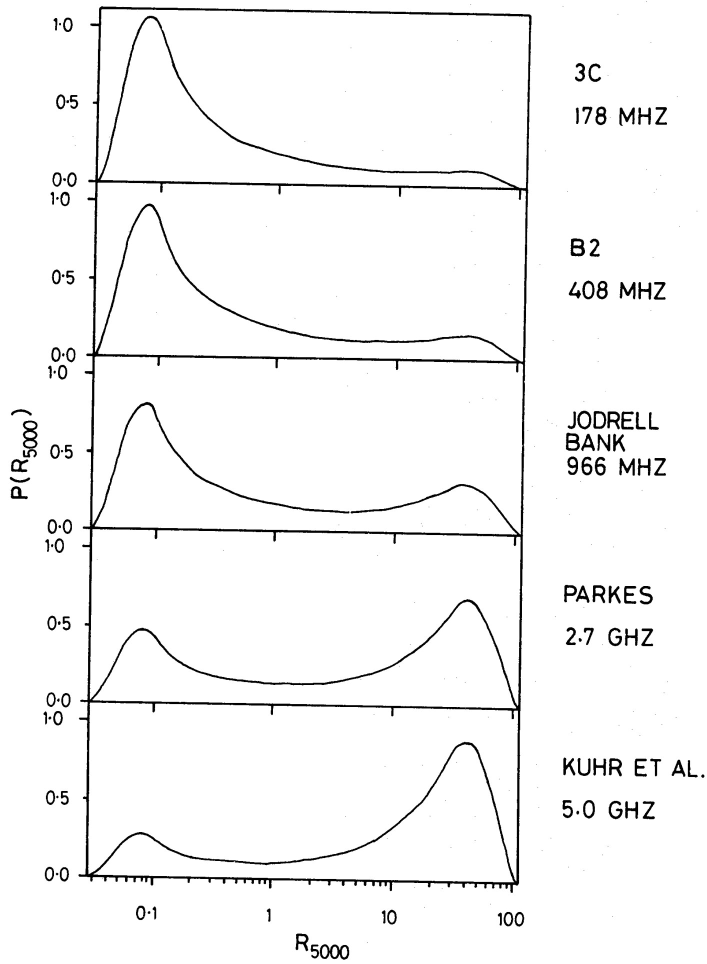

Figure 2. The expected probability distribution of R_{5000} for quasars selected from surveys done at different frequencies. $\gamma = 4.7$ and $R_T = 0.024$. The probability is in units of per unit logarithmic interval of R_{5000}.

Because quasars with high redshift have relatively large values of R_T, more high-, than low-redshift quasars, will receive sufficient Doppler-boosting to be converted into flat-spectrum sources. There will therefore be a slight excess of high-redshift objects in flux-limited samples selected at high frequencies compared to those at low ones. In addition, as expressed in equation (8), the fraction of flat-spectrum quasars will be a function of z.

Ulvestad *et al.* (1981) have suggested that the excess they find of unresolved sources amongst high-redshift quasars is a consequence of relativistic beaming. We can quantify this effect by calculating $F(z)$ for their quasars, taking $\gamma = 4.7$ and using their criterion for an

Table 2. The observed and predicted fraction of unresolved quasars as a function of redshift in the sample of Ulvestad *et al.* (1981).

$\log z$	Fraction unresolved (per cent)	Fraction predicted $R_{5000} > 9$ (per cent)
< -0.5	58 ± 22	38
-0.5 – -0.3	64 ± 21	46
-0.3 – -0.1	45 ± 12	53
-0.1 – 0.1	59 ± 12	61
0.1 – 0.3	78 ± 18	70
> 0.3	79 ± 20	77

unresolved source, $R_{5000} > 9$. In Table 2 we compare our predictions of $F(z)$ with their observed values. Clearly their results are consistent with the predictions of the unified scheme.

4 The log N/log S relations for quasars

Several recent papers (Condon & Ledden 1982; Kellermann & Pauliny-Toth 1981; Wall, Pearson & Longair 1981) have discussed the form of the source counts at high frequencies, and particularly the counts of flat-spectrum objects. The unified scheme implies that the steep- and flat-spectrum counts are closely related, and in this section we make use of the observed quasar counts at low frequencies, assuming them to be a close approximation to the unbeamed counts, to predict the log$N$/log$S$ distribution for flat-spectrum quasars. Direct consideration of the effects of cosmological evolution of source properties is unnecessary as these are implicit in the starting information, namely the low-frequency counts. To obtain the quasar counts, samples of radio sources are required for which the quasar identifications are complete. Fortunately the available evidence suggests that most radio quasars, even those with very low flux densities have apparent magnitudes significantly brighter than $m_V \sim 20$ (Peacock *et al.* 1981; Perryman 1979a, b; de Ruiter, Willis & Arp 1977). We will therefore assume that, for the samples which have been studied, the quasar identifications are complete.

4.1 QUASAR COUNTS

To determine the log$N$/log$S$ counts for steep-spectrum quasars we started with the overall 408-MHz counts given by Wall, Pearson & Longair (1980) and then estimated the fraction of these sources which are quasars from published 408-MHz identifications of samples with a wide range of flux density limits. These samples are, in order of decreasing flux density limit: the all-sky survey (Robertson 1973), part of the B2 sample identified by Grueff & Vigotti (1973), part of the Molonglo survey identified by Hazard & Murdoch (1977) and the 5C6 and 5C7 surveys identified by Perrymann (1979a, b). Table 3 lists the percentages of these sources which are identified with quasars.

The lack of available samples with flux density limits between 10 and 450 mJy presents us with a problem. What we have done for our calculations is to adopt the following simplified variation of the fraction of sources which are quasars: below 10 mJy a constant 5 per cent, between 10 and 1 Jy a fraction increasing linearly with log$S$ from 5 to 20 per cent, and above 1 Jy a constant 20 per cent. This simplification is not in fact critical to the final outcome of the predictions, since large changes to the form of the variation, especially at low flux densities, only produce relatively small changes in the expected flat-spectrum

Table 3. Quasar identification percentages in 408-MHz surveys.

Reference/Survey	Flux limit (mJy)	per cent QSOs
Perryman (1979a, b)		
5C6/5C7	10	~4
Hazard & Murdoch (1977)		
MC2/MC3	450	21 ± 3
Grueff & Vigotti (1973)		
B2	900	21 ± 2
Robertson (1973)		
All sky	10 000	19 ± 4

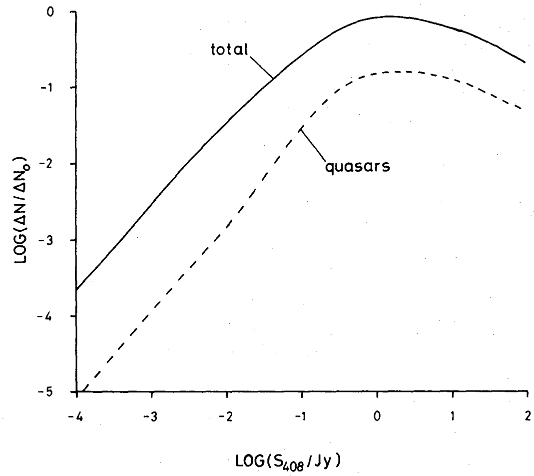

Figure 3. Differential 408-MHz source counts. The total count (continuous line) is taken from Wall *et al.* (1980). The dashed line shows the quasar counts used in Section 4. The Euclidean normalization is $N_0 = 1200\,S^{-1.5}$.

quasar counts. The quasar counts derived in this way from the overall counts (taken from Wall *et al.* 1980) are shown in Fig. 3. The overall counts have been extrapolated below 10 to 0.1 mJy by using model 5 of Wall *et al.*

At high frequencies we can combine information from 2.7 and 5 GHz to obtain the counts of flat-spectrum quasars since the $\log N/\log S$ curves at the two frequencies are the same within the errors (Peacock & Gull 1981). The information used is summarized in Table 4, which lists estimates of the normalized flat-spectrum quasar counts $\Delta N_{\mathrm{FQ}}/\Delta N_0$ derived from various identified samples. The flux density at which each estimate applies, S_{eff}, is the average flux density of the quasars in the sample from which the estimate is derived.

Table 4. Estimates of the $\log N$–$\log S$ relation for flat-spectrum quasars normalized to the expected Euclidean counts. The columns are (1) reference to identification of sample, (2) original selection frequency, (3) effective flux density of sample (see text), (4) estimate of $\Delta N_{\mathrm{FQ}}/\Delta N_0$ (where $N_0 = 60 S^{-1.5}$ sr^{-1}); errors are proportional to the square root of the number of FQs.

Reference	Selection frequency (GHz)	S_{eff} (Jy)	$\Delta N_{\mathrm{FQ}}/\Delta N_0$
Condon & Ledden 1982	5.0	0.041	0.017 ± 0.08
Condon, Balonek & Jauncey 1976	2.7	0.16	0.15 ± 0.05
Condon, Balonek & Jauncey 1975	5.0	0.16	0.19 ± 0.05
Condon, Jauncey & Wright 1978	2.7	0.45	0.28 ± 0.06
Condon *et al.* 1978	2.7	0.89	0.39 ± 0.07
Kühr *et al.* 1981	5.0	1.4	0.39 ± 0.03
Kühr *et al.* 1981	5.0	2.7	0.36 ± 0.05
Kühr *et al.* 1981	5.0	9.2	0.20 ± 0.05

4.2 THE MODEL PREDICTIONS

A quasar with (unbeamed) flux density S_{408} has unbeamed flux density $(408/\nu)\,S_{408}$ at ν MHz and therefore total flux density

$$S_\nu = S_{408}(408/\nu)(1 + R_\nu).$$

Then the high-frequency counts are given by

$$\Delta N_{FQ}(S_\nu) = \int_0^{z_{max}} \rho(z) \int_{R_\nu^{min}}^{R_\nu^{max}} \Delta N_Q \left[S_\nu \frac{\nu}{408}(1+R)^{-1}\right] P(R)\,dR\,dz, \tag{9}$$

where

$\Delta N_{FQ}(S_\nu)$ is the number of flat-spectrum quasars per steradian in a small logarithmic interval of flux density centred on S_ν,
$\Delta N_Q(S_{408})$ is the number of quasars per steradian at 408 MHz in the same small logarithmic interval centred on S_{408},
$\rho(z)$ is the distribution of redshifts,
$R_\nu^{max} = R_T(\nu, z)\gamma^2(2\gamma^2 - 1)$ and corresponds to $\theta = 0$,
R_ν^{min} is the minimum value of R consistent with a flat overall spectrum (Section 2.2) defined by $\alpha_{2700}^{5000} > -0.5$,
$R_T(\nu, z)$ is given by equation (4a),
$P(R)\,dR$ is given by equations (2) and (3).

A computer program was written to solve for $\Delta N_{FQ}(S_\nu)$ numerically for trial values of the only unknown parameter γ. Fig. 4 displays three solutions, normalized in the usual way to the static Euclidean counts, for $\gamma = 4$, 5 and 6, and $\nu = 2.7$ GHz. (The corresponding curves for $\nu = 5$ GHz are not significantly different.) Also shown are the observed values of $\Delta N_{FQ}/\Delta N_0$ from Table 4. Clearly, $\gamma \sim 5$, as found in Section 3, produces roughly the correct flat-spectrum quasar counts. The failure to fit in detail probably indicates that we are running into the limitations of the data (especially the uncertain 408-MHz quasar counts) and the limitations of the model. The model could be made more realistic by allowing for a dispersion in component spectral indices and in the value of R_T. No doubt such modifications could produce a better fit, but at this stage we feel that no useful purpose would be served by introducing such complications.

In contrast, inclusion of a dispersion in the value of γ is a realtively trivial modification of equation (9), simply requiring a third integration over a normalized γ-distribution, $G(\gamma)d\gamma$. Some trials were made with simple forms of $G(\gamma)$. Although not exhaustive, these showed that approximately the same solution $\Delta N_{FQ}(S_\nu)$ was obtained for a constant value of γ as for a distribution $G(\gamma)$ with the same mean. This is important because it means that the value of γ we determine from solving equation (9) can be regarded as the average of an unknown distribution.

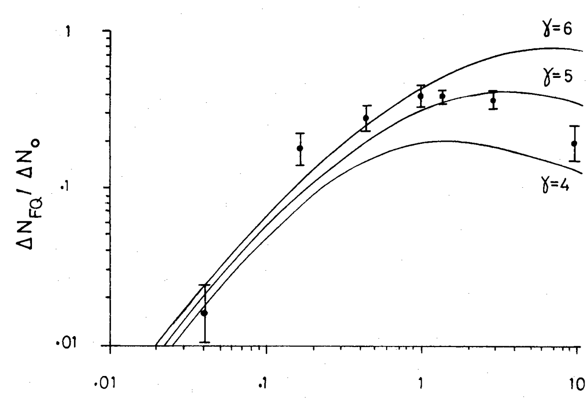

Figure 4. The observed and predicted number counts of flat-spectrum ($\alpha_{2700}^{5000} > -0.5$) quasars. The continuous lines show model predictions for $\gamma = 4$, 5 and 6, at 2.7 GHz. The 5.0-GHz predictions are very similar. The Euclidean normalization is $N_0 = 60\,S^{-1.5}$.

Table 5. Minimum Lorentz factors required to explain superluminal expansions on the basis of relativistic beam models. The Hubble constant has been taken as $100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

Source	γ_{min}	Reference
NRA0 140	5	Marscher & Broderick (1982)
3C 120*	3	Walker *et al.* (1982)
3C 179	4	Porcas (1981)
3C 273	5	Pearson *et al.* (1981)
3C 279	10	Cohen & Unwin (1982)
3C 345	8	Cohen & Unwin (1982)

* 3C 120 is a radio galaxy.

5 Discussion and conclusions

There are two features of our predictions which we wish to emphasize. The first is that, by using a simple model with a value for R_T deduced from the properties of steep-spectrum sources, we predict the right relative numbers of flat- and steep-spectrum quasars for a mean Lorentz factor of ~ 5. Such a value is consistent with the independent limits derived from the VLBI observations of superluminal motions (Cohen & Unwin 1982). Table 5 summarizes the available observations and the implied limits to γ. These are all within a factor of 2 of γ = 5, provided that the Hubble constant is ~ $100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

The second feature is that for the same value of γ which gives the correct fractions of flat-spectrum quasars, the source-count predictions have the correct overall shape; the declining number of flat-spectrum quasars at low flux densities is reproduced (Condon & Ledden 1982), so too is the rise in the slope of the counts above the Euclidean value at high flux densities, consistent with the results of Peacock *et al.* (1981). They find that both flat- and steep-spectrum quasars undergo similar degrees of cosmological evolution, which is exactly what is required by the unified scheme.

We also wish to discuss the place of optically selected quasars in the unified scheme. Contrary to the suggestion of Scheuer & Readhead (1979), optically selected quasars with very weak or undetectable radio emission are not simply the unaligned counterparts of strong flat-spectrum sources. In our picture, weak quasars, just like strong quasars, have a wide distribution of alignments; those whose axes point towards the observer have flat spectra, while those which do not, have steep. The success of the source-count predictions down to 15 mJy suggests that, at least to this flux density level, the scheme works. What is less clear is whether or not all optically selected quasars (i.e. those with $S_{5\,\mathrm{GHz}} < 15$ mJy) can also fit into the scheme. This is beyond the scope of the present paper, but recent evidence seems to indicate that radio-selected quasars are a subset of the optically selected ones, not a disjoint population (Condon & Ledden 1982).

If the selection effects present in the currently available samples of optical quasars were unimportant, it would be possible to use the radio observations of these to extend the quasar counts to flux densities as low as ~ 1 mJy. Unfortunately, there is clear evidence that such selection effects cannot be neglected. For example, if the spectral index distributions for samples of weak radio-selected quasars and for samples of optically selected quasars found to have similar radio flux densities are compared, striking differences are found. Condon *et al.* (1981) find that of 12 optically selected quasars which have been detected to have $S_{5\,\mathrm{GHz}} >$ 10 mJy only one has a steep radio spectrum. In contrast Condon & Ledden (1982) and Wall (private communication) find that in radio-selected samples to similar flux densities, about

60 per cent of quasars have steep spectra. The fact that the optical sample is incomplete, having very few objects with $z < 1.8$ and missing some faint objects, may account for the difference. The unified scheme, in fact, supplies at least a partial explanation of why this should be so. In Section 3.2 it was shown that, amongst high-redshift objects, a greater proportion of high-R quasars are expected and, as most of the optically selected quasars in Condon *et al.*'s sample have $z > 1.8$, more of them should have flat spectra than the missing low-redshift ones. Such an explanation is consistent with the finding by Shaffer, Green & Schmidt (1982) that, in an optically selected sample free from significant redshift bias, the *PG* quasars (Green 1976), many have extended structure indicating the presence of steep-spectrum emission.

Finally we summarize some of the arguments in favour of the unified scheme.

1. Relativistic motion in core-dominated sources explains superluminal motion and the asymmetry of structure on the VLBI scale.

2. The diffuse and presumably unbeamed emission seen in superluminal and other core-dominated sources is explained as the lobe emission of the normal double.

3. Radio variability is more pronounced in flat-spectrum sources with high values of R (Moore *et al.* 1981). These are the ones with the largest Doppler-enhancements and the time-scale for variability will be shortest in these.

4. The statistics of flat- and steep-spectrum quasars behave in the way consistent with the unified scheme. Specifically the scheme predicts:

(a) the correct form of the flat-spectrum quasar source counts.
(b) the correct proportions of flat- and steep-spectrum quasars in samples of radio sources selected at different frequencies
(c) the higher proportion of compact flat-spectrum quasars at high redshift.

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