Open access • Journal Article • DOI:10.1017/S0022377800019863

Relativistic nonlinear plasma waves in a magnetic field — Source link

Charles F. Kennel, R. Pellat

Institutions: University of California, Los Angeles, École Polytechnique

Published on: 01 Jun 1976 - Journal of Plasma Physics (Cambridge University Press)

Topics: Ion acoustic wave, Longitudinal wave, Relativistic particle, Wave propagation and Transverse wave

NGL
05-007-190

PLASMA PHYSICS GROUP

UNIVERSITY OF CALIFORNIA
LOS ANGELES

ENGINEERING

PHYSICS

METEOROLOGY

Relativistic Nonlinear Plasma Waves in a Magnetic Field

C. F. Kennel and R. Pellat

PPG-243 October 1975

RELATIVISTIC NONLINEAR PLASMA WAVES IN A MAGNETIC FIELD

C.F. Kennel*

R. Pellat

Centre de Physique Theorique de l'Ecole Polytechnique
17, rue Descartes 75230 Paris Cedex 05

Abstract. We study five relativistic plane nonlinear waves: circularly polarized waves and electrostatic plasma oscillations propagating parallel to the magnetic field, relativistic Alfvén waves, linearly polarized transverse waves propagating in zero magnetic field, and finally the relativistic analog of the extraordinary mode propagating at an arbitrary angle to the magnetic field. When the ions are driven relativistic, they behave like electrons, and the assumption of an "electron-positron" plasma guides us to equations which have the form of a one-dimensional potential well. Our solutions indicate that a large-amplitude superluminous wave determines the average plasma properties, and not *vice versa*. For example, linearly polarized waves impose a plasma number flux equal to the relativistic addition of Nc/β and $N\vec{V}_E$, where N is the density, c the speed of light, β (>1) the ratio of the phase speed to c and $\vec{V}_E$ the $\overline{E} \times \overline{B}$ speed measured in the frame moving with speed c/β with respect to the frame in which the phase speed is measured. The implications for cosmic ray acceleration in pulsar magnetospheres are considered.

*On leave from Department of Physics and Institute of Geophysics and Planetary Physics, University of California, Los Angeles.
Pub. No. 1499, Inst. Geophys. Planet. Phys., UCLA.

1. Introduction

With the discovery that a rotating magnetized neutron star energizes the electrons in the Crab nebula, a major astrophysical problem was solved, but a major new plasma physical problem posed. It is generally agreed that neutron stars rotate and have an immense magnetic field, since they must conserve both angular momentum and magnetic flux during their collapse from their pre-supernova state (Gold, 1968). Furthermore, plasma processes in the magnetic field ought to communicate rotational energy to the surrounding nebula.

Those rotating magnetized neutron stars observed as pulsars probably have misaligned magnetic dipole and rotational axes; otherwise there would be no rotationally asymmetric feature capable of producing a pulse. If such an "oblique rotator" is *in vacuo*, it will emit, according to Maxwell's equations, a strong magnetic dipole wave which carries off rotational energy and angular momentum (Pacini, 1968; Gunn and Ostriker, 1969). Even this simplest model of a pulsar magnetosphere does rather well, since the estimates of the surface magnetic field based on flux conservation arguments, $B \simeq 10^{11-12}$ Gauss, lead to rotational deceleration rates in rough agreement with the observed gradual lengthening of the time interval between pulses. At this point Ostriker and Gunn (1969) raised an important question. They found that a single charged particle dropped into the vacuum wave could be accelerated to extremely high energies. Could the pulsar wave accelerate cosmic rays? At the same time, Goldreich and Julian (1969) observed that a rotating magnetized neutron star would never find itself *in vacuo*: Its electric field has such a large component $E_{\parallel}$ parallel to the magnetic field near the star that field emission from the solid surface of the neutron star would be inevitable. They argued that the magnetosphere would fill up with plasma until sufficient densities would be reached that $E_{\parallel} \simeq 0$, and the hydromagnetic approximation becomes a valid way to describe the magnetosphere.

The fact that the plasma density would not be negligible in pulsar magnetospheres started two new lines of research. First, relativistic versions of the solar wind were proposed (Michel, 1969). For mathematical simplicity, the dipole was assumed aligned and all time dependencies were neglected. Secondly, it became important to understand *self-consistent* plasma waves of relativistic amplitude. For this work, there existed the pioneering effort of Akhiezer and Polovin (1956) on electromagnetic waves which drive electrons relativistic. This work was extended for laser-plasma interactions by Kaw and Dawson (1970) and in part for pulsars by Max and Perkins (1971, 1972) and Max (1973). We will discuss in more detail a recent paper by Clemmow (1974) later.

At present, the question of whether the outer magnetospheres of pulsars are "winds" or "waves", or, as is conceivable for oblique rotators, a mixture of winds and waves, has not been resolved by a clearcut theoretical delineation between the regimes, and consequently by observation. Kennel, Schmidt and Wilcox (1973) found that when ions as well as electrons are driven relativistic by a plane wave propagating in an unmagnetized plasma, there is an upper limit to the cosmic ray number flux which can be transported by a wave of a given amplitude, above which the wave encounters a cutoff. The plasma wave cutoff flux corresponds to the lower limit on density for which the hydromagnetic approximation is valid. Asseo, Kennel and Pellat (1975) reached a similar conclusion for the more realistic case of a spherical wave. Thus, wind and wave solutions may eventually be distinguishable observationally on the basis of density.

This paper concentrates upon extending our basic understanding of relativistic nonlinear waves. We focus on linearly polarized superluminous waves in a magnetic field. We will argue shortly that only waves with phase speeds exceeding that of light can have arbitrarily large amplitudes, and that linear polarization leads to a unique relation between the cosmic ray flux transported by the wave and the wave amplitude, whereas circular polarization does not. We include a magnetic field to gain some insight into possible wind-wave solutions. In particular, we ask how the limiting cosmic ray flux is affected by the magnetic field. While our motivation is primarily astrophysical, our work might eventually be applicable to laser-plasma interactions, since in the near future lasers will be sufficiently powerful to drive at least the electrons relativistic. Here again, the inclusion of the magnetic field might prove interesting.

2. Basic Equations

The fluid equations for a two-species ($j = 1, 2$) cold collisionless plasma are

$$\left(\frac{\gamma_j}{c}\frac{\partial}{\partial t} + \vec{U}_j \cdot \vec{\nabla}\right)\vec{U}_j = \frac{e_j\gamma_j}{M_j c^2}\left(\vec{E} + \frac{\vec{U}_j \times \vec{B}}{\gamma_j}\right) \tag{2.1}$$

$$\frac{1}{c}\frac{\partial}{\partial t}\left(n_j\gamma_j\right) + \vec{\nabla}\cdot\left(n_j\vec{U}_j\right) = 0 \tag{2.2}$$

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho \tag{2.3}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \tag{2.4}$$

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \tag{2.5}$$

$$\vec{\nabla}\times\vec{B} = 4\pi\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t} \tag{2.6}$$

$$\rho = \sum_j N_j e_j = \sum_j n_j e_j \gamma_j \tag{2.7}$$

$$\vec{j} = c\sum_j n_j e_j \vec{U}_j \tag{2.8}$$

Equations (2.1) - (2.8) are written in an arbitrary reference frame which we will henceforth designate as the laboratory frame. The notation above is standard (e_j = particle charge, M_j = rest mass, $\gamma_j = \sqrt{1 + \vec{U}_j}$, where U_j is the reduced momentum, and c the speed of light), n_j denotes the proper density and N_j the laboratory frame density.

We now specialize (2.1) - (2.8) to a plane wave propagating in the x-direction. In addition, following Kennel et al. (1973), we change variables from (x, t) to phase $\eta = \omega\left(t - \frac{x}{\beta c}\right)$, where ω is the frequency and β the normalized phase velocity.

Writing (2.1) in components, defining $\vec{U} = (U, V, W)$ and suppressing the species index where the meaning is obvious, we find

$$\Delta\frac{dU}{d\eta} = \gamma\nu_x + V\Omega_z - W\Omega_y \tag{2.9a}$$

$$\Delta\frac{dV}{d\eta} = \gamma\nu_y + W\Omega_x - U\Omega_z \tag{2.9b}$$

$$\Delta\frac{dW}{d\eta} = \gamma\nu_x + U\Omega_y - V\Omega_x \tag{2.9c}$$

$$\Delta\frac{d\gamma}{d\eta} = U\nu_x + V\nu_y + W\nu_z \tag{2.9d}$$

where in (2.9) we have normalized the electric and magnetic fields relativistically by defining

$$\vec{\nu}_j = \frac{e_j\vec{E}}{M_j\omega c} \qquad \vec{\Omega} = \frac{e_j\vec{B}}{M_j\omega c} \tag{2.10}$$

The equation of continuity (2.2) becomes

$$\frac{d}{d\eta}(n\Delta) = 0 \tag{2.11}$$

where we have defined the relativistic Landau function Δ

$$\Delta \equiv \gamma - \frac{U}{\beta} \tag{2.12}$$

$$\Delta \equiv \gamma - \frac{U}{\beta} \tag{2.12}$$

When $\Delta = 0$, a particle moves with the wave phase velocity in the x-direction. Equation (2.11) can be integrated to yield

$$n = \frac{n_0 \Delta_0}{\Delta} \tag{2.13}$$

where subscript zero denotes the arbitrary phase point η_0 where all boundary conditions specifying the plasma are imposed.

Equation (2.4) and the x-component of (2.5) combine to yield

$$\Omega_x = \text{constant} \tag{2.14}$$

The remaining two components of (2.5) are

$$\frac{d}{d\eta}\left(\Omega_y + \frac{\nu_z}{\beta}\right) = \frac{d}{d\eta}\left(\Omega_z - \frac{\nu_y}{\beta}\right) = 0 \tag{2.15}$$

Similarly, the y and z components of (2.6) become

$$\beta \frac{dB_z}{d\eta} = \frac{dE_y}{d\eta} = -\frac{4\pi c}{\omega}\gamma_*^2 \sum_j n_j e_j V_j \tag{2.16a}$$

$$-\beta \frac{dB_y}{d\eta} = \frac{dE_z}{d\eta} = -\frac{4\pi c}{\omega}\gamma_*^2 \sum_j n_j e_j W_j \tag{2.16b}$$

Equation (2.3) and the x-component of (2.6) yield equivalent expressions

$$\frac{dE_x}{d\eta} = -\frac{4\pi c}{\omega}\sum_j n_j e_j U_j = -\frac{4\pi c}{\omega}\beta\sum_j n_j e_j \gamma_j \tag{2.17}$$

Equation (2.17) describes a fundamental property of relativistic electromagnetic waves. Even if the proper densities of electrons and ions are equal, there is in general an electrostatic field component E_x, which disappears only under special assumptions. In this paper, we will exploit the fact that when the ions are driven relativistic by the wave, the plasma acts like an electron-positron plasma (or like a gas of charged photons) to eliminate the electrostatic field E_x.

It is illuminating to consider the properties of equations (2.9) - (2.17) under Lorentz transformation to a frame moving in the x-direction with respect to the laboratory frame. Let us denote the normalized x-momentum vector of the transformation by $(\bar{U}, \bar{\gamma})$ and transformed quantities by superscript tilde. Under transformation, the phase η becomes in this frame

$$\eta = \frac{\omega}{c}\left[c\tilde{t}\left(\tilde{\gamma} - \frac{\tilde{\bar{U}}}{\beta}\right) + \tilde{x}\left(\bar{U} - \frac{\tilde{\gamma}}{\beta}\right)\right] \tag{2.18}$$

while Faraday's law, equation (2.15), leads to

$$\frac{d}{d\eta}\left[\tilde{B}_z\left(\tilde{\gamma} - \frac{\bar{U}}{\beta}\right) + \tilde{E}_y\left(\bar{U} - \frac{\tilde{\gamma}}{\beta}\right)\right] = \frac{d}{d\eta}\left[\tilde{B}_y\left(\tilde{\gamma} - \frac{\bar{U}}{\beta}\right) - \tilde{E}_z\left(\bar{U}_T - \frac{\tilde{\gamma}}{\beta}\right)\right] \tag{2.19}$$

The other quantities transform in standard fashion. It is evident that there are two particularly convenient transformations. For subluminous waves, $\beta < 1$, we may choose $\tilde{\gamma} = \bar{U}/\beta$. In this frame, which moves at the wave phase velocity, the phase variable

η becomes the transformed space variable x, and the transformed transverse components of the electric field are constant. For superluminous waves, $\beta > 1$, we can choose $\tilde{\gamma}/\beta = \tilde{U}$, a frame which moves with speed c/β. Here η becomes the transformed time, and all components of the magnetic field are entirely constant. We will denote quantities pertaining to the transformation to this space-independent frame characteristic of superluminous waves by subscript star, in other words

$$\gamma_* = \frac{\beta}{\sqrt{\beta^2 - 1}}, \qquad U_* = \frac{\gamma_*}{\beta} = \frac{1}{\sqrt{\beta^2 - 1}} \tag{2.20}$$

Our strategy will be to seek special wave polarizations which render the wave equations (2.9) - (2.17) simple to solve and to exploit the special Lorentz transformations (2.18) - (2.20) to illuminate the physics of the special solutions so obtained. The simplest polarization--circular--leads to an algebraic dispersion relation. We review this solution, already obtained by many authors, in Chapter 3. Barring an algebraic dispersion relation, the next best thing is a second-order ordinary differential equation integrable once by quadrature, so that we can exploit the analogy with the equations of classical mechanics for a particle in a potential well. Such techniques have been successful for nonlinear waves and solitons in non-relativistic plasmas, and we shall seek cases where they can be applied to nonlinear waves of relativistic amplitude. In Chapter 4 we discuss the simplest possible linearly polarized wave, an electrostatic plasma oscillation propagating parallel to an external magnetic field. In Chapters 3 and 4, ions and electrons make equal contributions to the dispersion relations when the ions are driven relativistic by the wave. In other words, the masses of the particles are determined by the kinetic energy acquired from the wave, and we can neglect their rest masses, and more importantly, the differences between rest masses. This suggests that setting $M_i = M_e$ produces a set of equations valid in the large amplitude limit which can guide us to differential equations in potential form. Sturrock (1971) has suggested that an electron-positron plasma will in fact be injected into the wind or wave zone of a pulsar magnetosphere.

In Chapter 3 we will find that circularly polarized waves do not impose a unique mean number or energy flux on the background plasma. No unique statements concerning cosmic ray transport can be made for circularly polarized waves. Thus we concentrate upon linearly polarized waves in an electron-positron plasma containing a uniform magnetic field. In Chapter 5 we derive the equations for such a wave propagating at an arbitrary angle to the magnetic field. We then transform this laboratory frame equation to the space- and time-independent frames for the subluminous Alfvén and superluminous extraordinary modes, respectively. In Chapter 6 we study the subluminous relativistic Alfvén solitary wave. We find that it "breaks" at relatively low amplitudes, when the plasma encounters the $\Delta = 0$ Landau singularity. On the other hand, when $\beta > 1$, Δ must always be positive. Thus superluminous waves can reach arbitrarily large amplitudes without encountering a fundamental difficulty within cold fluid theory. In Chapter 7 we specialize to the case of superluminous waves in zero-average electric or magnetic fields, first treated by Akhiezer and Polovin (1956) in the limit $M_i \to \infty$. In the limit $M_i \to M_e$ we find an exact solution valid at all amplitudes. In Chapter 8 we consider the extraordinary mode in the large amplitude limit; we find a dispersion relation independent of the particle mass, justifying a posteriori our assumption of equal masses, or better, of a charged photon gas. We will find that superluminous linearly polarized waves impose characteristic number fluxes on the background plasma in the large amplitude limit.

3. Transverse Circularly Polarized Waves

We choose $E_x = 0$, but E_y and E_z non-zero. We keep B_z non-zero, but require that the phase-averaged y and z magnetic field components $\langle B_y\rangle$, $\langle B_z\rangle = 0$, which implies that $B_y = -E_z/\beta$ and $B_z = E_y/\beta$. Equation (2.9) then reduces to

$$\Delta\frac{dU}{d\eta} = \frac{V\nu_y + W\nu_z}{\beta} = \frac{1}{\beta}\frac{d\gamma}{d\eta} \tag{3.1a}$$

$$\Delta\frac{dV}{d\eta} = \Delta\nu_y + W\Omega_z; \qquad \Delta\frac{dW}{d\eta} = \Delta\nu_z - V\Omega_x \tag{3.1b}$$

where the particle species index has been suppressed. For a circularly polarized wave $\left(E_y = E\cos\eta,\ E_z = E\sin\eta\right)$, γ, Δ, U, and n are constant. In addition, for each species

$$\left(V^2 + W^2\right)^{\frac{1}{2}} = \frac{\nu}{\left(1 \pm \Omega_x/\Delta\right)} \tag{3.2}$$

where $\nu = \frac{e|E|}{M\omega c}$. Substituting (3.2) into (2.16a) leads to the dispersion relation

$$1 = \gamma_*^2 \sum_j \frac{\omega_{pj}^2}{\omega\left(\omega \pm \omega_{ij}/\Delta_j\right)} = \gamma_*^2 \sum_j \frac{W_{pj}^2/\gamma_j}{\omega\left(\omega \pm \omega_{ij}/\Delta_j\right)} \tag{3.3}$$

where $\omega_{pj}^2 = \frac{4\pi n_j e_j^2}{M_j}$ and $W_{pj} = \gamma_j\omega_{pj}^2$ denote the squares of the proper and laboratory frame plasma frequencies respectively, and $\omega_{ij} = \frac{e_j}{M_j c}B_x$ is the signed cyclotron frequency. The $\pm$ signs in (3.2) and (3.3) distinguish right and left circular polarizations. We are free to choose all the U_j identically zero, whereupon (3.3) is formally identical to the dispersion relation for circularly polarized waves of non-relativistic amplitude, with the rest masses multiplied by the appropriate γ_j factors. The γ_j in turn are calculated by substituting (3.2) into $\gamma_j = \sqrt{1 + V_{\perp j}^2}$, which gives a quartic for γ_j

$$\gamma_j^2 = 1 + \nu_j^2\left(1 + \omega_{ij}/\omega\gamma_j\right)^{-2} \tag{3.4}$$

when $|\nu_j| \gg 1$, $\gamma_j \gg 1$, an approximate solution to (3.4) is

$$\gamma_j = |\nu_j - \Omega_{xj}| \tag{3.5}$$

so that the dispersion relation (3.3) is independent of the particle rest masses.

Equation (3.3) does not necessarily describe superluminous waves, but in the limit $\omega \gg \omega_{ij}/\Delta_j$ it does so. In this limit (3.3) is also the dispersion relation for a circularly polarized wave propagating in a plasma with zero magnetic field. It is interesting to note the differences in dispersion relation created by the change from circular to linear polarization, as shown by equation (7.7). An even more fundamental difference is that the circularly polarized wave, unlike that in equation (7.7) or the others to be studied, does not fix the mean number and energy flux. Because of this, it is difficult to make any unique and meaningful statements concerning cosmic ray transport by circularly polarized waves.

4. Longitudinal Superluminous Relativistic Plasma Oscillations

Here we choose $E_y = E_z = B_y = B_z = V = W = 0$ everywhere. We solve for the electrostatic potential defined by

$$E_x = -\frac{d\Phi}{dx} = \frac{\omega}{\beta c}\frac{d\Phi}{d\eta} \tag{4.1}$$

Normalizing Φ relativistically, $\Psi_j = \dfrac{e_j\Phi}{M_j c^2\beta}$, the particle equations of motion reduce to

$$\Delta\frac{dU}{d\eta} = \gamma\frac{d\Psi}{d\eta}; \qquad \Delta\frac{d\gamma}{d\eta} = U\frac{d\Psi}{d\eta} \tag{4.2}$$

for both species. We solve (4.2) for the dependence of γ upon Ψ, using $\gamma = \sqrt{1+U^2}$ and $\Delta = \gamma - U/\beta$. Omitting the species subscript

$$\begin{aligned}\gamma &= \gamma_*\left\{U_*\left(\Psi+\delta_0\right) + \sqrt{1+\gamma_*^2\left(\Psi+\delta_0\right)^2}\right\}\\ &= \gamma_*^2\left\{\frac{1}{\beta}\left(\Psi+\delta_0\right)+\Delta\right\}\end{aligned} \tag{4.3a}$$

$$U = \frac{\gamma_*^2}{\beta}\left\{\beta\left(\Psi+\delta_0\right)+\Delta\right\} \tag{4.3b}$$

$\delta_0 = U_0 - \gamma_0/\beta$, where subscript zero denotes the phase points where $\Psi = 0$. One more equation relating γ and Φ can be found by solving (4.2) for γ/Δ (or U/Δ) and inserting into Poisson's equations (or the equation for conservation of charge), equation (2.17), using the definition (4.1), and integrating once

$$\frac{1}{2}\left(\frac{d\Phi}{d\eta}\right)^2 = E - \frac{\beta^2c^2}{\omega^2}\sum_j 4\pi n_{0j}\Delta_{0j}M_jc^2\gamma_j \tag{4.4}$$

where E is an arbitrary constant of integration.

Equations (4.3) and (4.4), a second-order differential equation integrable once by quadrature, present a strong analogy with particle mechanics. If $(d\Phi/d\eta)^2$ represents the kinetic energy of a "particle", then the E represents "total energy" and the remaining term in (4.4) "potential energy". We seek periodic nonlinear solutions to (4.4); therefore we adjust E so that the "particle" bounces back and forth between zeros of $(d\Phi/d\eta)^2$. There must be at least two zeros for a periodic solution to exist. One zero can be fixed by choosing E properly. Let $\hat{\Phi}$ be the maximum positive potential in the wave. Then, if

$$E = \frac{\beta^2c^2}{\omega^2}\sum_j 4\pi n_{0j}\Delta_{0j}M_jc^2\hat{\gamma}_j; \qquad \hat{\gamma}_j \equiv \gamma_j(\hat{\Phi}) \tag{4.5}$$

$\frac{d\Phi}{d\eta}(\hat{\Phi}) = 0$. The other zero of $d\Phi/d\eta$, $\hat{\Phi}'$, can only be determined from the explicit form of $d\Phi/d\eta$ which we deduce inserting (4.3a) into (4.4), using (4.5). In so doing we encounter the quantity

$$\sum_j 4\pi n_{0j}\Delta_{0j}M_jc^2\Psi_j = \frac{4\pi\Phi}{\beta}\sum_j 4\pi n_{0j}\Delta_{0j}e_j \tag{4.6}$$

which we assume vanishes.

In (4.4) we have used a mixed notation, with Φ on the left hand side and Ψ_j on the right, through γ_j, equation (4.3a).

We now specialize to a two-species plasma, electrons (e) and ions (i), and write (4.4) in terms of the relativistically normalized electron potential Ψ_e.

We note that $\Psi_e = -R\Psi_i$, where

$$R = M_i/ZM_e$$

and Z is the ionic charge. Thus

$$\frac{1}{2}\left(\frac{d\Psi_e}{d\eta}\right)^2 = \Delta_0\gamma_*\frac{\omega_{p0}^2}{\omega^2}\Big\{R\Big[\Big(1+\gamma_*^2\big(\hat{\Psi}_e/R-\delta_0\big)^2\Big)^{\frac{1}{2}}\Big] + \Big[\Big(1+\gamma_*^2\big(\hat{\Psi}_e+\delta_0\big)^2\Big)^{\frac{1}{2}} - \Big[1+\gamma_*^2\big(\Psi_e-\delta_0\big)^2\Big]^{\frac{1}{2}}\Big\} \quad (4.7)$$

where, consistent with assumption (4.6), we choose $\delta_{0i} = \delta_{0e}$.
The dispersion relation is the condition that η changes by π when the solution passes between two zeroes of $d\Psi_e/d\eta$:

$$\int_{\hat{\Psi}_e'}^{\hat{\Psi}_e} d\Psi_e\left(\frac{d\Psi_e}{d\eta}\right)^{-1} = \pi \quad (4.8)$$

Equations (4.7) and (4.8), while mathematically satisfactory, are not physically complete because they involve ω_{p0}, the proper electron plasma frequency at the point $\Psi = 0$, rather than the average proper or laboratory plasma frequency. With the techniques used in this paper, it is more convenient to deduce phase averaged quantities <u>after</u> the dispersion relation has been derived, since the phase average $\langle f\rangle$ of any quantity $f(\Psi_e)$ is clearly

$$\langle f\rangle = \frac{1}{\pi}\int_{\hat{\Psi}_e'}^{\hat{\Psi}_e} d\Psi_e f\left(\frac{d\Psi_e}{d\eta}\right)^{-1} \tag{4.9}$$

We can then re-express all the initial quantities (subscript 0) in the dispersion relation in terms of phase-averaged quantities.

We now turn to finding $\hat{\Psi}_e'$, which in general depends upon δ_0. We note immediately that $\delta_0 = 0$ makes $d\Psi_e/d\eta$ an even function of Ψ_e and $\hat{\Psi}_e' = -\hat{\Psi}_e$. Furthermore, if $d\Psi_e/d\eta$ is even, $\langle\Psi_e\rangle = 0$, which means the average plasma potential is zero.

The phase-averaged laboratory frame electron density is then

$$\langle N_e\rangle = n_{0e}\Delta_{0e}\langle\frac{\gamma_e}{\Delta_e}\rangle = n_{0e}\Delta_{0e}\gamma_*^2\langle 1 + \frac{\Psi_e/\beta}{\left(1+\gamma_*^2\Psi_e^2\right)^{\frac{1}{2}}}\rangle = n_{0e}\Delta_{0e}\gamma_*^2 \tag{4.10}$$

Similarly, $\langle N_i\rangle = n_{0i}\Delta_{0i}\gamma_*^2$. Thus, if we choose $n_{0i} = n_{0e}$, charge neutrality on the average is ensured.

An entirely similar calculation for the averaged lab frame ion and electron fluxes $\langle J_x^i\rangle$ and $\langle J_x^e\rangle$ respectively yields

$$\langle J_x^e\rangle = n_{0e}\Delta_{0e}c\langle\frac{U_e}{\Delta_e}\rangle = \langle n_e\rangle c/\beta = \langle J_x^i\rangle \tag{4.11}$$

Therefore, the choice $\delta_0 = 0$ which sets the speed of the plasma equal to c/β at $\Psi = 0$ ensures that the mean speed remains c/β for both species. Note that $\delta_0 = 0$ implies $\Delta_0\gamma_* = 1$, and requires $\beta > 1$.

In the small amplitude limit, $|\Psi_e| << 1$, (4.7) reduces to a harmonic oscillator

$$\left(d\Psi_e/d\eta\right)^2 = \frac{\Omega_{p0}^2 - \omega_{p0}^2}{\Delta_0^2\omega^2}\left(\hat{\Psi}_e^2 - \Psi_e^2\right) \tag{4.12}$$

where $\Omega_{p0} = \frac{4\pi n_0 e^2}{M_i}$, and

$$\omega = \frac{\sqrt{\omega_{p0}^2 + \Omega_{p0}^2}}{\Delta_0} = \gamma_* \sqrt{\omega_{p0}^2 + \Omega_{p0}^2} \tag{4.13}$$

Since $|\Psi_e| << 1$, the proper density n_0 equals the phase-averaged proper density $\langle n \rangle$, and so ω_{p0} and Ω_{p0} correspond to the conventional electron and ion proper plasma frequencies. Equation (4.13) therefore represents small amplitude plasma oscillations in the proper frame. In the laboratory frame they are observed to be Doppler-shifted by the factor Δ_0.

In the large amplitude limit $\left|\frac{\Psi_e}{R}\right| >> 1$, the electron and ion terms in (4.7) are identical, and (4.7) reduces to

$$\frac{1}{2}\left(\frac{d\Psi_e}{d\eta}\right)^2 = 2\frac{W_{pe}^2}{\omega^2}\left\{\left|\hat{\Psi}_e\right| - \left|\Psi_e\right|\right\}; \; W_{pe}^2 = \frac{4\pi\langle N\rangle e^2}{M_e} \tag{4.14}$$

where we used (4.10). Inserting (4.14) into (4.8) and integrating leads to the dispersion relation

$$\omega^2 = \left(\pi W_{pe}\right)^2 / \hat{\Psi} = 4\pi^3 \langle N\rangle e c^2 \beta / \hat{\Phi} \tag{4.15}$$

which is independent of the particles' rest masses. Note that (4.15) does not give the plasma cutoff, (since when $\beta \to \infty$, $\hat{\Psi} \to 0$ for all finite wave amplitudes), which is obtained from (4.13) in the limit $\beta \to \infty$

$$\omega_{\text{cutoff}} = \left[4\pi \langle N \rangle e \left(\frac{1}{M_i} + \frac{1}{M_e}\right)\right]^{\frac{1}{2}} \tag{4.16}$$

Thus the plasma cutoff frequency is always the laboratory plasma frequency, regardless of wave amplitude.

We did not begin our calculation in the space-independent frame, as did Clemmow (1974), but nonetheless we found a posteriori that choosing a plasma streaming velocity equal to c/β enabled us to find a particular solution for relativistic nonlinear plasma oscillations which preserved charge neutrality. A laboratory observer would find a plasma number flux Nc/β of each species as well as a relativistic wave. Of course, these observables could be suitably Lorentz-transformed to other reference frames.

Suppose we had not chosen $\delta_0 = 0$. Clearly our small amplitude dispersion relation would have been modified by the Doppler shift factor Δ_0. But it is not nearly so evident that the large amplitude result would have changed, for expanding (4.7) assuming $\left|\hat{\Psi}_e / R\delta_0\right| \gg 1$ leads to (4.14) in first approximation. Is it true then that a relativistic amplitude wave imposes upon the plasma a net speed c/β? The absence of detailed physically sound solutions for $\delta_0 \neq 0$ leaves this an interesting speculation, which we confirm more rigorously for a relativistic electromagnetic wave in Chapter 7.

5. Electromagnetic waves in a plasma with $M_i = M_e$

In Chapters 3 and 4 we showed that the electron and ion contributions to the dispersion relations for circularly polarized electromagnetic waves and longitudinal plasma oscillations become equal when the wave amplitude is relativistically large for both species. Kennel et al. (1973) found exactly the same thing for transverse electromagnetic waves. In their solution, the effective equality of ion and electron masses eliminated the electric field E_x parallel to the direction of wave propagation, which has complicated attempts at solution since the original work of Akhiezer and Polovin (1956).

Of course, it is reasonable that the particle inertia be determined by the wave amplitude, not by the rest mass in the super-relativistic limit. Furthermore, this suggests that assuming equal ion and electron masses at the outset can guide us to simple equations valid in this limit. Clemmow (1974) has pointed out that certain solutions become very simple when $M_i = M_e$, but we feel that rather than being curiosities, they represent the super-relativistic limit well.

Let us discuss first the conditions under which the electrostatic field E_x can be eliminated. Rewriting (2.17) we find

$$\frac{dE_x}{d\eta} = -\frac{4\pi c}{\omega}\sum_j n_{0j}\Delta_{0j}U_j/\Delta_j = -\frac{4\pi c\beta}{\omega}\sum_j n_{0j}\Delta_{0j}\gamma_j/\Delta_j \qquad (5.1)$$

assuming $n_{0e}\Delta_{0e} = n_{0i}\Delta_{0i}$. Charge neutrality can be preserved in a two-species plasma only if $U_i = U_e$ and $\gamma_i = \gamma_e$ everywhere.

Henceforth we will make this assumption. This of course implies that $V_i = \pm V_e$ and $W_i = \pm W_e$. Referring to (2.16ab) we see that assuming $V_i = -V_e$ and $W_i = W_e$ excites the field components $\left(E_y, B_z\right)$, and assuming $W_i = -W_e$, $V_i = V_z$ excites $\left(E_z, B_y\right)$. Either assumption leads to a one-dimensional potential well, and the two waves have identical properties, one being spatially rotated with respect to the other. Henceforth we pick $V_i = -V_e$, $E_x = B_y = E_x = 0$; $E_y \neq 0$ and B_z, $B_x \neq 0$. Then equations (2.9ad) reduce to (5.2) below, where the species index is suppressed

$$dU/d\eta = V\Omega_z/\Delta \tag{5.2a}$$

$$dV/d\eta = \left(\gamma\nu_y - U\Omega_z + W\Omega_x\right)/\Delta \tag{5.2b}$$

$$dW/d\eta = -V\Omega_x/\Delta \tag{5.2c}$$

$$d\gamma/d\eta = V\nu_y/\Delta \tag{5.2d}$$

Finally we have, integrating (2.15) once

$$\Omega_z - \nu_y/\beta = \Omega_{z0} - \nu_{y0}/\beta \tag{5.3}$$

$$\Omega_x = \text{constant} \tag{5.4}$$

Henceforth we will solve our equations in terms of ion quantities $\left(\nu_{yi}, V_i, \text{etc}\right)$. Then, using $V_i = -V_e$, equation (2.15a) can be rewritten

$$d\nu_y/d\eta = \beta d\Omega_z/d\eta = -V/(\alpha\Delta) \qquad (5.5)$$

where

$$\alpha \equiv \omega^2/\left(\omega_{p0}^2 \Delta_0 \gamma_*^2\right) \qquad (5.6)$$

and ω_{p0} is the proper plasma frequency at the reference phase denoted by subscript zero.

Combining (5.5) with (5.2a) leads to

$$U = V_0 + \alpha\beta\left(\Omega_{z0}^2 - \Omega_z^2\right)/2; \qquad (5.7)$$

Combining (5.5) with (5.2d), to

$$\gamma = \gamma_0 + \gamma\left(V_0^2 - \nu_y^2\right) \qquad (5.8)$$

where henceforth the y subscript on the electric field will be suppressed; and finally (5.5) combined with (5.2c) gives

$$W = W_0 - \alpha\left(\nu_0 - \nu\right)\Omega_x \qquad (5.9)$$

Squaring equation (5.5) gives us

$$\alpha^2 (d\nu/d\eta)^2 = \left(\gamma^2 - U^2 - W^2 - 1\right)/\Delta^2 \qquad (5.10)$$

which is the equation of motion we seek. In addition, we impose the condition $V_0 = 0$, in other words $\gamma_0^2 - U_0^2 - W_0^2 = 1$, which is necessary to preserve our assumption $V_i = -V_e$.

Our task is now to express γ, U, W, Δ in terms of ν alone. After some algebra we arrive at the following equation for the normalized electric field $y = \nu/\nu_0$:

$$\left(\frac{\alpha\nu_0}{\gamma_*}\frac{dy}{dn}\right)^2 = \frac{(1-y)^2\left[(y-y_1)(y-y_2)-q\right]+(1-y)qQ}{\left[q/2+(y-y_3)(1-y)\right]^2} \tag{5.11}$$

where

$$Q = 2\left[1-\frac{U_0\bar{B}_z-\gamma_*W_0B_x}{\gamma_*\Delta_0E_{y0}}\right] = \frac{2\gamma_0}{\Delta_0}\frac{\left(\vec{E}+\vec{U}\times\vec{B}/\gamma\right)_{0y}}{E_{0y}}$$

The quantities y_1, y_2, y_3 are defined by

$$y_3 = -1+2U_*\left(\gamma_*B_{z0}-U_*E_{y0}\right)/E_{y0} = -1+\left(2U_*\bar{B}_z\right)/E_{y0} \tag{5.12}$$

$$y_1 = y_3+2\gamma_*\left(\frac{B_x^2+\left(\gamma_*B_{z0}-U_*E_{y0}\right)^2}{E_{y0}^2}\right)^{\frac{1}{2}} = y_3+2\gamma_*\bar{B}/E_{y0} \tag{5.13}$$

$$y_2 = y_3-2\gamma_*\bar{B}/E_{y0} \tag{5.14}$$

In the derivation of (5.11) - (5.14) the characteristic quantity $\gamma_*B_{z0}-U_*E_{y0}$ appears often. It is the z-component of the magnetic field in the space-independent frame $\bar{B}_z$, and we have so indicated in the second forms of (5.12) - (5.14). Similarly $\bar{B} = \left(B_x^2+\bar{B}_z^2\right)^{\frac{1}{2}}$. Since the magnetic field is a constant in this frame, if $\bar{B} = 0$, there will be no <u>average</u> electric and magnetic fields in this or any other frame. On the other hand, choosing $\widetilde{B} \neq 0$ leads to averaged electric and magnegic fields in the laboratory frame.

If y_1, y_2, y_3 define the averaged electric and magnetic fields, then the parameter q defines the strength of the wave, for

$$q = \left(2\Delta_0 \gamma_*^2\right)^2 4\pi n_0 Mc^2 \Big/ E_0^2 \geq 0$$

The case $q \gg 1$ corresponds to the small and the case $q \ll 1$ to the large amplitude limit. Note that q can be much larger than unity if $\gamma_0 \gg 1$ and $\Delta_0 \gg 1$. Then, even if $E_0^2 / 4\pi n_0 Mc^2 \gg 1$, the wave makes a small perturbation on the particle trajectories; on the other hand, the large amplitude limit, $q \ll 1$, is one where the particle motion is determined by the wave. The value of the particle rest mass enters (5.11) only through q. Thus, when $q \ll 1$ the rest mass will disappear entirely from the dispersion relation in leading order.

It is illuminating to consider the properties of the non-linear wave equation (5.11) - (5.14) under the special Lorentz transformations discussed briefly in Chapter 2. For the relativistic non-linear Alfvén wave, $\beta < 1$, we arrive at

$$-\left(\frac{B_{z0}}{\sqrt{8\pi n_0 M c^2 U_0^2}} \frac{c}{\omega_{p0}} \frac{db}{dx}\right)^2$$

$$= \frac{\left(\frac{b+1}{2}\right)^2 \left\{\left(\frac{b+1}{2}\right)^2 - \frac{8\pi n_0 M c^2 U_0^2 + E_{y0}^2 - B_x^2}{B_{z0}^2}\right\} - \left(\frac{b-1}{2}\right) \left[\frac{\gamma_0 (E + U \times B/\gamma)_0}{U_0 B_{z0}}\right] \frac{8\pi n_0 M c^2 U_0^2}{B_{z0}^2}}{\left[\frac{4\pi n_0 M c^2 U_0^2}{B_{z0}^2} + \frac{\left(1 - b^2\right)}{4}\right]} \quad (5.15)$$

We have written in (5.15) in physical quantities for physical clarity, and the tilde superscript denoting transformed quantities has been dropped. ω_{p0} is the proper plasma frequency at the phase point zero, and $b = \mathcal{B}_z/\mathcal{B}_{z0}$.

For the relativistic nonlinear extraordinary mode, we find

$$\left(\frac{E_{y0}}{\sqrt{8\pi n_0 Mc^2}}\,\frac{1}{\omega_{p0}}\,\frac{de}{dt}\right)^2 = \frac{\left(\frac{e-1}{2}\right)^2\left\{\left(\frac{e+1}{2}\right)^2 - \frac{8\pi n_0 Mc^2 + B_x^2 + B_{z0}^2}{E_{y0}^2}\right\} - \left(\frac{e-1}{2}\right)\left\{\frac{(\bar{E} + \bar{U} \times \bar{B}/\gamma)_{0y}}{E_{0y}}\right\}\frac{8\pi n_0 Mc^2}{E_{y0}^2}}{\left[\frac{4\pi n_0 Mc^2}{E_{y0}^2} + \frac{\left(1-e^2\right)}{4}\right]} \tag{5.16}$$

where $e \equiv \tilde{E}_x / E_{y0}$ and the tilde or star notation denoting transformation has been suppressed.

Comparing (5.15) and (5.16), we see that for $\beta < 1$, the independent variable is time, the dependent variable is the magnetic field, and the electric field is constant. For $\beta > 1$, the independent variable is x, the dependent variable the electric field, and the magnetic field is constant. The denominators in (5.15) and (5.16) are simply Δ^2. The more profound differences between (5.15) and (5.16) stem from the differences in sign of the derivative terms, and from the fact that Δ^2 can never be zero for a superluminous wave, while zeros are possible for a subluminous wave.

In Chapters 6 - 8 we study special solutions to equations (5.11), (5.15) and (5.16). In Chapter 6 we study the relativistic subluminous fast mode solitary wave propagating across the magnetic field. We find that it breaks, due to the $\Delta = 0$ singularity, at a relatively small amplitude. In Chapter 7 we specialize to the special case of a superluminous plasma wave with zero-averaged electric and magnetic fields, and in Chapter 8 we consider the relativistic nonlinear extraordinary mode.

6. Relativistic Alfvénic Solitary Wave

The effective potential in (5.15) has a double root at $b = 1$ if $(E + U \times B/\gamma)_{0y} = 0$. Then it may easily be shown that the integral for the wave phase becomes singular as $b \to 1$, the classic condition for a solitary wave (Sagdeev, 1966; Tidman and Krall, 1971). Here the amplitude passes reversibly from $b = 1$ at $x \to +\infty$ to a certain maximum value b_{max}, to be determined below, at $x = 0$ and back to $b \to 1$ as $x \to -\infty$. We now rewrite (5.15) for the $B_x = 0$ solitary wave in the standard notation used for the non-relativistic solitary wave:

$$\frac{1}{2}\left(\frac{db}{d\lambda}\right)^2 = \frac{\dfrac{\gamma_\infty^2}{2M^2}\left\{\dfrac{M^2 + \gamma_\infty^2 - 1}{\gamma_\infty^2} - \left(\dfrac{b+1}{2}\right)^2\right\}(b-1)^2}{\left[1 - \dfrac{\gamma_\infty^2\left(b^2 - 1\right)^2}{2M^2}\right]^2} \tag{6.1}$$

where $\gamma_\infty = \left(1 + U_\infty^2\right)^{\frac{1}{2}}$ is the relativistic Lorentz factor of the plasma at $x \to \infty$, and λ is a normalized distance, $\lambda = \omega_{p\infty}x/c$ where $\omega_{p\infty}$ is the $\underline{\text{proper}}$ plasma frequency at $x \to \infty$. M is a relativistic Mach number:

$$M^2 = \frac{8\pi n_\infty M\gamma_\infty c^2 U_\infty^2}{B_\infty^2} = \frac{8\pi N_\infty\left(M\gamma_\infty\right)c^2U_\infty^2}{B_\infty^2} = \frac{U_\infty^2c^2}{C_A^2} \tag{6.2}$$

where C_A is the Alfvén speed at $x \to \infty$.

If $\gamma_\infty \to 1$, then $M^2 = \left(V_{x\infty}/C_A\right)^2$, the usual non-relativistic Mach number. When C_A^2/C^2 is of order unity or greater, the small amplitude dispersion relation for hydromagnetic waves propagating perpendicular to the magnetic field yields

$$\left(\frac{V_{ph}}{c}\right)^2 = \frac{C_A^2\ c^2}{1 + C_A^2/c} \tag{6.3}$$

where V_{ph} denotes the phase velocity. It is convenient to define an effective Lorentz factor γ_p, based upon the phase velocity V_{ph}:

$$\gamma_p \equiv \left(1 - V_{ph}^2/c^2\right)^{-1} = 1 + C_A^2/c^2 \tag{6.4}$$

In the super-relativistic limit, $\gamma_\infty^2 >> 1$, $U_\infty^2 \to \gamma_\infty^2$ and $\gamma_p^2 \to C_A^2/c^2$, so that $M^2 \to \gamma_\infty^2/\gamma_p^2$. Thus, M in this limit measures the ratio of the particle to wave Lorentz factors.

In the non-relativistic limit, $\gamma_\infty \to 1$ and $C_A^2/c^2 << 1$, (6.1) reduces to the standard form for solitary waves in a relativistic plasma (Sagdeev, 1966; Tidman and Krall, 1971). In fact, the relativistic form (6.1) is identical to the non-relativistic form, so that we need not dwell overlong on the properties of relativistic solitary waves.

A nonlinear solution is possible $(db/d\lambda)^2 > 0$, if $M^2 > 1$, since $b \geq 1$. The amplitude rises from b = 1 at x = + ∞ to a maximum b_{max} at x = 0 and then returns to b = 1 at x = - ∞, where

$$b_{max} = 2\left[\frac{M^2 + \gamma_\infty^2 - 1}{\gamma_\infty^2}\right]^{\frac{1}{2}} - 1 \tag{6.5}$$

is found from the condition $\frac{db}{d\lambda} = 0$.

The solution is well-behaved so long as the singularity in the denominator of (6.1) is avoided. The denominator is simply $\left(U(x)/U_\infty\right)$, so the singularity corresponds to $U \to 0$ and $n \to \infty$, the equivalent in this case of the condition $\Delta \to 0$. Choosing $b = b_{max}$, we can solve for the Mach number which sets $U = 0$

$$M_*^2 = 2\left(1 + \gamma_\infty\right) \tag{6.6}$$

Evidently, (6.6) defines the maximum Mach number permissible. When $\gamma_\infty \to 1$, (6.6) yields $M_* = 2$, the usual result, whereas in the relativistic limit, $M_* = \sqrt{2\gamma_\infty} >> 2$.

We can solve for b^*_{max}, the largest possible magnetic field amplitude, based on the limit $M = M_*$

$$b^*_{max} = \frac{\gamma_\infty + 2}{\gamma_\infty} \tag{6.7}$$

Equation (6.7) reduces to $b^*_{max} = 3$ when $\gamma_\infty = 1$, the usual result. While the relativistic limit permits an apparently much larger range of permissible Mach numbers

$$1 < M < \sqrt{2\gamma_*} \tag{6.8}$$

than the non-relativistic limit, this whole range corresponds to solitary waves of very small maximum amplitudes:

$$1 < b^*_{max} < 1 + 2/\gamma_\infty \tag{6.9}$$

Thus, it may be that fast waves propagating perpendicular to the magnetic field are restricted to low amplitudes in the relativistic limit $C_A^2/C^2 >> 1$. Since when $\beta < 1$, it seems likely that the $\Delta = 0$ conditions can be satisfied for many nonlinear waves, it appears that superluminous waves may be the only ones that can have the large amplitudes required by pulsar theory.

7. Transverse Superluminous Waves with Zero Average Field

In this chapter we study the special case of zero background magnetic field, $\bar{B} = 0$. Then, in (5.11) - (5.14) $y_1 = y_2 = y_3 = 1$ and (5.11) reduces to

$$\left(\frac{\alpha\nu_0}{\gamma_*}\frac{dy}{d\eta}\right)^2 = \frac{\left(1-y^2\right)\left[\left(1-y^2\right)+q\right]}{\left[q/2+\left(1-y^2\right)^2\right]^2} \tag{7.1}$$

There can be no average electric or magnetic field, since the right hand side of (7.1) is even in y. The dispersion relation corresponding to (7.1) is

$$1 = \frac{2}{\pi}\frac{\alpha\nu_0}{\gamma_*}\int_0^1 \frac{K\left(K'^2/2K^2 + 1 - y^2\right)}{\left[\left(1-y^2\right)\left(1-K^2y^2\right)\right]^{\frac{1}{2}}}$$

$$= \left(\frac{2\alpha\nu_0}{\pi\gamma_*}\right)\left[\frac{2E(K) - K'^2F(K)}{2K}\right] \tag{7.2}$$

where $K^2 = \frac{1}{1+q}$, $K'^2 = \frac{q}{1+q}$, $K'^2\ K^2$, and $F(K)$ and $E(K)$ are complete elliptic integrals of the first and second kind.

While mathematically complete, (7.2) is physically incomplete, because α involves n_0 and not the averaged laboratory density $\langle N\rangle$. To get $\langle N\rangle$ we compute the moment $\langle\gamma/\Delta\rangle$, since

$$\langle N\rangle = n_0\Delta_0\langle\tfrac{\gamma}{\Delta}\rangle = n_0\Delta_0\gamma_*^2\left[\frac{1+\left(\frac{\gamma_0}{\Delta_0\gamma_*^2}-1\right)K'^2F}{2E-K'^2F}\right] \tag{7.3}$$

So that our final form of the dispersion relation is

$$\left(\frac{2}{\pi}\,\frac{\omega^2\nu_0}{\gamma_* W_p}\right)\left[\frac{2E - K'^2F + \left(\frac{\gamma_0}{\Delta_0\gamma_*^2} - 1\right)K'^2F}{2K}\right] = 1 \qquad (7.4)$$

In addition, we compute the flux of each species

$$J_x = cn_0\Delta_0\langle U/\Delta\rangle = \frac{\langle N\rangle c}{\beta}\left\{1 + \frac{\left(\frac{U_0}{U_*\Delta_0\gamma_*} - 1\right)K'^2F}{2E - K'^2F}\right\} \qquad (7.5)$$

Exactly as for electrostatic plasma oscillations, the choice $U_0 = \gamma_0/\beta$ is a special one, for then $\Delta_0\gamma_*^2 = \gamma_0$ and $\langle\gamma/\Delta\rangle = \gamma_*^2$ <u>independent</u> of wave amplitude q. Similarly, $J_x = \langle Nc/\beta\rangle$ independent of q. But with (7.3) - (7.5) we can study other choices of U_0 and γ_0 rigorously.

First, we touch briefly upon the small amplitude limit $K^2 \to 0$, $K'^2 \to 1$. Here we have $\langle\gamma/\Delta\rangle = \gamma_0/\Delta_0$ and $\langle U/\Delta\rangle = \langle U_0/\Delta_0\rangle$, as expected. The dispersion relation is

$$\gamma_*^2 = \omega^2/\omega_{p0}^2 \qquad (7.6)$$

which is invariant to choice of plasma streaming velocity, or, equivalently, frame of reference. In the small amplitude limit, the wave has the form of a sine function.

In the large amplitude limit, $K^2 \to 1$, $K'^2 \to 0$, we find, using standard expansions for F and E and keeping the first two significant terms

$$1 = \frac{2}{\pi}\frac{\omega^2\nu_0}{\gamma_* W_p^2}\left[1 + \left(\frac{\gamma_0}{\Delta_0\gamma_*^2} - 1\right)\frac{K'^2}{2}\ln\frac{4}{K'}\right] \tag{7.7}$$

$$\langle N\rangle = n_0\Delta_0\gamma_*^2\left[1 + \left(\frac{\gamma_0}{\Delta_0\gamma_*^2} - 1\right)\frac{K'^2}{2}\ln\frac{4}{K'}\right] \tag{7.8}$$

$$J_x = \langle\frac{NC}{\beta}\rangle\left\{1 + \left(\frac{U_0}{U_*\Delta_0\gamma_*} - 1\right)\frac{K'^2}{2}\ln\frac{4}{K'}\right\} \tag{7.9}$$

In the limit $K'^2 \to 0$ (7.7) corresponds exactly to the dispersion relation obtained by Kennel et al. (1973). This wave is not a sine wave but a "sawtooth"; $dE_y/d\eta$ is virtually constant between $\eta = 0$ and $\eta = \pi$ and then abruptly changes sign. In addition, we see from (7.8) - (7.9) that for sufficiently large amplitude, small K'^2, the choice of initial conditions $(\Delta_0, \gamma_0, U_0)$ simply dpes not matter. When its amplitude is sufficiently large, the wave determines the mean properties of the plasma, and not vice versa. The most significant conclusion is that the wave imposes upon the plasma a flux of energetic particles $\langle Nc/\beta\rangle$.

It is straightforward but complicated to compute the particle energy flux, which involves $\langle\frac{U\gamma}{\Delta}\rangle$. We shall not write the results here, but simply state that as Kennel et al. (1973) noticed, the electromagnetic and particle energy fluxes are equipartitioned in the limit $q \gg 1$.

The energy flux F equals the particle energy flux F_p plus the Poynting flux F_{em}, and both equal $cE_{y0}^2/12\pi\beta$, so that $F = cE_{y0}^2/6\pi\beta$. We may then rewrite the $q \gg 1$ dispersion relation in terms of the particle number flux J and the total energy flux F, a form useful to pulsar physics

$$\gamma_* = \frac{\beta}{\sqrt{\beta^2 - 1}} = \frac{3}{\pi \nu_0} \frac{F}{JMc^2} \tag{7.10}$$

It is evident that (7.10) is independent of the value of the particle mass, consistent with our assumption of a charged photon gas in the $q \ll 1$ limit.

All the results of this chapter apply equally well to the relativistic nonlinear ordinary mode propagating perpendicular to the magnetic field, with E_z and $\langle B_y \rangle$ non-zero.

8. Transverse Waves with Non-Zero Average Electric and Magnetic Fields

Here we pass immediately to the $q \to 0$ large amplitude limit of the full set of nonlinear equations (5.11) - (5.14). Our first task is to classify the zeros of the denominator and numerator of (5.11). The zeros of the denominator correspond to points where $\Delta = 0$ and therefore where the proper density would be infinite. Of the four zeros of the numerator, two must be chosen to specify the maximum and minimum electric field so that the singularity at $\Delta = 0$ is avoided. Having chosen a pair of zeroes of the numerator, we then integrate (5.11) approximately to produce a dispersion relation. Then we compute mean values of the plasma parameters of interest and re-express our results in terms of them.

$\Delta = 0$ at the points given by (8.1) below

$$y = \frac{1+y_3}{2} \pm \sqrt{\left(\frac{1-y_3}{2}\right)^2 + q/8} \tag{8.1}$$

Since $q > 0$, (8.1) indicates that one zero of Δ occurs for $y > 1$ and the other for $y < y_3$.

While (8.1) is valid for arbitrary values of q, we solve for the four zeros of the numerator only in the limit $q \to 0$, keeping corrections to order q; the four zeros are given by (8.2ad) below

$$y = 1 \tag{8.2a}$$

$$y = 1 + qQ\Big/\left(1 - y_1\right)\left(1 - y_2\right) \tag{8.2b}$$

$$y = y_1 + q\left(1 - y_1 + Q\right)\Big/\left(1 - y_1\right)\left(y_1 - y_2\right) \tag{8.2c}$$

$$y = y_2 - q\left(1 - y_2 + Q\right)\Big/\left(1 - y_2\right)\left(y_1 - y_2\right) \tag{8.2d}$$

where Q is defined following (5.11).

Of the four roots (8.2ad) we must choose two which lie between the two roots of $\Delta = 0$ given by (8.1). It is clear, from equations (5.15) and (5.16), that $y_1 > y_3$ if $B/E_{y0} > 0$ and $y_2 > y_3$ if $\bar{B}/E_{y0} > 0$. Henceforth we will only consider the case $y_1 > y_3$, since the case $y_2 > y_3$ can be treated by exact analogy. A consistent choice is to take the smaller of (8.2ab) and the larger of (8.2cd), which when $\bar{B}/E_{y0} > 0$ is (8.2c), so that the electric field oscillates between the approximate limits $y_1 < y < 1$. If $y_1 = -1$ we recover the case treated in Chapter 7. All other values of y_1 correspond to non-zero mean fields. If $y_1 < 0$ the electric field oscillates between a positive maximum and a negative minimum; if $y_1 = 0$, the electric field oscillates between a positive maximum and zero minimum and the mean electric field is positive. As $y_1 \rightarrow 1$, the amplitude of the electric field oscillation approaches zero, whereas the mean electric field approaches E_{y0}. $y_1 \rightarrow 1$ corresponds to small amplitude waves. When $y_1 > 1$, no oscillations are possible, since $(dy/d\eta)^2 < 0$ for $1 < y < y$.

Assuming $W_0 = 0$, it is easy to show for superluminous waves that the condition $y_1 \leq 1$ also requires $|E_{y0}/B_{z0}| > 1$, whereas

$Q = 0$ requires $|E_{y0}/B_{z0}| > 1$ when $W_0 = 0$. Since we may then assume $Q \neq 0$, even though when $q \ll 1$ the phase integral may very nearly be singular near $y = 1$, we are certain that it is never truly singular. Only periodic nonlinear waves are possible.

In the limit $q \to 0$, therefore, we make no significant error in approximating (5.11) by

$$\left(\frac{\alpha\nu_0}{\gamma_*}\frac{dy}{d\eta}\right)^2 \simeq (y - y_1)(y - y_2)/(y - y_3)^2 \tag{8.3}$$

and the dispersion relation by

$$\left(\frac{\alpha\nu_0}{\gamma_*\pi}\right)\int_{y_1}^{1}\frac{(y - y_3)dy}{\sqrt{(y - y_1)(y - y_2)}} = 1 \tag{8.4}$$

Equations (8.3) and (8.4) have solutions which are independent of the rest mass

$$\eta = \pi\left\{\left[(y - y_3)^2 - (y_1 - y_3)^2\right]/\left[(1 - y_3)^2 - (y_1 - y_3)^2\right]\right\} \tag{8.5}$$

$$(\alpha\nu_0/\gamma_*\pi)\sqrt{(1 - y_3)^2 - (y_1 - y_3)^2} = 1 \tag{8.6}$$

The formal solution (8.5) - (8.6) indicates that both oscillatory <u>and</u> mean properties of the plasma are determined solely by the choice of minimum and maximum electric field amplitudes, together with the magnetic field in the space-independent frame and the plasma properties $(n_0, u_0, \gamma_0, W_0)$ at $y = 1$. However, it is more illuminating physically to describe the plasma and its wave in terms of the mean laboratory frame density, electric and magnetic field, and velocity.

Let us compute first the mean laboratory frame plasma density $\langle N\rangle$

$$\langle N\rangle = n_0\Delta_0\gamma_*^2\left\{1+\frac{\mathcal{B}_z}{\beta\mathcal{B}}G(r)\right\} \tag{8.7}$$

where we have used the definitions (5.12) - (5.14), defined the ratio of roots r

$$r \equiv \frac{1-y_3}{y_1-y_3} = \frac{E_{y0}-U_*\mathcal{B}_z}{\gamma_*\mathcal{B}} = \frac{\mathcal{E}_{y0}}{\mathcal{B}} \tag{8.8}$$

and the function G(r) which emerges from the integration of γ/Δ

$$G(r) = \frac{\sinh^{-1}\sqrt{r^2-1}}{\sqrt{r^2-1}} \tag{8.9}$$

We note that the condition $y_1 \le 1$ ensures that $r \ge 1$. When $y_1 = -1$, $G(r) \to 0$, and (8.7) reduces to the result (7.9) in the limit $K'^2 \to 0$.

We now compute the average laboratory frame electric and magmagnetic field components, again using the definitions (5.11) - (5.14):

$$\langle E_y\rangle = U_*\mathcal{B}_z + \gamma_*\mathcal{B}G(r) \tag{8.10}$$

$$\langle B\ \rangle = \gamma_*\mathcal{B}_z + U^*\mathcal{B}G(r) \tag{8.11}$$

Equations (8.10) and (8.11) lead to an immediate interpretation of the function G. If we choose

$$G = \langle \mathcal{E}_y \rangle \big/ \langle \mathcal{B} \rangle \tag{8.12}$$

Equations (8.10) and (8.11) reduce to the Lorentz transformation of the mean electric and magnetic fields between the laboratory and space-independent frames. We also note that if we choose $\mathcal{B} = 0$ in the space-independent frame, $\langle B \rangle = 0$ in the laboratory frame, as it should be, since for $\mathcal{B} \to 0$, $r \to 1$ and $G \to 0$. Similarly $\langle E_y \rangle = 0$ when $\mathcal{B} = 0$.

Finally it is illuminating to compute the mean laboratory frame flux vector $\vec{J}$, which involves $\langle \frac{\gamma W}{\Delta} \rangle$, using (8.7) - (8.12)

$$\vec{J} = \langle N \rangle \left\{ \left(\frac{c}{\beta} \hat{e}_x + \vec{\mathcal{V}}_E \right) \Big/ \left(1 + \mathcal{V}_{Ex}/c\beta \right) \right\} \tag{8.13}$$

where

$$\mathcal{V}_E = \frac{c \langle \mathcal{E} \rangle \times \mathcal{B}}{\mathcal{B}^2} \tag{8.14}$$

is the mean $\vec{E} \times \vec{B}$ plasma drift measured in the space-independent frame. Equation (8.13) is a generalization of our previous results, where there were no mean fields. A superstrong wave imposes upon the plasma a mean drift which is the relativistic addition of the characteristic velocity $c/\beta \vec{e}_x$ and the mean $\vec{E} \times \vec{B}$ drift in the space-independent frame.

Using (5.15) - (5.16) and (8.7) - (8.14), we rewrite partially in terms of mean quantities

$$\left\{\frac{\gamma_* \tilde{\Omega}}{\nu_0}\sqrt{r^2 - 1} + \frac{U_* \tilde{\Omega}_z}{\nu_0} \sinh^{-1}\sqrt{r^2 - 1}\right\} = \frac{\pi}{2} \frac{W_p^2}{\omega^2 \nu_0} \gamma_* \qquad (8.15)$$

where

$$\tilde{\Omega} = \frac{e}{Mc\omega}\sqrt{B_x^2 + \tilde{B}_z^2} \qquad \text{and} \qquad \tilde{\Omega}_z = e\tilde{B}_z\ Mc\omega.$$

when $B \to 0$, the left hand side of (8.15) reduces to unity and (8.15) therefore reduces to (7.8) in the limit $K'^2 \to 0$.

In general, (8.15) is a complicated transcendental dispersion relation for β, especially since $\tilde{\Omega}$, $\tilde{\Omega}_z$ and r contain β. One simple result can be retrieved from (8.15) however; the cutoff frequency at which $\beta \to \infty$. Holding all other quantities finite while letting β approach infinity leads to

$$\left(\frac{\pi}{2} \frac{W_{pe}^2}{\omega^2 \nu_0}\right)^2 = 1 - \omega_c^2 \big/ \omega^2 \nu_0^2 \qquad (8.16)$$

where $\omega_c = \frac{e}{Mc}\left(B_x^2 + \langle B_z \rangle^2\right)^{\frac{1}{2}}$ is the cyclotron frequency based upon the average laboratory frame magnetic field. In addition, $\langle E_y \rangle \to 0$ as $\beta \to \infty$. A strong magnetic field <u>lowers</u> the cutoff density for a wave of given laboratory frame frequency ω.

9. Summary and Discussion

When the wave energy density greatly exceeds the rest mass energy density, electrons and ions behave alike, since their inertia is determined not by their rest masses, but by the kinetic energy they acquire from the wave itself. In other words, if we can neglect the rest mass energy relative to the kinetic, it seems reasonable also to neglect differences in rest mass energy between species. In the two cases where we can easily calculate with different ion and electron masses, namely electrostatic plasma oscillations and circularly polarized electromagnetic waves, the dispersion relation does become independent of rest mass in the limit of large amplitudes. This suggests than an "electron-positron" plasma may be a convenient model to describe super-relativistic plasma waves.

In an electron-positron plasma, the classic problem posed by Akhiezer and Polovin (1956) has a simple complete solution in terms of elliptic functions. Beyond assuming $M_i = M_e$, no other approximations need be made. In this case, the simplification stems from the elimination of the electrostatic electric field component parallel to the direction of wave propagation. The existence of a simple solution ought to facilitate other investigations, such as that of the stability of super-relativistic waves, or the inclusion of radiation reaction in the equations of motion.

When non-zero average electric and magnetic fields are added to the Akhiezer-Polovin (1956) problem, the electron-positron model enables us to show that the super-relativistic extraordinary mode satisfies a second-order ordinary differential equation with a first integral in potential form, a fact which would only have emerged after a complex limiting process if we had started with unequal ion and electron masses and a multidimensional potential.

From the various investigations reported in this paper we have been able to abstract several apparently general conclusions. First, it seems that only $\beta > 1$, superluminous, waves can attain arbitrarily large amplitudes, at least within the present cold two-fluid theory. Subluminous waves, with $\beta < 1$, can encounter a density singularity at finite amplitude. An example of such a case--an Alfvénic soliton propagating perpendicular to the magnetic field--was discussed in Chapter 6. Secondly, just as a transformation to the time-independent frame $U = \gamma\beta$ is useful in treating subluminous waves, the transformation to the space-independent frame $U = \gamma/\beta$ is useful for superluminous waves, as Clemmow (1974) has recently emphasized. Indeed the averaged electric and magnetic fields naturally expressed themselves in this frame in our treatment of the super-relativistic extraordinary mode.

Finally, when the wave amplitude is truly large, $q \ll 1$, the wave determines the average properties of the plasma, and not vice versa. This conclusion is most forcefully expressed by our computations of the laboratory frame particle fluxes associated with linearly polarized waves. There, we found that the particle flux is uniquely determined by the wave in the large amplitude limit. In general the flux is the relativistic addition of the characteristic flux Nc/β associated with the space-independent frame and the flux associated with the average $\overline{E} \times \overline{B}$ drift measured in that frame.

As far as pulsar magnetospheres and cosmic ray acceleration are concerned, we have reached the following speculative conclusions. First, only superluminous plasma waves are likely to have the large amplitudes suggested in the original pulsar theories. Second, associated with each superluminous linearly polarized mode is a characteristic cosmic ray number and energy flux. Third, our results do not vitiate the conclusion that, due to the $\beta \to \infty$ cutoff, the wave solution is restricted to relatively low plasma densities, or equivalently to a small flux of high energy cosmic rays (Kennel et al., 1973; Asseo et al., 1975). Thus the only way a pulsar can deliver a larger number flux to its nebula is through a radial outflow "wind" solution. Finally, since the addition of an average magnetic field lowers the cutoff density, it may not be terribly realistic to think of a mixed "wind-wave" solution, at least involving the extraordinary mode.

<u>Acknowledgements.</u> It is a pleasure to acknowledge the help of Estelle Asseo, who, in addition to joining many conversations, worked actively with us to produce the solution presented in Chapter 6. C.F. Kennel acknowledges the gracious hospitality of the Centre de Physique Théorique, Ecole Polytechnique, where he was a visitor during 1974 - 1975. This work was supported

by NASA grant NGL 05-007-190 and by the Alfred P. Sloan Foundation.

References

Akhiezer A.I. and Polovin R.V. 1956 Sov. Phys. JETP 3, 696.

Asseo E., Kennel C.F. and Pellat R. 1975 Astron. Astrophys. submitted.

Clemmow P.C. 1974 J. Plasma Phys. 12, 297.

Gold T. 1968 Nature 218, 731.

Goldreich P. and Julian W. 1969 Ap. J. 157, 869.

Gunn J.E. and Ostriker J. 1969 Phys. Rev. Lett. 22, 728.

Kaw P. and Dawson J. 1970 Phys. Fluids 13, 472.

Kennel C.F., Schmidt G. and Wilcox T. 1973 Phys. Rev. Lett. 31, 1364.

Max C. 1973 Phys. Fluids 16, 1277.

Max C. and Perkins F. 1971 Phys. Rev. Lett. 27, 1342.

Max C. and Perkins F. 1972 Phys. Rev. Lett. 29, 1731.

Michel F.C. 1969 Phys. Rev. Lett. 23, 248.

Ostriker J. and Gunn J.E. 1969 Ap. J. 157, 1395.

Pacini F. 1968 Nature 219, 145.

Sagdeev R.Z. 1966 In Reviews of Plasma Physics 4: M.A. Leontovich, ed., Consultants Bureau, N.Y.

Sturrock P. 1971 Ap. J. 164, 529.

Tidman D.A. and Krall N. 1971 Shock Waves in Collisionless Plasmas, Wiley, N.Y.

PRECEDING PAGE BLANK NOT FILMED

ORIGINAL PAGE IS OF POOR QUALITY

UCLA PLASMA PHYSICS GROUP REPORTS

PPG-1 "Propagation of Ion Acoustic Waves Along Cylindrical Plasma Columns", A.Y. Wong (July 1965). Phys. Fluids 9, 1261 (1966).

PPG-2 "Stability Limits for Longitudinal Waves in Ion Beam-Plasma Interaction", B.D. Fried and A.Y. Wong (August 1965). Phys. Fluids 9, 1084 (1966).

PPG-3 "The Kinetic Equation for an Unstable Plasma in Parallel Electric and Magnetic Fields", B.D. Fried and S.L. Ossakow (November 1965). Phys. Fluids 9, 2428 (1966).

PPG-4 "Low-Frequency Spatial Response of a Collisional Electron Plasma", B.D. Fried, A.N. Kaufman and D.L. Sachs (August 1965). Phys. Fluids 9, 292 (1966).

PPG-5 "Effects of Collisions on Electrostatic Ion Cyclotron Waves", A.Y. Wong, D. Judd and F. Hai (December 1965). Phys. Letters, 21, 157 (1966).

PPG-6 "Interaction Between Ion Beams and Plasmas", R. Rowberg, A.Y. Wong and J.M. Sellen (April 1966). APS Bull. 10, 1182 (1965).

PPG-7 "Observation of Cyclotron Echoes from a Highly Ionized Plasma", D.E. Kaplan and R.M. Hill (May 1966) Phys. Lett. 21, 157 (1966).

PPG-8 "Excitation and Damping of Drift Waves", A.Y.Wong and R. Rowberg (July 1966). Proceedings of 1st Int. Conf. on Quiescent Plasma, Frascati, Rome, 1967. Phys. Rev. Lett. 18, 526 (1967).

PPG-9 "The Guiding Center Approximation in Lowest Order", A. Banos, Jr. (Sept. 1966). J. Plasma Phys. 1, 305 (1967).

PPG-10 "Plasma Streaming into a Magnetic Field", S.L. Ossakow (Nov. 1966) Dissertation.

PPG-11 "Cooperative Effects in Plasma Echo Phenomena", A.Y. Wong (March 1967). Proc. of 1st Inter. Conf. on Quiescent Plasmas, Frascati, Rome, 1967.

PPG-12 "A Quantum Mechanical Study of the Electron Gas Via the Test Particle Method", M.E. Rensink (March 1967). Dissertation

PPG-13 "Linear and Nonlinear Theory of Grid Excitation of Low Frequency Waves in a Plasma", G.L. Johnston (April 1967). Dissertation.

PPG-14 "The Expansion and Diffusion of an Isolated Plasma Column", J. Hyman (May 1967). Dissertation

PPG-15 "Two-Pole Approximation for the Plasma Dispersion Function", B.D. Fried, C.L. Hedrick and J. McCune (Aug. 1967). Phys. Fluids 11, 249 (1968).

PPG-16 "Experimental Investigation of Electron Runaway Phenomena", J.S. DeGroot (Aug. 1967). Dissertation

PPG-17 "Parametric Coupling Between Drift Waves", F. Hai, R. Rowberg and A.Y. Wong (Oct. 1967). Proc. of 2nd Int. Symp. on Fluctuations and Diffusion in Plasmas, June, 1967.

PPG-18 "Cyclotron Echoes from Doppler Effects", A.Y. Wong (March 1968).

PPG-19 "Ion Wave Echoes", D.R. Baker, N.R. Ahern and A.Y. Wong (Nov. 1967). Phys. Rev. Lett. 20, 318 (1968).

PPG-20 "Cyclotron Echoes in Plasmas", D. Judd (March 1968). Dissertation

PPG-21 "Test Particle Theory for Quantum Plasmas", M.E. Rensink (Oct. 1967). Phys. Rev. 164, 175 (1967).

PPG-22 "Artificial Van Allen Belt", C.F. Kennel (Nov. 1967).

PPG-23 "Landau Damping of Ion Acoustic Waves in a Cesium Plasma with Variable Electron-Ion Temperature Ratio", K.B. Ranjangam (Oct. 1967). Dissertation

PPG-24 The Inhomogeneous Two-Stream Instability", G. Knorr (Sept. 1967).

PPG-25 "Magnetic Turbulence in Shocks", C.F. Kennel and H.E. Petschek (Dec. 1967). in Physics of the Magnetosphere, R. Carovillano, J.F. McClay, and H.R. Rudoski, eds. 485-513, D. Reidel, Dordrecht, Holland, 1968.

PPG-26 "Small Amplitude Waves in High Beta Plasmas", V. Formisano and C.F. Kennel (Feb. 1968). J. Plasma Phys. 3, 55 (1969).

PPG-27 "Low Beta Plasma Penetration Across a Magnetic Field", B.D. Fried and S. Ossakow (Mar. 1968). Phys. Fluids 12, 702 (1969)

PPG-28 "Annual Status Report", Feb. 1, 1967-Jan. 31, 1968, Principal Investigators, A. Banos, Jr., B.D. Fried, C.F. Kennel.

PPG-29 "The Theorist's Magnetosphere", C.F. Kennel (April 1968).

PPG-30 "Electromagnetic Pitch Angle Instabilities in Space", C.F. Kennel and F.L. Scarf (April 1968) in Plasma Waves in Space and in the Laboratory, ed. J.O. Thomas and B.J. Landmark, Edinburgh U. Press, Edinburgh, Vol. II, 1969.

PPG-31 "Electromagnetic Echoes in Collisionless Plasmas," A.Y. Wong (April 1968). Phys. Fluids 12, 866 (1969).

PPG-32 "Parametric Excitation of Drift Waves in a Resistive Plasma," G. Weyl and M. Goldman (June 1968). Phys. Fluids 12, 1097 (1969).

PPG-33 "Parametric Excitation from Thermal Fluctuations at Plasma Drift Wave Frequencies," A.Y. Wong, M.V. Goldman, F. Hai, R. Rowberg (May 1968). Phys. Rev. Letters 21, 518 (1968).

PPG-34 "Current Decay in a Streaming Plasma Due to Weak Turbulence," S.L. Ossakow and B.D. Fried (June 1968). Phys. Fluids 11, 274 (1968).

PPG-35 "Temperature Gradient Instabilities in Axisymmetric Systems," C.S. Liu (Aug. 1968). Phys. Fluids 12, 1489 (1969).

PPG-36 "Electron Cyclotron Echo Phenomena in a Hot Collisionless Plasma," D. Judd (Aug. 1968). Dissertation.

PPG-37 "Transverse Plasma Wave Echoes," B.D. Fried and C. Olson (Oct. 1968). Phys. Rev. 180, 214 (1969). also presented at APS Plasma Physics Div. Ann. Meeting, Nov. 1968

PPG-38 "Low Frequency Interchange Instabilities of the Ring Current Belt," C.S. Liu (Jan. 1969). J. Geophys. Res. 75, 3789 (1970

PPG-39 "Drift Waves in the Linear Regime," R.E. Rowberg and A.Y. Wong (Feb. 1969). Phys. Fluids 13, 661 (1970).

PPG-40 "Parametric Mode-Mode Coupling Between Drift Waves in Plasmas," F. Hai and A.Y. Wong (Jan. 1969). Phys. Fluids 13, 672 (1970).

PPG-41 "Nonlinear Oscillatory Phenomena wtth Drift Waves in Plasmas," F. Hai and A.Y. Wong (Sept. 1970).

PPG-42 "Ion-Burst Excited by a Grid in a Plasma," H. Ikezi and R.J. Taylor (Feb. 1969). J. Appl. Phys. 41,738 (1970).

PPG-43 "Measurements of Diffusion in Velocity Space from Ion-Ion Collisions," A. Wong and D. Baker (March 1969). Phys. Rev. 188, 1 (1969).

PPG-44 "Nonlinear Excitation in the Ionosphere," A.Y. Wong (March 1969).

PPG-45 "Observation of 1st order Ion Energy Distribution in Ion Acoustic Waves," H. Ikezi, R. Taylor, Phys. Rev. Lett. 22, 923, (1969

PPG-46 "A New Representation for the Conductivity Tensor of a Collisionless Plasma in a Magnetic Field," B.D. Fried and C. Hedrick (March 1969). Festschrift for Gregor Wetzel, U. of Chicago Press, 1969.

PPG-47 "Direct Measurements of Linear Growth Rates and Nonlinear Saturation Coefficients," A.Y. Wong and F. Hai (April 1969). Phys. Rev Lett. 23, 163 (1969).

PPG-48 "Electron Precipitation Pulsations," F. Coroniti and C.F. Kennel (April 1969). J. Geophys. Res. 75, 1279 (1970).

PPG-49 "Auroral Micropulsation Instability," F. Coroniti and C.F. Kennel (May 1969). J. Geophys. Res. 75, 1863 (1970).

PPG-50 "Effect of Fokker-Planck Collisions on Plasma Waves Echoes," G. Johnston (June 1969). Phys. of Fluids 13, 136 (1970).

PPG-51 "Linear and Nonlinear Theory of Grid Excitation of Low Frequency Waves in a Plasma," G. Johnston (July 1969).

PPG-52 "Theory of Stability of Large Amplitude Periodic (BGK) Waves in Collisionless Plasmas," M.V. Goldman (June 1969). Phys. Fluid 13, 1281 (1970).

PPG-53 "Observation of Strong Ion Wave-Wave Interaction," R. Taylor and H. Ikezi (Aug. 1969).

PPG-55 "Optical Mixing in a Magnetoactive Plasma," G. Weyl (Aug. 1969). Phys. Fluids 13, 1802 (1970).

PPG-56 "Trapped Particles and Echoes," A.Y. Wong and R. Taylor (Oct. 1969). Phys. Rev. Lett. 23, 958 (1969).

PPG-57 "Formation and Interaction of Ion-Acoustic Solitons," H. Ikezi, R.J. Taylor and D.R. Baker, Phys. Rev. Lett. 25, 11, (1970

PPG-58 "Observation of Collisionless Electrostatic Shocks," R. Taylor, D. Baker and H Ikezi, Phys. Rev. Lett. 24, 206 (1970).

PPG-59 "Turbulent Loss of Ring Current Protons," J.M. Cornwall, F.V. Coroniti and R.M. Thorne (Jan 1970). J. Geophys. Res. 75, 4699 (1970).

PPG-60 "Efficient Modulation Coupling Between Electron and Ion Resonances in Magnetoactive Plasmas," A. Wong, D.R. Baker, N. Booth (Dec. 1969). Phys. Rev. Lett. 24, 804 (1970).

PPG-61 "Interaction of Qusi-Transverse and Quasi-Longitudinal Waves in an Inhomogeneous Vlasov Plasma," C.L. Hedrick (Jan. 1970) Dissertation.

PPG-62 "Observation of Strong Ion-Acoustic Wave-Wave Interaction," R.J. Taylor and H. Ikezi (Jan. 1970).

PPG-63 "Perturbed Ion Distributions in Ion Waves and Echoes," H. Ikezi and R.J. Taylor (Jan. 1970). Phys. Fluids 13, 2348 (1970)

PPG-64 "Propagation of Ion Cyclotron Harmonic Wave," E.R. Ault and H. Ikezi (Nov. 1970). Phys. Fluids 13, 2874 (1970).

PPG-65 "The Analytic and Asymptotic Properties of the Plasma Dispersion Function," A. Banos, Jr. and G. Johnston (Feb. 1970).

PPG-66 "Effect of Ion-Ion Collision and Ion Wave Turbulence on the Ion Wave Echo," D. Baker (June 1970).

PPG-67 "Dispersion Discontinuities of Strong Collisionless Shocks," F.V. Coroniti (Mar. 1970). J. Plasma Phys. 4, 265 (1970)

PPG-68 "An Ion Cyclotron Instability," E.S. Weibel (Apr. 1970). Dissertation.

PPG-69 "Turbulence Structure of Finite-Beta Perpendicular Fast Shocks," F.V. Coroniti (Apr. 1970). J Geophys. Res. 75, 7007 (1970)

PPG-70 "Steepening of Ion Acoustic Waves and Formation of Collisionless Electrostatic Shocks," R. Taylor (April 1970)

PPG-71 "A Method of Studying Trapped Particles Behavior in Magnetic Geometries," C.S. Liu and A.Y. Wong (Apr. 1970). Phys. Rev. Lett. 25, 1702 (1970).

PPG-72 "A Note on the Differential Equation $g'' + x^2g = 0$''," E.S. Weibel (April 1970).

PPG-73 "Plasma response to a Step Electric Field Greater than the Critical Runaway Field, With and Without an Externally Applie Magnetic Field," J.E. Robin (June 1970). Dissertation.

PPG-74 "The UC Mathematical On-Line Systems as a Tool for Teaching Physics," B.D. Fried and R.B. White (Aug. 1970). Proc. of Computer Graphics, 136-154, 1970.

PPG-75 "High Frequency Hall Current Instability," K. Lee, C.F. Kennel, J.M. Kindel (Aug. 1970). Radio Sci. 6, 209 (1971).

PPG-76 "Laminar Wave Train Structure of Collisionless Magnetic Slow Shocks," F.V. Coroniti (Sept. 1970). Nucl. Fusion 11, 261 (1971).

PPG-77 "Field-Aligned Current Instabilities in the Topside Ionosphere," J.M. Kindel and C.F. Kennel (Aug. 1970). J. Geophys. Res. 76, 3005, (1972).

PPG-78 "Spatial Cyclotron Damping," Craig Olson (Sept. 1970). Phys. Fluids 15, 160 (1972).

PPG-79 "Electromagnetic Plasma Wave Propagation Along a Magnetic Field, C.L. Olson (Sept. 1970). Dissertation.

PPG-80 "Electron Plasma Waves and Free-Streaming Electron Bursts," H. Ikezi, P.J. Barrett, R.B. White and A.Y. Wong (Nov. 1970) Phys. Fluids 14, 1997 (1971).

PPG-81 "Relativistic Electron Precipitation During Magnetic Storm Main Phase," R.M. Thorne and C.F. Kennel (Nov. 1970). J. Geophys. Res. 76, 4446 (1971).

PPG-82 "A Unified Theory of SAR-Arc Formation at the Plasmapause," J.M. Cornwall, F.V. Coroniti and R.M. Thorne (Nov. 1970). J. Geophys. Res. 76, 4428 (1971).

PPG-83 "Nonlinear Collisionless Interaction Between Electron and Ion Modes in Inhomogeneous Magnetoactive Plasmas," N. Booth (Dec. 1970). Dissertation.

PPG-84 "Observation of Parametrically Excited Ion Acoustic Waves," R. Stenzel (March 1971).

PPG-85 "Remote Double Resonance Coupling of Radar Energy to Ionospheric Irregularities," C.F. Kennel (Jan. 1971). Comm. Astro. and Space Phys. 3, 87 (1971).

PPG-86 "Ion Acoustic Waves in a Multi-Ion Plasma," B.D. Fried, R.B. White, T. Samec (Jan. 1971). Phys. Fluids 14, 2388 (1971).

PPG-87 "Current-Driven Electrostatic and Electromagnetic Ion Cyclotron Instabilities," D.W. Forslund, C.F. Kennel, J.M. Kindel (Feb. 1971).

PPG-88 "Locating the Magnetospheric Ring Current," C.F. Kennel and Richard Thorne (Mar. 1961). Comm. on Astrophys. and Space Phys. C3, 115 (1971).

PPG-89 "Ion Acoustic Instabilities Due to Ions Streaming Across Magnetic Field," P.J. Barrett, R.J. Taylor (March 1971).

PPG-90 "Evolution of Turbulent Electronic Shocks," A.Y. Wong and R. Means (July 1971). Phys. Rev. Lett. 27, 973 (1971).

PPG-91 "Density Step Production of Large Amplitude Collisionless Electrostatic Shocks and Solitons," D.B. Cohen (June 1971). Submitted Phys. Rev. Lett. (1973).

PPG-92 "Turbulent Resistivity, Diffusion and Heating," B.D. Fried, C.F. Kennel, K. MacKenzie, F.V. Coroniti, J.M. Kindel, R. Stenzel, R.J. Taylor, R.B. White, A.Y. Wong, W. Bernstein, J.M. Sellen, Jr., D. Forslund and R.Z. Sagdeev (June 1971) Proc. of the 4th Conf. on Plasma Phys. and Cont. Nucl. Fusion Res., Madison, Wis., IAEA-CN-28/E-4, Plasma Physics and Cont. Nucl. Fus. Res., Vol II, Vienna 1971.

PPG-93 "Nonlinear Evolution and Saturation of an Unstable Electrostatic Wave," B.D. Fried, C.S. Liu, R.W. Means and R.Z. Sagdeev (Aug. 1971). Submitted Doklady, 1973.

PPG-94 "Cross-Field Current-Driven Ion Acoustic Instability," P.J. Barrett, B.D. Fried, C.F. Kennel, J.M. Sellen and R.J. Taylo (Dec. 1971). Phys. Rev. Lett. 28, 337 (1972).

PPG-95 "3-D Velocity Space Diffusion in Beam-Plasma Interaction Without Magnetic Field," P.J. Barrett, D. Gresillon and A.Y. Wong (Sept. 1971). Proc. of 3rd Int. Conf.on Quiescent Plasmas, 291, Elsinore, Calif. 1971. Submitted Phys. Rev. Lett.

ORIGINAL PAGE IS OF POOR QUALITY

PPG-96 "Dayside Auroral Oval Plasma Density and Conductivity Enhancements Due to Magnetosheath Electron Precipitation," C.F. Kennel and M.H. Rees (Sept. 1971). J. Geophys. Res. 77, 2294 (1972).

PPG-97 "Collisionless Wave-Particle Interactions Perpendicular to the Magnetic Field," A.Y. Wong, D.L. Jassby (Sept. 1971). Phys. Rev. Lett. 29, 41 (1972).

PPG-98 "Magnetospheric Substorms," F.V. Coroniti and C.F. Kennel (Sept. 1971). in Cosmic Plasma Physics, Proc. of the Conf. on Cosmic Plasma Phys., held at the European Space Res. Inst., Frascati, Italy, Sept. 1971. Plenum Press, 1972, ed. Karl Schindler.

PPG-99 "Magnetopause Motions, DP-2, and the Growth Phase of Magnetospheric Substorms," F.V. Coroniti and C.F. Kennel (Sept. 1971)

PPG-100 "Structure of Ion Acoustic Solitons and Shock Waves in a Two-Component Plasma," R.B. White, B.D. Fried and F.V. Coroniti (Sept. 1971). Phys. Fluids 15, 1484 (1972).

PPG-101 "Solar Wind Interaction with Lunar Magnetic Field," G. Siscoe (Meteorology Dept.) and Bruce Goldstein (JPL) (Nov. 1971).

PPG-102 "Changes in Magnetospheric Configuration During Substorm Growth Phase," F.V. Coroniti and C.F. Kennel (Nov. 1971). J. Geophys. Res. 77, 3361 (1972).

PPG-103 "Trip Report--1971 Kiev Conference on Plasma Theory and Visits to Lebedev and Kurchatov Institutes," B.D. Fried (Oct. 1971).

PPG-104 "Pitch-Angle Diffusion of Radiation Belt Electrons Within the Plasmasphere," L.R. Lyons, R.M. Thorne, C.F. Kennel (Jan. 1972). J. Geophys. Res. 77, 3455 (1972).

PPG-105 "Remote Feedback Stabilization of a High-Beta Plasma," F.F. Chen, D. Jassby and M. Marhic (Dec. 1971). Phys. Fluids 15, 1864 (1972).

PPG-106 "Remote Plasma Control, Heating Measurements of Electron Distribution and Trapped Particles by Nonlinear Electromagnetic Interaction," A.Y. Wong, F.F. Chen, N. Booth, D.L. Jassby, R. Stenzel, D. Baker and C.S. Liu (June 1971). J. of Plasma Phys. and Cont. Nucl. Fus. Res. 1, 335 (1971)

PPG-107 "Computational and Experimental Plasma Physics for Theoreticians," B.D. Fried (Jan. 1972). Advances in Plasma Physics 5 211, 1974.

PPG-108 "Threshold and Saturation of the Parametric Decay Instability," R. Stenzel and A.Y. Wong (Nov. 1971). Phys. Rev. Lett. 28, 274 (1972).

PPG-109 "Laser Amplification in an Inhomogeneous Plasma," R. White (Jan. 1972).

PPG-110 "External Production and Control of Electrojet Irregularities," K. Lee, P.K. Kaw and C.F. Kennel (Jan. 1972). J. Geophys Res. 77, 4197 (1972).

PPG-111 "Ion Heating Via Turbulent Ion Acoustic Waves," R.J. Taylor and F.V. Coroniti (Feb. 1972). Phys. Rev. Lett. 29, 34 (1972

PPG-112 "Polarization of the Auroral Electrojet," F.V. Coroniti and C.R. Kennel (Feb. 1972). J. Geophys. Res. 77, 2835 (1972).

PPG-113 "Mode Coupling and Wave Particle Interactions for Unstable Ion Acoustic Waves," P. Martin and B.D. Fried (Feb., 1972). Phys. Fluids 15, 2275 (1972).

PPG-114 "Parallel Magnetic Multi-pole Confinement of a Magnetic Field-Free Plasma," R. Limpaecher (Mar. 1972) Dissertation. Submitted Rev. Sci. Instr., 1973.

PPG-115 "Turbulence in Electrostatic Collisionless Shock Waves," R.W. Means (Apr. 1972). Dissertation.

PPG-116 "Large Diameter, Quiescent Plasma in a Magnetospheric Field," E. Ault (Apr. 1972). Dissertation.

PPG-117 "Parasitic Pitch-Angle Diffusion of Radiation Belt Particles by Ion-Cyclotron Waves," L.R. Lyons and R.M. Thorne (May 1972). J. Geophys. Res. 77, 5608 (1973)

PPG-118 "A New Role for Infrared Lasers," F.F. Chen (May 1972). Comm. Plasma Phys. and Cont. Fus. 1, 81 (1972).

PPG-119 "Electrostatic Instability of Ring Current Protons Beyond the Plasmapause During Injection Events," F.V. Coroniti, R.W. Fredricks and R.B. White (May 1972). J. Geophys. Res. 77, 6243 (1972).

PPG-120 "Magnetospheres of the Outer Planets," C.F. Kennel (May 1972). Space Scie. Rev. 14, 511 (1973).

PPG-121 "Measurement of Transverse and Longitudinal Heat Flow in a Laser-Heated, Magnetically Confined Arc Plasma," S.W. Fay (June 1972). Dissertation. Condensed version by S.W. Way, F. Chen and D. Jassby, Phys. Lett. 42A, 261 (1972).

PPG-122 "Plasmaspheric Hiss," R.M. Thorne, E.J. Smith, R.K. Burton, R.E. Holzer (July 1972).J. Geophys. Res. 78, 1581 (1973).

PPG-123 "Magnetospheric Electrons," F.V. Coroniti and R.M. Thorne (July 1972). Ann. Rev. of Earth and Planetary Science, 1, 107, 1973.

PPG-124 "Calculation of Reflection and Transmission Coefficients for a Class of One-Dimensional Wave Propagation Problems in Inhomogeneous Media," Alfredo Baños, Jr. (September 1972). J. Math. Phys. 14, 963 (1973).

PPG-125 "Electromagnetic Wave Functions for Parabolic Plasma Density Profiles," Alfredo Baños, Jr. and Daniel L. Kelly (September 1972). Physics of Fluids 17, 2275, 1974.

PPG-126 "Amplification of Electromagnetic Waves in Overdense Plasmas," F.F. Chen and R.B. White (September 1972, revised August 1973). J. Plasma Physics 16, 565 (1974).

PPG-127 "Abstracts presented at the American Physical Society Division of Plasma Physics Annual Meeting, Monterey, November 13-16, 1972".

PPG-128 "Can the Ionosphere Regulate Magnetospheric Convection?" F.V. Coroniti and C.F. Kennel (October 1972). J. Geophys. Res. 78, 2837 (1973).

PPG-129 "Nonlinear Stabilization of Oscillating Two-Stream Instability," K. Nishikawa, Y.C. Lee and P.K. Kaw (October 1972). Physics of Fluids 16, 1380 (1973).

PPG-130 "Drift Waves in Finite Beta Plasmas," Morrell S. Chance (October 1972). Thesis.

PPG-131 "Wave Packet Formulation of Nonlinear Plasma Wave Kinetics," K. Nishikawa and B.D. Fried (October 1972). Physics of Fluids 16, 1321 (1973).

PPG-132 "Electron Cyclotron Drift Instability Experiment," B.H. Ripin and R.L. Stenzel (October 1972). Phys. Rev. Letters 30, 45 (1973).

PPG-133 "Resonant Excitation of Electrostatic Modes with Electromagnetic Waves," G. Schmidt (October 1973). Physics of Fluids 16, 1676 (1973).

PPG-134 "Energetic Ion Beam Source and Free-Stream Beam Diagnostic Techniques," R.L. Stenzel and B.H. Ripin (November 1972). Rev. Sci. Instr. 44, 617 (1973).

PPG-135 "Electron Plasma Waves in an Unbounded Uniform Magnetoplasma," R.L. Stenzel (November 1972). Physics of Fluids 16, 565 (1973).

PPG-136 "Convective Amplification of Type I Irregularities in the Equatorial Electrojet," K. Lee and C.F. Kennel (November 1972), J. Geophys. Res. 78, 4619 (1973).

PPG-137 "Effects of Propagation Parallel to the Magnetic Field on the Type I Electrojet Irregularity Instability," K. Lee and C.F. Kennel (November 1972). Planetary and Space Sciences 21, 1339 (1973).

PPG-138 "Analog Computer Simulation of Parametric Instabilities," R.L. Stenzel (November 1972).

PPG-139 "Theory of Double Resonance Parametric Excitation in Plasmas," D. Arnush, B.D. Fried, C.F. Kennel, K. Nishikawa and A.Y. Wong (November 1972). Physics of Fluids 16, 2270 (1973).

PPG-140 "Filamentation and Trapping of Electromagnetic Radiation in Plasmas," P. Kaw, G. Schmidt and T. Wilcox (December 1972). Physics of Fluids 16, 1522 (1973).

PPG-141 "Finite Beta Drift Alfven Instability," M.S. Chance, F.V. Coroniti and C.F. Kennel (January 1973). J. Geophys. Res. 78, 7521 (1973).

PPG-142 "The Formation of Ion Acoustic Shocks," R.B. White, B.D. Fried and F.V. Coroniti (January 1973). Physics of Fluids 17, 211, 1974.

PPG-143 "Experiments on Parametric Instabilities," A.Y. Wong (March 1973).

PPG-144 "On Cosmic Ray Generation by Pulsars," C.F. Kennel, G. Schmidt and T. Wilcox (March 1973). Phys. Rev. Letters 31, 1364 (1973).

PPG-145 "On the Marginally Stable Saturation Spectrum of Unstable Type I Equatorial Electrojet Irregularities," K. Lee, C.F. Kennel and F.V. Coroniti (April 1973). J. Geophys. Research 79, 249, 1974.

PPG-146 "Spatial Growth Properties of Parametric and Backscattering Plasma Instabilities," B.D. Fried, R. Gould and G. Schmidt (April 1973). Submitted to Phys. Rev. Letters.

PPG-147 "Evolution of BGK-Like Modes with Trapped Electrons," A.Y. Wong, B.H. Quon and B. Ripin (April 1973). Phys. Rev. Letters 30, 1299 (1973).

PPG-148 "Stabilization of Ion Acoustic Waves by Electron Trapping," N. Albright (April 1973). Physics of Fluids 17, 206, 1974.

PPG-149 "Turbulence in Electrostatic Ion Acoustic Shocks," R.W. Means, F.V. Coroniti, A.Y. Wong and R.B. White (May 1973). Physics of Fluids 16, 2304, 1973.

PPG-150 "Theory of Dielectric Function in a Magnetized Plasma," Y.C. Lee and C.S. Liu (June 1973). Submitted to Physics of Fluids.

PPG-151 "Physical Interpretation of the Oscillatory Two-Stream Instability," A.Y. Wong and G. Schmidt (June 1973). Submitted to Physics of Fluids.

PPG-152 "Relativistic Particle Motion in Nonuniform Electromagnetic Waves," G. Schmidt and T. Wilcox (June 1973). Phys. Rev. Letters 31, 1380, 1973.

PPG-153 "The Ring Current and Magnetic Storms," F.V. Coroniti (July 1973). Radio Science 8, 1007, 1973.

PPG-154 "Energetic Electrons in Jupiter's Magnetosphere," F.V. Coroniti (July 1973). Astrophysical Journal 27, 261, 1974.

PPG-155 "Stably Trapped Proton Fluxes in the Jovian Magnetosphere," F.V. Coroniti, C.F. Kennel and R.M. Thorne (July 1973). Astrophysical Journal 189, 383, 1974.

PPG-156 "Absolute Raman Scattering Instabilities in an Inhomogeneous Plasma," J.F. Drake and Y.C. Lee (July 1973). Physical Review Letters 31, 1197 (1973).

PPG-157 "Growth and Saturation of the Absolute Electron Cyclotron Drift Instability," R.L. Stenzel and B.H. Ripin (July 1973). Phys. Rev. Letters 31, 1545 (1973).

PPG-158 "Parametric Instabilities of Electromagnetic Waves in Plasmas," J. Drake, P.K. Kaw, Y.C. Lee, G. Schmidt, C.S. Liu and M.N. Rosenbluth (July 1973). Physics of Fluids 17, 778, (1974).

PPG-159 "Nonlinear Optics of Plasmas," F.F. Chen (August 1973). Survey Lectures. International Congress on Waves and Instabilities in Plasmas, Innsbruck, Austria, 1973 (Institute for Theoretical Physics, Innsbruck) pp. C1 - C19.

PPG-160 "Physical Mechanisms for Laser and Plasma Heating - Parametric Instabilities," F.F. Chen (August 1973). In Laser Interaction and Related Plasma Phenomena, ed. H.J. Schwartz and H. Hora, Vol. 3A p. 291 - 313 (Plenum Press 1974).

PPG-161 "Trip Report on the Sixth European Conference on Controlled Fusion and Plasma Physics, July 30 - August 4, 1973, Moscow," B.D. Fried (August 1973).

PPG-162 "Abstracts presented at the Philadelphia Meeting of the American Physical Society, Division of Plasma Physics, October 31 - November 3, 1973".

PPG-163 "Enhancement of Plasma DC Currents by Intense AC Fields," A.T. Lin and J.M. Dawson, October 1973. Physics of Fluids 17, 987, 1974.

PPG-164 "Temporal Electrostatic Instabilities in Inhomogeneous Plasmas," Y.C. Lee and P.K. Kaw, November, 1973. Physical Review Letters 32, 135 (1974).

PPG-165 "Nonlinear Schrodinger Equation Model of the Oscillating Two-Stream Instability," G.J. Morales, Y.C. Lee and R.B. White, December 1973. Phys. Rev. Letters 32, 457 (1974).

PPG-166 "Backscattering Decay Processes in Electron Beam-Plasma Interactions Including Ion Dynamics," B.H. Quon, A.Y. Wong and B.H. Ripin, December 1973. Phys. Rev. Letters 32, 406, 1974.

PPG-167 "Conversion of Electromagnetic Waves to Electrostatic Waves in Inhomogeneous Plasmas," R. Stenzel, A.Y. Wong and H.C. Kim, December 1973. Phys. Rev. Letters 32, 654, 1974.

PPG-168 "Langmuir Wave Turbulence - Condensation and Collapse," Y.C. Lee, C.S. Liu and K. Nishikawa, January 1974. To appear in Comments on Plasma Physics and Controlled Fusion.

PPG-169 "The Consequences of Micropulsations on Geomagnetically Trapped Particles," R.M. Thorne, January 1974. Reviews of Space Science 16, 443, 1974.

PPG-170 "Linear Wave Conversion in Inhomogeneous Plasmas," D.L. Kelly and A. Baños, Jr., March, 1974.

PPG-171 "The Cause of Storm After Effects in the Middle Latitude D-Region Ionosphere," W.N. Spjeldvik and R.M. Thorne, March 1974. Accepted by J. of Atmospheric and Terrestrial Physics.

PPG-172 "Application of an Electromagnetic Particle Simulation Code to the Generation of Electromagnetic Radiation," A.T. Lin, J.M. Dawson and H. Okuda, March 1974. Physics of Fluids 17, 1995, 1974.

PPG-173 "The Ponderomotive Force Exerted on a Plasma by an Infrared Laser Beam," M. Marhic, March 1974 (Dissertation)

OOG-174 "What we have Learned from the Magnetosphere," C.F. Kennel, April 1974. Submitted to Comments on Astrophysics and Space Science.

PPG-175 "Observation of the Ponderomotive Force and Oscillating Two-Stream Instability," H.C. Kim, R. Stenzel and A.Y. Wong, April 1974.

PPG-176 "Electron Beam Plasma Interaction Including Ion Dynamic," B.H. Quon, June 1974. Thesis.

PPG-177 "Linear Conversion and Parametric Instabilities in a Non-Uniform Plasma," H.C. Kim, R. Stenzel and A.Y. Wong, June 1974. Thesis.

PPG-178 "Stimulated Compton Scattering of Electromagnetic Waves in Plasma," A.T. Lin and J.M. Dawson, June 1974. The Physics of Fluids 18, 201, 1975.

PPG-179 "Equatorial Spread F: Low Frequency Modes in a Collisional Plasma," M.K. Hudson, July 1974 (Dissertation).

PPG-180 "Effect of the Ponderomotive Force in the Interaction of a Capacitor RF Field with a Nonuniform Plasma," G.J. Morales and Y.C. Lee, July 1974. Phys. Rev. Letters 33, 1016, 1974.

PPG-181 "Deducation of Ionospheric Tidal Winds by Dynamo Simulation," J.P. Schieldge and S.V. Venkataswaran, August, 1974 Submitted to J. of Atmospheric and Terrestrial Physics.

PPG-182 "Response of the Middle Latitude D-Region to Geomagnetic Storms," W. Spjeldvik and R.M. Thorne, August 1974. Accepted by J. of Atmospheric and Terrestrial Physics.

PPG-183 "Production of Negative lons and Generation of Intense Neutral Beams," A.Y. Wong, J.M. Dawson and W. Gekelman, August 1974

PPG-184 "Development of Cavitons and Trapping of RF Fields," H.C. Kim, R. Stenzel and A.Y. Wong, August 1974. Phys. Rev. Letters 33, 886 (1974).

PPG-185 "Albuquerque Abstracts: Papers presented at Albuquerque Meeting of the American Physical Society Division of Plasma Physics, October 28-31, 1974."

PPG-186 "Localized Quasi-Stationary Plasma Modes in One, Two and Three Dimensions," J. Zitkova Wilcox and T.J. Wilcox, September 1974. Accepted by Physical Review Letters.

PPG-187 "Denouement of Jovian Radiation Belt Theory," F.V. Coroniti, September 1974. Proceedings of Conference on Magnetospheres of the Earth and Jupiter, Frascati, Italy, May 28 - June 1, 1974.

PPG-188 "Is Jupiter's Magnetosphere Like a Pulsar's or Earth's?" C.F. Kennel and F.V. Coroniti, September 1974. Ibid.

PPG-189 "Parametric Instability of the Sheath Plasma Resonance," R. Stenzel, H.C. Kim and A.Y. Wong, September 1974. Bull. Am. Phys. Soc., October 1974.

PPG-190 "Effect of Localized Electric Fields on the Evolution of the Velocity Distribution Function," G.J. Morales and Y.C. Lee, September 1974. Phys. Rev. Letters 33, 1534 (1974).

PPG-191 "Parametric Instabilities in Plasma," J.M. Dawson and A.T. Lin, September 1974.

PPG-192 "Surmac - a Large Surface Magnetic Confinement Device," A. Y. Wong, September 1974.
PPG-193 "Extraction of Energy From High Intensity Ion Beams," A. T. Forrester, September 1974. Presented at the IInd Symposium on Ion Sources and Formation of Ion Beams, Berkeley, Calif. 22-25 October, 1974.
PPG-194 "A Large Quiescent Magnetized Plasma for Wave Studies," W. Gekelman and R. L. Stenzel, December 1974 Accepted Rev. Sci Instr.
PPG-195 "Electrostatic Waves Near the Lower Hybrid Frequency," R. Stenzel and W. Gekelman, October 1974. Accepted Phys. Rev. A
PPG-196 "A Corrugated Mirror-Cyclotron Frequency Direct Conversion System (Comi-Cyfer)," A.T. Forrester, J. Busnardo-Neto and J.T. Crow, October 1974. IEEE Transactions on Plasma Science.
PPG-197 "The Study of Comparative Magnetospheres: The Future of Space Physics," F.V. Coroniti and C.F. Kennel, October 1974. Presented to the NASA Study Group On "Outlook for Space", Goddard Space Flight Center, September 10, 1974.
PPG-198 "Application of the Fokker-Planck Numerical Method to Anisotropic and Energy-Dependent Electron Precipitation," W. Spjeldvik, October 1974.
PPG-199 "Self-focusing and Filamentation of Laser Light in Plasmas," Y.C. Lee, C.S. Liu, H.H. Chen and K. Nishikawa, October 1974. To appear in Proceedings of IAEA Sixth Conference on Plasma Physics, held in Tokyo, Nov. 1974.
PPG-200 "Stimulated Brillouin Backscatter in the Equatorial Electrojet," D. D. Barbosa and C.F. Kennel, November 1974. Submitted to Planetary and Space Sciences.
PPG-201 "The Electromagnetic Interchange Mode in a Partially Ionized Collisional Plasma," M. K. Hudson and C. F. Kennel, December 1974. Submitted to J. of Plasma Physics.
PPG-202 "The Collisional Drift Mode in a Partially Ionized Plasma," M. K. Hudson and C. F. Kennel, December 1974. Submitted to J. of Plasma Physics.
PPG-203 "High Density Constraint on the Entropy Instability," M. K. Hudson and C. F. Kennel, December 1974. Submitted to Physics of Fluids.
PPG-204 "Excitation of Zero-Frequency Instabilities and Spatial Collapse of Beam Driven Plasma Waves," A.Y. Wong and B.H. Quon, December 1974. Submitted to Phys. Rev. Letters.
PPG-205 "A Recursive Numerical Method to Solve the Pure Pitch Angle Diffusion Equation, A Technical Report," W. N. Spjeldvik, December 1974.
PPG-206 "The Equilibrium Radiation Belt Electron Pitch Angle Distribution and its Dependence on the Radial Diffusive Source," W.N. Spjeldvik, January 1975. Submitted to Geophys. Res. Letters.
PPG-207 "Optimization of Plasma Confinement with Permanent Magnet Multipoles," K. N. Leung, T. K. Samec and A. Lamm January 1975. Submitted to Physics Letters.
PPG-208 "Anomalous Electron Transport and Lower-Hybrid Wave Damping," C. Chu, J. M. Dawson and H. Okuda, January 1975. Submitted to Phys. Fluids
PPG-209 "Plasma Confinement by a Picket-Fence", K. N. Leung, N. Hershkowitz and T. Romesser, January 1975. Submitted to Physics Letters.
PPG-210 "Secular Mode Coupling and Anomalous Drag: A Theory of Plasma Turbulence with Application to the Electron Beam-Plasma Instability," J. F. Drake, February 1975. Dissertation.
PPG-211 "Nonlinear Generation of Intense Localized Electric Fields in Plasmas," G. J. Morales and Y. C. Lee, February 1975.
PPG-212 "Plasma Electron Heating by Injection of Low Energy Electrons," N. Hershkowitz and K. N. Leung, February 1975.
PPG-213 "Ion Confinement by Electrostatic Potential Well in Magnetic Multiple Device," Y. Nakamura, B. Quon and A.Y. Wong, February 1975. Submitted to Physics Letters A.
PPG-214 "Scattering of Electromagnetic Waves into Plasma Oscillations via Plasma Particles," A.T. Lin and J.M. Dawson, March 1975. Submitted to Physics of Fluids.

PPG-215 "Surface Magnetic Confinement", A. Y. Wong, Y. Nakamura and B. Quon, March 1975.

PPG-216 "Experimental Facilities of Plasma Physics Research Laboratories at UCLA", March 1975.

PPG-217 "A Modified Surmac Configuration", A. T. Forrester, March 1975.

PPG-218 "Some New Ideas on Wet Wood Burners", J. M. Dawson and A. T. Lin, April 1975. Submitted to Phys. Rev. Letters.

PPG-219 "Plasma Heating at Frequencies Near the Lower Hybrid", C. Chu, J. M. Dawson and H. Okuda, April 1975. Submitted to Phys. Fluids.

PPG-220 "Unstable High Frequency Electrostatic Modes Induced by a Loss Cone Distribution of Electrons", Burton D. Fried and David E. Baldwin. May 1975.

PPG-221 "ICRH Heating of a Magnetized Plasma and Associated Longitudinal Cooling", J. Busnardo-Neto, J. Dawson, T. Kamimura and A. T. Lin. Submitted to Phys. Rev. Lett. May 1975.

PPG-222 "Plasma Confinement by Localized Magnetic Cusps" K. Leung, May 1975. Thesis.

PPG-223 "The Generation of Spiky Turbulence", G. J. Morales and Y. C. Lee. Submitted to Phys. Rev. Letters.

PPG-224 "A Multipole Containment-Single Grid Extraction Ion Source", A. T. Forrester, J. T. Crow, N. A. Massie and D. M. Goebel.

PPG-225 "Q-Machines by Robert W. Motley" A book review by Francis F. Chen. Submitted to Physics Today.

PPG-226 "Plasma Leakage Through a Low-Beta Line Cusp", N. Hershkowitz, K. N. Leung, T. Romesser. Submitted to Phys. Rev. Letters.

PPG-227 "Localized Fields and Density Perturbations in Nonlinear Lower Hybrid Waves", W. Gekelman and R. L. Stenzel. Submitted to Phys. Rev. Letters.

PPG-228 "Effect of Mirroring on Convective Transport in Plasmas", T. Kamimura and J.M. Dawson. Submitted to Phys. Rev. Letters. June 1975.

PPG-229 "Parametric Excitation of Ion Fluctuations in the Relativistic Beam-Plasma Interaction", H. Schamel, Y.C.Lee, & G. J. Morales. Submitted to Phys. Rev. Letters. June 1975.

PPG-230 UCLA Plasma Physics & Controlled Fusion Research, Presented for ERDA Program Review, July 18, 1975, B. Fried, J. Dawson, A. Wong, T. Forrester, and F. Chen. (Not printed yet)

PPG-231 "The Nonlinear Filamentation of Lower-Hybrid Cones", G. J. Morales and Y. C. Lee. Submitted to Phys.Rev. Letters. July 1975

PPG-232 "Fusion Reactor with Picket Fence Walls", N. Hershkowitz and J. Dawson. Submitted to Phys. Rev. Lett., August, 1975.

PPG-233 "Shock Formation in Large Amplitude Waves Due to Relativistic Electron Mass Variation", J. F. Drake, Y. C. Lee, K. Nishikawa, and N. L. Tsintsadze. July 1975.

PPG-234 "Characteristics of a Large Volume RF Grid Discharge Plasma", R. Schumacher, N. Hershkowitz and K.R. MacKenzie, submitted to Journal of Applied Physics, July 1975.

PPG-235 "Surface Magnetic Confinement - a Review - A. Y. Wong, July 1975. (Reports-not submitted for publication).

PPG-236 "Anomalous Electron Transport and Lower-Hybrid Wave Damping", C. Chu, J. M. Dawson and H. Okuda, August 1975.

PPG-237 St. Petersburg Abstracts - Papers to be Presented at St. Petersburg Meeting of the American Physical Society, Div. of Plasma Physics, Nov. 10-14, 1975. By the UCLA Plasma Physics Group.

PPG-238 "Parametric Instabilities in Strongly Relativistic, Plane Polarized Electromagnetic Waves", J. F. Drake, Y. C. Lee, and N. L. Tsintsadze, submitted to Phys. Rev. Lett., September 1975.

PPG-239 "Detection of Brillouin Backscattering in Underdense Plasmas", John J. Turechek and Francis F. Chen, submitted to Phys. Rev. Lett., September 1975.

PPG-240 "Adiabatic Invariance and Charged Particle Confinement in a Geometric Mirror", T.K. Samec, Y.C. Lee, and Burton D. Fried, September 1975.

PPG-241 "Magnetic Fields for Surface Containment of Plasmas, A. T. Forrester and J. Busnardo-Neto, September 1975.

PPG-242 "Electrostatic Parametric Instabilities Arising from Relativistic Electron Mass Oscillations", A. T. Lin and N. L. Tsintsadze, October 1975.

PPG-243 "Relativistic Nonlinear Plasma Waves in a Magnetic Field", C. F. Kennel and R. Pellat, October 1975.

PPG-244 "Flux Limits on Cosmic Acceleration by Strong Spherical Pulsar Waves", E. Asseo, C. F. Kennel and R. Pellat, October 1975.