Progress of Theoretical Physics, Vol. 46, No. 5, November 1971

Relativistic Quantum Mechanics of One-Dimensional Mechanical Continuum and Subsidiary Condition of Dual Resonance Model

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(Received May 10, 1971)

Relativistic quantum mechanics of a finite one-dimensional continuum is studied in the framework of Dirac's generalized Hamiltonian dynamics. It is shown that the wave equation and subsidiary conditions found by Virasoro in the dual resonance model are equivalent to relativistic quantum mechanics in our system. Interaction with external fields is also studied briefly.

§ 1. Introduction

After the dual resonance model was formulated in terms of infinitely many oscillators by Nambu, Veneziano and Fubini,[1] it has been frequently suggested that the underlying string model of hadrons furnishes the multiparticle dual amplitudes.[2] One of the most crucial problems in the dual resonance model, however, is the existence of ghosts which are unphysical states having negative norms or space-like momenta. For eliminating ghosts, Fubini and Veneziano have found a Ward-like identity which has been generalized by Virasoro.[3] However, the Ward-like identity in their form is abstract and its relation to the so-called string model of hadrons is obscure. On the other hand, Takabayasi has proposed new relativistic quantum mechanical equations of one-dimensional string which are defined at each material point on the string.[4] Following Takabayasi's, formalism, subsidiary conditions proposed by Virasoro are contained in his new quantum mechanical equations. It is, however, not clear whether his new formulation is equivalent to ordinary quantum mechanics or not. Recently, Hara[5] has pointed out that Virasoro's condition is derived from the invariance under a general coordinate transformation of the Lagrange coordinates which specify each material point on the string. He has also shown that Virasoro's algebra is derived from the algebra of the general coordinates transformation.

In this note, we would like to show that relativistic quantum mechanics of a one-dimensional object with uniform mass density is equivalent to the so-called "string" model of hadrons with Virasoro's subsidiary conditions. Our argument is as follows: Starting from a Lagrangian which is invariant under a general coordinate transformation of the Lagrange parameters and local time transformations, we put the Lagrangian formalism into the canonical formalism. Since the Lagrangian possesses the above mentioned invariance, we should make use of the homogeneous canonical formalism extensively developed by Dirac.[6] In this way, we can obtain two "weak" equations which are replaced by constraints on physical states in quantized theory. These constraints satisfy a closed algebra which is equivalent to the algebra given by Virasoro and Takabayasi. It is also obvious that these are generators of general coordinate transformations of the Lagrange parameter and the local time variation because constraints are derived from invariance of the Lagrangian under these transformations. In § 2, a canonical formalism of our dynamical system is developed. In § 3, it is shown that Virasoro's condition is equivalent to the subsidiary conditions derived in § 2. Interacting cases are briefly discussed in § 4 and § 5 is devoted to giving additional comments.

§ 2. Canonical formalism

Let us consider a finite one-dimensional continuous medium. Relativistic motion of this medium is represented by a two dimensional world sheet in four-dimensional Minkowski space (as shown in Fig. 1). The positional coordinates x^μ on the world sheet are given in terms of two invariant parameters τ and σ:

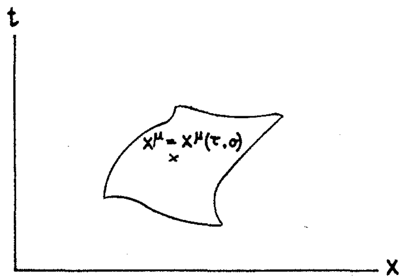

Fig. 1.

$$x^\mu = x^\mu(\tau, \sigma). \tag{2·1}$$

Hereafter, we call these parameters Lagrange coordinates (or L-coordinates in short). The four-velocity v^μ of a point designated by $x^\mu(\tau, \sigma)$ on the world sheet can be defined by

$$v^\mu = \frac{1}{\sqrt{g_{00}}} \frac{\partial x^\mu}{\partial \tau}, \tag{2·2}$$

where

$$g_{00} = \frac{\partial x^\mu}{\partial \tau} \frac{\partial x_\mu}{\partial \tau} > 0 . \tag{2·3}$$

Condition (2·3) means that τ plays a role of a parameter describing time development of the system. Infinitesimal separation dx^μ between a point (σ, τ) and a point $(\sigma + d\sigma, \tau + d\tau)$ on the world sheet is

$$dx^\mu = \frac{\partial x^\mu}{\partial \tau} d\tau + \frac{\partial x^\mu}{\partial \sigma} d\sigma . \tag{2·4}$$

If we choose $d\tau$ so that the separation dx^μ is orthogonal to v^μ and denote it as $d_\perp x^\mu$, $d_\perp x^\mu$ is written as follows:

$$d_\perp x^\mu = \left[\frac{\partial x^\mu}{\partial \sigma} - v^\mu\left(v_\nu \frac{\partial x^\nu}{\partial \sigma}\right)\right] d\sigma$$

$$= \left(\frac{\partial x^\mu}{\partial \sigma} - \frac{g_{01}}{g_{00}}\frac{\partial x^\mu}{\partial \tau}\right) d\sigma\,, \tag{2·5}$$

where

$$g_{01} = \frac{\partial x^\mu}{\partial \tau}\frac{\partial x_\mu}{\partial \sigma}\,. \tag{2·6}$$

Therefore, infinitesimal invariant length dl at a point $P(\tau, \sigma)$ on the world sheet can be defined as follows:

$$dl = \sqrt{-d_\perp x^\mu d_\perp x_\mu}$$

$$= \sqrt{\frac{g_{01}^2 - g_{00}g_{11}}{g_{00}}}\, d\sigma = \sqrt{\frac{-\det g}{g_{00}}}\, d\sigma\,, \tag{2·7}$$

where

$$g_{11} = \frac{\partial x^\mu}{\partial \sigma}\frac{\partial x^\mu}{\partial \sigma} \tag{2·8}$$

and

$$\det g = \begin{vmatrix} g_{00} & g_{01} \\ g_{01} & g_{11} \end{vmatrix}. \tag{2·9}$$

The proper time of a point $P(\sigma, \tau)$ is written by

$$ds = \sqrt{g_{00}}\, d\tau\,. \tag{2·10}$$

Now, let us suppose that the mass density of our one-dimensional continuum is uniform and denoted by κ_0. Then, the kinetic energy (including rest mass) of this system is simply given as follows:

$$T = \int_0^{l_1} \kappa_0 dl = \int_{\sigma_0}^{\sigma_1} d\sigma \kappa_0 \sqrt{\frac{-\det g}{g_{00}}}\,. \tag{2·11}$$

If there is no internal force such as an elastic force, the action integral becomes

$$\bar{L} = \iint ds dl \kappa_0 = \iint d\tau d\sigma \kappa_0 \sqrt{-\det g}\,. \tag{2·12}$$ *),[7]

It should be noticed that Eq. (2·12) is invariant under the general L-coordinate transformations

$$\sigma \to \sigma' = \sigma'(\tau, \sigma)\,,$$

*) The same Lagrangian and the algebra (3·10) are also found by Nambu[7] from a different point of view. After the completion of this work, Prof. J. Iizuka pointed out Nambu's work. He thanks Prof. J. Iizuka for his kind communication.

$$\tau \to \tau' = \tau'(\tau, \sigma). \tag{2·13}$$

In order to obtain the canonical formalism, we must define canonical momenta $p_\mu(\sigma)$ corresponding to positional coordinates $x^\mu(\sigma)$ in the medium. $p_\mu(\sigma)$ is defined in the following way:

$$p_\mu(\sigma) = \frac{\partial L}{\partial(\partial x^\mu/\partial\tau)} = \frac{\kappa_0}{\sqrt{-\det g}}\left[-g_{11}\frac{\partial x^\mu}{\partial\tau} + g_{01}\frac{\partial x^\mu}{\partial\sigma}\right], \tag{2·14}$$

where

$$L = \int d\sigma \kappa_0 \sqrt{-\det g} \equiv \int d\sigma L_0 .$$

Multiplying (2·14) by $\partial x^\mu/\partial\sigma$, we can easily see that the following equation holds:

$$T(\sigma) \equiv p_\mu \frac{\partial x^\mu}{\partial\sigma} = 0 . \tag{2·15}$$

From (2·14) and (2·15), we obtain the equation

$$p_\mu \frac{\partial x^\mu}{\partial\tau} - \kappa_0\sqrt{-\det g} \equiv p_\mu \frac{\partial x^\mu}{\partial\tau} - L_0 = 0 . \tag{2·16}$$

This shows that Hamiltonian is weakly zero. From (2·14), (2·15) and (2·16), it is not difficult to find the following relation:

$$H(\sigma) \equiv p_\mu p^\mu + \kappa_0^2 g_{11} = 0 . \tag{2·17}$$

Since Eqs. (2·15) and (2·17) are weak relations among canonical variables, they are interpreted as constraints on physical states in quantum theory. Consistency between these weak relations is easily examined by calculating Poisson brackets (or commutators in the quantized theory) of $H(\sigma)$ and $T(\sigma')$. In fact, the following commutation relations are obtained:

$$[H(\sigma), H(\sigma')] = 8\kappa_0^2 i\delta'(\sigma-\sigma')T(\sigma) + 4i\kappa_0^2\delta(\sigma-\sigma')\frac{\partial T}{\partial\sigma}, \tag{2·18a}$$

$$[T(\sigma), T(\sigma')] = 2i\delta'(\sigma-\sigma')T(\sigma) + i\delta(\sigma-\sigma')\frac{\partial T}{\partial\sigma}, \tag{2·18b}$$

$$[T(\sigma), H(\sigma')] = 2i\delta'(\sigma-\sigma')H(\sigma) + i\delta(\sigma-\sigma')\frac{\partial H}{\partial\sigma}, \tag{2·18c}$$

where $\delta'(\sigma-\sigma') = \partial\delta(\sigma-\sigma')/\partial\sigma$ and $T(\sigma)$ is symmetrized as $T(\sigma) = \frac{1}{2}(p_\mu \cdot \partial x^\mu/\partial\sigma + \partial x^\mu/\partial\sigma \cdot p_\mu)$. From commutation relations (2·18), it can be seen that $T(\sigma)$'s and $H(\sigma)$'s form a closed algebra.

Following Dirac's generalized canonical formalism, Hamiltonian $\mathcal{H}$ is given as follows:

$$\mathcal{H}=\int d\sigma\left[\frac{1}{2}\lambda_0 H(\sigma)+\lambda_1 T(\sigma)\right], \tag{2·19}$$

where λ_0 and λ_1 are arbitrary functions of σ. Hamilton's equations are, now, easily derived as follows:

$$\begin{aligned}\frac{\partial x^\mu}{\partial\tau}&=\frac{1}{i}[\mathcal{H}, x^\mu(\sigma)]^{*)}\\&=\lambda_0 p^\mu+\lambda_1\frac{\partial x^\mu}{\partial\sigma}\,,\end{aligned} \tag{2·20a}$$

$$\begin{aligned}\frac{\partial p^\mu}{\partial\tau}&=\frac{1}{i}[\mathcal{H}, p^\mu]\\&=-\kappa_0{}^2\frac{\partial}{\partial\sigma}\left(\lambda_0\frac{\partial x^\mu}{\partial\sigma}\right)-\frac{\partial}{\partial\sigma}(\lambda_1 p)\,.\end{aligned} \tag{2·20b}$$

On the other hand, Euler's equation from Lagrangian (2·12) is

$$\begin{aligned}&-\frac{\partial}{\partial\tau}\left[\frac{\kappa_0}{\sqrt{-\det g}}\left(-g_{11}\frac{\partial x^\mu}{\partial\tau}+g_{01}\frac{\partial x^\mu}{\partial\sigma}\right)\right]=0\,,\\&-\frac{\partial}{\partial\sigma}\left[\frac{\kappa_0}{\sqrt{-\det g}}\left(-g_{00}\frac{\partial x^\mu}{\partial\sigma}+g_{01}\frac{\partial x^\mu}{\partial\tau}\right)\right]=0\,.\end{aligned} \tag{2·21}$$

If we put

$$\begin{aligned}&\frac{1}{\lambda_0}=-\frac{\kappa_0 g_{11}}{\sqrt{-\det g}}\,,\\&\frac{\lambda_1}{\lambda_0}=-\frac{\kappa_0 g_{01}}{\sqrt{-\det g}}\quad\text{or}\quad\lambda_1=\frac{g_{01}}{g_{11}}\,,\end{aligned} \tag{2·22}$$

we can show by employing (2·14) that Euler's equation is equivalent to Hamilton's equation (2·20). It should also be mentioned that, since λ_0 and λ_1 are arbitrary functions, Hamiltonian (2·19) can be interpreted as generators of general L-coordinate transformations, and constraints (2·15) and (2·17) show that physical states are invariant under the transformation. Therefore, the situation is quite analogous to the case of quantum electrodynamics where the Lorentz condition means that the generators of gauge transformation are zero on physical states.

§ 3. Virasoro's subsidiary conditions for dual resonance model

Since our mechanical continuum is finite, we can choose the range of σ so as to be $[0,\pi]$ without loss of generality. Now, let us suppose that $x^\mu(\sigma)$ can be expanded by Fourier cosine series, that is,

*) $1/i[A,B]$ should be understood as Poisson brackets in classical theory.

$$x_\mu(\sigma)=\sqrt{\frac{2}{\pi}}\sum_{r=0}^{\infty}\xi_\mu{}^r\cos r\sigma,$$
$$p_\mu(\sigma)=\sqrt{\frac{2}{\pi}}\sum_{r=0}^{\infty}\pi_\mu{}^r\cos r\sigma\ ,\tag{3·1}$$

where

$$[\pi_\mu{}^r,\xi_\nu{}^s]=-ig_{\mu\nu}\delta^{rs}\tag{3·2}$$

and

$$[p_\mu(\sigma),x_\nu(\sigma')]=-ig_{\mu\nu}\delta(\sigma-\sigma').$$
$$(g_{00}=-g_{11}=-g_{22}=-g_{33}=1)$$

As is well known, we can introduce oscillator variables $(C_\mu{}^r, C_\mu{}^{r+})$ by making use of $(\pi_\mu{}^r,\xi_\mu{}^r)$ as follows:

$$C_\mu{}^r=\frac{1}{\sqrt{2}}[r\xi_\mu{}^r+i\pi_\mu{}^r],$$
$$C_\mu{}^{r\dagger}=\frac{1}{\sqrt{2}}[i\xi_\mu{}^r-i\pi_\mu{}^r],\tag{3·3}$$
$$C_\mu{}^0=-C_\mu{}^{0\dagger}=\frac{1}{\sqrt{2}}\pi_\mu{}^0=\frac{i}{\sqrt{\pi}}P_\mu,$$

where $P_\mu=\int_0^\pi d\sigma p_\mu(\sigma)$ and is interpreted as the total four-momentum of our system. Commutation relations of C and C^+ are

$$[C_\mu{}^r,C_\nu{}^{s\dagger}]=-r\delta^{rs}g_{\mu\nu}\tag{3·4}$$

and, therefore, the usual oscillator variables $(a_\mu{}^r, a_\mu{}^{r\dagger})$ are given by

$$a_\mu{}^r=\frac{1}{\sqrt{r}}C_\mu{}^r,\qquad r\geq 1,$$
$$a_\mu{}^{r\dagger}=\frac{1}{\sqrt{r}}C_\mu{}^{r\dagger},\qquad r\geq 1.\tag{3·5}$$

Throughout this section, we choose a suitable unit such that $\kappa_0=1$. (See (3·8).)

Instead of considering $H(\sigma)$ and $T(\sigma)$, it is sufficient to study their Fourier coefficients defined as follows:

$$H^n=\frac{1}{2}\int_0^\pi d\sigma\cos n\sigma H(\sigma),\tag{3·6a}$$

$$T^n=\int_0^\pi d\sigma\sin n\sigma T(\sigma).\tag{3·6b}$$

It is more convenient to consider the following quantities:

$$U^n=H^n+iT^n,\quad (n\geq 1)\tag{3·7a}$$

$$U^{n\dagger}=H^n-iT^n,\quad (n\geq 1) \tag{3·7b}$$

$$U^0=H^0=P^2-\sum_{r=1}^{\infty}C_\mu{}^{r\dagger}C^{r,\mu}-\omega_0\,, \tag{3·8}$$

where ω_0 is a constant. The explicit form of U^n is as follows:

$$U^n=-i\sqrt{2}P_\mu C^{n,\mu}+\sum_{r=1}C_\mu{}^{r\dagger}C^{n+r,\mu}-\frac{1}{2}\sum_{r=1}^{n-1}C_\mu{}^{r}C^{n-r,\mu}, \quad (n\geq 1) \tag{3·9}$$

and the commutation relations of these quantities are

$$[U^n, U^{n'}]=(n-n')\,U^{n+n'}, \tag{3·10a}$$

$$\begin{aligned}[U^n, U^{n'\dagger}]&=(n+n')\,U^{n-n'}, \quad n-n'>0\\ &=(n+n')\,U^{n'-n\dagger}, \quad n-n'<0\,.\end{aligned} \tag{3·10b}$$

Equations (3·8), (3·9) and (3·10) are equivalent to those obtained by Virasoro and Takabayasi. Constraints (2·15) and (2·17) are now replaced by the following:

$$H^0\Psi=0\,, \tag{3·11}$$

$$U^n\Psi=0\,,\quad n\geq 1\,, \tag{3·12a}$$

$$U^{n\dagger}\Psi=0\,,\quad n\geq 1\,. \tag{3·12b}$$

Equation (3·11) is interpreted as a master equation describing the motions of our system and Eqs. (3·12) are constraints on physical states. It is worthy to notice that we do not use an indefinite metric in the preceeding discussions and we understand the zeroth component $a_0{}^r$ of $a_\mu{}^r$ as a creation operator and consequently $a_0{}^{r\dagger}$ as a destruction operator. If, however, we use an indefinite metric formalism in analogy with quantum electrodynamics, $a_\mu{}^r$ and $a_\mu{}^{r\dagger}$ are the creation and destruction operators, respectively. Moreover, it may be sufficient to keep (3·12a) as constraints, because the expectation values of subsidiary conditions are required to hold, i.e.,

$$\langle U^n\rangle=\langle U^{n\dagger}\rangle=0\,. \tag{3·13}$$

Therefore, the indefinite metric formalism of our quantum mechanical system is equivalent to the dual resonance model with Virasoro's condition or Takabayasi's proposal of a new relativistic wave equation. It is also expected in analogy with quantum electrodynamics that the indefinite metric formalism is equivalent to the formalism without use of an indefinite metric.

§ 4. Interaction with external field

Now, let us consider briefly the following interacting case. The interaction

Lagrangian density $\mathcal{L}_1$ is given, for example, as follows:

$$\mathcal{L}_{1s}=g\phi(x(\sigma))\sqrt{g_{00}}\delta(\sigma), \tag{4·1a}$$

(for the case of external scalar field)

$$\mathcal{L}_{1v}=g\phi_\mu\frac{\partial x^\mu}{\partial\tau}\delta(\sigma). \tag{4·1b}$$

(for the case of external vector field)

Here, we assume that the interactions with external fields occur at the end point of our rod. It is also possible to assume the charge (i.e., the coupling constant) is distributed uniformly over the rod. In this case, the interaction Lagrangian density becomes

$$\mathcal{L}_2=g\phi(x(\sigma))\sqrt{-\det g}\,. \tag{4·1c}$$

(for the case of external scalar field)

In the case of (4·1a), weak equations corresponding to (2·15) and (2·17) are obtained as follows:

$$p_\mu p^\mu+\kappa_0 g_{11}=-2\kappa_0 g\phi(x(\sigma))\delta(\sigma)+[g\phi(x(\sigma))\delta(\sigma)]^2, \tag{4·2a}$$

$$p_\mu\frac{\partial x^\mu}{\partial\sigma}=-g\phi(x(\sigma))\delta(\sigma)\sqrt{1+g_{11}}\,, \tag{4·2b}$$

where we required $\sqrt{-\det g/g_{00}}=1$ at the end point ($\sigma=0$). Therefore, if we disregard the last term of (4·2a) and expansion (3·1) of Fourier cosine series is still valid, our result is equivalent to that obtained by Virasoro and Takabayasi. However, it is doubtful whether expansion (3·1) remains valid and the invariance of the L-coordinate transformation requires the second order term in Eq. (4·2a). Unfortunately, this characteristic second order term is very pathological because of presence of the factor $[\delta(\sigma)]^2$. The condition $[\sqrt{-\det g/g_{00}}]_{\sigma=0}=1$ is necessary to obtain such a simple result as (4·2).

In the case of (4·1b), the situation is similar to the above. Corresponding to (4·2a) and (4·2b), one obtains the following:

$$p^2+\kappa_0^2 g_{11}=-\kappa_0 g[p_\mu\phi^\mu+\phi^\mu p_\mu]\delta(\sigma)$$
$$+g^2\phi_\mu\phi^\mu[\delta(\sigma)]^2, \tag{4·3a}$$

$$p_\mu\frac{\partial x^\mu}{\partial\sigma}=g\phi_\mu\frac{\partial x^\mu}{\partial\sigma}\delta(\sigma). \tag{4·3b}$$

In this case, it is not necessary to require such a condition as $(\sqrt{-\det g/g_{00}})_{\sigma=0}=1$. However, there appears again a pathological term $[\delta(\sigma)]^2$.

In the case of (4·1c), there is no mathematical trouble. In fact, weak equations are now given as follows:

$$p_\mu p^\mu + \kappa_0^2 g_{11} = -[2\kappa_0 g\phi(x) + g^2\phi^2(x)] g_{11}, \tag{4·4a}$$

$$p_\mu \frac{\partial x^\mu}{\partial \sigma} = 0. \tag{4·4b}$$

Unfortunately, in this case one cannot obtain such a complete *s-t* symmetric scattering amplitude as that given by Veneziano. The problems in the interacting case discussed above remain to be investigated further.

§ 5. Discussion

Usually, such a model as discussed in this paper is called an elastic string model. However, as shown in § 2, there is only one parameter κ_0 which is naturally interpreted as a mass density and we have no parameter which characterizes elastic property of the medium. Although Euler's Equation (2·21) governs motions of our system, it is not very easy to solve it in general. In the static case (i.e., all the time derivatives vanish), we can choose τ so that Eq. (2·21) becomes

$$\frac{\partial}{\partial \sigma}\left[\frac{1}{\sqrt{(\partial \boldsymbol{x}/\partial \sigma)^2}} \frac{\partial \boldsymbol{x}}{\partial \sigma}\right] = 0, \tag{5·1}$$

where $\partial x^\mu/\partial \tau = (1, 000)$ and $\partial x^\mu/\partial \sigma = (0, \partial \boldsymbol{x}/\partial \sigma)$. Equation (5·1) shows that in the static case our system is a straight line. Therefore, it may be more suitable to call it a linear *rod* than an elastic *string*.

Formulation developed in this paper is also applicable formally to the case of three-dimensionally extended object. Let us take, for example, the following Lagrangian:

$$\mathcal{L} = \kappa_0 \int d\xi_0 d^3\xi \sqrt{-\det g}, \tag{5·2}$$

where

$$g_{\alpha\beta} = \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x_\mu}{\partial \xi^\beta}. \quad (\alpha, \beta = 0, 1, 2, 3)$$

Then, we obtain the following constraints corresponding to (2·15) and (2·17),

$$p_\mu \frac{\partial x^\mu}{\partial \xi^a} = 0, \quad a = 1, 2, 3, \tag{5·3a}$$

$$p_\mu p^\mu + \kappa_0^2 \det \bar{g} = 0, \tag{5·3b}$$

where

$$\det \bar{g} = \begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{12} & g_{22} & g_{23} \\ g_{13} & g_{23} & g_{33} \end{vmatrix}. \tag{5·3c}$$

Though Hamiltonian dynamics can be obtained in this way, it is not easy to handle Eq. (5·3b) because of (det $\bar{g}$) which is not bilinear in x^μ. Situation is similar even in the case of two-dimensional sheet. Therefore, a one-dimensional rod is a particular example which one can treat rather easily.

The author wishes to thank Professor O. Hara for his kind and suggestive discussions and encouragement. He is also indebted to Dr. S. Ishida and Dr. J. Otokozawa for their critical comments.

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