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©Relativistic Statistical Mechanics and Particle Spectroscopy*

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Abstract

We review the formulation of manifestly covariant relativistic statistical mechanics as the description of an ensemble of events in spacetime parametrized by an invariant proper time τ. We discuss the linear and cubic mass spectra which result from this formulation (the latter with the inclusion of anti-events) as the actual spectra of an individual hadronic multiplet and hot hadronic matter, respectively. These spectra allow one to predict the masses of particles nucleated to quasi-levels in such an ensemble. As an example, the masses of the ground state mesons and baryons are considered, the results are in excellent agreement with the measured hadron masses. Additivity of inverse Regge slopes is established and shown to be consistent with available experimental data on the D^* meson and Λ_c baryon production.

Key words: special relativity, mass spectrum, hot hadronic matter, mesons, baryons, meson production, baryon production

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1 Introduction

A manifestly covariant form of relativistic statistical mechanics has more general structure than the standard forms of relativistic statistical mechanics, but reduces to those theories in a certain limit, to be described precisely below. These theories, which are characterized classically by mass-shell constraints, and the use, in quantum field theory, of fields which are constructed on the basis of mass-shell free fields, are associated with the statistical treatment of *world lines* and hence, considerable coherence (in terms of the macroscopic structure of whole world lines as the elementary objects of the theory) is implied. In nonrelativistic statistical mechanics, the elementary objects of the theory are points. The relativistic analog of this essentially structureless foundation for a statistical theory is the set of points in spacetime, i.e., the so-called *events*, not the world lines (Currie, Jordan and Sudarshan [1] have discussed the difficulty of constructing a relativistic mechanics on the basis of world lines).

The mass of particles in a mechanical theory of events is necessarily a dynamical variable, since the classical phase space of the relativistic set of events consists of the spacetime and energy-momentum coordinates $\{\mathbf{q}_i, t_i;\ \mathbf{p}_i, E_i\}$, with no *a priori* constraint on the relation between the $\mathbf{p}_i$ and the E_i, and hence such theories are "off-shell". It is well known from the work of Newton and Wigner [2] that on-shell relativistic quantum theories such as those governed by Klein-Gordon or Dirac type equations do not provide local descriptions (the wave functions corresponding to localized particles are spread out); for such theories the notion of ensembles over local initial conditions is difficult to formulate. The off-shell theory that we shall use here is, however, precisely local in both its first and second quantized forms [3, 4].

We finally remark that the standard formulations of quantum relativistic statistical mechanics, and quantum field theory at finite temperature, lack manifest covariance on a fundamental level. As for nonrelativistic statistical mechanics, the partition function is described by the Hamiltonian, which is not an invariant object, and hence thermodynamic mean values do not have tensor properties. [One could consider the invariant $p_\mu n^\mu$ in place of the Hamiltonian [5], where n^μ is a unit four-vector; this construction (supplemented by a spacelike vector othogonal to n^μ) implies an induced representation for spacetime. The quantity that takes the place of the parameter t is then $x_\mu n^\mu$; in the corresponding quantum mechanics, the space parts of (induced form of) the momentum do not commute with this time variable. Some of the problems associated with this construction are closely related to those pointed out by Currie, Jordan and Sudarshan [1], for which different world lines are predicted dynamically by the change in the form of the effective Hamiltonian in different frames.] Since the form of such a theory is not constrained by covariance requirements, its dynamical structure and predictions may be different than for a theory which satisifies these requirements. For example, the canonical distribution of Pauli [6] for the free Boltzmann gas has a high temperature limit in which the energy is given by $3k_BT$, which does not correspond to any known equipartition rule, but for the corresponding distribution for the manifestly covariant theory, the limit is $2k_BT$, in agreement with $\frac{1}{2}k_BT$ for each of the four relativistic degrees of freedom [7, 8]. For the quantum field theories at finite temperature, the path integral formulation [9] replaces the

Hamiltonian in the canonical exponent by the Lagrangian due to the infinite product of factors $\langle\phi|\pi\rangle$ (transition matrix element of the canonical field and its conjugate required to give a Weyl ordered Hamiltonian its numerical value). However, it is the t variable which is analytically continued to construct the finite temperature canonical ensemble, completely removing the covariance of the theoretical framework. One may argue that some frame has to be chosen for the statistical theory to be developed, and perhaps even for temperature to have a meaning, but as we have remarked above, the requirement of relativistic covariance has dynamical consequences (note that the model Lagrangians used in the non-covariant formulations are established with the criterion of relativistic covariance in mind), and we argue that the choice of a frame, if necessary for some physical reason, such as the definition and measurement of temperature, should be made in the framework of a manifestly covariant structure.

The standard formulation of a finite-temperature field theory is also known to experience difficulties of a different kind: Consider the breaking of Lorentz invariance in matter or temperature states [10]. It leads to the Narnhofer-Thirring theorem stating the impossibility of a perturbation theory with quasiparticles at finite temperature [11]. In other words, the entities to be used for a perturbative description in statistical systems must have a continuous *mass spectrum* – they cannot be quasiparticles. The formulation of a finite-temperature perturbation theory for quantum fields with continuous mass spectrum has been developed in [12], and generalized to time-dependent [13, 14], as well as spacially inhomogeneous [15] nonequilibrium situations. Transport equations for quantum fields with continuous mass spectrum have been derived in [16]. The framework of a manifestly covariant relativistic statistical mechanics, to be discussed in what follows, eliminates two fundamental drawbacks of the standard formulation discussed above: It has a manifestly covariant structure, and operates quasiparticles.

In the framework of a manifestly covariant relativistic statistical mechanics, the dynamical evolution of a system of N particles, for the classical case, is governed by equations of motion that are of the form of Hamilton equations for the motion of N *events* which generate the space-time trajectories (particle world lines) as functions of a continuous Poincaré-invariant parameter τ [17, 18], usually referred to as a "proper time". These events are characterized by their positions $q^\mu = (t, \mathbf{q})$ and energy-momenta $p^\mu = (E, \mathbf{p})$ in an $8N$-dimensional phase-space. For the quantum case, the system is characterized by the wave function $\psi_\tau(q_1, q_2, \ldots, q_N) \in L^2(R^{4N})$, with the measure $d^4q_1 d^4q_2 \cdots d^4q_N \equiv d^{4N}q$, $(q_i \equiv q_i^\mu; \quad \mu = 0,1,2,3; \quad i = 1,2,\ldots,N)$, describing the distribution of events, which evolves with a generalized Schrödinger equation [18]. The collection of events (called "concatenation" [19]) along each world line corresponds to a *particle*, and hence, the evolution of the state of the N-event system describes, *a posteriori*, the history in space and time of an N-particle system.

For a system of N interacting events (and hence, particles) one takes [18]

$$K = \sum_i \frac{p_i^\mu p_{i\mu}}{2M} + V(q_1, q_2, \ldots, q_N), \tag{1.1}$$

where M is a given fixed parameter (an intrinsic property of the particles), with the dimension of mass, taken to be the same for all the particles of the system. The Hamilton equations are

$$\frac{dq_i^\mu}{d\tau} = \frac{\partial K}{\partial p_{i\mu}} = \frac{p_i^\mu}{M},$$
$$\frac{dp_i^\mu}{d\tau} = -\frac{\partial K}{\partial q_{i\mu}} = -\frac{\partial V}{\partial q_{i\mu}}. \tag{1.2}$$

In the quantum theory, the generalized Schrödinger equation

$$i\frac{\partial}{\partial \tau}\psi_\tau(q_1, q_2, \ldots, q_N) = K\psi_\tau(q_1, q_2, \ldots, q_N) \tag{1.3}$$

describes the evolution of the N-body wave function $\psi_\tau(q_1, q_2, \ldots, q_N)$. To illustrate the meaning of this wave function, consider the case of a single free event. In this case (1.3) has the formal solution

$$\psi_\tau(q) = (e^{-iK_0\tau}\psi_0)(q) \tag{1.4}$$

for the evolution of the free wave packet. Let us represent $\psi_\tau(q)$ by its Fourier transform, in energy-momentum space:

$$\psi_\tau(q) = \frac{1}{(2\pi)^2}\int d^4p e^{-i\frac{p^2}{2M}\tau} e^{ip\cdot q}\psi_0(p), \tag{1.5}$$

where $p^2 \equiv p^\mu p_\mu$, $p \cdot q \equiv p^\mu q_\mu$, and $\psi_0(p)$ corresponds to the initial state. Applying the Ehrenfest arguments of stationary phase to obtain the principal contribution to $\psi_\tau(q)$ for a wave packet at p_c^μ, one finds (p_c^μ is the peak value in the distribution $\psi_0(p)$)

$$q_c^\mu \simeq \frac{p_c^\mu}{M}\tau, \tag{1.6}$$

consistent with the classical equations (1.2). Therefore, the central peak of the wave packet moves along the classical trajectory of an event, i.e., the classical world line.

It is clear from the form of (1.3) that one can construct relativistic transport theory in a form analogous to that of the nonrelativistic theory; a relativistic Boltzmann equation and its consequences, for example, was studied in ref. [20].

2 Ideal relativistic gas of events

To describe an ideal gas of events in the grand canonical ensemble, we use the expression for the number of events given in [7] (in the following we shall use the system of units in which $c = k_B = 1$, unless otherwise specified),

$$N = \sum_{p^\mu} n_{p^\mu} = \sum_{p^\mu} \frac{1}{e^{(E-\mu-\mu_K\frac{m^2}{2M})/T} \mp 1}, \tag{2.1}$$

where $E \equiv p^0$, $m^2 \equiv -p^2 = -p^\mu p_\mu$, and the sign in the denominator of (2.1) is determined by the event statistics in the usual way; μ_K is an additional mass potential [7], which arises in the grand canonical ensemble as the derivative of the free energy with respect to the value of the dynamical evolution function K, interpreted as the invariant mass of the system. In the kinetic theory [7], μ_K enters as a Lagrange multiplier for the equilibrium distribution for K, as μ is for N and $1/T$ for E. In order to simplify subsequent considerations, we shall take it to be a fixed parameter.

Since the new mass potential is atypical of the standard formulation of statistical mechanics, in cintrast to μ, we shall dwell upon the question of the additive properties of μ_K (and μ.)

2.1 Additiveness of μ and $1/\mu_K$

To clarify the additive properties of the both potentials, consider a model two-phase system. According to the Gibbs criteria, the equilibrium conditions for two phases consisting of identical particles are the equalities of temperature, chemical potential and pressure in both phases [21]. For, e.g., a system the baryon content of which is partly in quarks and partly in nucleons, the conservation of the total baryon number yields

$$\frac{N_q}{3} + N_N = \text{const}, \text{ and hence } dN_q = -3dN_N, \tag{2.2}$$

if there are N_q quarks more than antiquarks, and N_N nucleons more than antinucleons, since there are three quarks in a baryon. In order to establish a relation between the chemical potentials of quarks and nucleons at phase equilibrium, we consider the Gibbs free energy

$$G(T,P,N) = E - TS + pV. \tag{2.3}$$

The use of the second law of thermodynamics in the generalized form [7]

$$dE = TdS - pdV + \mu dN + \mu_K dK \tag{2.4}$$

in the differential of (2.3) leads to

$$dG = -SdT + Vdp + \mu dN + \mu_K dK, \tag{2.5}$$

from which [7]

$$\left(\frac{\partial G}{\partial N_q}\right)_{T,p,K} = \mu^q, \quad \left(\frac{\partial G}{\partial N_N}\right)_{T,p,K} = \mu^N. \tag{2.6}$$

By minimizing the Gibbs free energy at fixed temperature, pressure and total mass squared (K) and using (2.2),(2.6), we obtain

$$0 = \frac{\partial G}{\partial N_N} + \frac{\partial G}{\partial N_q}\frac{\partial N_q}{\partial N_N} = \frac{\partial G}{\partial N_N} - 3\frac{\partial G}{\partial N_q} = \mu^N - 3\mu^q, \tag{2.7}$$

and therefore

$$\mu^N = 3\mu^q. \tag{2.8}$$

Similarly,

$$\left(\frac{\partial G}{\partial K_q}\right)_{T,p,N} = \mu_K^q, \quad \left(\frac{\partial G}{\partial K_N}\right)_{T,p,N} = \mu_K^N. \tag{2.9}$$

Since the analog of (2.2) for the conservation of the total mass squared (K) reads, obviously,

$$K_N + 3K_q = \text{const}, \text{ and hence } dK_N = -3dK_q, \tag{2.10}$$

the minimization of the Gibbs free energy at fixed temperature, pressure and total particle number yields

$$0 = \frac{\partial G}{\partial K_N} + \frac{\partial G}{\partial K_q}\frac{\partial K_q}{\partial K_N} = \frac{\partial G}{\partial K_N} - \frac{1}{3}\frac{\partial G}{\partial K_q} = \mu_K^N - \frac{1}{3}\mu_K^q, \tag{2.11}$$

and therefore

$$\frac{1}{\mu_K^N} = \frac{3}{\mu_K^q}. \tag{2.12}$$

Following the same logics, one can easily show that, for a general type of many-species equilibrium,

$$A_1 + A_2 + \ldots + A_N \longleftrightarrow B_1 + B_2 + \ldots + B_M, \tag{2.13}$$

the chemical and mass potentials satisfy the following types of additivity:

$$\mu^{A_1} + \mu^{A_2} + \ldots + \mu^{A_N} = \mu^{B_1} + \mu^{B_2} + \ldots + \mu^{B_M}, \tag{2.14}$$

$$\frac{1}{\mu_K^{A_1}} + \frac{1}{\mu_K^{A_2}} + \ldots + \frac{1}{\mu_K^{A_N}} = \frac{1}{\mu_K^{B_1}} + \frac{1}{\mu_K^{B_2}} + \ldots + \frac{1}{\mu_K^{B_M}}. \tag{2.15}$$

3 Linear mass spectrum

In the following, we restrict ourselves to the case of the events obeying Bose-Einstein statistics and use, therefore, the relation (2.1) with the minus sign in the denominator. Similar analysis can be easily made in the case of the Fermi-Dirac events, which we skip here for brevity.

To ensure a positive-definite value for n_{p^μ}, the number density of bosons with four-momentum p^μ, we require that

$$m - \mu - \mu_K\frac{m^2}{2M} \geq 0. \tag{3.1}$$

The discriminant for the l.h.s. of the inequality must be nonnegative, i.e.,

$$\mu \leq \frac{M}{2\mu_K}. \tag{3.2}$$

For such μ, (3.1) has the solution

$$m_1 \equiv \frac{M}{\mu_K}\left(1 - \sqrt{1 - \frac{2\mu\mu_K}{M}}\right) \leq m \leq \frac{M}{\mu_K}\left(1 + \sqrt{1 - \frac{2\mu\mu_K}{M}}\right) \equiv m_2. \tag{3.3}$$

For small $\mu\mu_K/M$, the region (3.3) may be approximated by

$$\mu \leq m \leq \frac{2M}{\mu_K}. \tag{3.4}$$

One sees that μ_K plays a fundamental role in determining an upper bound of the mass spectrum, in addition to the usual lower bound $m \geq \mu$. In fact, small μ_K admits a very large range of off-shell mass, and hence can be associated with the presence of strong interactions [22]. For our present purposes it will be sufficient to assume that the mass distribution has a finite range $m_1 \leq m \leq m_2$ around the on-shell value $m_c = M/\mu_K$ corresponding to the limiting value for which the inequality (3.2) becomes an equality.

In order to show that our results hold independent of the dimensionality of spacetime, we shall consider our ensemble in one temporal and D spatial dimensions, $D \geq 1$.

Replacing the sum over p^μ (2.1) by an integral,

$$\sum_{p^\mu} \Longrightarrow \frac{V^{(1+D)}}{(2\pi\hbar)^{1+D}} \int d^{1+D}p,$$

where $V^{(1+D)}$ is the system's $(1+D)$-volume, and using the relation ($p^\mu = (p^0, \mathbf{p})$)

$$d^{(1+D)}p = \frac{d^D p}{2p^0} dm^2, \quad m^2 \equiv -p^\mu p_\mu, \quad \mu = 0, 1, \ldots, D,$$

one obtains for the density of events per unit $(1+D)$-volume, $n \equiv N/V^{(1+D)}$,

$$n = \int_{m_1}^{m_2} \frac{dm\, m}{2\pi\hbar} \int \frac{d^D\mathbf{p}}{(2\pi\hbar)^D p^0} f(p), \tag{3.5}$$

with $f(p) \equiv n_{p^\mu}$, as given in Eq. (2.1). Typical average values are given by the relations

$$\langle p^\mu \rangle = \frac{1}{n} \int_{m_1}^{m_2} \frac{dm\, m}{2\pi\hbar} \int \frac{d^D\mathbf{p}}{(2\pi\hbar)^D p^0}\, p^\mu f(p), \tag{3.6}$$

$$\langle p^\mu p^\nu \rangle = \frac{1}{n} \int_{m_1}^{m_2} \frac{dm\, m}{2\pi\hbar} \int \frac{d^D\mathbf{p}}{(2\pi\hbar)^D p^0}\, p^\mu p^\nu f(p), \quad \text{etc.} \tag{3.7}$$

To find the expressions for the pressure and energy density in our ensemble, we study the particle energy-momentum tensor defined by the relation[1] [20]

$$T^{\mu\nu}(q) = \sum_i \int d\tau \frac{p_i^\mu p_i^\nu}{m_c} \delta^{1+D}(q - q_i(\tau)), \tag{3.8}$$

in which m_c is the value around which the mass of the events making up the ensemble is distributed. Upon integrating over a small $(1+D)$-volume $\triangle V$ and taking the ensemble average, (3.8) reduces to [20]

$$\langle T^{\mu\nu} \rangle = \frac{T_{\triangle V}}{m_c} n \langle p^\mu p^\nu \rangle. \tag{3.9}$$

[1]The corresponding relation of ref. [20] is given in four-dimensional spacetime.

In this formula, $n = N/V$, and $T_{\triangle V}$ is the average passage interval in τ for the events which pass through $\triangle V$, which we discussed above. The formula (3.9) reduces, through Eq. (3.7), to

$$\langle T^{\mu\nu}\rangle = \frac{T_{\triangle V}}{2\pi\hbar m_c}\int_{m_1}^{m_2} dm\ m \int \frac{d^D\mathbf{p}}{(2\pi\hbar)^D p^0}\ p^\mu p^\nu f(p). \tag{3.10}$$

Using the standard expression

$$\langle T^{\mu\nu}\rangle = p g^{\mu\nu} - (p+\rho)u^\mu u^\nu, \tag{3.11}$$

where p and ρ are the particle pressure and energy density, respectively, we obtain

$$\rho = \langle T^{00}\rangle, \quad p = \frac{1}{D} g^{ii}\langle T_{ii}\rangle, \quad i = 1,2,\ldots,D,$$

and therefore, through (3.10),

$$p = \frac{T_{\triangle V}}{2\pi\hbar m_c}\int_{m_1}^{m_2} dm\ m \int \frac{d^D\mathbf{p}}{(2\pi\hbar)^D}\frac{\mathbf{p}^2}{Dp^0} f(p), \tag{3.12}$$

$$\rho = \frac{T_{\triangle V}}{2\pi\hbar m_c}\int_{m_1}^{m_2} dm\ m \int \frac{d^D\mathbf{p}}{(2\pi\hbar)^D}\ p^0 f(p). \tag{3.13}$$

We now calculate the particle number density per unit D-volume. The particle $(1+D)$-current is given by the formula [20]

$$J^\mu(q) = \sum_i \int d\tau \frac{p_i^\mu}{m_c}\delta^{1+D}(q - q_i(\tau)), \tag{3.14}$$

which upon integrating over a small $(1+D)$-volume and taking the average reduces to

$$\langle J^\mu\rangle = \frac{T_{\triangle V}}{m_c} n\langle p^\mu\rangle; \tag{3.15}$$

then the *particle* number density [5, 23] is

$$N_0 \equiv \langle J^0\rangle = \frac{T_{\triangle V}}{m_c} n\langle E\rangle, \tag{3.16}$$

so that, with the help of (3.6),

$$N_0 = \frac{T_{\triangle V}}{2\pi\hbar m_c}\int_{m_1}^{m_2} dm\ m \int \frac{d^D\mathbf{p}}{(2\pi\hbar)^D}\ f(p). \tag{3.17}$$

Since

$$p = \int \frac{d^D\mathbf{p}}{(2\pi\hbar)^D}\frac{\mathbf{p}^2}{Dp^0} f(p) \equiv p(m), \tag{3.18}$$

$$\rho = \int \frac{d^D\mathbf{p}}{(2\pi\hbar)^D}\ p^0 f(p) \equiv \rho(m) \tag{3.19}$$

and

$$N_0 = \int \frac{d^D\mathbf{p}}{(2\pi\hbar)^D}\, f(p) \equiv N_0(m) \tag{3.20}$$

are the standard expressions for the pressure, energy density and particle number density in $1 + D$ dimensions, respectively [24, 25, 26], we have the following relations:

$$p = \frac{T_{\triangle V}}{2\pi\hbar m_c} \int_{m_1}^{m_2} dm\; m\; p(m), \tag{3.21}$$

$$\rho = \frac{T_{\triangle V}}{2\pi\hbar m_c} \int_{m_1}^{m_2} dm\; m\; \rho(m), \tag{3.22}$$

$$N_0 = \frac{T_{\triangle}}{2\pi\hbar m_c} \int_{m_1}^{m_2} dm\; m\; N_0(m). \tag{3.23}$$

It is seen in these relations that the manifestly covariant framework provides *a linear mass spectrum, independent of the dimensionality of spacetime.* In order to obtain the expressions for the basic thermodynamic quantities, one has to integrate the standard (on-shell) results over this spectrum within the range of the mass distribution.

On the other hand, Eqs. (3.21)-(3.23) imply that the statistical ensemble provides a linear distribution. If we assume that the number of degrees of freedom is conserved in what might be considered as a sequence of nucleation to quasi-discrete levels, it follows that the degeneracy should be spread linearly on the interval $m_1 \leq m \leq m_2$, with the linear mass spectrum

$$\tau(m) = Cm, \tag{3.24}$$

which leads to the formula

$$p = \int_{m_1}^{m_2} dm\; \tau(m)\; p(m), \tag{3.25}$$

and analogous relations for ρ and N_0 (similar to the treatment of a strongly interacting system by means of a particle resonance spectrum [28]). In fact, the linear mass spectrum finds its confirmation in the experimental hadronic resonance spectrum. If one calculates, for example, the pressure in the hadronic resonance gas by summing up the individual contributions of hadronic species within a given multiplet, e.g., the vector meson nonet ($\rho(770)$, $\omega(783)$, $K^*(892)$, $\phi(1020)$), baryon $J = 1/2$ octet ($N(939)$, $\Lambda(1115)$, $\Sigma(1190)$, $\Xi(1320)$), or baryon $J = 3/2$ decuplet ($\Delta(1232)$, $\Sigma^*(1385)$, $\Xi(1530)$, $\Omega(1672)$), with the corresponding degeneracies,

$$p = \sum_i g_i\, p(m_i), \quad p(m_i) = \frac{T^2 m_i^2}{2\pi^2} K_2\left(\frac{m_i}{T}\right), \tag{3.26}$$

and by using the formula (3.25), in which m_1 and m_2 are the masses of the lightest and the heaviest species, respectively, the results coincide. The details of this analysis, as well as conclusions concerning the overall mass spectrum in the ensemble composed by the different hadronic multiplets, and the thermodynamics of this ensemble, are discussed elsewhere [29, 30, 31]). Here, for brevity, we only show that the linear mass spectrum leads the Gell-Mann–Okubo mass formula for a meson octet [32].

Let us place 8 isospin degrees of freedom of a meson octet (3 of an isovector, 4 of an isodoublet and 1 of an isoscalar; e.g., 3 ρ, 4 K^* and 1 ω_8) in the mass interval $(m_{I=1}, m_{I=0})$ in such a way that ensures the equality of the average mass squared as calculated either with the help of the linear mass spectrum or directly by definition,

$$\frac{3m_{I=1}^2 + 4m_{I=1/2}^2 + m_{I=0}^2}{8} = \langle m^2 \rangle = \frac{\int_{m_{I=1}}^{m_{I=0}} dm\; m\; m^2}{\int_{m_{I=1}}^{m_{I=0}} dm\; m} = \frac{m_{I=0}^2 + m_{I=1}^2}{2}, \tag{3.27}$$

which corresponds to the conservation of the total mass squared (K) in what might be considered as a sequence of nucleation to the quasi-discrete mass levels $m_{I=1}$, $m_{I=1/2}$, $m_{I=0}$. It then follows from (3.27) that

$$4m_{I=1/2}^2 = m_{I=1}^2 + 3m_{I=0}^2, \tag{3.28}$$

which is the standard Gell-Mann–Okubo mass formula for a meson octet [32].

In general, the normalization constant C of (3.24) is determined by the condition

$$\int_{m_1}^{m_2} \tau(m)\; dm = \nu, \tag{3.29}$$

where ν is the number of states in the mass interval (m_1, m_2); therefore

$$C = \frac{2\nu}{m_2^2 - m_1^2}. \tag{3.30}$$

It then follows that

$$\tau(m) = \frac{d\nu(m)}{dm}, \tag{3.31}$$

i.e., $\tau(m)$ represents the density of states per unit mass interval, which is linear in mass in the case of an individual hadronic multiplet [29, 30, 31].

4 Cubic mass spectrum

As discussed in a previous Section, the manifestly covariant framework provides a good model for the nucleation of individual hadronic multiplets, in terms of the linear mass spectrum. It is interesting to ask whether this framework can be suitable for the description of hadronic resonance gas which is known to be composed of different multiplets which populate linear rising Regge trajectories. As is clear from what follows, the description of hadronic resonance gas in the manifestly covariant framework necessitates the introduction of *anti-events*, i.e., the events moving in the opposite direction of a proper time. They were first discussed in [33]. It turns out that the account for the anti-events results in the opposite signs of both μ_K in the expression for the anti-event distribution function and the density of anti-events per unit $(1 + D)$-volume, as compared to those for the events. (The situation is similar to the account for antiparticles in the standard framework which results in the opposite signs of the antiparticle chemical potential and the antiparticle number density per unit space volume, as comparted to those for particles.) The use of

$$\exp\left(\frac{\mu_K m^2}{2MT}\right) - \exp\left(-\frac{\mu_K m^2}{2MT}\right) = 2\sinh\left(\frac{\mu_K m^2}{2MT}\right) \simeq \frac{\mu_K m^2}{MT}.$$

for small μ_K [22], in the expressions of Section 3, will result in the new, *cubic*, mass spectrum:

$$p = \frac{T_{\triangle V}}{2\pi\hbar m_c^2 T}\int_{m_1}^{m_2} dm\; m^3\; p(m), \tag{4.1}$$

$$\rho = \frac{T_{\triangle V}}{2\pi\hbar m_c^2 T}\int_{m_1}^{m_2} dm\; m^3\; \rho(m), \tag{4.2}$$

$$N_0 = \frac{T_{\triangle V}}{2\pi\hbar m_c^2 T}\int_{m_1}^{m_2} dm\; m^3\; N_0(m). \tag{4.3}$$

To see how this cubic mass spectrum complies with the actual mass spectrum of hadrons which populate linear trajectories, we dwell upon the latter.

4.1 Mass spectrum of linear Regge trajectories

It is very easy to show that the mass spectrum of an individual Regge trajectory is cubic. Indeed, consider, e.g., a model linear trajectory with negative intercept:

$$\alpha(t) = \alpha^{'} t - 1. \tag{4.4}$$

The integer values of $\alpha(t)$ correspond to the states with integer spin, $J = \alpha(t_J)$, the masses squared of which are $m^2(J) = t_J$. Since a spin-J state has multiplicity $2J+1$, the number of states with spin $0 \leq J \leq \mathcal{J}$ is

$$N(\mathcal{J}) = \sum_{J=0}^{\mathcal{J}}(2J+1) = (\mathcal{J}+1)^2 = \alpha^{'2} m^4(\mathcal{J}), \tag{4.5}$$

in view of (4.4), and therefore the density of states per unit mass interval (the mass spectrum) is

$$\tau(m) = \frac{dN(m)}{dm} = 4\alpha^{'2} m^3. \tag{4.6}$$

It is also clear that for a finite number of collinear trajectories, the resulting mass spectrum is

$$\tau(m) = 4N\alpha^{'2} m^3, \tag{4.7}$$

where N is the number of trajectories, and does not depend on the numerical values of trajectory intercepts, as far as its asymptotic form $m \to \infty$ is concerned.

4.2 Mass spectrum of an individual hadronic multiplet

It then follows that the form (4.7) of the cubic spectrum of the family of collinear Regge trajectories allows one to establish the normalization constant of the linear spectrum of an individual hadronic multiplet.

Consider the family of hadronic multiplets with spin 0,1,..., which populate collinear trajectories. Then the total number of states can be obtained in two ways: summing up individual trajectories for every fixed value of isospin, or summing up individual multiplets for every fixed value of spin. Either way should lead to the cubic spectrum, as discussed above. In the case of meson multiplets (similar analysis may be done in the case of baryon multiplets, of course), in Eq. (3.26) $g_i = (2J_i + 1)(2I_i + 1)^{'}$, where J_i and I_i are the values of individual spin and isospin, respectively ("$'$" means that for $I_i = 1/2$, the above expression for g_i should be multiplied by 2). Then, in view of (3.26),(4.4),

$$p = \sum_i (2J_i + 1)(2I_i + 1)^{'} p(m_i) \simeq 4N\alpha^{'2} \int dm\; m^3\; p(m). \tag{4.8}$$

Since also $J_i \simeq \alpha^{'} m_i^2$, it follows from the above expression that[2]

$$\sum_i (2I_i + 1) p(m_i) \simeq 2N\alpha^{'} \int dm\; m\; p(m), \tag{4.9}$$

i.e., one sees that the mass spectrum of an individual meson multiplet is indeed linear, and its normalization constant is $C = 2N\alpha^{'}$.

Thus, the manifestly covariant framework can be a good model for the nucleation of hadronic resonance gas composed of different multiplets which populate collinear Regge trajectories, since, as we have seen above, it does provide the cubic spectrum in its event–anti-event version. Now it becomes of special interest to ask if this cubic spectrum of the family of multiplets, or the linear spectrum of an individual multiplet, can predict the masses of the states.

5 Particle spectroscopy

Let us start with meson spectroscopy. To establish the masses of the states in the model of collinear Regge trajectories discussed above, one has to know the intramultiplet mass splitting $m^2_{I=1/2} - m^2_{I=1}$ and the mass of the lowest-lying isovector, $m_{I=1}$. The former can be easily found with the help of (4.9), for 9 isospin degrees of freedom of a meson nonet placed in the mass interval[3] $(m_{I=1}, m_{I=0})$, with $m^2_{I=0} - m^2_{I=1} = 4/3\;(m^2_{I=1/2} - m^2_{I=1}) \equiv 4/3\;\triangle$:

$$9 = 2N\alpha^{'} \int_{m_{I=1}}^{m_{I=0}} dm\; m = 2N\alpha^{'} \frac{1}{2}\frac{4}{3}\triangle; \tag{5.1}$$

[2] This results may be rigorously proven by the use of, e.g., the Euler-Maclaurin summation formula which relates a sum to an integral.

[3] We assume that the remaining ninth isoscalar belongs to this interval; As established in [29], for idealized meson nonets, its mass is equal to $(2m^2_{I=1/2} + m^2_{I=1})/3$ which coincides with the center of mass squared of a meson octet, in view of (3.27),(3.28).

therefore

$$\triangle = \frac{27}{4N\alpha'}. \tag{5.2}$$

To determine the number of collinear trajectories, we note that there are four different meson multiplets for every partial wave, except r S-wave, which in the standard spectroscopic notation are[4]

$$\begin{array}{cccccc} {}^1S_0 & {}^1P_1 & {}^1D_2 & {}^1F_3 & \dots \\ & {}^3P_0 & {}^3D_1 & {}^3F_2 & \dots \\ & {}^3P_1 & {}^3D_2 & F_3 & \dots \\ {}^3S_1 & {}^3P_2 & {}^3D_3 & {}^3F_4 & \dots \end{array}$$

(note that two missing S-wave nonets can be replaced by the radial excitations of ${}^1S_0$ and ${}^3S_1$), each of which contains 9 isospin states; therefore, the total number of different collinear meson trajectories is

$$N = 4 \times 9 = 36.$$

Hence, as follows from (5.2),

$$\triangle = \frac{3}{16\alpha'}. \tag{5.3}$$

It is well known that two isoscalar states of an idealized bare meson nonet mix with each other to form the p' ·sical states the masses of which are [29]

$$m^2_{I=0'} = m^2_{I=1}, \quad m^2_{I=0''} = 2m^2_{I=1/2} - m^2_{I=1}. \tag{5.4}$$

Therefore, one has

$$m^2_{K^*} = m^2_\rho + \frac{3}{16\alpha'}, \quad m^2_\phi = m^2_\rho + \frac{3}{8\alpha'}, \quad \text{etc.}, \tag{5.5}$$

and also

$$m^2_K = m^2_\pi + \frac{3}{16\alpha'}. \tag{5.6}$$

It is widely believed that pseudoscalar mesons are the Goldstone bosons of broken SU(3)×SU(3) chiral symmetry of QCD, and that they should be massless in the chirally-symmetric phase. Therefore, it is not clear how well would the framework that we discuss here be suitable for the description of the pseudoscalar nonet. Indeed, as we have tested in [34], this nonet is *not* described by the linear spectrum. Moreover, pseudoscalar mesons are extrimely narrow (zero width) states to fit into a resonance description. Probably, the manifestly covariant framework cannot predict the mass of the pion, although the formula (5.6) is consistent with data, as we shall see below.

Thus, the resonance description should start with vector mesons, and the cubic spectrum of a linear trajectory enables one to determine the mass of the ρ meson, as follows:

[4] In a constituent quark model, these multiplets correspond to spin-singlet and spin-triplet states of a bound system of two quarks.

Since the ρ meson has the lowest mass which the resonance description starts with. let us locate this state by normalizing the ρ trajectory to one state in the characteristic mass interval $(\sqrt{m_\rho^2 - 1/(2\alpha^{'})}, \sqrt{m_\rho^2 + 1/(2\alpha^{'})})$. With the cubic spectrum (4.6) of a linear trajectory, one has[5]

$$1 = 4\alpha^{'2} \int_{\sqrt{m_\rho^2 - 1/(2\alpha^{'})}}^{\sqrt{m_\rho^2 - 1/(2\alpha^{'})}} m^3 \, dm = 2\alpha^{'} m_\rho^2; \tag{5.7}$$

therefore

$$m_\rho^2 = \frac{1}{2\alpha^{'}}, \tag{5.8}$$

and, through (5.5),

$$m_{K^*}^2 = \frac{11}{16\alpha^{'}}, \quad m_\phi^2 = \frac{7}{8\alpha^{'}}, \quad \text{etc.} \tag{5.9}$$

Similar analysis can be easily done for baryons. Here, for brevity, we skip this analysis and only refer to [30] where preliminary discussion on the baryon spectroscopy can be found. Let us just write down the final expressions:

$$m_N^2 = \frac{3}{4\alpha^{'}}, \quad m_{\Sigma^{'}}^2 = \frac{9}{8\alpha^{'}}, \quad m_\Xi^2 = \frac{3}{2\alpha^{'}}, \tag{5.10}$$

$$m_\Delta^2 = \frac{5}{4\alpha^{'}}, \quad m_{\Sigma^*}^2 = \frac{13}{8\alpha^{'}}, \quad m_{\Xi^*}^2 = \frac{2}{\alpha^{'}}, \quad m_\Omega^2 = \frac{19}{8\alpha^{'}}, \quad \text{etc.} \tag{5.11}$$

In (5.10), $m_{\Sigma^{'}}^2 \equiv (m_\Lambda^2 + m_\Sigma^2)/2$ [30].

It is seen in (5.10),(5.11) that the mass squared splitting within an individual baryon multiplet is twice as large as that for an individual meson multiplet; e.g., $m_{\Sigma^*}^2 - m_\Delta^2 = 3/(8\alpha^{'})$, as compared to (5.5),(5.6). The mass squared splitting between multiplets which differ by one unit of spin remains, however, the same: since $m_\pi \ll m_\rho$, it follows from (5.8),(5.10),(5.11) that $m_\rho^2 - m_\pi^2 \approx m_\rho^2 = 1/(2\alpha^{'}) = m_\Delta^2 - m_N^2$. Also, the relation $m_N^2 = 3/2 \; m_\rho^2$, as follows from (5.5),(5.10), is definitely related to the valence quark structure interpretation of the two states.

5.1 Comparison with data

Now we wish to compare the formulas (5.5),(5.6),(5.10),(5.11) with available experimental data on the particle masses [35].

It is seen that the particle masses are solely determined by the value of $\alpha^{'}$. Although this parameter is known to coincide for both light mesons and baryons, it is also known to have a weak flavor dependence for light mesons [36]: $\alpha_\rho^{'} \cong 0.88$ GeV^{-2}, $\alpha_{K^*}^{'} \cong 0.85$ GeV^{-2}, α $\cong 0.81$ GeV^{-2}. Since here we are not concerned with accuracies of better than 1% (i.e., on the level of electromagnetic corrections), it would be enough to neglect the flavor dependence of $\alpha^{'}$ and take

$$\alpha^{'} = 0.85 \text{ GeV}^{-2}, \tag{5.12}$$

[5]Since the ρ trajectory starts with a spin-1 isospin-1 state (ρ), it corresponds to the spectrum $\tau(m) = 9 \times 4\alpha^{'2} m^3$. There is therefore no difference in normalizing this trajectory to 9 states, or (4.6) to one state, in the vicinity of the ρ mass.

which is the average of the above three values.

Let us start with (5.6). The use of $m_\pi = (m_\pi^0 + m_\pi^\pm)/2 = 137.3$ MeV [35] in this formula leads, via (5.12), to

$$m_K = 489.3 \text{ MeV}, \quad \text{vs.} \quad m_K = 495.7 \pm 2.0 \text{ MeV [34]}. \tag{5.13}$$

Similar comparison of the hadron masses predicted by (5.5),(5.11),(5.12) with data is presented in Table I.

State	Mass from (5.5),(5.10),(5.11), MeV	Mass from ref. [34], MeV
ρ	767.0	768.5 ± 0.6
K^*	899.3	893.9 ± 2.3
ϕ	1014.6	1019.4
N	939.3	938.9
Σ	1150.5	1155 ± 2
Ξ	1328.4	1318 ± 3
Δ	1212.7	1232 ± 2
Σ^*	1382.7	1385 ± 2
Ξ^*	1533.9	1533.5 ± 1.5
Ω	1671.6	1672.4

Table I. Comparison of the particle masses predicted by the formulas (5.5),(5.10),(5.11) with experimental data from ref. [35].

One sees excellent agreement with experiment for all states, except for Δ. We however note that this state has largest width among the ground state baryons (~ 120 MeV; for comparison, the Σ^* has largest width of ~ 9.5 MeV among the remaining ground state baryons), and therefore its mass is poorly known. Indeed, the pole position of Δ, as indicated in [35], is 1210 ± 1 MeV, and hence the prediction of Eq. (5.10) for the Δ mass is in excellent agreement with the pole position of Δ.

One can easily obtain expressions similar to (5.5),(5.10),(5.11) for other hadronic multiplets, assuming that they populate liniar trajectories; e.g.,[6] $m_{a_2}^2 = m_\rho^2 + 1/\alpha' = 3/(2\alpha') =$ 1328.4 MeV, vs. 1318 ± 1 MeV [35].

6 Intramultiplet mass relations

By comparing the mass spectra provided by the manifestly covariant framework and hadron phenomenology, Eqs. (3.21)-(3.23) and (4.9), and (4.1)-(4.3) and (4.7), respectively, one can establish the following relations (we recover $m_c = M/\mu_K$ in Eqs. (3.21)-(3.23) and (4.1)-(4.3)):

$$\frac{T_{\triangle V}\mu_K}{2\pi\hbar M} = 2N\alpha', \tag{6.1}$$

[6] It is interesting to note that, although the numerical values of $m_{a_2}^2 = m_\Xi^2 = 3/(2\alpha')$, as calculated from our formulas, do not coincide with data for $\alpha' = 0.85$ GeV^{-2}, they do coincide with each other: $m_{a_2} = m_\Xi = 1318$ MeV.

$$\frac{T_{\triangle V}\mu_K^2}{2\pi\hbar T M^2} = 4N\alpha'^2. \quad (6.2)$$

Eq. (6.1) establishes a relation of α' to the thermodynamic parameters of the off-shell system. It follows from (6.1),(6.2) that

$$T_{\triangle V} \cdot T = 2\pi\hbar N. \quad (6.3)$$

This relation is in agreement with naive expecations, since $T_{\triangle V}$ is related to the (average) extent of the ensemble along the time axis. The low-temperature limit, $T \to 0$, of the theory we are discussing here is known to coincide in form with the Galilean limit $c \to \infty$ [37], in which the theory passes over to a nonrelativistic statistical mechanics of on-shell particles. Also, the limit $N \to \infty$ means that more and more events become nucleated to on-shell particles. Each of these two limits is associated with $T_{\triangle V} \to \infty$, according to (6.3). This means that the ensemble fills a longer tube in phase space, and its displacement with τ is not as important. It is in this limit alone that a direct comparison with the usual on-shell relativistic ensemble can be made, since in this case the system possesses a stationarity in spacetime but not a non-trivial evolution in τ [7, 37].

6.1 Additiveness of inverse Regge slopes

Since M is the same for different species (which only differ by the values of μ_K, or, equivalently, by the on-shell mass values $m_c = M/\mu_K$), and $T_{\triangle V}$ is a characteristic time scale for the particles to remain very close to its mass shell (in the interpretation of a strongly interacting system as a distribution of free particles which temporarily go off-shell while undergoing an interaction [38]), which is common to every species, Eq. (6.1) really implies that particle trajectory slope is proportional to μ_K. This in turn implies *additivity of inverse slopes,* in view of additivity of μ_K's (see Section 2).

A typical example of such inverse slope additivity is "flavor equilibration",

$$A\bar{A} + B\bar{B} \longleftrightarrow A\bar{B} + B\bar{A},$$

described by

$$\frac{1}{\mu_K^{A\bar{A}}} + \frac{1}{\mu_K^{B\bar{B}}} = \frac{1}{\mu_K^{A\bar{B}}} + \frac{1}{\mu_K^{B\bar{A}}}, \quad (6.4)$$

in view of (2.15), which may be rewritten in terms of inverse Regge slopes, as follows (with $\alpha'_{A\bar{B}} = \alpha'_{B\bar{A}}$),

$$\frac{1}{\alpha'_{A\bar{A}}} + \frac{1}{\alpha'_{B\bar{B}}} = \frac{2}{\alpha'_{A\bar{B}}}. \quad (6.5)$$

Eq. (6.5) represents a crucial difference between the manifestly covariant framework which leads to (6.5) and the standard framework which operates zero width resonances, in terms of Veneziano amplitudes [39] (which contain linear trajectories), and leads to *factorization of slopes,*

$$\alpha'_{A\bar{A}} \cdot \alpha'_{B\bar{B}} = \left(\alpha'_{A\bar{B}}\right)^2, \quad (6.6)$$

which follows from the factorization of residues of the t-channel poles [40].

If one assumes linear Regge trajectories for hadrons with identical J^{PC} quantum numbers (i.e., which form a common multiplet), one will obtain for the states with spin J

$$\begin{aligned} J &= \alpha'_{A\bar{A}} m^2_{A\bar{A}} + a_{A\bar{A}}(0), \\ J &= \alpha'_{A\bar{B}} m^2_{A\bar{B}} + a_{A\bar{B}}(0), \\ J &= \alpha'_{B\bar{B}} m^2_{B\bar{B}} + a_{B\bar{B}}(0). \end{aligned}$$

Further, the following relation among the intercepts exists:

$$a_{A\bar{A}}(0) + a_{B\bar{B}}(0) = 2a_{A\bar{B}}(0). \tag{6.7}$$

This relation was first derived for $u(d)$- and s-quarks in the dual-resonance model [41]. It is satisfied in two-dimensional QCD [42], the dual-analytic model [43], and the quark bremsstrahlung model [44]. Also, it saturates inequalities for Regge trajectories [45] which follow from the s-channel unitarity condition. Hence, it may be considered as firmly established and may transcend specific models.

Then, with (6.7), one obtains from the above three relations, the quadratic intramultiplet mass formula

$$\alpha'_{A\bar{A}} m^2_{A\bar{A}} + \alpha'_{B\bar{B}} m^2_{B\bar{B}} = 2\alpha'_{A\bar{B}} m^2_{A\bar{B}}. \tag{6.8}$$

In the light hadron sector, where all slopes almost coincide and there is no difference between (6.5) and (6.6), the above formula reduces to known relations ot the type $m^2_\rho + m^2_\varphi = 2m^2_K$, which describe ideally mixed meson nonets.

However, in the heavy, or heavy-light, sector, (6.5) and (6.6) will have different implications for hadron spectroscopy.

In a series of publications [36, 46, 47, 48] we chose Eq. (6.5), since it is much more consistent with (6.8) than is Eq. (6.6), when tested by using measured quarkonia masses in Eq. (6.8). By eliminating the values of the Regge slopes from Eqs. (6.5),(6.8), we derived new (higher power) mass relations which hold with high accuracy for all well established meson multiplets, and may be reduced to quadratic formulas by fitting the values of the slopes [36, 47, 48], and new quadratic baryon mass relations [46].

Here, for brevity, we only compare predictions of (6.5) and (6.6) for the inclusive production of heavy flavors. We choose the D meson production, according to the reaction

$$\pi + N \to D + X, \tag{6.9}$$

and the Λ_c baryon production, according to the reaction

$$\pi + N \to \Lambda_c + X, \tag{6.10}$$

for both of which experimental data exist: [49, 50, 51, 52] and [53], respectively. We note that no experimental data yet exist for, e.g., the B meson production.

The following parametrization of the differential production cross section, in terms of the Feynman-x (x_F) and transverse momentum (p_T) variables is commonly used:

$$\frac{d^2\sigma}{dx_F\, dp_T^2} \approx (1 - x_F)^n e^{-bp_T^2}, \tag{6.11}$$

where b is constant, and $n = 1 - 2a(0)$, where $a(0)$ is the value of intercept of the trajectory exchanged in the production reaction.

In both the reactions (6.9) and (6.10), the D^* meson trajectory is exchanged. Therefore, for these reactions, Eq. (6.11) contains $n = 1 - 2a_{D^*}(0)$. Since the value of the D^* trajectory intercept implies the corresponding value of n which may be directly compared with available experimental data, one can choose between Eqs. (6.5),(6.6) by their predictions for $a_{D^*}(0)$.

Consider Eqs. (6.5),(6.6) with $A = n(= u, d)$, $B = c$. Introducing $x \equiv \alpha'_{c\bar{c}}/\alpha'_{n\bar{n}}$, we can rewrite Eq. (6.8) as

$$m^2_{n\bar{n}} + x m^2_{c\bar{c}} = \frac{4x}{1+x} m^2_{c\bar{n}}, \quad (6.12)$$

$$m^2_{n\bar{n}} + x^2 m^2_{c\bar{c}} = 2x m^2_{c\bar{n}}, \quad (6.13$$

respectively, using (6.5) or (6.6). The use of the measured vector and tensor meson masses in (6.12),(6.13) allows one to extract the following values of x in both cases:

$$x = 0.50 \pm 0.01 \quad \text{for Eq. (6.5)}, \quad (6.14)$$

$$x = 0.77 \pm 0.01 \quad \text{for Eq. (6.6)}. \quad (6.15)$$

Further calculation of the value of intercept of the D^* trajectory, with the formula

$$a_{D^*}(0) + \alpha'_{D^*} m^2_{D^*} = 1,$$

where α'_{D^*} is determined from (6.5),(6.12) or (6.6),(6.13) and $\alpha'_\rho = 0.88$ GeV^{-2}, gives, respectively,

$$a_{D^*}(0) = \begin{cases} -1.365 \pm 0.035 & \text{for Eq. (6.5)}, \\ -1.73 \pm 0.04 & \text{for Eq. (6.6)}. \end{cases} \quad (6.16)$$

Therefore,

$$n = \begin{cases} 3.73 \pm 0.07 & \text{for Eq. (6.5)}, \\ 4.46 \pm 0.08 & \text{for Eq. (6.6)}. \end{cases} \quad (6.17)$$

Since the above values of n differ by $\sim 20\%$, it becomes crucial to compare both of them with experiment. Available experimental data are presented in Table II.

Reference	[49]	[50]	[51]	[52]	[53]
Value of n	3.80 ± 0.63	3.74 ± 0.23	3.9 ± 0.3	3.5 ± 0.5	3.52 ± 0.5

Table I. Comparison of the values of n given in (6.17) for both Eqs. (6.5) and (6.6) with available experimental data on the D meson and Λ_c baryon production.

One sees that the value of n given by (6.5) is supported by existing experimental data, while that given by (6.6) is in apparent disagreement with these data. This confirms the conclusion drawn before on the basis of particle spectroscopy [36, 46, 47, 48] that it is additivity of inverse Regge slopes, Eq. (6.5), which is realized in the real world, not factorization of slopes, Eq. (6.6). Since additivity of inverse slopes follows clearly from additivity of inverse mass potentials in the manifestly covariant framework, experimental confirmation of additivity of inverse slopes gives extra credibility, in addition to apparent success of the linear and cubic spectra, as discussed in Sections 3-5, to manifestly covariant relativistic statistical mechanics as a framework for the description of realistic strongly interacting physical systems.

7 Concluding remarks

We have discussed manifestly covariant relativistic statistical mechanics as the description of an ensemble of events in spacetime parametrized by an invariant proper time τ. We have shown that the linear and cubic mass spectra result from this formulation (the latter with the inclusion of anti-events). We have presented evidence that these spectra are the actual spectra of an individual hadronic multiplet and hot hadronic matter, respectively. These spectra allow one to predict the masses of particles nucleated to quasi-levels in such an ensemble. As an example, we have calculated the masses of the ground state mesons and baryons, in excellent agreement with the measured hadron masses. We have established additivity of inverse Regge slopes, through additivity of the mass potentials, and shown that this additivity is consistent with available experimental data on the D^* meson and Λ_c baryon production, while factorization of slopes, as given in the standard framework, is in apparent disagreement with data. All this supports manifestly covariant relativistic statistical mechanics as a framework for the description of realistic strongly interacting physical systems.

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