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Relativistic Properties of a Lagrangian and a Hamiltonian in Quantum Theories

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ABSTRACT

Relativistic properties of a Dirac Lagrangian density are compared with those of a Dirac Hamiltonian density. Differences stem from the fact that a Lagrangian density is a Lorentz scalar, whereas a Hamiltonian density is a 00-component of a second rank tensor, called the energy-momentum tensor. This distinction affects the form of an interaction term of a Dirac particle. In particular, a tensor interaction term of a Dirac Lagrangian density transforms to a difference between a vector and an axial vector of the corresponding Hamiltonian density. This outcome shows that fundamental principles can prove the V-A attribute of weak interactions. A further analysis supports these results. Inherent problems of the electroweak theory are discussed.

Keywords: Dirac Lagrangian density; Dirac Hamiltonian density; Dirac generalized momentum; weak interactions.

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1 INTRODUCTION

There is now a common agreement concerning the crucial role of the variational principle and of its Lagrangian density $\mathcal{L}(\psi(x), \psi(x)_{,\mu})$ in the structure of a quantum field theory (QFT) of a given elementary quantum particle. This Lagrangian density is a Lorentz scalar. For example: "All field theories used in current theories of elementary particles have Lagrangians of this form" (see [1], p. 300). Another support for this approach states that the variational principle is "the foundation on which virtually all modern theories are predicated" (see [2], p. 353). The Euler-Lagrange equations of a given Lagrangian density are a vital element of this theoretical structure. These equations are partial differential equations that describe the time-evolution of the relevant quantum particle.

The Noether theorem is an important element of this theoretical structure. This theorem connects a symmetry of a Lagrangian density with a conservation law that the relevant theory satisfies. For example, the Noether theorem proves that a Lagrangian density that does not depend explicitly on the space-time coordinates yields a theory that conserves energy, momentum, and angular momentum (see [3], pp. 17-19). An important part of the proof of the Noether theorem is that the quantum function satisfies the Euler-Lagrange equations of the Lagrangian density.

Evidently, the validity of a given physical theory is based on the goodness of its predictions of relevant experimental results. The bottom line of measuring an experimental effect is the transition of a measuring device from an initial state at an initial time to a different state at a later time. An interaction term of the Lagrangian density connects a given quantum particle to external fields which eventually affect the state of a measuring device. Hence, the Lagrangian density of a given quantum particle should have an interaction term.

The above-mentioned issues are used as the basis for the discussion that is presented in this work.

The electroweak theory is the Standard Model sector that describes electromagnetic and weak processes. This is an example of a QFT theory of several quantum particles (see [4], chapter 21.3). The factor $(1 \pm \gamma^5)$ is an important quantity of the electroweak theory, and it agrees with a massless neutrino. The literature substantiates the relation between a massless neutrino and the electroweak theory. Indeed, the factor $(1 \pm \gamma^5)$ is associated with "a neutrino which travels exactly with the velocity of light" [5]. A review article restates the neutrino masslessness attribute of the electroweak theory: "Two-component left-handed massless neutrino fields play crucial role in the determination of the charged current structure of the Standard Model" (see the Abstract of [6]). Similarly, a textbook says: "Neutrino masses are exactly zero in the Standard Model" (see [7], p. 533).

It turns out that experimental progress has provided results that disagree with a massless neutrino. Indeed, it is now recognized that "neutrinos can no longer be considered as massless particles in the Standard Model, representing perhaps the first significant alteration to the theory" (see [8]). This experimental evidence proves that the electroweak theory has been based on an erroneous assumption concerning the neutrino mass. This is not a trivial issue. Thus, Wigner has analyzed the irreducible representations of the inhomogeneous Lorentz group (see [1, 9, 10, 11]). An important result of his work states that a massive quantum particle has a well-defined mass and spin. Massless particles belong to a different category. Instead of spin, they have helicity and they travel at the speed of light in every Lorentz frame.

The experimentally confirmed neutrino mass indicates that the structure of the electroweak theory is likely to have intrinsic problems. Evidently, a theoretical analysis of a physical topic is always welcome, because it aims to shed a new light on the relevant theory. The main objective of this work is to carry out an analysis of weak interaction theories.

This work uses units where Planck's constant and the speed of light are $\hbar = c = 1$. Greek indices run from 0 to 3. Most formulas take the standard form of a relativistic covariant expression. The metric is diagonal and its entries are (1,-1,-1,-1). The second section shows how the Dirac γ matrices affect the form of a Lorentz transformation of terms of a Dirac Lagrangian density, and that of the corresponding terms of the Hamiltonian density. The third section contains a further analysis of this issue. The fourth section shows new inconsistencies in the electroweak theory. The last section contains conclusions of this work.

2 CONSEQUENCES OF LORENTZ TRANSFORMATION OF DIRAC γ MATRICES

It is explained in the first section why the variational principle requires that the Lagrangian density of a given quantum particle should have an interaction term that is a Lorentz scalar. In the case of a Dirac particle, this Lorentz scalar takes the form of the scalar product of Dirac γ matrices with an external field. For example, the electromagnetic interaction of an electron is described by the Dirac Lagrangian density

$$\mathcal{L}_D = \bar{\psi}[\gamma^\mu(i\partial_\mu) - m]\psi - e\bar{\psi}\gamma^\mu A_\mu\psi \tag{1}$$

where m, e are the electron's mass and charge, respectively, and $A^\mu = (V, \boldsymbol{A})$ are the components of the electromagnetic 4-potential (see [3], p. 84; [12], p. 78). The last term of (1) represents interaction, and it contains the scalar product of γ^μ with the external 4-potential. Furthermore, the product of the Dirac functions

$$I = \bar{\psi}\psi \tag{2}$$

is a Lorentz scalar (see [12], p. 43; [13], p. 26). Therefore, an interaction term that is enclosed within the functions $\bar{\psi}$ ψ should also be a Lorentz scalar.

Products of the Dirac γ matrices can be organized in five sets, where each set comprises γs that undergo the same Lorentz transformation. These sets are:

$$\begin{array}{rl} 1 & \text{scalar} \\ \gamma^\mu & \text{vector} \\ \sigma^{\mu\nu} & \text{tensor} \\ \gamma^\mu\gamma^5 & \text{pseudo-vector} \\ \gamma^5 & \text{pseudo-scalar,} \end{array} \tag{3}$$

where $\sigma^{\mu\nu} \equiv i(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)/2$ and $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$ (see [12], p. 50; [13], p. 26).

The idea that a term that is based on the tensor $\sigma^{\mu\nu}$ of (3) can be applied to the electron's electromagnetic interaction was examined a long time ago (see [1], pp. 14, 517, 520; [14], p. 223). The corresponding interaction, which is called the Pauli term, takes the form

$$\mathcal{L}' = \mathrm{d}\bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\psi, \tag{4}$$

where $F^{\mu\nu}$ is the electromagnetic field tensor, and the coefficient d has the dimension of length. The interaction (4) alters the Dirac expression for the electron's dipole moment (see [1], p. 14; [14], p. 223). However, the ordinary Dirac Lagrangian density (1), which contains no term like (4), yields a very good prediction for the electron's magnetic dipole moment. Hence, the Pauli term (4) has been abandoned as a term that pertains to the electron's *electromagnetic interaction*.

As a matter of fact, it is argued that "the term (4) is consistent with all accepted invariance principles, including Lorentz invariance and gauge invariance, and so there is no reason why such a term should *not* be included in the field equations" (see [1], p. 14). Therefore, one may wonder why Nature has not applied the Pauli term (4).

It is proved here how the distinction between the form of the Dirac Lagrangian density and the corresponding Hamiltonian density illuminates the merits of the Pauli term (4). An application of the following transformation to the Dirac Lagrangian density (1) yields the required expression for the Dirac Hamiltonian density

$$\mathcal{H} = \frac{\partial \mathcal{L}}{\partial \dot{\psi}}\dot{\psi} - \mathcal{L}, \tag{5}$$

where the upper dot denotes a time derivative (see [3], p. 55; [12], p. 16). This expression proves that if the (relativistic form of the) interaction term is derivative-free then the interaction term of the Hamiltonian is the same as that of the Lagrangian, but with an opposite sign.

A general law says that the Hamiltonian is a function of coordinates and their generalized momenta. An expression for the generalized momentum which is conjugate to the coordinate ψ of the Dirac Lagrangian density is obtained from this expression (see [3], p. 55; [12], p. 52)

$$\pi_D = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = i\bar{\psi}\gamma^0 = i\psi^\dagger. \quad (6)$$

The form of the Dirac generalized momentum (6) points out the different structures of the Dirac Lagrangian density and that of its Hamiltonian density. The Dirac Lagrangian density (1) is written in terms of $\bar{\psi}$, whereas the corresponding Hamiltonian density is written in terms of $\psi^\dagger$, where $\bar{\psi} \equiv \psi^\dagger\gamma^0$. Evidently, the additional γ^0 is the 0-component of the 4-vector γ^μ. Hence, a transition to the Hamiltonian density entails a modification of the relativistic form of terms of the Dirac Lagrangian density. In particular, interaction terms take a different form. For example, in the case of the electromagnetic interaction, one has the Dirac Hamiltonian density

$$\mathcal{H}_D = \psi^\dagger[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta m - e\boldsymbol{\alpha}\cdot\boldsymbol{A} + eV]\psi. \quad (7)$$

Here $\boldsymbol{\alpha}$, β denote the four Dirac matrices, $\psi^\dagger\psi$ is the Dirac density, and the terms inside the square brackets are the Dirac Hamiltonian (see [13], p. 48). The interaction terms of the Lagrangian density (1) do not take the same form as those of the Hamiltonian density (7). For example, the scalar component $V \equiv A_0$ of the electromagnetic 4-vector of the Lagrangian density (1) is multiplied by the Dirac γ^0 matrix, whereas no Dirac matrix multiplies the term eV of the Hamiltonian (7).

The corresponding changes of the Pauli tensor interaction (4) are more dramatic. Thus, the substitution of $\bar{\psi} \equiv \psi^\dagger\gamma^0$ transforms (4), and the interaction term of the Hamiltonian density is

$$\begin{aligned}\mathcal{H}_{int} &= -\mathrm{d}\psi^\dagger\gamma^0\sigma_{\mu\nu}F^{\mu\nu}\psi \\ &= -2\mathrm{d}\psi^\dagger(i\gamma_i E^i - \gamma^5\gamma_i B^i)\psi,\end{aligned} \quad (8)$$

where E^i, B^i are components of the external field tensor (see [15]). Here one obtains two terms. One term contains the spatial components γ_i of the γ_μ 4-vector, and the second term contains the corresponding components of a pseudo-vector.

The Pauli term has recently been rediscovered, and it is shown that it describes weak interactions, where parity violation is *proved* (see [15, 16, 17]). Obviously, the electromagnetic field tensor $F^{\mu\nu}$ of (4) is replaced by an analogous tensor of weak interactions fields which is denoted by $\mathcal{F}^{\mu\nu}$. This is a Maxwellian-like fields' tensor that is associated with an external weak dipole. Here the transition from the Lagrangian density to the Hamiltonian density adds a γ^0 factor, and (4) splits into a sum of a vector and an axial vector. The Pauli term (4) shows the flexibility of the first-order Dirac theory, where the dimensionless $\sigma^{\mu\nu}$ 4-tensor of (3) enables to write down a consistent expression for an interaction with a second rank antisymmetric tensor that takes the form of $F^{\mu\nu}$. Like the electromagnetic interaction term of (1), also the Pauli term is free of derivatives of the Dirac functions.

The dependence of the weak field $F^{\mu\nu}$ of (8) on an external weak dipole means that (8) is a dipole-dipole interaction. The dimension of the weak interaction Fermi constant G is $[L^2]$ (see [18], p. 212). This property agrees with the dipole-dipole interaction of the Pauli term (4), where the coefficient d has the dimension of length. This dimensional agreement and the universality of the Fermi constant G (see [19], p. 256) is another experimental support for the dipole-dipole weak interactions theory (see [15, 16, 17]).

Remark: The dipole-dipole weak interactions theory is based on a consistent Lagrangian density. It explains the important parity nonconservation attribute of weak interactions. However, this is not the final word because details like flavor nonconservation processes, generation-dependent effects, and the CKM matrix require further elaboration.

3 DISCUSSION

The previous section emphasizes the effect of the additional γ^0 on the form of the interaction term of a Dirac Hamiltonian H_D. This is an important issue because the time-evolution of a Dirac particle is determined by the Dirac Hamiltonian

$$i\frac{d\psi}{dt} = H_D\psi \quad (9)$$

(see [1], p. 8). It is mentioned in the first section of this work that the transition of a measuring device from an initial state at an initial time to a different state at a later time establishes a physical effect. This time-dependence means that the Dirac Hamiltonian is required for this purpose. In particular, in the case of weak interactions, one should not examine the tensorial form of the Pauli term (4) of a Dirac Lagrangian density, but the corresponding vector and pseudo-vector terms (8), which belong to a Dirac Hamiltonian. It turns out that the Hamiltonian's interaction (8) shows a unique success because it *proves* the $V-A$ property of weak interactions (see e.g. [18], pp. 217-220). Here V denotes a vector interaction and A denotes an axial vector interaction.

Authors of mainstream literature have overlooked the effect of the γ^0 matrix on the different forms of the interaction term of a Dirac Lagrangian density and the corresponding term of a Dirac Hamiltonian density. This is the primary reason for the rejection of the Pauli term (4) (namely, the tensor interaction $\sigma_{\mu\nu}$) as a candidate for a description of weak interactions of a Dirac Lagrangian density (see e.g. [18], pp. 217-220; [20]).

The discussion of the previous section explains why the relativistic covariance form of a Dirac Lagrangian density differs from that of a Dirac Hamiltonian density. This result is derived from the Hamiltonian's dependence on the generalized momentum (6). It is proved below that this is a more general property. Thus, the Lagrangian density is a Lorentz scalar. On the other hand, the Hamiltonian is an energy operator, and energy is the 0-component of the energy-momentum 4-vector $(E, \boldsymbol{p})$. Furthermore, density is the 0-component of the 4-current $(\rho, \boldsymbol{j})$ (see [21] pp. 73-78). Hence, the Hamiltonian density is the 00-component of a second rank tensor, called the energy-momentum tensor.

The standard construction of the energy-momentum tensor sheds light on how in different circumstances, one and the same term does not undergo the same covariant transformations. Let $\mathcal{L}$ be a Lagrangian density which is a Lorentz scalar. The standard expression of its energy momentum tensor is:

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \psi_{,\nu}} g^{\mu\alpha}\psi_{,\alpha} - g^{\mu\nu}\mathcal{L} \tag{10}$$

(see [12], p. 310; [21], p. 83). The tensor (10) satisfies energy-momentum conservation

$$T^{\mu\nu}_{,\nu} = 0. \tag{11}$$

The last term of (10) shows how every Lorentz scalar term of a Lagrangian density $\mathcal{L}$ appears as a (positive or negative) diagonal entry of the second rank energy-momentum tensor (10). This argument proves that *all terms of a Lagrangian density and corresponding terms of a Hamiltonian density have different relativistic properties.*

It is explained above how the entire electroweak theory has been based on an erroneous concept, which identifies relativistic properties of terms of a Lagrangian density with relativistic properties of corresponding terms of its Hamiltonian density (see e.g. [18], pp. 217-220; [20]). The required coherence of the mathematical structure of a physical theory is the basis for the expectation that the erroneous basis of the electroweak theory is likely to yield other specific errors. This approach is true. Thus, the second section of [15] discusses several uncorrectable electroweak errors. The following list describes briefly these errors.

Er.1 The $(1 \pm \gamma^5)$ electroweak factor is inconsistent with a massive neutrino.

Er.2 The previous error means that the electroweak theory cannot explain the V-A attribute of weak interactions.

Er.3 The electroweak theory regards the $W^\pm$ bosons as elementary charged particles. Even though the electroweak theory is about 50 years old, it still has no consistent expression for the $W^\pm$ electromagnetic interaction.

Er.4 Contradictions arise from the lack of a coherent expression for the electroweak Z boson density.

It turns out that this list does not exhaust the erroneous elements of the electroweak theory. Several other issues are mentioned in the next section.

4 FURTHER INCONSISTENCIES OF THE ELECTROWEAK THEORY

The crucial role of the Lagrangian density in the structure of QFT is pointed out in the first section of this work. In particular, solutions of the Euler-Lagrange equations of this Lagrangian density describe the physical properties of a given quantum system.

The Dirac equation of spin-1/2 particles abides by this requirement, and textbooks present this equation together with some of its solutions (see e.g. [13], pp. 2-13, 28-60). By contrast, electroweak textbooks refrain from showing the partial differential equations of the $W^{\pm}$, Z particles of this theory. A fortiori, no specific solution of these equations is shown and discussed.

One explanation for this shortcoming is probably the fact that the full Lagrangian density of these particles is terribly complicated, and the form of their Euler-Lagrangian equations should be even worse. For example, an expansion of the full Lagrangian density of the electroweak bosons (see [22], p. 518), yields dozens of terms. This is just a part of the electroweak Lagrangian density because one should also examine the electroweak fermionic fields whose interaction with the electroweak bosons contains the $(1 \pm \gamma^5)$ factor. Obviously, the number of terms of the respective Euler-Lagrangian equations is even larger. By contrast, the Lagrangian density of electromagnetic interactions together with the dipole-dipole weak interaction theory of [15] comprises four terms – the three terms of the Dirac electromagnetic Lagrangian density (1) and the tensor interaction term (4). Hence, even if one ignores the above mentioned inherent electroweak contradictions, the Occam razor principle [23], which favors the relative simplicity of theories, provides another support for the weak interaction theory of [15].

The factor $(1 \pm \gamma^5)$ is a crucial element of the electroweak theory (see e.g. [4], pp. 305-313). Let us use the γ matrix notation of [13], p. 17. The matrix $(1 \pm \gamma^5)$ is a special case of the matrix $(1 \pm \lambda\gamma^5)$, where $\lambda > 0$ is a real number. The explicit form of an application of this matrix to the spinor of a motionless spin-up Dirac particle is:

$$\begin{pmatrix} 1 & 0 & \pm\lambda & 0 \\ 0 & 1 & 0 & \pm\lambda \\ \pm\lambda & 0 & 1 & 0 \\ 0 & \pm\lambda & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \pm\lambda \\ 0 \end{pmatrix}. \tag{12}$$

The result of (12) is unacceptable. Indeed, if $\lambda > 1$ then the result is a negative energy spinor; if $\lambda = 1$ then it describes a particle that moves at the speed of light; if $\lambda < 1$ then it describes a Dirac particle that moves parallel to the z-axis (see [13], p. 30). The latter case violates energy conservation. Therefore, a factor of the form $(1 \pm \lambda\gamma^5)$ is unacceptable for a massive spinor.

The electroweak theory aims to combine electromagnetic and weak interactions. Let us examine the relative strength of these interactions. The electromagnetic electron-electron cross section decreases rapidly with energy (see [18], p. 193). On the other hand, a neutrino participates only in weak interaction, and its cross section increases with energy (see [24], p. 3). Hence, one should not ignore weak interactions in cases of high enough energy. The data of the decay of the $W^{\pm}$, Z bosons and of the top quark support this conclusion (see [25]). Thus, the decay channels of these particles contain many products having a new flavor. It means that these channels are a weak interaction process. The width of these particles is about 2 GeV. This width indicates that weak interactions are stronger than strong interactions in the energy region which is greater than 80 GeV. It is explained above that the relative strength of weak interactions is an important effect that cannot be ignored at high enough energy. This issue is used below in an examination of an elastic electron-electron collision (see Fig. 1). Two incoming electrons collide elastically at point O, exchange momentum and depart from each other. The arrows denote the direction of the motion of the incoming and the outgoing electrons. This is certainly a process that should be described by electromagnetic *and* weak interaction theories. The process of Fig. 1. comprises electrons and is free of neutrinos. An electron is a well-known *massive* Dirac particle, and it is shown above that errors emerge from an application of

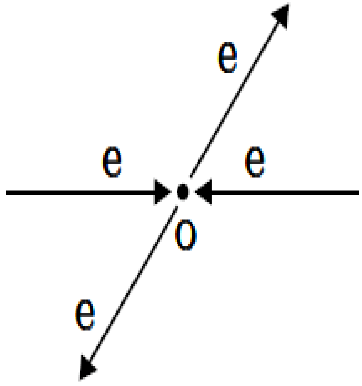

Fig. 1. Two electrons collide elastically at point O (see text)

the factor $(1 \pm \gamma^5)$ to a massive Dirac particle. This factor is an inherent element of the electroweak theory. For this reason, the electroweak theory cannot properly describe an electron-electron scattering process.

The experiment of Fig. 1 proves that the electroweak neutrino mass problem is just a rediscovery of the inherent contradiction associated with the $(1 \pm \gamma^5)$ factor. Indeed, the same problem holds for an electron, which is a well-known *massive* Dirac particle.

5 CONCLUSIONS

This work compares relativistic properties of the Lagrangian density of a spin-1/2 Dirac particle with those of its Hamiltonian density. It proves that these theoretical concepts have different relativistic attributes, and that important physical consequences are derived from this distinction. The main results are

1. The Lagrangian density is a Lorentz scalar, whereas the Hamiltonian density is a 00-component of a second rank energy-momentum tensor.
2. This distinction is consistent with the fact that the Lagrangian density is written in terms of $\bar{\psi}$. By contrast, the Hamiltonian density is written in terms of $\psi^\dagger$, where $\bar{\psi} == \psi^\dagger\gamma^0$. The additional γ^0 factor is the 0-component of the 4-vector γ^μ, and it means that terms of a Lagrangian density and corresponding terms of the associated Hamiltonian density undergo a different Lorentz transformation.
3. The foregoing outcome entails that a tensor interaction term of a Lagrangian density (called a Pauli term) yields a Hamiltonian density that comprises two terms – a vector and an axial vector. This result explains the V-A attribute of weak interactions.
4. An overlook of the meaning of items 1-3 is the reason for the formulation of the electroweak theory.
5. The erroneous basis of the electroweak theory is the origin of several specific errors of this theory. Here are several examples:

   A. Electroweak textbooks do not show fundamental quantum requirements. Thus, the Dirac equation of motion of a spin-1/2 particle is shown in every relevant textbook. By contrast, the electroweak theory is about 50 years old, but textbooks still do not explicitly display the quantum equations of motion of its particles.

B. The electroweak theory claims that the $W^{\pm}$ are electrically charged elementary particles, but these particles still have no expression for the electromagnetic interaction that is consistent with Maxwellian electrodynamics.

C. The factor $(1 \pm \gamma^5)$ is a vital element of the electroweak theory. It is now recognized that this factor is inconsistent with a massive neutrino. The paper proves that this factor is also inconsistent with the electron.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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