Astronomy Letters, Vol. 31, No. 5, 2005, pp. 340–349. Translated from Pis'ma v Astronomicheskiĭ Zhurnal, Vol. 31, No. 5, 2005, pp. 378–387.

Relativistic Effects and Solar Oblateness from Radar Observations of Planets and Spacecraft

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Received September 3, 2004

Abstract—We used more than 250 000 high-precision American and Russian radar observations of the inner planets and spacecraft obtained in the period 1961–2003 to test the relativistic parameters and to estimate the solar oblateness. Our analysis of the observations was based on the EPM ephemerides of the Institute of Applied Astronomy, Russian Academy of Sciences, constructed by the simultaneous numerical integration of the equations of motion for the nine major planets, the Sun, and the Moon in the post-Newtonian approximation. The gravitational noise introduced by asteroids into the orbits of the inner planets was reduced significantly by including 301 large asteroids and the perturbations from the massive ring of small asteroids in the simultaneous integration of the equations of motion. Since the post-Newtonian parameters and the solar oblateness produce various secular and periodic effects in the orbital elements of all planets, these were estimated from the simultaneous solution: the post-Newtonian parameters are $\beta = 1.0000 \pm 0.0001$ and $\gamma = 0.9999 \pm 0.0002$, the gravitational quadrupole moment of the Sun is $J_2 = (1.9 \pm 0.3) \times 10^{-7}$, and the variation of the gravitational constant is $\dot{G}/G = (-2 \pm 5) \times 10^{-14}$ yr^{-1}. The results obtained show a remarkable correspondence of the planetary motions and the propagation of light to General Relativity and narrow significantly the range of possible values for alternative theories of gravitation.

Key words: *celestial mechanics, cosmology, Sun.*

INTRODUCTION

Radar observations of planets began in 1961 and have been widely used in astronomical practice ever since. High-precision radar measurements spanning a time interval of more than forty years allow not only the orbital elements of the planets, but also other constants of the planetary theory, including the relativistic parameters, to be determined with a high accuracy.

Of the three main tests of General Relativity in the Solar system (the secular motions of the planetary perihelia, the signal delay, and the deflection of light in a gravitational field), the first two tests have been performed using radar observations of planets and spacecraft.

The main and best determined relativistic effect in the Solar system is the secular motion of Mercury's perihelion that was discovered by Le Verrier in 1859. For him, this was a major problem of the discrepancy between theoretical predictions and observations, and it was explained in 1915 by Einstein's theory of General Relativity. However, the secular motion of Mercury's perihelion is known to depend on a linear combination of the post-Newtonian parameters (β, γ) and the gravitational quadrupole moment of the Sun (J_2). Papers (see, e.g., Pireaux and Rozelot 2003) arguing that only this combination rather than the three parameters themselves could be determined from current observations have appeared in recent years. However, the post-Newtonian parameters and the solar oblateness cause different secular and periodic perturbations both for different orbital elements (and not just for the perihelia) and for different planets. In addition, the parameter γ can also be determined from Shapiro's effect, which allows all three parameters to be estimated. Since these parameters can in most cases be obtained by analyzing the secular variations of orbital elements, the errors of their determination decrease with increasing time interval of observations. At the same time, the errors in the secular variation of the gravitational constant ($\dot{G}/G$), one of the most interesting parameters, decrease even faster: as the square of the time interval. This allows $\dot{G}/G$ to be estimated, thereby basically verifying the strong equivalence principle, since many theories of gravitation predict a variation of the locally measured Newtonian gravitational constant with time on the evolutionary scale of the Universe.

Some of the recent post-Newtonian-parameter

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determinations, e.g., $\gamma = 1.000021 \pm 0.000023$, from Cassini radar observations (Bertotti *et al.* 2003) reach a high accuracy. However, the improvement in quality and the increase in the number of current radar observations of planets and spacecraft as well as the increase in the time interval of observations have allowed not only γ, but also β, $\dot{G}/G$, and the gravitational quadrupole moment of the Sun to be estimated independently and from other data.

THE METHOD, EPM EPHEMERIDES

We used the following method to calculate the relativistic parameters and the solar oblateness. First, we constructed a numerical theory for the motion of the planets and the Moon, EPM2004—Ephemerides of Planets and the Moon (Pitjeva 2004, 2005), by using more than 317 000 observations (1913–2003) of various types. These included radiometric measurements of planets and spacecraft, astrometric CCD observations of the outer planets and their satellites, and meridian and photographic observations. Apart from the planetary ephemerides, we also constructed the ephemerides of the orbital and rotational motion of the Moon that were improved by processing the 1970–2003 LLR observations (Krasinsky 2002). The ephemerides of the planets and the Moon were constructed by the simultaneous numerical integration of the equations of motion for all planets, the Sun, the Moon, 301 largest asteroids, rotation of the Earth and the Moon, including the perturbations from the solar oblateness and the asteroid ring that lies in the plane of the ecliptic and consists of the remaining smaller asteroids. The equations of motion for bodies were taken in the post-Newtonian approximation in the Schwarzschild gravitational field described by a three-parameter (α, β, γ) metric in a harmonic coordinate system with $\alpha = 0$; all versions of the ephemerides were constructed for General Relativity: $\beta = \gamma = 1$. The general equations of motion for bodies in a nonrotating barycentric coordinate system are

$$\ddot{\mathbf{r}}_i = A + B + C + D,$$

where A are the Newtonian gravitational accelerations, B are the relativistic terms (Newhall *et al.* 1983), C are the terms attributable to the solar oblateness, and D are the terms attributable to the asteroid ring (Krasinsky *et al.* 2002).

Below, we provide brief information about the EPM2004 theory and its construction (Pitjeva 2005).

First, a physical model that includes all of the significant factors and that adequately reflects the actual planetary motions underlies this theory. In particular, including the perturbations from the several largest asteroids, as was done in previous versions of our EPM or Jet Propulsion Laboratory (JPL) DE ephemerides, was shown (Krasinsky *et al.* 2001; Standish and Fienga 2002) to be insufficient. In EPM2004, the gravitational perturbations that are introduced into the orbits of the inner planets by asteroids and that make it difficult to determine the parameters were reduced significantly by including 301 large asteroids and the perturbations from the massive ring of small asteroids in the simultaneous integration of the equations of motion and by estimating their masses when processing the observations.

Second, the accuracy of the numerical integration itself was checked by comparing the results of the forth and back integrations on a hundred-year time interval. The emerging errors were at least an order of magnitude smaller than the observational errors. Thus, the accuracy of the ephemerides is determined mainly by the accuracy of the observations and their reductions.

Third, producing the ephemerides is an iterative process of comparing the constructed ephemerides with observations, improving the parameters by the least-squares method (LSM), introducing these in the theory, and constructing a new version of the ephemerides.

In the main improvement of the planetary part of the EPM2004 ephemerides, we determined about 200 parameters: the orbital elements of all planets and the 13 satellites of the outer planets the observations of which were used to improve the orbits of these planets; the astronomical unit in kilometers; three orientation angles of the ephemerides relative to the International Celestial Reference Frame (ICRF); the rotation parameters of Mars (two orientation angles of the equator of Mars relative to its orbit and their secular variations, the velocity, and eight coefficients of the seasonal rotation terms of the Martian axis) and the coordinates of three landers on the Martian surface; the masses of the bodies (Jupiter and the six asteroids that perturb Mars most strongly), the mean densities for three taxonomic classes of asteroids (C, S, M), the mass and radius of the asteroid ring, the ratio of the Earth's and Moon's masses; the gravitational quadrupole moment of the Sun (J_2) and twelve parameters of the solar corona for different conjunctions with the Sun; eight coefficients of Mercury's topography and the corrections to the level surfaces of Venus and Mars relative to which the topographies of these planets were calculated; five parameters for calculating the additional phase effect in the optical observations of the outer planets; and the constant shifts for six groups of observations that were interpreted as systematic errors or calibration errors of the instrumentation.

Once the EPM2004 ephemerides were constructed from all radar observations of the inner planets, spacecraft passing by or orbiting these planets, and

Martian landers, we improved the parameters, including the relativistic ones (β, γ, $\dot{G}/G$, the secular motions of the planetary perihelia), by the LSM.

The partial derivatives of the observed quantities (e.g., the delay time τ) with respect to the parameters being improved, $\frac{\partial\tau(t)}{\partial q(t_0)}$, must be known to determine the parameters of the theory by the LSM. In this case, the expression $\frac{\partial\tau(t)}{\partial s(t)}\frac{\partial s(t)}{\partial q(t_0)}$ is commonly used, since calculating the derivatives of the observed quantities with respect to the coordinates and velocities or orbital elements of the object $s(t)$ using the analytical formulas of the two-body problem involves no difficulty. At the same time, the derivatives $\frac{\partial s(t)}{\partial q(t_0)}$ are calculated either analytically or by integrating the variational equations. The derivative $\frac{\partial\tau(t)}{\partial\dot{G}/G}$ is calculated via the partial derivatives of τ with respect to the differences and sums of the mean longitudes of the observed object and the Earth. We derived expressions for the partial derivatives of the orbital elements with respect to the post-Newtonian parameters (β and γ) using the analytical formulas for the relativistic perturbations of the elements, including the secular and principal periodic terms, given in the monograph by Brumberg (1972). The derivative $\frac{\partial\tau(t)}{\partial\gamma}$ calculated from Shapiro's effect should also be added for γ.

Thus, improving the parameters can be reduced to the following:

(1) Numerical integration of the equations of motion for the planets and some of the partial derivatives;

(2) Computing the model observations (time delays) from the produced ephemerides for each time of observations, calculating the residuals and the required partial derivatives;

(3) Obtaining the values of the parameters being determined and deriving the residuals of the observations after the improvement.

As experience shows, the formal accuracy of determining the parameters by the LSM is overly optimistic. The actual accuracy could be an order of magnitude lower due to the deviation of the distribution of observations from a Gaussian law and due to the systematic errors in the observations, often of an unknown nature. The actual accuracies of the parameters given below were estimated by comparing the values obtained in dozens of different test LSM solutions that differed by the sets of observations, their weights, and the sets of parameters included in the solution.

OBSERVATIONS, THEIR REDUCTION AND ERRORS

We used all the available radar observations of planets (58 116, 1961–1997), spacecraft and landers (195 271, 1971–2003) that were retrieved from the JPL database (http:/ssd.jpl.nasa.gov/iaucomm4/) created and maintained by Dr. Standish and that were supplemented by series of American and Russian radar observations of planets in the period 1961–1995 taken from different sources. The Russian radar observations of planets together with references to the sources are stored at the site of the Institute of Applied Astronomy, Russian Academy of Sciences, //www.ipa.nw.ru/ PAGE/DEPFUND/LEA/ENG/englea.htm. A brief description of all astrometric radar observations can be found in Table 2 from Pitjeva (2005).

The accuracy of the first time-delay (τ) measurements for planets performed in 1961–1962 was 200–500 μs. The accuracy of the 1964–1969 measurements was higher (30 μs). The current accuracy of the radar observations of planets and spacecraft reaches a few hundredths of a microsecond, which corresponds to an error of several meters.

The reductions of the radar observations, including the relativistic corrections—the delay of radio signals near the Sun (Shapiro's effect) and the transition from the coordinate time, the argument of the ephemerides, to the observer's proper time as well as the delay of radio signals in the Earth's troposphere and in the solar coronal plasma,—are well known and were described, for example, by Standish (1990). The observations of Mars and Venus were corrected for topography using the currently available hypsometric maps of the surfaces of these planets and the representation of the topography as a decomposition into spherical harmonics of degrees 16–18. Details on the corrections for the topographies of Venus and Mars can be found in our previous paper (Pitjeva 1996). The topography of Mercury was represented as a decomposition into spherical harmonics up to the second order inclusive; the harmonic coefficients were determined from Mercury's radar observations (Pitjeva 2000). The shortcomings of the reduction include the inability to allow for the rapid change of the surface relief using the harmonics of degrees 16–18 and the limited size of the grid cell in the correction for the surface topography using hypsometric maps. Therefore, unfortunately, the topography errors remain in the observations of planets and are $\sim$100 m. Accordingly, high-precision observations of spacecraft orbiting planets and Martian landers, which are free from these topography errors, are of particularly great importance.

The time delay τ for the Viking-1 and-2 landers on Mars were measured at JPL in the period 1976–1982.

For 20 years, these measurements had been most accurate (an *a priori* accuracy of 7 m) among the positional observations of the major planets; in 1997, the new Pathfinder Martian lander was observed for three months. The differenced range $d\tau$ was measured simultaneously with the time delay. P. Wimberly managed to restore the differenced range for the Viking-1 lander observations in 1976–1978. To compute the lander positions on the Martian surface in the reference frame of the ephemerides, it was necessary to use a theory of the Martian rotation that included not only the precession and nutation of the Martian axis, but also the seasonal terms in the Martian rotation (Pitjeva 1999). The lander observations allowed not only the orbital elements of the Earth and Mars to be accurately determined, but also the Martian rotation parameters and, in particular, such an important (for understanding the geophysics of Mars) parameter as the Martian precession rate. Since the orbit of Mars is perturbed by Jupiter and asteroids, these observations can also be used to improve the masses of Jupiter and the largest asteroids.

The observations of Martian orbiters, Mariner-9 (1971–1972), Mars Global Surveyor (MGS, 1998–2003), and Odyssey (2002–2003), are given in the form of normal points of distances between the antennae of observational stations and the Martian center of mass and could contain systematic errors due to the insufficiently accurate elimination of the spacecraft orbit when producing the normal points. These systematic errors, which exceed the 2-m *a priori* errors, were seen in the original MGS data; these have now been reduced considerably. Unfortunately, in contrast to the two-frequency Viking observations, which made it possible to completely allow for the delay in the solar corona, the Mariner-9 observations as well as the MGS and Odyssey measurements were performed in one band. Therefore, the effect of the solar corona was significant, particularly near superior conjunctions with the Sun. We used the following model of the solar corona to reduce these observations:

$$N_e(r) = \frac{A}{r^6} + \frac{B + \dot{B}t}{r^2},$$

where $N_e(r)$ is the electron density; the parameters B and $\dot{B}$ were determined from observations and were different for different conjunctions. Although the residuals in the observations decrease significantly after this correction for the solar corona, the remaining influence of the corona is still noticeable in them. Moreover, the parameters of the corona correlate with other parameters being determined and adversely affect their determination.

In addition, for some of the series of observations, it was necessary to introduce constant shifts that were interpreted as systematic errors of an unknown origin or calibration errors of the instrumentation. We introduced the following constant shifts for six groups of observations: 6.9 km for the Goldstone observations of Venus in 1964, 2.9 km for the Crimean observations of Venus in 1969, 7.3 km for the Crimean observations of Mercury in 1986–1989, about 20 m for the Viking-1 and -2 observations, and 2.5 m for Odyssey. The possibility of such errors in the Crimean observations of Venus in 1969 and the Viking-1, -2 and Odyssey observations was pointed out by the observers themselves; the existence of systematic errors in the observations of Venus in 1964 in Goldstone and of Mercury in 1986–1989 in Crimea follows from a comparison with other radar measurements in the same period.

All of the above errors reduce significantly the accuracy of the parameters.

It should also be noted that in those cases where additional information about any parameters could not be obtained, the observations performed during a day or within one session for MGS and Odyssey were combined into normal points after applying all the necessary corrections. During the combination, we assigned a weight to all measurements according to their *a priori* accuracy that is generally given in the publications.

DETERMINING THE RELATIVISTIC PARAMETERS AND THE SOLAR OBLATENESS

The Secular Variation of the Gravitational Constant

Finding the possible secular variation of the gravitational constant is of crucial importance, since, basically, the strong equivalence principle is verified. If the cosmology of the Universe affects the local physical processes, then one could expect the coupling coefficients between the various physical fields to vary with cosmological time scale, and no single fundamental natural clocks are possible; i.e., the gravitational and atomic clocks are incommensurable. According to Canuto *et al.* (1979), the secular difference between the atomic time scale in which the observations are performed and the dynamical time scale in which the General Relativity equations of motion are valid can be interpreted in terms of the variation of the gravitational constant G. The directly observed effect in the planetary longitudes depends on the time interval quadratically, and one might expect the error in $\dot{G}/G$ from the *a priori* errors to be $\sim 10^{-12}$ per year or less. This is approximately the level at which the G variability is expected, as implied by certain physical arguments, for example, by Dirac's hypothesis of large numbers.

Table 1. Secular variation of the gravitational constant

$\dot{G}/G$ (10^{-11} per year)	
15 ± 9	Reasenberg and Shapiro (1978)
14 ± 2	Anderson *et al.* (1978)
0.2 ± 0.4	Hellings *et al.* (1983, 1989)
1.10 ± 1.07	Damour and Taylor (1991)
0.00 ± 0.11	Williams *et al.* (2002)
4.1 ± 0.8	Pitjeva (1986)
0.28 ± 0.32	Pitjeva (1993)
-0.002 ± 0.005	Pitjeva (this paper)

The parameter $\dot{G}/G$ was improved simultaneously with all the major parameters of the theory and the additional parameters β, γ, $\dot{G}/G$, and J_2 of the Sun. In addition, we calculated the test versions of the solutions where other unknowns or sets of observations were included in or excluded from the parameters to be determined. Just as Reasenberg *et al.* (1979), we calculated the masking factor: $\mu(\dot{G}/G) = \sigma(\dot{G}/G)/\sigma^*(\dot{G}/G)$, where σ and σ^* are, respectively, the standard deviations of the $\dot{G}/G$ estimates when all parameters are estimated simultaneously (σ) and when only one parameter is estimated (σ^*). The closer the value of μ to unity, the more stable the estimate of the parameter. In the case of a strong correlation between the parameters, μ can reach large values (several hundred). For $\mu(\dot{G}/G)$, we obtained a value of 28, which shows the achieved stability of the derived value ($\mu(\dot{G}/G) = 80$ in the paper by Reasenberg and Shapiro (1978)).

The possible variation of the gravitational constant can in principle be determined by analyzing lunar (including ancient) eclipses, lunar laser-ranging data, radar observations of planets and spacecraft, and pulsar timing data. Table 1 gives the values of $\dot{G}/G$ obtained by different methods. The first two values were independently obtained in 1978 by two groups by analyzing radar observations of planets and spacecraft on a relatively short time interval; the accuracy of the best observations was $\sim$1 μs at that time. A zero $\dot{G}/G$ was obtained when the 6-yr-long series of much more accurate Viking lander observations were included in the data analysis by Hellings *et al.* in 1983 and confirmed in 1989. Williams *et al.* determined $\dot{G}/G$ in 2002 by processing the 1970–2000 lunar laser-ranging data. Damour and Taylor (1991) derived $\dot{G}/G$ by analyzing the rate of change in the orbital period of the binary pulsar PSR 1913+16 and assumed that the variation of the gravitational constant could be determined most accurately only by this method. However, it subsequently emerged that the accuracy of this parameter is limited for pulsar timing and depends on the equation of state for a neutron star and the theory of gravitation in strong fields. Our values obtained in different years by processing radar observations of planets and spacecraft are given at the bottom of Table 1. The nonzero $\dot{G}/G$ in 1986 can probably be attributed to the systematic errors of the earliest radar observations. Substantial progress in the accuracy of estimating this parameter and a decrease in the possible range of the $\dot{G}/G$ variation can be seen from Table 1.

Parameters of the PPN Formalism

The quantities β and γ are the parameters of the PPN formalism that describe the metric theories of gravitation; β represents the degree of nonlinearity of gravitation, and γ characterizes the curvature of space produced by the rest mass. In General Relativity, $\beta = \gamma = 1$. The two classical relativistic tests, the deflection of light by the Sun and the delay of a signal as it passes near the Sun, measure the same effect, the propagation of photons in curved space near the Sun, and depend on the parameter γ. This parameter can be estimated with a high accuracy by measuring the deflection of light during VLBI observations of quasars. In Table 2, these are the values obtained by Robertson *et al.* (1991), Lebach *et al.* (1995), and Eubanks *et al.* (1997). The value of γ estimated by Eubanks *et al.* (1997) was combined with the latest values of Nordtvedt's parameter from lunar laser-ranging observations and the correction to the advance of Mercury's perihelion from radar observations of planets and spacecraft, which allowed them to also estimate β and the solar oblateness. Froeschle *et al.* (1997) estimated γ by analyzing optical Hipparcos observations.

After Shapiro *et al.* discovered the theoretical effect of the delay of radio signals as they pass near the Sun, in 1968, this effect has been measured several times using radar observations of planets and spacecraft: Anderson *et al.* (1975) (Mariner-6,7) and Reasenberg *et al.* (1979) (Viking). The most recent and accurate estimate (Bertotti *et al.* 2003) was obtained by measuring the frequency shift of radio photons to and from the Cassini spacecraft.

The possibilities for estimating β are much fewer. This parameter can be determined from Nordtvedt's effect ($4\beta - \gamma - 3$) when processing laser-ranging observations (Williams *et al.* 2002) or from the analysis of radar observations of the inner planets and spacecraft using the relativistic perturbations that produce periodic and secular variations in the orbital

Table 2. Parameters of the PPN formalism

$\gamma - 1$		$\beta - 1$	
0.00 ± 0.03	Anderson *et al.* (1975)		
0.000 ± 0.002	Reasenberg *et al.* (1979)		
0.0002 ± 0.0010	Robertson *et al.* (1991)		
-0.0004 ± 0.0017	Lebach *et al.* (1995)		
-0.003 ± 0.003	Froeschle *et al.* (1997)		
-0.00006 ± 0.00031	Eubanks *et al.* (1997)	-0.00019 ± 0.00026	Eubanks *et al.* (1997)
0.002 ± 0.004	Williams *et al.* (2002)	-0.001 ± 0.004	Williams *et. al.* (2002)
-0.0015 ± 0.0021	Anderson *et al.* (2002)	-0.0010 ± 0.0012	Anderson *et al.* (2002)
0.000021 ± 0.000023	Bertotti *et al.* (2003)		
-0.13 ± 0.06	Pitjeva (1986)	0.24 ± 0.12	Pitjeva (1986)
0.006 ± 0.037	Pitjeva (1993)	0.014 ± 0.070	Pitjeva (1993)
-0.0001 ± 0.0002	Pitjeva (this paper)	0.0000 ± 0.0001	Pitjeva (this paper)

elements of planets (Anderson *et al.* 2002); in particular, the relativistic secular motion of the perihelia depends on $(2 + 2\gamma - \beta)/3$. In these cases, the two parameters β and γ can be simultaneously estimated by also taking into account Shapiro's effect. It should be noted, however, that the correlation between β and γ is rather strong; it is 95% and 84% in the papers by Williams *et al.* (2002) and Pitjeva (2005), respectively.

Our estimates of β and γ obtained in different years by processing radar observations of the inner planets and spacecraft similar to Anderson *et al.* (2002) are given at the bottom of Table 2. Compared to the paper by Anderson *et al.* (2002), our 2005 results were obtained by including a large number of high-precision radar and VLBI observations of the MGS and Odyssey spacecraft (1998–2003) and some of the other series of observations, for example, the Russian radar observations (1961–1995), in the data analysis. In addition, the dynamic model of planetary motions was improved significantly by including 301 large asteroids and the perturbations from the asteroid ring with their masses estimated from observations in the simultaneous numerical integration; in this way, we reduced significantly the asteroid noise that deteriorates the accuracy of the solution parameters. The higher accuracy achieved in the last paper can probably be explained by these two factors.

The results show that the motions of the inner planets are in excellent agreement with General Relativity and leave increasingly few possibilities for alternative theories of gravitation.

The Secular Motions of Planetary Perihelia

Detecting the advance motions of planetary perihelia and, subsequently, their explanation in terms of General Relativity effects was one of the first relativistic tests. Indeed, the corrections to the motions of planetary perihelia are clearly revealed from observations; the masking factor $\mu(\Delta\delta_i)$ is only within the range 1.1–1.8 when determining these parameters from currently available observations.

The Schwarzschild advance of a planetary perihelion in a century is (Brumberg 1972)

$$\Delta\pi = \frac{3R_\mu n}{a(1-e^2)},$$

where R_μ and a are, respectively, the gravitational radius of the Sun and the semimajor axis of the planet in the same units; e is the eccentricity, n is the mean motion of the planet in arcsecs per 100 yr. The relativistic advances of the perihelia $\Delta\pi$ estimated for the inner planets are given in Table 3. However, the orbital elements of the planets vary with time due to the mutual perturbations of all objects in the Solar system; therefore, the precise advances of the perihelia cannot be given. Standish (2000) determined the mean secular relativistic advance of Mercury's perihelion, which is $42''.980$ on the interval 1800–2200, by comparing Mercury's perihelia every 400 days in two ephemerides obtained by integration and distinguished by the presence or absence (i.e., $\beta = \gamma = 0$) of the relativistic terms of General Relativity in the equations of motion for the planets.

Table 3. Secular motions of the planetary perihelia (arcsecs per century)

Mercury	Venus	Earth	Mars	Source
42.98	8.62	3.84	1.35	Brumberg (1972)
0.11 ± 0.22	-3.03 ± 0.71	-0.12 ± 0.16	-0.35 ± 0.24	Pitjeva (1986)
-0.017 ± 0.052	–	–	–	Pitjeva (1993)
-0.0036 ± 0.0050	0.53 ± 0.30	-0.0002 ± 0.0004	0.0001 ± 0.0005	Pitjeva (this paper)

For alternative theories of gravitation, the principal term in the advance of the perihelion is

$$\frac{1}{3}(2+2\gamma-\beta)\Delta\pi.$$

The second term for nonconservative theories of gravitation, which appears with the coefficient $M_\odot M_p/(M_\odot + M_p)$ ($M_\odot$ and M_p are the masses of the Sun and the planet, respectively), is negligible for the inner planets and is not considered below.

The situation is complicated by the fact that the solar oblateness also causes the secular advance of the planetary perihelia. Thus, the total advance of the perihelia (δ) is a linear combination of the post-Newtonian parameters and the gravitational quadrupole moment of the Sun (J_2):

$$\delta = \Delta\pi\left[\frac{1}{3}(2+2\gamma-\beta) - \frac{1}{2}\frac{R_\odot^2}{R_\mu a(1-e^2)}J_2(3\sin^2 i - 1)\right],$$

where i is the orbital inclination of the planet.

By comparing the model observations computed using the constructed ephemerides with actual observations, we can obtain the correction $\Delta\delta$ that can be interpreted as a correction to the combination of post-Newtonian parameters $2+2\gamma-\beta$ or as a correction to J_2, or as a correction to both. The accuracy and the number of existing observations in the 1960s–1970s were not enough to determine the individual parameters β, γ, and J_2; only the correction $\Delta\delta$ to their linear combination and only for Mercury could be determined. The actual corrections to the motions of the perihelia of other planets could not be determined at that time. At present, as a test, we can determine not β, γ, and J_2, but the corrections to the motions of the planetary perihelia, which allows us to judge whether the values of β, γ, and J_2 used to construct the ephemerides are valid.

Table 3 gives our corrections to the secular motions of the planets obtained in different years. We see from Table 3 that the accuracies of these parameters for all the planets, except Venus, has increased significantly due to the increase of the time interval on which the planets are observed and owing to the high-precision MGS and Odyssey observations. Table 3 shows that the parameters $\beta = 1$, $\gamma = 1$, and $J_2 = 2 \times 10^{-7}$ used to construct the EPM2004 ephemerides are in excellent agreement with the observations. Although the correction to the advance of Mercury's perihelion is within the error limits, a small negative correction to the combination δ may be required. Assuming that $\beta = 1$ and $\gamma = 1$, we obtain a new estimate of the solar oblateness, $J_2 = (1.7 \pm 0.5) \times 10^{-7}$. The solar oblateness is discussed in more detail below.

The Gravitational Quadrupole Moment of the Sun

Determining the dynamical oblateness of the Sun is of great importance, since the solar oblateness serves as a check for the theories that describe the interior structure of the Sun and its rotation and is one of the parameters required to construct high-precision theories of planetary and lunar motions. As yet, there is no universally accepted and satisfactorily determined value of the dynamical oblateness of the Sun. This parameter can be determined indirectly from various astrophysical observations of the Sun. However, such observations involve many problems: the rotation of the Sun around its axis is fairly complex, the outer layers have different angular velocities at different latitudes, the information about the rotation of inner layers is insufficient; the brightness of the solar limb depends on the latitude as well as on the solar cycle, the number of faculae, and the number of sunspots; the calibration of ground-based data for the atmosphere causes great difficulties. The initial estimates of the solar oblateness (before approximately 1970) using heliometers and photographic plates were often erroneous, but using new techniques, improving the theory of the solar interior structure, and performing satellite observations allow the gravitational quadrupole moment of the Sun (J_2) to be determined with a higher accuracy. Some of the J_2 values obtained from astrophysical observations are given in the upper part of Table 4. The value by Hill *et al.* (1982) was one of the most accurate for his time, and it was commonly used in celestial mechanics for various estimates. The table also gives the recent and (probably) most accurate estimates obtained from astrophysical observations (Paterno *et al.* 1996; Pijpers 1998; Godier and Rozelot 2000). A good overview of all the available estimates of the solar quadrupole moment was given by Pireaux and Rozelot (2003).

The dynamic oblateness of the Sun can be determined independently during the construction of a theory for the motion of bodies in the Solar system when determining the parameters of this theory from observations. The solar oblateness produces secular trends in all the elements of the planets, except their semiaxes and eccentricities. The secular trends are inversely proportional to the square of the semiaxes; the largest secular trend due to the solar oblateness arises in Mercury's perihelion. According to Brumberg (1972), the rate of secular motion of the perihelion is given by

$$d\pi_S = \frac{3}{2}\left(\frac{R_\odot}{a}\right)^2 \frac{n}{(1-e^2)^2} J_2.$$

In constructing the JPL versions of the DE405 ephemerides and our EPM2000 based on the estimates by Duvall *et al.* (1984) and Brown *et al.* (1989) obtained from helioseismometric measurements (under certain additional assumptions) (see Table 4), a nonzero solar oblateness, $J_2 = 2 \times 10^{-7}$, has been used for the first time in the integration.

In this case, the main problem lies in the smallness of the parameter J_2 and in its separation from the post-Newtonian parameters β and γ. This could not be done before a large number of high-precision MGS and Odyssey data appeared in recent years, and the dynamic estimate of the solar oblateness was obtained from the estimates of the motion of Mercury's perihelion that included a linear combination of post-Newtonian parameters and the solar oblateness. Some of these estimates are given in the middle part of Table. 4. The estimate by Eubanks *et al.* (1997) was obtained by combining the resent estimates of γ, Nordtvedt's parameter, and the advance of Mercury's perihelion.

In 1990, the dynamic oblateness of the Sun was determined by the methods of celestial mechanics from the analysis of radar and optical (1960–1986) observations by Afanasieva *et al.* 1990. Unfortunately, most of the modern high-precision American radar observations of planets, spacecraft, and landers was inaccessible at that time, and the accuracy of the estimate obtained was not high enough. It has become possible to simultaneously estimate all three parameters J_2, β, and γ only in recent years. These determinations are given in the lower part of Table 4.

Table 4. The gravitational quadrupole moment of the Sun

$J_2 \times 10^{-7}$	
55 ± 13	Hill *et al.* (1982)
1.7 ± 0.4	Duvall *et al.* (1984)
1.7 ± 0.2	Brown *et al.* (1989)
2.08 ± 0.14	Paterno *et al.* (1996)
2.18 ± 0.06	Pijpers (1998)
2.0 ± 1.4	Godier and Rozelot (2000)
13.9 ± 24.7	Shapiro *et al.* (1972)
26.3 ± 16.5	Anderson *et al.* (1978)
12.3 ± 11.5	Anderson *et al.* (1992)
-1.8 ± 4.5	Eubanks *et al.* (1997)
-11.7 ± 9.5	Pitjeva (1986)
-1.3 ± 4.1	Pitjeva (1993)
2.4 ± 0.7	Pitjeva (2001)
6.6 ± 9.0	Afanasieva *et al.* (1990)
-5 ± 10	Williams *et al.* (2002)
2.3 ± 5.2	Anderson *et al.* (2002)
1.9 ± 0.3	Pitjeva (this paper)

The value by Williams *et al.* (2002) was obtained from the analysis of lunar laser-ranging observations. The estimates by Anderson *et al.* (1992) and our estimates were obtained by analyzing radar observations of the inner planets and spacecraft (the masking factor of the solar oblateness $\mu(J_2)$ is 17). The last, more accurate estimate agrees well with the estimate deduced from the advance of Mercury's perihelion in the previous section.

A test version of the EPM ephemerides similar to EPM2004, but with $J_2 = 6.52 \times 10^{-7}$ deduced from S. Lefevre's figure theory, was constructed at the request of the colleagues involved in preparing the European BepiColombo mission to Mercury. Subsequently, these ephemerides were improved using observations with an improvement of all the parameters, except J_2. As might be expected, the representation of the observations of Venus and Mars did not change, because this parameter is small. Although the usual accuracy of Mercury's observations is only about 1 km, a deterioration of Mercury's representation in the test version is still noticeable: the rms error of the residual of Mercury's observations was 1228 m compared to 1192 m for the main ephemerides, where $J_2 = 2 \times 10^{-7}$ was used for the integration. Improving the parameter J_2 for the test version of the theory yielded $J_2 = 1.6 \times 10^{-7}$, which again agrees with the values obtained when improving the EPM2004 ephemerides.

Thus, we conclude that the gravitational quadrupole moment of the Sun is probably close to or slightly smaller than 2.0×10^{-7}.

CONVERSION FROM THE EPHEMERIDES IN THE TDB TIME SCALE TO THE EPHEMERIDES IN THE TCB SCALE

According to IAU resolutions, the ICRS should be considered as a four-dimensional coordinate system with an independent variable—the TCB coordinate time in the scale of which the planetary ephemerides should be given. For comparison with the widely used JPL DE ephemerides, our EPM ephemerides have been constructed until the present time with the TDB time scale (as an independent variable) close to $T_{\rm eph}$ (Standish 1998), which is used to construct the DE ephemerides. Since, according to IAU recommendations, the planetary ephemerides constructed in the TCB scale are required for the users that process VLBI measurements and observations of Earth satellites, we constructed an additional version of the EPM ephemerides in the TCB scale. The following transformations should be made to make a transition from the TDB scale to the TCB scale (see, e.g., Brumberg and Groten 2001):

(1) The initial epoch of integration JD = 2 448 800.5 TDB is expressed in terms of TCB:

$$\text{date(TCB)} = (\text{date(TDB)} - 2\,443\,144.5) \times L_B + \text{date(TDB)},$$

(2) The coordinates are multiplied by $(1 + L_B)$:

$$x_i(\text{TCB}) = x_i(\text{TDB}) \times (1 + L_B),$$

(3) The masses are multiplied by $(1 + L_B)$:

$$GM_i(\text{TCB}) = GM_i(\text{TDB}) \times (1 + L_B),$$

(4) The round-trip light time of the radar observations calculated in the TCB scale should be expressed in terms of the proper time, i.e., first

$$\tau_{\text{TDB}} = \tau_{\text{TCB}} \times (1 - L_B),$$

and then transformed to the proper time scale in a standard way.

Since the EPM ephemerides are close to DE405, we used

$$L_B = 1.55051976772 \times 10^{-8},$$

obtained for the relationship between TCB and TDB of the DE405 ephemerides.

The conversion to the TCB coordinate time scale should not and did not cause the accuracy of the ephemerides and the parameters being improved to increase. The residuals in the observations are identical for these two versions of the EPM ephemerides. As might be expected, the formal standard accuracies of all parameters and their values (except the orbital elements of the planets) are equal within the formal uncertainties.

CONCLUSIONS

The passage of photons and the motion of planets in the gravitational field of the Sun allow the Solar system to be considered as a convenient laboratory for testing various theories of gravitation. The currently available radar observations of planets and spacecraft with a meter accuracy (a relative error of 10^{-11}–10^{-12}) make it possible to test the relativistic effects and to estimate the solar oblateness. The uncertainties in these parameters (see Tables 1–4) has decreased significantly since 1993: by more than an order of magnitude for the gravitational quadrupole moment of the Sun and the advance of Mercury's perihelion and even by one and a half orders of magnitude or more for the remaining parameters (β, γ, $\dot{G}/G$, the advance of the perihelia of the Earth and Mars).

Substantial progress can be explained by several factors: an increase in the accuracy of observational data reduction procedures and dynamical models of motion as well as an improvement in the quality of observational data and an increase in their accuracy and in the length of the time interval on which these observations were obtained. As the uncertainties in these parameters decrease, the domain of possible values of the relativistic parameters narrows, imposing increasingly stringent constraints on the theories of gravitation alternative to General Relativity.

In conclusion, note that the numerical EPM2004 ephemerides of all planets and the Moon are available via FTP: flp://quasar.ipa.nw.ru/incoming/ EPM2004 or via the web site of the Institute of Applied Astronomy, Russian Academy of Sciences.

REFERENCES

1. E. I. Afanasieva, M. D. Kislik, Yu. F. Kolyuka, and V. F. Tikhonov, Astron. Zh. **67**, 1326 (1990) [Sov. Astron. **34**, 670 (1990)].
2. J. D. Anderson, J. K. Campbell, R. F. Jurgens, *et al.*, *6th Marcel Grossman Meeting on General Relativity*, Ed. by H. Sato and T. Nakamura (World Sci., 1992), p. 353.
3. J. D. Anderson, P. B. Esposito, W. Martin, *et al.*, Astrophys. J. **200**, 221 (1975).
4. J. D. Anderson, M. S. Keesey, E. L. Lau, *et al.*, Acta Astronautica **5**, 43 (1978).
5. J. D. Anderson, E. L. Lau, S. Turyshev, *et al.*, Bull. Am. Astron. Soc. **34**, 660 (2002).

6. B. Bertotti, L. Iess, and P. Tortora, Nature **425**, 374 (2003).
7. T. M. Brown, J. Christensen-Dalsgaad, W. A. Dziembowski, *et al.*, Astrophys. J. **343**, 526 (1989).
8. V. A. Brumberg, *Relativistic Celestial Mechanics* (Nauka, Moscow, 1972).
9. V. A. Brumberg and E. Groten, Astron. Astrophys. **367**, 1070 (2001).
10. V. M. Canuto, S.-H. Hsieh, and J. R. Owen, Mon. Not. R. Astron. Soc. **188**, 829 (1979).
11. T. Damour and J. H. Taylor, Astrophys. J. **366**, 501 (1991).
12. T. L. Duvall, W. A. Dziembowski, P. Goode, *et al.*, Nature **310**, 22 (1984).
13. T. M. Eubanks, D. N. Matsakis, J. O. Martin, *et al.*, *American Physical Society APS/AAPT Joint Meeting* (1997).
14. M. Froeschle, F. Mignard, and F. Arenou, *Hipparcos—Venice 97, ESA-402* (Venice, 1997), p. 49.
15. S. Godier and J. P. Rozelot, Astron. Astrophys. **355**, 365 (2000).
16. R. W. Hellings, P. J. Adams, J. D. Anderson, *et al.*, Phys. Rev. Lett. **51**, 1609 (1983).
17. R. W. Hellings, P. J. Adams, J. D. Anderson, *et al.*, Int. J. Theor. Phys. **28**, 1035 (1989).
18. H. A. Hill, R. J. Bos, and P. R. Goode, Phys. Rev. Lett. **49**, 1794 (1982).
19. G. A. Krasinsky, Commun. of IAA RAS **148**, 1 (2002).
20. G. A. Krasinsky, E. V. Pitjeva, M. V. Vasilyev, and E. I. Yagudina, Commun. of IAA RAS **139**, 1 (2001).
21. G. A. Krasinsky, E. V. Pitjeva, M. V. Vasilyev, and E. I. Yagudina, Icarus **158**, 98 (2002).
22. D. E. Lebach, B. E. Corey, I. I. Shapiro, *et al.*, Phys. Rev. Lett. **75**, 1439 (1995).
23. X. X. Newhall, E. M. Standish, Jr., and J. G. Williams, Astron. Astrophys. **125**, 150 (1983).
24. L. Paterno, S. Sofia, and M. P. Di Mauro, Astron. Astrophys. **314**, 940 (1996).
25. F. P. Pijpers, Mon. Not. R. Astron. Soc. **297**, L76 (1998).
26. S. Pireaux and J.-P. Rozelot, Astrophys. Space Sci. **284**, 1159 (2003).
27. E. V. Pitjeva, Byull. Inst. T. A., Ross. Akad. Nauk **15**, 538 (1986).
28. E. V. Pitjeva, Celest. Mech. **55**, 333 (1993).
29. E. V. Pitjeva, *Third International Workshop on Position Astronomy and Celestial Mechanics*, Ed. by G. A. Lopez, E. I. Yagudina, U. M. Martinez, and B. A. Condero (Observ. Astron. Univ. Valencia, 1996), p. 583.
30. E. V. Pitjeva, Tr. Inst. P. A., Ross. Akad. Nauk **4**, 22 (1999).
31. E. V. Pitjeva, Tr. Inst. P. A., Ross. Akad. Nauk **5**, 58 (2000).
32. E. V. Pitjeva, Celest. Mech. **80**, 249 (2001).
33. E. V. Pitjeva, Tr. Inst. P. A., Ross. Akad. Nauk **10**, 112 (2004).
34. E. V. Pitjeva, *Transit of Venus: New Views of the Solar System and Galaxy, IAU Coll. 196*, Ed. by D. W. Kurtz (Cambridge University Press, 2005).
35. R. D. Reasenberg and I. I. Shapiro, *On the Measurement of Cosmological Variations of the Gravitational Constant*, Ed. by L. Halpern (Florida Univ. Press, USA, 1978), p. 71.
36. R. D. Reasenberg, I. I. Shapiro, P. E. MacNeil, *et al.*, Astrophys. J. **234**, L219 (1979).
37. D. S. Robertson, W. E. Carter, and W. H. Dillinger, Nature **349**, 768 (1991).
38. I. I. Shapiro, G. H. Pettengill, M. E. Ash, *et al.*, Phys. Rev. Lett. **28**, 1594 (1972).
39. I. I. Shapiro, G. H. Pettengill, M. E. Ash, *et al.*, Phys. Rev. Lett. **20**, 1265 (1968).
40. E. M. Standish, Bull. Am. Astron. Soc. **32**, 870 (2000).
41. E. M. Standish, Astron. Astrophys. **233**, 252 (1990).
42. E. M. Standish, Astron. Astrophys. **336**, 381 (1998).
43. E. M. Standish and A. Fienga, Astron. Astrophys. **384**, 322 (2002).
44. J. G. Williams, D. H. Boggs, J. O. Dickey, and W. M. Folkner, *Ninth Marcel Grossman Meeting*, Ed. by V. G. Gurzadyan, R. T. Jantzen, and R. Ruffini (World Sci., 2002), p. 1797.

Translated by V. Astakhov