Relativistic wave equations for interacting, massive particles with arbitrary half-integer spins

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New relativistic wave equations (RWE) for massive particles with arbitrary half-integer spins s interacting with external electromagnetic fields are proposed. They are based on wave functions which are irreducible tensors of rank $2n$ $(n=s-\frac{1}{2})$ antisymmetric with respect to n pairs of indices, whose components are bispinors. The form of RWE is straightforward and free of inconsistencies associated with the other approaches to equations describing interacting higher spin particles.

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I. INTRODUCTION

Over the years many relativistic wave equations (RWE) for the description of particles with arbitrary spin s have been proposed and studied in detail by the field- or group-theoretical methods (see, e.g., Refs. [1–11] and surveys in [12–14]). It turns out that the various proposed RWE are more or less equivalent as far as free particles are concerned but differ essentially in the physically more relevant cases, i.e., whenever interactions of particles with an external electromagnetic or other field are taken into account. In fact it has been discovered that several difficulties arise for RWE describing higher spin particles interacting with external fields. They are related to several mutually dependent facts and can be briefly summarized as follows.

First, the wave function which is a solution of a given first order RWE describing a particle with higher spin s $(s>1/2)$ must necessarily have more components than are theoretically required [i.e., more than $2(2s+1)$]. Hence the RWE should be provided with the appropriate number of constraints to ensure the right number of independent components of the wave function. While this requirement can be met in the case of the RWE for free particles, the introduction of interactions with an external electromagnetic field may cause a failure in this respect. It leads either to too many constraints on the components of wave function or to not enough of them (for details see [15], [16] or [18]), or it yields to an unacceptable restriction on the external field already discussed by Fierz and Pauli in [3] (see also [16]). The algebraic criteria which determine whether or not the above mentioned difficulties will arise, can be found in [10].

Second, the wave function describing a higher spin particle interacting with external fields can propagate acausally since the corresponding RWE may not be hyperbolic or the propagation speed of the wave function can be larger than that of the velocity of light in the vacuum. This phenomenon which was first discovered to the surprise of theoreticians by Velo and Zwanziger [15] in 1969 (see however also paper by Johnson and Sudarshan of [17]) reopened the problem of RWE once again—the problem that, after the papers of Salam and Mathews [19] and by Schwinger [20] had been considered as completely solved.

Third, unacceptable changes in the anticommutation rules for field components can occur when interactions with an electromagnetic (or other) field are introduced [17] (see also [13]).

In the fourth place, modes of complex frequency (i.e., the complex energy levels) may appear for a system of higher spin particle interacting with a strong external magnetic field (for details see [21]).

Fifth, starting with spin s RWE for a free particle and introducing to it interactions via minimal coupling a charged particle is described whose gyromagnetic ratio g is equal to $1/s$ instead of the desired $g=2$ (see, e.g., [8]). The other inconsistency of RWE with minimal interaction consists in the absence of spin-orbit coupling [22].

Let us remark that the above mentioned difficulties are in addition to those which appear already in the free particle theory, namely, that the charge of integer spin particle and energy of half-integer spin particle are indefinite (see, e.g., [7,22]).

In order to complete this brief survey let us mention the main disadvantages of the most frequently used approaches.

In the Bargman-Wigner formulation [5] in which the wave function has $2s$ bispinorial indices and satisfies the Dirac equation for each of them the main disadvantage consists in the impossibility to introduce a minimal interaction because the resultant equations have trivial solutions only. The same is true for covariant systems of equations proposed by Bakri [23].

In the Bhabha approach [24] the corresponding equations admit the minimal interaction [25]. But these equations describe multiplets of particles with spins equal to $s,s-1,\ldots,s_0$ where $s_0=\frac{1}{2}$ or $s_0=0$ for half-integer and integer spins, respectively.

The Lomont-Moses [26], Hagen-Hurley [27], and Dirac-like equations with differential constraints [28] are causal in the case of anomalous interaction, but yield complex energies for a particle interacting with crossed electric and magnetic fields [14].

The Weinberg equations [29] for particles of spin s contain time derivatives of order $2s$, and, as a result, admit nonphysical solutions. For the recent analysis of these equations, see Ref. [30].

The relativistic Schrödinger equations without redundant components [31] admit reasonable quasirelativistic approximations [14], however, make troubles to introduce minimal interaction since they are formulated in terms of integro-differential operators.

These inconsistencies of RWE for particles with higher spin s are especially provoking due to the following well-known experimental facts: (i) that many baryonic resonances with spins equal to $s=\frac{1}{2},\frac{3}{2},\ldots$ up to $\frac{13}{2}$ have been found and are well established [32]; (ii) that relatively stable and massive vector bosons W^+ and W^- mediating weak interactions were discovered and has been studied in great details; (iii) that there exists a number of composite systems (e.g., exotic atoms [33], or excited states of the Helium nuclei) whose energy states and other properties should be described by the RWE for particles of higher spin.

Moreover, in connection with the idea of unification of fundamental particle interactions and of quantum theory with gravity in contemporary particle physics (i.e., in string theories, supergravity, M theory, etc.) many interacting higher spin particles or other objects (p-branes) have been introduced and must be consistently described (and not only in 3+1 dimensions).

In the present paper we propose new equations for charged *massive* particles with arbitrary half-integer spins interacting with an electromagnetic (or other) external field. In fact we propose two kinds of models: one for a single interacting particle and the second one for a pair of particles or more precisely for a parity doublet. Our approach is based on wave functions with well defined tensorial and spinorial properties. Namely, our wave functions describing an interacting massive particle with higher spin s is an irreducible skew-symmetric tensor of rank $2n$ with $n=s-\frac{1}{2}$ each component of which is a bispinor.

Our approach is simple and straightforward when going from, say, $s=\frac{3}{2}$ to a general half-integer spin s, is causal, describes the anomalous interaction of a particle having spin s and preferred value $g=2$ of the gyromagnetic ratio, has a suitable nonrelativistic limit, etc.

The appearance of RWE which consistently describe pairs of higher spin particles (parity doublets) instead of single particles might be advantageous of our approach since most of above mentioned observed resonances with higher spins have been found to be parity doublets [32]. Mathematically, each of these RWE actually define a carrier space of irreducible representation of the *complete* Poincaré group (i.e., the Poincaré group including discrete transformations P, T and C) which, when considered as a representation space of the *proper* Poincaré group, corresponds to the carrier space of a reducible representations isomorphic to a direct sum of two equivalent irreducible representations.

We shall restrict ourselves to massive interracting particles since for massless ones there are no-go theorems which state that it is impossible to build a consistent theory of interaction of such particles with electromagnetic [34] or gravitational [35] fields in space-time which is assymptotically flat. However, we present a brief discussion of the massless limit of free particle equations which appears to be well defined and generates consistent equations for massless fields with arbitrary spins.

In Sec. II we outline the Rarita-Schwinger theory [36] for particles of spin $s=\frac{3}{2}$ and discuss the troubles with interaction problems. It was noticed in [37] that, contrary to the statement of paper [38] these troubles cannot be overcome with using the Singh-Hagen approach [8] (for a simple proof see Appendix A).

In Secs. III–V we introduce a new formulation of equations for particles with spin $\frac{3}{2}$ (which effectively are equations for parity doublets) which are causal. The massless limit of these equations is discussed in Sec. VI. In Sec. VII we present equations for single particle states, causality aspects of which are discussed in Appendix B.

II. RARITA-SCHWINGER EQUATION

We begin with the most popular formulation of RWE for particle of spin $\frac{3}{2}$ proposed by Rarita and Schwinger [36]. The wave function is a 16-component fourvector-bispinor $\psi^{\mu}_{(\alpha)}$ with $\mu=0,1,2,3$ being a four-vector index and $\alpha=1,2,3,4$ a bispinor index which will be usually omitted. Then the RS equation can be written in the form [36]

$$(\gamma_\lambda p^\lambda + m)\psi^\mu = 0,$$

$$\gamma_\mu \psi^\mu = 0, \tag{2.1}$$

where γ_λ are the Dirac matrices acting on the bispinor index in the following way: $(\gamma_\lambda \psi^\mu)_{(\alpha)} = \Sigma_{\sigma=1}^4 (\gamma_\lambda)_{(\alpha)(\sigma)} \psi^\mu_{(\sigma)}$.

Contracting the first of Eqs. (2.1) with γ_μ we obtain the compatibility condition for the system (2.1):

$$p_\mu \psi^\mu = 0. \tag{2.2}$$

The RS system (2.1), (2.2) can be rewritten as a single equation

$$\mathcal{F}_\mu = L_{\mu\lambda}\psi^\lambda = 0 \tag{2.3}$$

with operator $L_{\mu\nu}$ of the form

$$L_{\mu\lambda} = (\gamma^\nu p_\nu + m) g_{\mu\lambda} - \gamma_\mu p_\lambda - \gamma_\lambda p_\mu + \gamma_\mu (\gamma^\nu p_\nu - m)\gamma_\lambda . \tag{2.4}$$

Reducing Eq. (2.3) with γ_μ and p_μ we get Eqs. (2.1).

Equation (2.3) admits the Lagrangian formulation. The corresponding Lagrangian L can be written as

$$L = \bar{\psi}^\mu L_{\mu\nu} \psi^\nu, \tag{2.5}$$

where $\bar{\psi}^\mu = \psi^{\mu\dagger}\gamma_0$.

Let us discuss now the RS equation with interaction. The minimal interaction with the external e.m. field can be introduced replacing

$$p_\mu \rightarrow \pi_\mu = p_\mu - eA_\mu \tag{2.6}$$

in the considered free equation. In order to be sure that this change does not break the compatibility of our equations we have to make a change (2.6) in the Lagrangian (2.5) whose variation with respect to $\overline{\psi^{\mu}}$ generates the following equation:

$$(\gamma^{\nu}\pi_{\nu}+m)\psi^{\mu}-\gamma^{\mu}\pi_{\alpha}\psi^{\alpha}-\pi^{\mu}\gamma_{\alpha}\psi^{\alpha}+\gamma^{\mu}(\gamma^{\nu}\pi_{\nu}-m)\gamma_{\lambda}\psi^{\lambda}=0. \quad (2.7)$$

Contracting Eq. (2.7) with γ_{μ} and π_{μ} we obtain two conditions, namely

$$\gamma_{\mu}\psi^{\mu}=f^{\nu}\psi_{\nu} \quad (2.8)$$

and

$$\pi_{\mu}\psi^{\mu}=\left(\gamma_{\nu}\pi^{\nu}-\frac{3}{2}m\right)f^{\nu}\psi_{\nu}. \quad (2.9)$$

Here

$$f^{\nu}=\frac{2ie}{3m^{2}}\gamma_{\mu}\tilde{F}^{\mu\nu}, \quad \tilde{F}^{\mu\nu}=\frac{1}{2}\gamma_{5}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

with $\gamma_5=\gamma_0\gamma_1\gamma_2\gamma_3$ and $F^{\nu\sigma}=-(i/e)[\pi^{\nu},\pi^{\sigma}]$ is the strength tensor of the electromagnetic field.

Using conditions (2.8), (2.9), Eq. (2.7) reduces to the form

$$(\gamma_{\nu}\pi^{\nu}+m)\psi^{\mu}-\left(\pi^{\mu}-\frac{m}{2}\gamma^{\mu}\right)f^{\nu}\psi_{\nu}=0, \quad (2.10)$$

which together with Eq. (2.8) is equivalent to Eq. (2.7). Equation (2.10) has a nonsingular matrix coefficient for the time derivative and is called the "true motion equation."

There are two important physical requirements which have to be imposed to any RWE for a particle of spin s. Namely, that (a) the related Cauchy initial value problem must possess a unique solution depending on $2(2s+1)$ initial data functions, and that (b) the velocity of propagation of the wave solutions must not exceed the velocity of light in vacuum.

For $F_{\mu\nu}F^{\mu\nu}<2(3m^2/2e)^2$ condition (a) for the the RS equation is fulfilled due to the following facts. First, evolution equation (2.10) is supplemented by constraint (2.8). One more constraint is generated by Eq. (2.7) for $\mu=0$:

$$\pi_a\psi_a+(\gamma_a\pi_a-m)\gamma_b\psi_b=0, \quad (2.11)$$

where summation is understood over the repeated indices $a,b=1,2,3$.

Relations (2.8) and (2.11) are compatible with Eq. (2.10) and reduce the 16 components ψ^{μ} to 8 [i.e., $2(2s+1)$ with $s=3/2$] independent ones.

However, the RS equation does not satisfy requirement (b). To show this it is sufficient to consider Eqs. (2.8)–(2.10) in the eikonal approximation $\Psi^{\mu}=\hat{\psi}^{\mu}\exp(i\tau n_{\nu}x^{\nu}),\tau\rightarrow\infty$ where $\hat{\psi}^{\mu}$ are constants and n_{μ} is a constant four-vector. This actually means to substitute the characteristic four-vector n_{μ} to the covariant derivatives and keep only leading terms in n_{μ}. Then Eq. (2.10) reduces to a system of linear homogeneous algebraic equations. Equating to zero the determinant of matrix defining this system we obtain an algebraic equation for n_{μ}. Then, if all n_0 satisfying this equation are real, the system (2.7) is hyperbolic and if all n_0 satisfy $n_0^2/\mathbf{n}^2\leqslant 1$ or $n_0^2-\mathbf{n}^2\leqslant 0$ where $\mathbf{n}^2=n_1^2+n_2^2+n_3^2$, the theory is causal [15,39].

However it seems that the simplest way to prove acausality of Eqs. (2.8), (2.10) is to choose *ad hoc* special solutions of the form $\Psi^{\mu}=p^{\mu}\phi$ and show that it is acausal. In the eikonal approximation such solutions satisfy Eq. (2.10) identically provided Eq. (2.8) is satisfied. On the other hand, Eq. (2.8) is reduced to the following form:

$$\left(\gamma_{\nu}n^{\nu}-\frac{2ie}{3m^{2}}\gamma_{\nu}\tilde{F}^{\nu\sigma}n_{\sigma}\right)\phi=0. \quad (2.12)$$

Choosing $n_{\mu}=(n,0,0,0)$ we conclude that Eq. (2.12) admits nontrivial solutions for time-like n_{μ} which evidently are acausal. Moreover, it is possible to show that acausal solutions appear even for very small (but nonzero) $F^{\mu\nu}$ [15].

Thus the minimally coupled RS equation admits faster-than-light solutions and is not in this sense satisfactory. It was shown in [40] that the RS equation with anomalous interaction is acausal too (for the most recent analysis of this problem see Ref. [37]).

It is, therefore, still current to search for consistent formulations of RWE for a particle with spin $\frac{3}{2}$ and higher. They will be described in Secs. III–VII.

III. EQUATIONS FOR PARITY DOUBLETS

The *RS* equation with spin $\frac{3}{2}$ and its generalizations have been formulated in terms of fourvector-bispinor and symmetric tensor-bispinor wave functions respectively [3,8].

We shall propose here an approach valid for particles with arbitrary higher half-integer spins s in which the spin-s fermionic field is described by $\Psi^{[\mu_1\nu_1][\mu_2\nu_2]\ldots[\mu_n\nu_n]}$—an *anti-symmetric* irreducible tensor-spinor of rank $2n$[1] ($n=s-\frac{1}{2}$) satisfying the condition

$$\gamma_{\mu_1}\gamma_{\nu_1}\Psi^{[\lambda\sigma][\mu_1\nu_1]\ldots[\mu_n\nu_n]}=0, \quad (3.1)$$

where γ_{λ} and γ_{σ} are the Dirac matrices. Moreover, field $\Psi^{[\mu_1\nu_1][\mu_2\nu_2]\ldots}$ is supposed to satisfy the Dirac equation

$$(\gamma^{\lambda}p_{\lambda}-m)\Psi^{[\mu_1\nu_1][\mu_2\nu_2]\ldots[\mu_n\nu_n]}=0. \quad (3.2)$$

A mere consequence of Eqs. (3.1) and (3.2) is the following relation:

$$\gamma_{\lambda}\pi_{\sigma}\Psi^{[\lambda\sigma][\mu_2\nu_2]\ldots[\mu_n\nu_n]}=0. \quad (3.3)$$

[1]That is, the tensor antisymmetric with respect to permutations μ_i with ν_i and symmetric with respect to permutations of $[\mu_i,\nu_i]$ with $[\mu_j,\nu_j]$ and, moreover, having zero all contractions with $g_{\mu_i\nu_j}$ and $\varepsilon_{\mu_i\nu_i\mu_j\nu_j}$, $i,j=1,2,\ldots,n$.

We shall see that antisymmetric tensors are in many respects more convenient for constructing RWEs than the usually used symmetric tensors [3,8], since they more naturally lead to causal equations.

In accordance with its definition, field $\Psi^{[\mu_1\nu_1]\dots[\mu_n\nu_n]}$ transforms according to the representation

$$[D(s-\tfrac{1}{2},0)\oplus D(0,s-\tfrac{1}{2})]\otimes[D(\tfrac{1}{2},0)\oplus D(0,\tfrac{1}{2})] = D(s,0)\oplus D(0,s)\oplus D(s-\tfrac{1}{2},\tfrac{1}{2})\oplus D(\tfrac{1}{2},s-\tfrac{1}{2}) \oplus D(s-1,0)\oplus D(0,s-1) \quad (3.4)$$

of the Lorentz group, so that it has $16s$ components.

Relation (3.1) defines a static constraint, i.e., the constraint which does not involve derivatives. Expressing $p_0\Psi^{[0\nu_1][\mu_2\nu_2]\dots[\mu_n\nu_n]}$ in terms of derivatives with respect to the space variables in Eq. (3.3) we get the second, dynamical constraint.

Static constraint (3.1) suppresses the states corresponding to the representations $D(s-1,0)$ and $D(0,s-1)$ and relation (3.3) reduces half of the remaining states, so that we have exactly $4(2s+1)$ independent components, i.e., twice more than necessary.

Equations (3.1)–(3.3) can be replaced by the following equation:

$$L^{[\mu_1\nu_1][\mu_2\nu_2]\dots[\mu_n\nu_n][\lambda_1\sigma_1][\lambda_2\sigma_2]\dots[\lambda_n\sigma_n]} \times\Psi_{[\lambda_1\sigma_1][\lambda_2\sigma_2]\dots[\lambda_n\sigma_n]} \equiv(\gamma_\lambda p^\lambda-m)\Psi^{[\mu_1\nu_1][\mu_2\nu_2]\dots[\mu_n\nu_n]} -\frac{1}{4s}\sum_{\wp}(\gamma^{\mu_1}\gamma^{\nu_1}-\gamma^{\nu_1}\gamma^{\mu_1})p_\lambda\gamma_\sigma\Psi^{[\lambda\sigma][\mu_2\nu_2]\dots[\mu_n\nu_n]} =0, \quad (3.5)$$

where the symbol $\Sigma_\wp$ denotes the sum over permutations of subindices $(1,2,\dots n)$ and tensor $\Psi^{[\mu_1\nu_1][\mu_2\nu_2]\dots[\mu_n\nu_n]}$ is supposed to satisfy relation (3.1).

Contracting Eq. (3.5) with $\gamma_\mu\gamma_\nu$ we get an identity while contraction (3.5) with $p_\mu\gamma_\nu$ yields relation (3.3).

It is important to notice that Eqs. (3.5) can be derived from a Lagrangian of the form

$$L=\overline{\Psi}_{[\mu_1\nu_1][\mu_2\nu_2]\dots[\mu_n\nu_n]} \times L^{[\mu_1\nu_1][\mu_2\nu_2]\dots[\mu_n\nu_n][\lambda_1\sigma_1][\lambda_2\sigma_2]\dots[\lambda_n\sigma_n]} \times\Psi_{[\lambda_1\sigma_1][\lambda_2\sigma_2]\dots[\lambda_n\sigma_n]}, \quad (3.6)$$

with

$$L^{[\mu_1\nu_1][\mu_2\nu_2]\dots[\mu_n\nu_n][\lambda_1\sigma_1][\lambda_2\sigma_2]\dots[\lambda_n\sigma_n]} \times\Psi_{[\lambda_1\sigma_1][\lambda_2\sigma_2]\dots[\lambda_n\sigma_n]}$$

defined in Eq. (3.5) and $\Psi^{[\lambda\sigma][\mu_1\nu_1]\dots[\mu_n\nu_n]}$ assumed to satisfy Eq. (3.1).

In the case $s=\frac{3}{2}$ this Lagrangian is of the form

$$L=\overline{\Psi}_{[\mu\nu]}L^{[\mu\nu][\lambda\sigma]}\Psi_{[\lambda\sigma]}, \quad (3.7)$$

where

$$L^{[\mu\nu][\lambda\sigma]}=\tfrac{1}{2}(\gamma^\alpha p_\alpha-m)(g^{\mu\lambda}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\lambda}) -\tfrac{1}{12}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)(p^\lambda\gamma^\sigma-p^\sigma\gamma^\lambda). \quad (3.8)$$

We notice that it is always possible to chose such Lagrangian which generates also simultaneously Eq. (3.1) (so that validity of this equation is not necessary to be assumed *a priori*). For $s=3/2$ it has the form

$$L^{[\mu\nu][\lambda\sigma]}=\tfrac{1}{2}(\gamma^\alpha p_\alpha-m)(g^{\mu\lambda}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\lambda})+\tfrac{1}{12}(p^\mu\gamma^\nu -p^\nu\gamma^\mu)(\gamma^\lambda\gamma^\sigma-\gamma^\sigma\gamma^\lambda)-\tfrac{1}{12}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu) \times(p^\lambda\gamma^\sigma-p^\sigma\gamma^\lambda)+\tfrac{1}{24}(\gamma\mu\gamma^\nu-\gamma^\nu\gamma^\mu)\gamma_\rho p^\rho(\gamma^\lambda\gamma^\sigma -\gamma^\sigma\gamma^\lambda). \quad (3.9)$$

The corresponding propagator is given by

$$G^{[\mu\nu][\lambda\sigma]}=\frac{\gamma^\alpha p_\alpha+m}{p_\lambda p^\lambda-m^2}\Bigg[(g^{\mu\lambda}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\lambda})+\frac{1}{6m}(p^\mu\gamma^\nu -p^\nu\gamma^\mu)(\gamma^\lambda\gamma^\sigma-\gamma^\sigma\gamma^\lambda)-\frac{1}{6m}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu) \times(p^\lambda\gamma^\sigma-p^\sigma\gamma^\lambda)+\frac{1}{12m}(\gamma^\mu\gamma^\nu -\gamma^\nu\gamma^\mu)\gamma_\rho p^\rho(\gamma^\lambda\gamma^\sigma-\gamma^\sigma\gamma^\lambda)\Bigg]. \quad (3.10)$$

Let us remark that solutions of both our Eqs. (3.10) and those of Lomont and Moses [26] (see also [27] and [28]) transform according to the same representation of the Lorentz group specified in Eq. (3.4), and in this respect the mentioned equations are equivalent. However, due to their different forms they essentially differ in the interaction context. Whereas our tensor-spinorial formulation (3.5) seems to be suitable and very convenient for systematic and consistent introduction of various types of interactions, the Lomont-Moses formulation is consistent for description of free particles only.

IV. MINIMAL AND ANOMALOUS INTERACTIONS

The minimal interaction with an external electromagnetic field can be introduced by using replacement (2.6) in the Euler-Lagrange equation (3.5). As a result we obtain

$$(\gamma_\lambda\pi^\lambda-m)\Psi^{[\mu_1\nu_1][\mu_2\nu_2]\dots[\mu_n\nu_n]}-\frac{1}{4s}\sum_{\wp}(\gamma^{\mu_1}\gamma^{\nu_1} -\gamma^{\nu_1}\gamma^{\mu_1})\pi_\lambda\gamma_\sigma\Psi^{[\lambda\sigma][\mu_2\nu_2]\dots[\mu_n\nu_n]}=0. \quad (4.1)$$

Contracting Eq. (4.1) with $\pi_\mu\gamma_\nu$ and using Eq. (3.1) we obtain the following relation:

$$\pi_\lambda \gamma_\sigma \Psi^{[\lambda\sigma][\mu_1\nu_1]\dots[\mu_{n-1}\nu_{n-1}]} = \frac{ie}{2m}(F_{\lambda\sigma} - \gamma^\nu \gamma_\lambda F_{\sigma\nu})\Psi^{[\lambda\sigma][\mu_1\nu_1]\dots[\mu_{n-1}\nu_{n-1}]}. \tag{4.2}$$

In view of Eqs. (3.1) and (4.2), Eq. (4.1) can be written as

$$(\gamma_\mu \pi^\mu - m)\Psi^{[\mu_1\nu_1][\mu_2\nu_2]\dots[\mu_n\nu_n]} = \frac{ie}{4sm}\sum_{\wp}(\gamma^{\mu_n}\gamma^{\nu_n} - \gamma^{\nu_n}\gamma^{\mu_n})(F_{\lambda\sigma} - \gamma^\lambda \gamma_\sigma F_{\alpha\lambda})\Psi^{[\sigma\alpha][\mu_1\nu_1]\dots[\mu_{n-1}\nu_{n-1}]}. \tag{4.3}$$

Equations (4.1), (3.1), and (4.3) are suitable for description of a particle with arbitrary half-integer spin s. We shall discuss these equations in detail for the simplest case $s=\frac{3}{2}$. However, the obtained results remain true for arbitrary s.

For $s=\frac{3}{2}$ the corresponding tensor-spinor function has only one pair of indices and thus Eqs. (4.3), (3.1) are reduced to the following form:

$$\mathcal{F}^{\mu\nu} = (\gamma_\mu \pi^\mu - m)\Psi^{[\mu\nu]} - \frac{ie}{6m}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)(F_{\lambda\sigma}\Psi^{[\lambda\sigma]} + \gamma^\lambda\gamma_\sigma F_{\alpha\lambda}\Psi^{[\sigma\alpha]}) \tag{4.4}$$

and

$$\gamma_\mu \gamma_\nu \Psi^{[\mu\nu]} = 0. \tag{4.5}$$

Equation (4.4) is equivalent to the system

$$\mathcal{F}_+^{\mu\nu} = \gamma_\lambda \pi^\lambda \Psi_+^{[\mu\nu]} - m\Psi_-^{[\mu\nu]} = 0, \tag{4.6}$$

and

$$\mathcal{F}_-^{\mu\nu} = \gamma_\lambda \pi^\lambda \Psi_-^{[\mu\nu]} - m\Psi_+^{[\mu\nu]} + \tfrac{1}{6}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\gamma_\lambda \pi_\sigma \Psi_-^{[\lambda\sigma]} = 0, \tag{4.7}$$

where $\mathcal{F}_\pm^{\mu\nu} = \mathcal{F}^{\mu\nu} \pm \frac{1}{2}\gamma_5 \varepsilon^{\mu\nu}{}_{\rho\sigma}\mathcal{F}^{\rho\sigma}$, $\Psi_\pm^{[\mu\nu]} = \Psi^{[\mu\nu]} \pm \frac{1}{2}\gamma_5 \varepsilon^{\mu\nu}{}_{\rho\sigma}F^{[\rho\sigma]}$.

Solving Eq. (4.6) for $\Psi_-^{[\mu\nu]}$ and using Eq. (4.7) we obtain the following relation:

$$\left(\pi_\lambda \pi^\lambda - \frac{ie}{2}\gamma_\lambda\gamma_\sigma F^{\lambda\sigma} - m^2\right)\Psi_+^{[\mu\nu]} - \frac{i}{6}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)F_{\lambda\sigma}\Psi_+^{[\lambda\sigma]} = 0. \tag{4.8}$$

Formula (4.8) presents a nice second-order hyperbolic differential equation whose solutions $\Psi_+^{[\mu\nu]}$ are causal. The same is true for components $\Psi_-^{[\mu\nu]}$, expressed in terms of $\Psi_+^{[\mu\nu]}$ via relation (4.7), as well as for $\Psi^{[\mu\nu]}$ which is the sum of $\Psi_+^{[\mu\nu]}$ and $\Psi_-^{[\mu\nu]}$.

Let us remark that Eq. (4.8) can be expressed in the form

$$\left(\pi_\mu \pi^\mu - m^2 - \frac{ige}{2}S^{\mu\nu}F_{\mu\nu}\right)\Psi_+^{[\lambda\sigma]} = 0, \tag{4.9}$$

where $g=\frac{2}{3}$, i.e., is reciprocal to s, and $S_{\mu\nu}$ are spin generators of the Lorentz group which act on the tensor-bispinor $\Psi_+^{[\lambda\sigma]}$ in the following way:

$$S^{\rho\sigma}\Psi_+^{[\mu\nu]} = \frac{i}{4}[\gamma^\rho,\gamma^\sigma]\Psi_+^{[\mu\nu]} + i(g^{\rho\mu}\Psi_+^{[\delta\nu]} - g^{\delta\mu}\Psi_+^{[\rho\nu]} - g^{\rho\nu}\Psi_+^{[\sigma\mu]} + g^{\sigma\nu}\Psi_+^{[\rho\mu]}) \equiv \frac{3i}{4}[\gamma^\rho,\gamma^\sigma]\Psi_+^{[\mu\nu]} + \frac{i}{2}[\gamma^\mu,\gamma^\nu]\Psi_+^{[\rho\sigma]}. \tag{4.10}$$

Formula (4.9) generalizes the Zaitsev-Feynman-Gell-Mann equation for electron [41] to the case of particles with spin $\frac{3}{2}$. It describes a charged particle whose gyromagnetic ratio g is $1/s=\frac{2}{3}$.

Following Pauli [42] we can generalize Eq. (3.5) to that with "anomalous" interaction by adding to it a term $L^{[\mu\nu][\rho\sigma]}(F)$ linear in $F^{\mu\nu}$, i.e., by changing

$$L^{[\mu\nu][\rho\sigma]} \rightarrow L^{[\mu\nu][\rho\sigma]}(\pi) + L^{[\mu\nu][\rho\sigma]}(F).$$

This term can be found as a linear combination of all antisymmetric tensors linear in $F^{\mu\nu}$. The complete set of such tensors can be derived in terms of tensors $F^{[\mu\nu]}$, $\varepsilon^{\mu\nu\rho\sigma}$, $g_{\mu\nu}$ and vectors γ^μ and is given by

$$\begin{aligned}
L_1^{[\mu\nu][\rho\sigma]} &= \gamma_\lambda\gamma_\alpha F^{\lambda\alpha}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}),\\
L_2^{[\mu\nu][\rho\sigma]} &= i\gamma_5\gamma_\lambda\gamma_\alpha F^{\lambda\alpha}\varepsilon^{\mu\nu\rho\sigma},\\
L_2^{[\mu\nu][\rho\sigma]} &= F^{\mu\rho}g^{\nu\sigma} - F^{\nu\rho}g^{\mu\sigma} - F^{\mu\sigma}g^{[\nu\rho]} + F^{\nu\sigma}g^{\mu\rho},\\
L_4^{[\mu\nu][\rho\sigma]} &= i\gamma_5(F^{\alpha\mu}\varepsilon_\alpha{}^{\nu\rho\sigma} - F^{\alpha\nu}\varepsilon_\alpha{}^{\mu\rho\sigma} + F^{\alpha\rho}\varepsilon_\alpha{}^{\sigma\mu\nu} - F^{\alpha\sigma}\varepsilon_\alpha{}^{\rho\mu\nu},\\
L_5^{[\mu\nu][\rho\sigma]} &= F^{\nu\rho}\gamma^\mu\gamma^\sigma - F^{\mu\rho}\gamma^\nu\gamma^\sigma - F^{\nu\sigma}\gamma^\mu\gamma^\rho + F^{\mu\sigma}\gamma^\nu\gamma^\rho,\\
L_6^{[\mu\nu][\rho\sigma]} &= \gamma^\mu\gamma_\lambda F^{\rho\lambda}g^{\rho\sigma} - \gamma^\nu\gamma_\lambda F^{\rho\lambda}g^{\mu\sigma} - \gamma^\mu\gamma_\lambda F^{\sigma\lambda}g^{\nu\rho} + \gamma^\nu\gamma_\lambda F^{\sigma\lambda}g^{\mu\rho},\\
L_7^{[\mu\nu][\rho\sigma]} &= F^{\mu\nu}(\gamma^\rho\gamma^\sigma - \gamma^\sigma\gamma^\rho) + F^{\rho\sigma}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu).
\end{aligned} \tag{4.11}$$

Hence the general form of $L^{[\mu\nu][\rho\sigma]}(F)$ can be written as

$$L^{[\mu\nu][\rho\sigma]}(F) = \sum_{n=1}^{7}\alpha_n L_n^{[\mu\nu][\rho\sigma]}, \tag{4.12}$$

where α_n are arbitrary constants.

A natural condition which can be imposed on $L^{[\mu\nu][\rho\sigma]}(F)$ is that the equation with anomalous interaction should be compatible with relation (3.1) which suppresses spin $\frac{1}{2}$ states. The *sufficient* conditions which guarantee this property of $L^{[\mu\nu][\rho\sigma]}$ are

$$\gamma_\mu\gamma_\nu L^{[\mu\nu][\rho\sigma]}(F)=0 \tag{4.13}$$

and

$$\pi_\mu\gamma_\nu L^{[\mu\nu][\rho\sigma]}(F)=0. \tag{4.14}$$

Substituting expression (4.12) into the conditions (4.13) and (4.14) we obtain

$$4\alpha_1=4\alpha_2=\alpha_3=\alpha_4=\frac{2k}{3}, \quad \alpha_5=\alpha_6=\alpha_7=0,$$

where k is so far an arbitrary parameter. Consequently the equation with anomalous interaction is of the form

$$(\gamma^\lambda\pi_\lambda-m)\Psi^{[\mu\nu]}-\frac{1}{6}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)\pi_\alpha\gamma_\sigma\Psi^{[\alpha\sigma]} + \frac{iek}{3m}\left(\frac{1}{4}\gamma_\alpha\gamma_\sigma F^{\alpha\sigma}\Psi_+^{[\mu\nu]}+F_\alpha^\mu\Psi_+^{[\nu\alpha]}-F_\alpha^\nu\hat{\Psi}_+^{[\mu\alpha]}\right). \tag{4.15}$$

Contracting Eq. (4.15) with $\gamma_\mu\gamma_\nu$ and $\pi_\mu\gamma_\nu$ we get again conditions (3.1) and (4.2), which enable us to write Eq. (4.15) as a system which consists of (4.6) and the equation

$$\gamma_\lambda\pi^\lambda\Psi_-^{[\mu\nu]}-m\Psi_+{}^{[\mu\nu]}-\frac{1}{6}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)\gamma_\lambda\pi_\sigma\Psi_-^{[\lambda\sigma]} + \frac{iek}{3m}\left(\frac{1}{4}\gamma_\alpha\gamma_\sigma F^{\alpha\sigma}\Psi_+^{[\mu\nu]}+F_\alpha{}^\mu\Psi_+^{[\nu\alpha]}-F_\alpha{}^\nu\hat{\Psi}_+^{[\mu\alpha]}\right)=0. \tag{4.16}$$

Solving Eq. (4.6) for $\Psi_-^{[\mu\nu]}$ and using Eq. (4.16) we obtain the second order equation (4.9) in which, however, $g=\frac{2}{3}(1+k)$.

Thus the anomalous interaction causes only one thing, namely, that the gyromagnetic ratio g in Eq. (4.8) which in minimal interaction case was fixed and equal to $1/s$ becomes arbitrary, but the form of the equation remains the same. The possibility of changing g without changing the form of the equation seems to be an attractive feature of the proposed approach.

We recall that even in the case of the Dirac equation introduction of the anomalous interaction leads to a very essential complication of the theory. Indeed, the Dirac equation with minimal interaction is mathematically equivalent to Zaitsev-Feynman-Gell-Mann equation, the explicit form of which can be obtained from Eq. (4.9) by changing $\Psi_+^{[\mu\nu]}\rightarrow\psi, g\rightarrow 2, S^{\mu\nu}\rightarrow(i/2)\sigma^\mu\sigma^\nu$, where ψ is a two-component spinor and σ^μ are the Pauli matrices. In the case of anomalous interaction the related second-order equation [i.e., the analog of Eq. (4.9)] includes a second order polynomial in $F^{\mu\nu}$ and derivatives of $F^{\mu\nu}$ with respect to x_λ as well, which does not happen in our approach.

Taking $k=2$ we can get the gyromagnetic ratio g equal to 2, i.e., to its "natural value" (see, e.g., [38]).

V. FOLDY-WOUTHUYSEN REDUCTION

In order to analyze a nonrelativistic approximation of Eq. (4.15) it is convenient to make the Foldy-Wouthuysen reduction and express the corresponding Hamiltonian in a power series of $1/m$. For this purpose we shall introduce the following notations:

$$\Psi=\text{column}(\Psi^{23},\Psi^{31},\Psi^{12},\Psi^{01},\Psi^{02},\Psi^{03}),$$

$$\tilde{S}_{\mu\nu}=I_4\otimes S_{\mu\nu}, \quad \hat{S}^{\mu\nu}=\tilde{S}_{\mu\nu}+\frac{i}{4}[\hat{\gamma}^\mu,\hat{\gamma}^\nu],$$

$$\hat{\gamma}_\mu=\gamma_\mu\otimes I_6, \quad \hat{\sigma}_2=\begin{pmatrix}0 & -iI_{12}\\ iI_{12} & 0\end{pmatrix}, \tag{5.1}$$

$$S_{ab}=\varepsilon_{abc}\begin{pmatrix}s_c & 0\\ 0 & s_c\end{pmatrix}, \quad S_{0a}=\begin{pmatrix}0 & -s_a\\ s_a & 0\end{pmatrix},$$

where I_{12}, I_6, and I_4 are the 12×12, 6×6 and 4×4 unit matrices, respectively, and s_c are 3×3 matrices elements of which are $(s_c)^{ab}=i\varepsilon_{abc}$.

Then Eq. (4.16) multiplied by $\tilde{\gamma}_0$ reads

$$i\frac{\partial}{\partial t}\Psi=H\Psi, \tag{5.2}$$

where

$$H=\hat{\gamma}_0\hat{\gamma}_a\pi_a+\hat{\gamma}_0 m+eA_0+\hat{\gamma}_0(1+i\hat{\gamma}_5\hat{\sigma}_2)\frac{e}{4m}(g\hat{S}_{\mu\nu} -i\hat{\gamma}_\mu\hat{\gamma}_\nu)F^{\mu\nu}. \tag{5.3}$$

To simplify calculations we suppose that $\partial F^{ab}/\partial x_c\ll 1$, $a,b,c=1,2,3$, and $g=2$. Then, transforming $H\rightarrow H'=VHV^{-1}+i(\partial V/\partial t)V^{-1}$ where $V=\exp(iS_3)\exp(iS_2)\exp(iS_1)$ with

$$S_1=-\frac{i}{m}\hat{\gamma}_0(1+i\hat{\sigma}_2\hat{\gamma}_5)\hat{\gamma}^a\pi^a,$$

$$S_2=-\frac{\gamma_5}{4m^2}(\pi^2-e\hat{S}_{\mu\nu}F^{\mu\nu})-i\frac{\gamma_0\gamma_5}{8m^3}\times\left[e(p_aE_a+E_ap_a)-2i\frac{\partial\hat{S}_{\mu\nu}F^{\mu\nu}}{\partial t}\right],$$

$$S_3=\frac{ig}{2}\gamma_0\varepsilon_{abc}\hat{S}^{ab}\pi^c$$

and omitting terms of the order of $1/m^3$ we finally obtain[2]

$$H'=\hat{\gamma}_0\left(m+\frac{\pi^2}{2m}-\frac{\pi^4}{8m^3}-\frac{e}{m}\vec{S}\cdot\vec{H}\right)+eA_0+\frac{e}{2m^2}\vec{S}\cdot(\vec{\pi}\times\vec{E}-\vec{E}\times\vec{\pi})-\frac{e}{12m^2}Q^{ab}\frac{\partial E_a}{\partial x_b}-\frac{es(s+1)}{6m^2}\vec{\nabla}\cdot\vec{E}. \tag{5.4}$$

Here $\vec{S}$ denotes a vector (S_1,S_2,S_3) with $S_a=\frac{1}{2}\varepsilon_{abc}\hat{S}_{bc}$, $Q_{ab}=3[S_a,S_b]_+-2s(s+1)\delta_{ab}(s=\frac{3}{2})$ is the quadrupole interaction tensor, and E_a and H_a denote components of the electric and magnetic fields vectors.

All terms of Hamiltonian (5.4) have a clear physical meaning. For positive energy solutions they have the following interpretation: $m+\pi^2/2m+eA_0$ represents the Schrödinger Hamiltonian with the rest energy term, $-\pi^4/8m^3$ the relativistic correction to the kinetic energy, $(e/m)\vec{S}\cdot\vec{H}$ is the Pauli coupling, $-(e/2m^2)\vec{S}\cdot(\vec{\pi}\times\vec{E}-\vec{E}\times\vec{\pi})$ is the spin-orbit coupling, $-(e/12m^2)Q^{ab}(\partial E_a/\partial x_b)$ is the quadrupole coupling and $-[e(s+1)/6m^2]\vec{\nabla}\cdot\vec{E}$ is the Darwin coupling.

Let us remark that all equations starting with Eq. (4.3) up to Eq. (5.4) can easily be extended to the case with *arbitrary* half-integer spin s. As a result we obtain the quasirelativistic Hamiltonian (5.4) which is of the same form but with $\vec{S}$ corresponding to appropriate spin matrices for the considered spin s.

VI. THE MASSLESS CASE

It is well known that relativistic wave equations for massless particles with higher spins cannot be generally obtained from those for massive particles by taking the limit $m\rightarrow 0$ [4]. Here we demonstrate that tensor-spinorial equations (3.1) and (3.5) have similiar properties like the Dirac equation, namely, that they have a clear physical meaning for $m=0$ provided some additional constraints are imposed on their solutions.

We begin with spin $s=\frac{3}{2}$. Taking Eq. (3.5) appropriate for this case, setting in it $m=0$ and supposing that the condition

$$\gamma_\nu\Psi^{[\mu\nu]}=0 \tag{6.1}$$

is true, we obtain the equation

$$\gamma^\alpha p_\alpha\Psi^{[\mu\nu]}=0 \tag{6.2}$$

which describes a massless field whose helicities are $\pm\frac{3}{2}$ and energy signs are ± 1. This can be shown in the following way.

Reducing Eq. (6.2) with γ_ν and using Eq. (6.1) we get the condition

$$p_\nu\Psi^{[\mu\nu]}=0. \tag{6.3}$$

It follows from Eqs. (6.1) and (6.3) that

$$\varepsilon_{\mu\nu\rho\sigma}\gamma^\nu\Psi^{[\rho\sigma]}=0 \tag{6.4}$$

and

$$\varepsilon_{\mu\nu\rho\sigma}p^\nu\Psi^{[\rho\sigma]}=0. \tag{6.5}$$

In other words field $\Psi^{[\mu\nu]}$ satisfies both the massless Dirac equation (6.2) and the Maxwell equations (6.3) and (6.5).

Condition (6.4) reduces the number of independent components of $\Psi^{\mu\nu}$ to 8 while relation (6.3) reduces this number to 4. To prove that solutions of Eqs. (6.2), (6.1) correspond to helicities $\pm\frac{3}{2}$ relations (4.10) and (6.5) should be used from which follow that

$$\varepsilon_{abc}S^{ab}p_c\Psi^{[\mu\nu]}=\frac{3}{4}i\varepsilon_{abc}\gamma^a\gamma^b p_c\Psi^{[\mu\nu]}\equiv\frac{3}{2}i\gamma_5\gamma_0\gamma^a p_a\Psi^{[\mu\nu]}. \tag{6.6}$$

In accordance with Eqs. (6.2) and (6.6) the eigenvalues of the related helicity operator coincide with eigenvalues of the energy sign operator multiplied by $\pm\frac{3}{2}$. Thus solutions of Eqs. (6.2) and (6.1) belong to the carrier space of the irreducible representation $D^+(\frac{3}{2})\oplus D^-(\frac{3}{2})\oplus D^+(-\frac{3}{2})\oplus D^-(-\frac{3}{2})$ of the Poincaré group, where $D^\epsilon(\lambda)$ denotes representation corresponding to energy sign ϵ and to helicity λ. Imposing the additional constraints $(1+i\gamma_5)\Psi^{[\mu\nu]}=0$ or $(1-i\gamma_5)\Psi^{[\mu\nu]}=0$ it is possible to reduce this representation to $D^+(\frac{3}{2})\oplus D^-(-\frac{3}{2})$ or $D^-(\frac{3}{2})\oplus D^+(-\frac{3}{2})$. In other words, relations (6.2) and (6.1) form a natural generalization of the massless Dirac equation to the case of spin $\frac{3}{2}$.

We note that the ansatz

$$\Psi^{[\mu\nu]}=p^\mu\Psi^\nu-p^\nu\Psi^\mu, \tag{6.7}$$

where Ψ^μ is a vector-spinor satisfying the condition $\gamma_\lambda\Psi^\lambda=0$ reduces Eqs. (6.1), (6.2) to the massless RS equation for Ψ^μ:

$$\gamma^\alpha p_\alpha\Psi^\mu=0, \qquad \gamma_\lambda\Psi^\lambda=0.$$

Equation (6.7) is invariant with respect to the gauge transformation $\Psi^\lambda\rightarrow\Psi^\lambda+\partial\varphi/\partial x_\lambda$, where φ is an arbitrary solution of the massless Dirac equation $\gamma^\alpha p_\alpha\varphi=0$.

Analogously, starting with Eqs. (3.1), (3.5) for arbitrary spin we come to the following equations for the massless field with spin $s=(2n+1)/2$:

$$\gamma^\alpha p_\alpha\Psi^{[\mu_1\nu_1][\mu_2\nu_2]\dots[\mu_n\nu_n]}=0,$$

$$\gamma_\alpha\Psi^{[\alpha\nu_1][\mu_2\nu_2]\dots[\mu_n\nu_n]}=0.$$

Like solutions of Eqs. (6.2) and (6.1), the related wave function $\Psi^{[\mu_1\nu_1][\mu_2\nu_2]\dots[\mu_n\nu_n]}$ has only four independent components corresponding to states with helicities $\pm s$ and energy signs ± 1.

[2]The only term in Eq. (5.4) which is of order $1/m^3$, i.e., the term $\pi^4/8m^3$, should be present in as much as it is of the same order $1/c^2$ as the last three terms (c is the speed of light). Using the Heaviside units in which $h=c=1$ leads to implicit dependence of the Hamiltonian on c.

VII. SINGLE PARTICLE EQUATIONS

As was shown, Eqs. (3.1) and (3.2) describe a doublet of relativistic particles with spin s. In order to find the Poincaré and parity invariant equation for a single particle it is necessary to impose on $\Psi^{[\mu_1\nu_1][\mu_2\nu_2]\dots[\mu_n\nu_n]}$ an additional condition which annuls half of the physical components. It can be taken in the form

$$p_\mu \Psi^{[\mu\nu_1][\mu_2\nu_2]\dots[\mu_n\nu_n]}=0. \tag{7.1}$$

The resulting system, i.e., Eqs. (3.1), (3.2), and (7.1), obviously satisfies all required invariance properties and describes a particle of arbitrary half-integer spin $s=2n+1/2$.

In the case $s=\frac{3}{2}$ this system is reduced to the equations

$$(\gamma^\lambda p_\lambda - m)\Psi^{[\mu\nu]}=0, \tag{7.2}$$

$$\gamma_\mu\gamma_\nu\Psi^{[\mu\nu]}=0, \tag{7.3}$$

$$p_\mu\Psi^{[\mu\nu]}=0. \tag{7.4}$$

In the rest frame $p=(m,0,0,0)$ relation (7.4) reduces to $m\Psi^{[oa]}=0$, which implies $\Psi^{[oa]}=0$. Thus Eq. (7.4) annuls half of the components of the wave function. On the other hand, in the rest frame condition (7.3) can be written as

$$\vec{S}^2\Psi = s(s+1)\Psi, \quad s=\frac{3}{2}, \tag{7.5}$$

where $\Psi=\text{column}(\Psi^{[23]},\Psi^{[31]},\Psi^{[12]})$ and $\vec{S}=(S_{23},S_{31},S_{12})$ is the total spin for the tensor-spinor wave function, components of which are given in Eq. (4.10).

The system of equations (7.2)–(7.4) can be replaced by one equivalent equation which is of the form

$$(\gamma^\lambda p_\lambda - m)\Psi^{[\mu\nu]}+\gamma^\nu p_\lambda\Psi^{[\lambda\mu]}-\gamma^\mu p_\lambda\Psi^{[\lambda\nu]}-\frac{1}{2}\Bigg[\gamma^\mu p^\nu - \gamma^\nu p^\mu - (\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)\left(\frac{1}{2}\gamma_\lambda p^\lambda - \frac{m}{3}\right)\Bigg]\gamma_\lambda\gamma_\sigma\Psi^{[\lambda\sigma]} = 0. \tag{7.6}$$

Indeed, reducing Eq. (7.6) with $\gamma_\mu\gamma_\nu$ and $p_\mu\gamma_\nu$ we get the system (7.2)–(7.4). On the other hand, reducing Eq. (7.6) with γ_ν and denoting $\gamma_\nu\Psi^{[\mu\nu]}$ by Ψ^μ we obtain the RS equation (2.3) as an algebraic consequence of Eq. (7.6). However, Eq. (7.6) is not of the Euler-Lagrange type.

In order to find a Lagrangian generating Eqs. (7.2)–(7.4) one should add an auxiliary field. Using this old idea of Fierz and Pauli [3] the desired Lagrangian is given by

$$L = L^{TS}+L^{RS}+L^{CR}, \tag{7.7}$$

where L^{TS} is the Lagrangian of the tensor-spinor field defined in Eqs. (3.7), (3.9), L^{RS} is the Rarita-Schwinger Lagrangian given in Eq. (2.5) and L^{CR} is the "crossed Lagrangian" of the form

$$L^{CR} = -\overline{\Psi}_{[\mu\nu]}p^\mu\Psi^\nu+\overline{\Psi}_\mu p_\nu\Psi^{[\mu\nu]} - \tfrac{1}{12}(\overline{\Psi}_{[\mu\nu]}\gamma^\mu\gamma^\nu(p_\lambda-\gamma_s p^s\gamma_\lambda)\Psi^\lambda - \overline{\Psi}^\lambda(p_\lambda-\gamma_s p^s\gamma_\lambda)\gamma^\mu\gamma^\nu\Psi_{[\mu\nu]}). \tag{7.8}$$

Variation of Lagrangian (7.7) with respect to $\tilde{\Psi}_{[\mu\nu]}$ and $\tilde{\Psi}_\mu$ yields two equations: namely,

$$(\gamma_\lambda p^\lambda - m)\Psi^{[\mu\nu]}-p^\mu\Psi^\nu+p^\nu\Psi^\mu+\tfrac{1}{12}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)(f - 2p_\lambda\gamma_\sigma\Psi^{[\lambda\sigma]})+\tfrac{1}{24}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)\times(\gamma_\lambda p^\lambda - m)\gamma_\lambda\gamma_\sigma\Psi^{[\lambda\sigma]}=0 \tag{7.9}$$

and

$$(\gamma_\lambda p^\lambda + m)\Psi^\mu - \gamma^\mu(f+m\gamma_\lambda\Psi^\lambda)-p^\mu\gamma_\lambda\Psi^\lambda+p_\nu\Psi^{[\mu\nu]} - (p^\mu-\gamma_\lambda p^\lambda\gamma^\mu)\gamma_\lambda\gamma_\sigma\Psi^{[\lambda\sigma]}=0, \tag{7.10}$$

in which f denotes the expression $p_\lambda\Psi^\lambda-\gamma_\lambda p^\lambda\gamma_\nu\Psi^\nu$.

Reducing Eq. (7.9) with $\gamma_\mu\gamma_\nu$ we obtain condition (7.3). Thus Eqs. (7.9) and (7.10) can be simplified to

$$(\gamma_\lambda p^\lambda - m)\Psi^{[\mu\nu]}-p^\mu\Psi^\nu+p^\nu\Psi^\mu+\tfrac{1}{12}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)\times(f-2p_\lambda\gamma_\sigma\Psi^{[\lambda\sigma]})=0 \tag{7.11}$$

and

$$(\gamma_\lambda p^\lambda + m)\Psi^\mu - \gamma^\mu(f+m\gamma_\lambda\Psi^\lambda)-p^\mu\gamma_\lambda\Psi^\lambda+p_\nu\Psi^{[\mu\nu]}=0, \tag{7.12}$$

respectively.

Reducing Eq. (7.11) successively with γ_μ, p_μ, and $p_\mu\gamma_\nu$, and Eq. (7.12) with γ_μ and p_μ, we obtain Eqs. (7.2)–(7.4) for $\Psi^{[\mu\nu]}$ and the condition $\Psi^\mu=0$. In other words, the equations of motion annul the auxiliary field Ψ^μ and are equivalent to the system (7.2)–(7.4) describing a particle of spin $\frac{3}{2}$ and mass m.

Taking into account relation (7.3) it is convenient to represent $\Psi^{[\mu\nu]}$ in the form

$$\Psi^{[\mu\nu]}=\chi^{\mu\nu}+\frac{1}{2}(\gamma^\mu A^\nu - \gamma^\nu A^\mu), \tag{7.13}$$

where $\chi^{\mu\nu}$ and A^μ is a γ-irreducible tensor and vector, respectively. They satisfy the conditions: $\chi^{\mu\nu}=-\chi^{\nu\mu}, \gamma_\nu\chi^{\mu\nu}=0$ and $\gamma_\nu A^\nu=0$. In view of Eq. (7.3) we easily find that $\chi^{\mu\nu}=\frac{1}{2}\Psi_+^{[\mu\nu]}$ and $A^\mu=-\gamma_\nu\Psi^{[\mu\nu]}$.

Using variables (7.13) and introducing a minimal interaction via replacement p_μ by π_μ we can write the related equations (7.3), (7.11) and (7.12) in the following equivalent form:

$$\pi^\mu(\Psi^\nu - A^\nu)-\pi^\nu(\Psi^\mu-A^\mu)+\tfrac{1}{12}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)\hat{f} + m(\chi^{\mu\nu}-\tfrac{1}{2}(\gamma^\mu A^\nu-\gamma^\nu A^\mu)+\gamma^\mu\Psi^\nu-\gamma^\nu\Psi^\mu)=0, \tag{7.14}$$

$$2\gamma_\lambda\pi^\lambda\chi^{\mu\nu}-m(\gamma^\mu A^\nu-\gamma^\nu A^\mu)-(\pi\cdot\Psi)_+^{\mu\nu}=0, \tag{7.15}$$

where the following notations have been used: $(\pi\cdot\Psi)^{\mu\nu}_{\pm}=\pi^{\mu}\Psi^{\nu}-\pi^{\nu}\Psi^{\mu}\pm\frac{1}{2}\gamma_5\varepsilon^{\mu\nu\rho\sigma}\pi_{\rho}\Psi_{\sigma}$, $\hat{f}=\pi_{\lambda}\Psi^{\lambda}-\gamma_{\lambda}\pi^{\lambda}\gamma_{\nu}\Psi^{\nu}$.

We show in Appendix B that for a rather extended class of external fields Eqs. (7.14) and (7.15) remain causal.

VIII. DISCUSSION

In the paper RWE for a massive interacting particle with arbitrary half integer spin s has been proposed and especially for $s=3/2$ discussed in detail.

RWE considered in Sec. III are causal and free of most inconsistences which are typical for equations for particles of spin greater than 1. Moreover, these equations have a physically suitable form in quasirelativistic approximation and are able to describe mostly used interactions such as Pauli, spin-orbit, quadrupole and Darwin couplings. We recall that even such popular equations as the Kemmer-Duffin-Petiau [43] one does not describe the spin-orbit coupling in the framework of the minimal interaction principle [22].

The other attractive feature of the tensor-spinorial wave equations consists in their hidden simplicity which can be recognized considering the second-order equation (4.9) for the physical components. This equation can be easily solved for many particular cases of the external fields as it was done in [28,44] for the special case of $g=1/s$. We plan to present these exact solutions elsewhere.

The considered equations have a reasonable zero mass limit for a free particle case and so can serve as a basis to formulate consistent equations for massless fields with arbitrary spin. Such equations were discussed briefly in Sec. VI.

Finally, introduction of anomalous interaction into the tensor-spinorial wave equations generates a surprisingly small complexity of the theory in comparison with the case of the minimal interaction. In this aspect the proposed equations are quite unique and are more convenient than even the Dirac equation.

We do not discuss specific kind of difficulties connected with the complex energy eigenvalues for the case of interaction with the constant magnetic field provided the gyromagnetic ratio g of the particle is equal to 2 [21]. This problem arises also for the tensor-spinorial wave equation, but it can be overcome using the approach proposed in [45].

For completeness notice that single particle equations for spin $\frac{3}{2}$ considered in Sec. VII correspond to the Harish-Chandra index 4 and thus belong to the class described by Labonté [46]. We believe that our tensor-spinorial formulation (7.6)–(7.12) and (7.14), (7.15) forms an appropriate basis for the theory of interacting particles of arbitrary half-integer spin and its various applications.

APPENDIX A: INCONSISTENCY OF SINGH-HAGEN EQUATIONS

A specific formulation of the RS equations was used by Singh and Hagen [8] who introduced an additional scalar-bispinor field ψ such that ψ^{μ} and ψ satisfy the following system:

$$\tilde{\mathcal{F}}^{\mu}=(\gamma^{\nu}p_{\nu}+m)\tilde{\psi}^{\mu}-\tfrac{1}{2}\gamma^{\mu}p_{\lambda}\tilde{\psi}^{\lambda}-\tfrac{2}{3}(p^{\mu}-\tfrac{1}{4}\gamma^{\mu}\gamma^{\nu}p_{\nu})\tilde{\psi}=0, \tag{A1}$$

$$\tilde{\mathcal{F}}=p_{\nu}\tilde{\psi}^{\nu}-(\gamma^{\nu}p_{\nu}-2m)\tilde{\psi}=0, \qquad \gamma_{\mu}\tilde{\psi}^{\mu}=0.$$

Equations (A1) are equivalent to the RS equations. Indeed, denoting in Eq. (2.1) $\tilde{\psi}^{\mu}+\frac{1}{3}\gamma^{\mu}\tilde{\psi}$ by ψ^{μ} we easily find that Eq. (A1) is an algebraic consequence of Eq. (2.1) and vice versa, because

$$\tilde{\mathcal{F}}=\tfrac{1}{2}\gamma_{\mu}\mathcal{F}^{\mu}, \quad \tilde{\mathcal{F}}^{\mu}=\mathcal{F}^{\mu}-\tfrac{1}{4}\gamma^{\mu}\gamma_{\lambda}\mathcal{F}^{\lambda},$$
$$\mathcal{F}^{\mu}=\tilde{\mathcal{F}}^{\mu}+\tfrac{1}{2}\gamma^{\mu}\tilde{\mathcal{F}}. \tag{A2}$$

In contradistinction to the RS equation, it was stated in [38] that the Singh-Hagen formulation (A1) is causal provided a nontrivial anomalous interaction is introduced. We think that this statement has no meaning since in the case of anomalous interaction proposed in [38] the Singh-Hagen equations became inconsistent. This can be easily seen in the following way. The equation proposed in [38] has the form

$$\tilde{\mathcal{F}}^{\mu}=(\gamma^{\nu}p_{\nu}+m)\tilde{\psi}^{\mu}-\tfrac{1}{2}\gamma^{\mu}p_{\lambda}\tilde{\psi}^{\lambda}-\tfrac{2}{3}(p^{\mu}-\tfrac{1}{4}\gamma^{\mu}\gamma^{\nu}p_{\nu})\tilde{\psi}+\alpha F^{\mu\nu}_{+}\tilde{\psi}_{\nu}=0, \tag{A3}$$

$$\tilde{\mathcal{F}}=p_{\nu}\tilde{\psi}^{\nu}-(\gamma^{\nu}p_{\nu}-2m)\tilde{\psi}=0, \qquad \gamma_{\mu}\tilde{\psi}^{\mu}=0,$$

where α is a coupling constant.

Using relations (A2) we reduce Eq. (A3) to the RS form

$$(\gamma^{\nu}\pi_{\nu}+m)\psi^{\mu}-\gamma^{\mu}\pi_{\alpha}\psi^{\alpha}-\pi^{\mu}\gamma_{\alpha}\psi^{\alpha}+\gamma^{\mu}(\gamma^{\nu}\pi_{\nu}-m)\gamma_{\lambda}\psi^{\lambda}+T^{\mu\nu}\psi_{\nu}=0, \tag{A4}$$

where

$$T^{\mu\nu}=\alpha(F^{\mu\nu}_{+}-\tfrac{1}{4}F^{\mu\lambda}_{+}\gamma_{\lambda}\gamma^{\nu}). \tag{A5}$$

It is easy to show that in contrast with Eq. (2.7), Eq. (A4) does not include required eight constraints but only four of them. Indeed, reducing Eq. (A4) with γ_{μ} and π_{μ} we obtain the correct number of constraints only for the case $T^{00}=0$ [40], which is compatible with Eq. (A5) only for the trivial anomalous interaction $\alpha F^{\mu\nu}_{+}=0$.

APPENDIX B: CONSISTENCY OF EQUATIONS FOR SINGLETS

Let us show that for some class of external fields Eqs. (7.14), (7.15) are consistent, i.e., include the correct number of constraints and are hyperbolic. To do this we will use also differential and algebraic consequences of these equations.

Contracting Eqs. (7.14), (7.15) with γ_{ν} and $\gamma_{\mu}\gamma_{\nu}$ and using Eqs. (7.14), (7.15) we come to the equivalent γ-irreducible set of equations:

$$(\pi\cdot\Psi)_{-}^{\mu\nu}-(\pi\cdot A)_{-}^{\mu\nu}+2m\chi^{\mu\nu}+\tfrac{1}{6}(\gamma^{\mu}\gamma^{\nu}-\gamma^{\nu}\gamma^{\mu})\times(\hat{f}+3m\gamma_{\lambda}\Psi^{\lambda})=0, \quad \text{(B1)}$$

$$\gamma_{\lambda}\pi^{\lambda}(\Psi^{\mu}-A^{\mu})-\pi^{\mu}\gamma_{\lambda}\Psi^{\lambda}+m(2\Psi^{\mu}-A^{\mu})+\gamma^{\mu}(\tfrac{1}{2}\hat{f}+m\gamma_{\lambda}\Psi^{\lambda})=0, \quad \text{(B2)}$$

$$2\pi_{\nu}\chi^{\mu\nu}+\gamma_{\lambda}\pi^{\lambda}A^{\mu}-2m(\Psi^{\mu}-A^{\mu})-\gamma^{\mu}(\hat{f}+m\gamma_{\lambda}\Psi^{\lambda})=0, \quad \text{(B3)}$$

$$\pi_{\nu}A^{\nu}-2\hat{f}-3m\gamma_{\lambda}\Psi^{\lambda}=0. \quad \text{(B4)}$$

The other (differential) consequences can be found by reducing Eqs. (B1)–(B3) with p_{μ}. In this way we obtain from Eqs. (B2), (B3) the following two relations:

$$\gamma_{\lambda}\pi^{\lambda}\hat{f}+6m^{2}\gamma_{\lambda}\Psi^{\lambda}-2i\gamma_{\lambda}\tilde{F}^{\lambda\sigma}(\Psi_{\sigma}-A_{\sigma})=0 \quad \text{(B5)}$$

and

$$m\hat{f}=i(\gamma_{\lambda}\tilde{F}^{\lambda\sigma}(\tfrac{1}{2}A_{\sigma}-\Psi_{\sigma})+\tfrac{1}{2}\tilde{F}_{\lambda\sigma}\chi^{\lambda\sigma}). \quad \text{(B6)}$$

One more consequence can be obtained reducing Eq. (B1) with π_{ν}, acting on Eq. (B2) by $\gamma_{\lambda}\pi^{\lambda}$ and adding the resultant expressions together. We get

$$m^{2}\Psi^{\mu}+\tfrac{1}{6}\pi^{\mu}\hat{f}+i(\tilde{F}^{\mu\sigma}-\tfrac{1}{3}\gamma^{\mu}\gamma_{\lambda}\tilde{F}^{\lambda\sigma})(\Psi_{\sigma}-A_{\sigma})=0. \quad \text{(B7)}$$

Reducing Eq. (B7) once more with π_{μ} we obtain a scalar consequence

$$\left(m^{2}-\frac{i}{12}\gamma_{\mu}\gamma_{\nu}F^{\mu\nu}\right)\hat{f}=-i\pi_{\lambda}\tilde{F}^{\lambda\sigma}(\Psi_{\sigma}-A_{\sigma}). \quad \text{(B8)}$$

Applying operator $\gamma_{\lambda}\pi^{\lambda}$ to Eq. (B7) and using Eqs. (B2), (B5) we come to one more consequence

$$m^{2}\left(\gamma_{\lambda}\pi^{\lambda}\Psi^{\mu}-\frac{i}{3}\pi^{\mu}\gamma_{\lambda}\Psi^{\lambda}\right)+\frac{i}{3}(\gamma^{\mu}\gamma_{\lambda}\pi^{\lambda}-\pi^{\mu})\gamma_{\lambda}\tilde{F}^{\lambda\sigma}(\Psi_{\sigma}-A_{\sigma})-i\gamma_{\lambda}\pi^{\lambda}\tilde{F}^{\mu\sigma}(\Psi_{\sigma}-A_{\sigma})+\frac{i}{6}F^{\mu\lambda}\gamma_{\lambda}\hat{f}=0. \quad \text{(B9)}$$

Finally, contracting Eq. (B7) with $F^{\mu\lambda}\pi_{\lambda}$ and using Eq. (7.15) we get the following important condition:

$$m^{2}F^{\mu\lambda}\pi_{\lambda}\Psi^{\mu}-\frac{i}{3}F^{\mu\lambda}\pi_{\lambda}\gamma^{\mu}\gamma_{\lambda}\tilde{F}^{\lambda\sigma}(\Psi_{\sigma}-A_{\sigma})-i\gamma_{5}C_{2}(\pi_{\sigma}\Psi^{\sigma}-2\hat{f}-3m\gamma_{\lambda}\Psi^{\lambda})-F^{\mu\lambda}\left(\frac{\partial}{\partial x^{\lambda}}\tilde{F}^{\mu\sigma}\right)\times(\Psi_{\sigma}-A_{\sigma})=0, \quad \text{(B10)}$$

where $C_{2}=F^{\mu\lambda}\tilde{F}_{\mu\lambda}$ is an invariant of the electromagnetic field.

Now we are ready to analyze the constraint context of Eqs. (7.14) and (7.15). First we note that the considered system includes nine dependent variables (each being a four-component spinor). To describe a particle of spin $\frac{3}{2}$ it is sufficient to have eight degrees of freedom and so we need seven constraints which do exist. Six of them are presented by Eqs. (7.14) for $\mu,\nu=1,2,3$ and Eq. (B7) for $\mu=1,2,3$. The seventh constraint is easily obtained from Eq. (B3) for $\mu=0$ and Eq. (B4):

$$2\pi_{a}\chi^{0a}-\gamma_{a}\pi_{a}A^{0}+\gamma^{0}(\pi_{a}A_{a}+2m\gamma_{\lambda}\Psi^{\lambda}+\hat{f})+2m(\Psi^{0}-A^{0})=0.$$

The next task is to find the true motion equations. They are given by Eq. (7.14) for $\mu=1,2,3,\nu=0$, by Eqs. (B3), (B9) for $\mu=1,2,3$ and by Eq. (B10). The related matrix with time derivatives is nonsingular provided

$$C_{2}\neq 0 \quad \text{or (and)} \quad \tilde{F}^{0a}\neq 0. \quad \text{(B11)}$$

On the other hand, if $C_{2}=0$ and $\tilde{F}^{0a}=0$ then relation (B7) for $\mu=0$ reduces to the constraint expressing ψ^{0} via other variables and so that in this case we do not need a motion equation for ψ^{0}.

To investigate causality we consider the true motion equations in the eikonal approximation. Substituting the characteristic four-vector n_{μ} to the covariant derivatives and keep only leading terms in n_{μ} we come to the following system:

$$n^{\mu}(\Psi^{\nu}-A^{\nu})-n^{\nu}(\Psi^{\mu}-A^{\mu})=0,\qquad 2n_{\nu}\chi^{\mu\nu}+\gamma_{\lambda}n^{\lambda}A^{\mu}=0, \quad \text{(B12)}$$

$$m^{2}(\gamma_{\lambda}n^{\lambda}\Psi^{\mu}-n^{\mu}\gamma_{\lambda}\Psi^{\lambda})+\frac{i}{3}(\gamma^{\mu}\gamma_{\lambda}n^{\lambda}-n^{\mu})\times\gamma_{\lambda}\tilde{F}^{\lambda\sigma}(\Psi_{\sigma}-A_{\sigma})=0,$$

$$m^{2}F^{\mu\lambda}n_{\lambda}\Psi_{\mu}-\frac{i}{3}F^{\mu\lambda}n_{\lambda}\gamma_{\mu}\gamma_{\alpha}\tilde{F}^{\alpha\sigma}(\Psi_{\sigma}-A_{\sigma})-i\gamma_{5}C_{2}n_{\sigma}\Psi^{\sigma}=0.$$

Setting $n_{\mu}=(n,0,0,0)$ in Eq. (B12) we easily find that $\chi^{\mu\nu}=A^{\mu}=\Psi^{\mu}=0$ provided n_{0} and C_{2} are not equal to zero. Thus Eqs. (7.14), (7.15) are causal provided the external electromagnetic field satisfies the covariant relation $C_{2}\neq 0$.

We remind that acausality of the RS equation is caused by noncovariance of its hyperbolicity condition [47].

[1] P.A.M. Dirac, Proc. R. Soc. London **A117**, 616 (1928).
[2] P.A.M. Dirac, Proc. R. Soc. London **A155**, 447 (1936).
[3] M. Fierz and W. Pauli, Proc. R. Soc. London **A173**, 211 (1939).
[4] W. Harish-Chandra, Proc. R. Soc. London **A186**, 502 (1946); Phys. Rev. D **74**, 883 (1948).
[5] V. Bargmann and E. Wigner, Proc. Natl. Acad. Sci. U.S.A. **34**, 221 (1948).
[6] H. Umezawa, *Quantum Field Theory* (North-Holland, Amsterdam, 1956).
[7] I.M. Gel'fand and A.M. Yaglom, Sov. Phys. JETP **18**, 703 (1948); see also I.M. Gel'fand, R.A. Minlos, and Z.Ya. Shapiro, *Representations of the Rotation and Lorentz Groups and Their Applications* (Pergamon, Oxford, 1963).
[8] L.P.H. Singh and C.R. Hagen, Phys. Rev. D **9**, 898 (1974).
[9] R. Kotecký and J. Niederle, Rep. Math. Phys. **12**, 237 (1977).
[10] P.M. Mathews, T.R. Govindarajar, and M. Seetharaman, J. Math. Phys. **21**, 1495 (1980).
[11] E.M. Corson, *Tensors, Spinors and Relativistic Wave Equations* (Blackie & Son, Glasgow, 1953).
[12] A.S. Wightman, *Troubles in the External Field Problems for Invariant Wave Equations* (Gordon and Breach, New York, 1971).
[13] A.S. Wightman, Lecture Notes in Physics Vol. 73 (Springer-Verlag, Berlin, 1978), p. 1.
[14] W. I. Fushchich and A.G. Nikitin, *Symmetries of Equations of Quantum Mechanics* (Allerton Press, Allerton, NY, 1994).
[15] G. Velo and D. Zwanziger, Phys. Rev. **186**, 1218 (1969).
[16] J.D. Jenkins, J. Phys. A **5**, 1461 (1972); J. Seetharaman, J. Prabhakaran, and P.M. Mathews, Phys. Rev. D **12**, 458 (1975).
[17] K. Johnson and E.C.G. Sudarshan, Ann. Phys. (N.Y.) **13**, 126 (1961).
[18] P. Federbush, Nuovo Cimento **19**, 572 (1961); see also [10] and C.R. Hagen, Phys. Rev. D **4**, 2204 (1971).
[19] A. Salam and P.T. Mathews, Phys. Rev. **90**, 690 (1953).
[20] J. Schwinger, Phys. Rev. **93**, 615 (1954).
[21] W. Tsai, Phys. Rev. D **7**, 1945 (1973); P.M. Mathews, *ibid.* **9**, 365 (1974).
[22] J.D. Bjorken and S.D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).
[23] M.M. Backri, J. Math. Phys. **10**, 298 (1969).
[24] H.J. Bhabha, Rev. Mod. Phys. **17**, 200 (1945).
[25] R.A. Kraicik and N.M. Nieto, Phys. Rev. D **14**, 418 (1976).
[26] L.S. Lomont and H.E. Moses, Phys. Rev. **118**, 337 (1960).
[27] C.R. Hagen and W.I. Hurley, Phys. Rev. Lett. **24**, 1381 (1970); W.J. Hurley, Phys. Rev. D **4**, 3605 (1971).
[28] A.G. Nikitin and W.I. Fushchich, Teor. Mat. Fiz. **34**, 319 (1978) [Theor. Math. Phys. **34**, 203 (1978)].
[29] S. Weinberg, Phys. Rev. **133**, B1318 (1964).
[30] D.V. Ahluwalia and D.J. Ernst, Phys. Rev. C **45**, 3010 (1992); V.V. Dvoeglazov, Int. J. Theor. Phys. **37**, 1915 (1998).
[31] D.L. Weaver, C.L. Hammer, and R.H. Good, Phys. Rev. **135**, B241 (1964); P.M. Mathews, Phys. Rev. **143**, 978 (1966); W.I. Fushchich, A.L. Grishchenko, and A.G. Nikitin, Teor. Mat. Fiz. **8**, 192 (1971).
[32] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).
[33] H.H. Giannini and H.I. Krivoruchenko, Phys. Lett. B **291**, 329 (1992).
[34] Yu.M. Zinoviev, in Proceeding of XVII Workshop "Problems of High Energy Physics and Field Theory," Protvino, 1994.
[35] C. Aragone and S. Deser, Phys. Lett. **86B**, 161 (1979).
[36] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).
[37] S. Deser, V. Pascalutsa, and A. Waldron, Phys. Rev. D **62**, 105031 (2000).
[38] S. Ferrara, M. Porrati, and V.L. Telegdi, Phys. Rev. D **46**, 3529 (1992).
[39] R. Courant and D. Gilbert, *Methods of Mathematical Physics* (Wiley-Interscience, New York, 1992), Vol. 2, Chap. VI.
[40] A. Shamali and A.Z. Capri, Ann. Phys. (N.Y.) **74**, 503 (1972).
[41] G.A. Zaitsev, Sov. Phys. JETP **28**, 530 (1955); R.P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[42] W. Pauli, Rev. Mod. Phys. **13**, 203 (1941).
[43] N. Kemmer, Proc. R. Soc. London **A173**, 91 (1939); R.J. Duffin, Phys. Rev. **54**, 1114 (1938); G. Petiau, Acad. Roy. Belgique, Classe Si., Mémoire Cod. in 8^0, **16** (1936) Fasc. 2.
[44] A.G. Nikitin, Teor. Mat. Fiz. **57**, 253 (1983); W.I. Fushchich, A.G. Nikitin, and V.V. Susloparow, Nuovo Cimento A **87**, 415 (1985).
[45] J. Beckers, N. Debergh, and A.G. Nikitin, Fortschr. Phys. **43**, 81 (1995).
[46] G. Labonté, Nuovo Cimento A **80**, 77 (1984).
[47] A.F. Ranada and G.S. Rodero, Phys. Rev. D **22**, 385 (1980).