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Paper

Relay Feedback Systems— Established Approaches and New Perspectives for Application

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Relay feedback systems belong, since long, to the rather classical topics of control theory and engineering. While they were first used as robust and simple switching elements in automatic control systems, over time, they have experienced different periods of renewed interest with regards to both theoretical analysis and applications. Examples include the automatic tuning of simple feedback controllers, delta-sigma modulators for analog-digital converters, robust relay-based controls, nonlinear systems analysis and identification, among others. One of the remarkable features of using relays in feedback is the appearance of stable limit cycles. The structural and parametric conditions for the stable limit cycles, their prediction, and controllability of magnitude and frequency have already been solved, to a large part, in previous researches, driven by both theoretical curiosity and application requirements. This brief tutorial paper summarizes the basic principles of relay feedback systems, discusses several characteristics that are interesting from an application perspective, and addresses some issues related to its further use for system identification. A case-specific study for estimating the unknown backlash, hidden within two-mass systems, is demonstrated along with an experimental example, based on provoking the controllable drifting limit cycles by the non-ideal relay in the velocity feedback loop.

Keywords: relay feedback systems, harmonic balance, nonlinearity, limit cycles, backlash identification, hysteresis

1. Introduction and Problem Overview

Relay feedback systems, that are belonging to rather classical topics of the control and system theory, are mostly understood as dynamic LTI (linear time-invariant) systems which additionally incorporate the discontinuous relay elements in feedback. Using a standard state-space notation, with the input vector $u \in \mathbb{R}^m$, output vector $y \in \mathbb{R}^m$, and state vector $x \in \mathbb{R}^n$, a relay feedback system can be written as

$$\begin{cases} \dot{x} = Ax + Bu, \\ y = Cx, \\ u = -H(y). \end{cases} \quad (1)$$

Note that while u can be understood as a control vector in case of the LTI system regulation, it can equally serve as a feedback coupling vector in case of the relay-type plant nonlinearities (for instance Coulomb friction [1][2]). Obviously, the LTI part of the relay feedback system (1) is parameterized by the system matrix $A \in \mathbb{R}^{n\times n}$ and input and output distribution matrices $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{m\times n}$ correspondingly. The vector-valued nonlinear function H includes the decoupled from each other, i.e. without cross-talk, relay operators h_i with $i = 1, \ldots, m$. Each of them can assume only two discrete states, ± 1 for convenience. Among different possible definitions, a relay operator can be written in the closed analytic form, according to [3], as

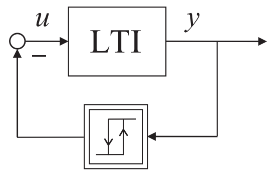

Fig. 1. SISO relay feedback system

$$h(y) = \min\Big[\mathrm{sign}(y+\epsilon), \max\big[h(y^-), \mathrm{sign}(y-\epsilon)\big]\Big]. \quad (2)$$

It switches between two discrete states upon the threshold values $\pm\epsilon$, while the output state at the previous time instant is denoted by y^-. Note that (2) is the general case of a symmetric non-ideal relay with hysteresis, sometimes also called hysteron, while an ideal relay (switching in both directions at zero) arises directly when setting the threshold value $\epsilon = 0$. For the rest of the paper, we will focus solely on the SISO (single-input-single-output) LTI systems, i.e. with $m = 1$, and that for the sake of simplicity and without loss of generality. A relay feedback loop is exemplified in Fig. 1.

From a historical perspective, the relay feedback systems have been intensively studied in the former works of Tsypkin [4] (originally published in Russian in 1974 [5]; an interesting fact is that the same Tsypkin's seminal work appeared already in 1958 in German [6]). At that time, intensive efforts on analyzing and reshaping dynamics of the systems incorporating the relay elements have been motivated by the versatile electromechanical devices, like e.g. thermal regulators, controlled motors and valves, gyroscope instruments, and others. A considerable collection of applications of the

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relay feedback systems can be found in the first chapter of [4], including the classical academic examples of an automatic temperature regulation with two-level controller (bang-bang controller), and centrifugal vibrational speed controller of an electric motor. Also the simplified relay-like transitions have been assumed for describing the discontinuous effects in the system behavior, like e.g. dry friction, mechanical play, and others. Some related analysis can be found also in an early work [7].

Over time, the initial relay use as switching and correspondingly amplifying elements for automatic control, became yet less interesting due to a rapid development of the integrated circuits and power electronic technologies. Nevertheless, the analysis of dynamic systems containing a relay feedback was continuously inspiring the theoretical works, due to a close relationship to such phenomena as self-oscillations and limit cycles, sliding and chattering modes, stability and solutions of differential equations with discontinuous right-hand side, and others. In other ways, the relay feedbacks have been used in self-oscillating adaptive controllers, see e.g. [8], developed for the flight control systems and missiles [9]. These self-oscillating-adaptive systems [10][11] benefited, already at that early stage, from the intentionally induced limit cycles, hereby allowing for the parameter adaptivity, correspondingly control synthesis. Later, the same theoretical principles came to flourishing in the developed automatic tuning of the simple feedback regulators [12][13]. The mostly known developments in the field of adaptive control around 60th (for the Western world) are summarized in [14].

Further developments in the digital computing and signal processing brought additional attention to the relay feedback systems incorporated into the integrated circuits and electronic elements. One of the most pronounced examples is the so-called delta-sigma modulator [15], developed as an alternative to the conventional parallel A/D (analog-to-digital) converters, and particularly successful in high-resolution applications at lower rates. An ideal relay, even though denoted in the delta-sigma modulation literature mostly as quantizer [16], is incorporated in a feedback loop which constitutes a l-bit quantization circuit. Here a detailed analysis and prediction of the limit cycles have been required to ensure a stable operation of the delta-sigma modulators [17]–[19]. Investigations showed that the appearance of the so-called saturation limit cycles is a primary problem that leads to instable behavior of the high-order systems with quantizer (i.e. relay) in feedback. Further analysis of the nonlinear behavior of delta-sigma modulators can be also found e.g. in [20].

The relay feedback systems became again topical in control theory and, over time, very common in the control engineering due to their explicit use for automatic tuning of the simple feedback controllers. This type of auto-tuning approach has certain similarity to the well-known Ziegler-Nichols closed-loop tuning method [21], for which the ultimate gain and ultimate period are experimentally determined from a sustained oscillation of the output variable under control. On the contrary to the Ziegler-Nichols method, which requires expert knowledge and presence when gradually increasing the control loop gain and can bear risks of destabilizing the process, the relay method generates a stable limit cycle. This, again, constitutes a sustained oscillation of the output variable, but with the amplitude restricted to a safe range and, a priory, adjustable to the require level by the relay gain. The introduction of the relay-based auto-tuners into practical applications, including the backgrounds and illustrative examples, can be credited to the seminal work [12]. A later tutorial review on the relay feedback auto-tuning of process controllers can be found in [13], while the consolidated details about the oscillations in systems with a relay feedback can be found in [22]. Since then, variations of relay feedback-based algorithms and approaches for auto-tuning have been implemented and used in different industrial controllers, while numerous related literature appeared, e.g. [23]–[25].

The most relevant properties of the limit cycles in relay feedback systems, and that the existence and analytic form of a stable solution, have been addressed already in the former work [26]. It has been shown that the related analysis can be performed by introducing two fictitious synchronous samplers, operating at the relay switching instants, and zero-order hold into the feedback loop, so that is is valid

$$Y(z)+U(z)G(z)=0 \quad \text{with} \quad G(z)=(1-z^{-1})\mathcal{Z}\left[P(s)s^{-1}\right]. \qquad (3)$$

The z-transform input/output (Y and U) across the relay in the loop are related to each other via the first-order sampling and hold transformation $\mathcal{Z}$ of the system transfer function $P(s)$. Balancing the terms of the above equation in the steady-state allows one obtaining the frequency of the limit cycles as a function of the relay and system process parameters. The necessary conditions for limit cycles and discussion on their stability can be found further in [22]. The global asymptotic stability of the unimodal limit cycles have been later addressed in detail and proved in [27], based on finding quadratic surface Lyapunov functions for Poincaré maps associated with relay feedback systems. For the case of ideal relays, i.e. with $\epsilon = 0$, there can be initial conditions for which no solution exists. Indeed, for an ideal relay feedback system with $CB > 0$ there can be intervals on the switching surface where the vector fields on the both sides of the switching surface are orthogonal to that, while having the same magnitude and opposite sign. Such situation is well-known and easily interpretable through the case of a linear feedback control system with dry friction represented by the discontinuous Coulomb friction law, see e.g. in [1]. There exists a certain dead-band in vicinity to the reference set point, within which the control system remains for always sticking and no trajectories proceed since being on zero velocity switching surface with $CB > 0$. For the non-ideal relays, i.e. with $\epsilon > 0$, the existence of the limit cycles reduces to the question of trajectory reaching, correspondingly crossing, the opposite switching surface. For any initial point x_0, the equilibrium point is given by $A^{-1} \cdot (\pm B)$, depending on the initial state ∓ 1 of the relay in feedback. In fact, it has been shown [27] as necessary to have $CA^{-1}B + \epsilon < 0$ for the globally stable limit cycles exist. Otherwise, a trajectory starting at $A^{-1} \cdot (\pm B)$ would not cross the opposite switching surface and, therefore, not converge to a steady limit cycle. Due to associated complexity of the analysis and methodology provided in [27] we refrain from giving further principles and conditions developed there and refer to the original work.

Since stable limit cycles of the relay feedback systems contain several steady characteristics like amplitude and period, an idea of using it for identification is natural. Therefore, already former works, like e.g. [28], tried to use for example biased-relay feedback for system identification. Later, a technique for frequency response identification from the relay feedback has been proposed [29] and demonstrated on a real-time simulator of the linear plants. Another related approach of a relay-based frequency response identification that, however, required extending, correspondingly restructuring, the closed-loop system has been proposed in [30].

Obviously, operating in the limit cycles, and that in a feedback control manner, can provide more reliable signature of the system behavior and is less influenced by unsystematic or transient perturbations than other open-loop or linear closed-loop methods. In addition, a discontinuous (i.e. step-wise due to the switching relay action) input can also ensure a sufficient excitation of the whole system dynamics. That means several minor, or even hidden, system characteristics can be detected at all. Otherwise, they are not always visible during a straight time-domain identification, or can be simply overlooked within an averaging steady-state identification in frequency-domain. In that way, a parameter identification based on limit cycles in the relay feedback systems can, to certain degree, inherit the advantages of both, time- and frequency-domain. While the estimation of general linear system characteristics, i.e. frequency response, has already benefited from a relay feedback approach, as referred above, the case-specific and especially nonlinear system parameters can still have a large undiscovered potential for the relay-based identification. For instance, an approach for system mass measurement using the relay feedback with hysteresis has been reported in [31]. The most recently developed method [32] of backlash identification in two-mass systems, by using the non-ideal relay in feedback, cf. Fig. 1, will be also discussed in detail in the last part of this paper.

The rest of the paper is organized as follows. In Section 2, we briefly revisit the use of the relay feedback for auto-tuning of simple linear feedback controllers. This part largely orients on the established works [12][13][33] and, additionally, provides illustrative explanations to the use of relay feedback with related properties. In Section 3, an original approach [32] of identifying the unknown backlash nonlinearity, hidden inside of the two-mass systems, by using the drifting limit cycles is presented. An experimental case study is accompanying the proposed method and is also presented. The conclusions are drawn in Section 4.

2. Relay Feedback for Controller Tuning

In this Section, we will discuss the basics behind a relay feedback auto-tuning of the standard process controllers, i.e. of a PID-type. We note that the below developments are close to those provided in more detail in [12][33].

Considering a continuous time process $P(s) = Y(s)/U(s)$, and that with only stable poles and, eventually, one free integrator, a linear transfer function regulator $R(s) = U(s)/E(s)$, which should be tuned, can be assumed. Note that, in a most simple case, $R(s)$ constitutes a PID controller which is a weighted superposition of proportional, integral, and derivative terms of the control error $E(s) = Y^*(s) - Y(s)$. A closed-loop control specification can be given, for instance, by the required phase margin ϕ_m of the open-loop transfer function. This case, the Nyquist plot of $R(j\omega) \cdot P(j\omega)$ should cross (in the complex plain) the unit circle at a particular point S satisfying $\phi_m = \arg(S) + \pi$. Then, a rather 'classical' procedure of tuning the feedback controller $R(s)$ requires first identifying, to say measuring, at least one point M, with the corresponding angular frequency ω_M, so that the complex equation

$$M \cdot R(j\omega) = S \quad\cdots\cdots (4)$$

can be solved with respect to the controller parameters. Note that solving (4) by varying, i.e. tuning, the control gains is in the same spirit as what one knows under the loop shaping [34][35], independent of regarded either the Nyquist or Bode plots. Once a point $M \mathrel{\widehat{=}} P(j\omega_M)$ is found, or more precisely determined from the measurements, the controller parameters have to be computed so as to move this point onto the unit cycle [33]. In doing so, the control specification requirements, such as phase and/or amplitude margins, should be met. It is worth noting that other closed-loop requirements, like for instance the control damping or bandwidth, can be considered based on the same principles, when determining the control parameters that will move M into S.

When the process loop is closed by the k-gained relay (2), arranged in a negative feedback instead of controller R, the system loop starts exhibiting sustainable oscillations, i.e. limit cycles, at the angular frequency $\omega_c = 2\pi/T_u$. Such loop configuration, used for controller tuning as developed in [12] and related works, is shown in Fig. 2 for the notations made above. Note that T_u is the ultimate period of the closed-loop harmonic oscillations – the same as one can detect when applying the well-known Ziegler-Nichols approach (cf. Section 1), in terms of finding the ultimate (also denoted as critical) gain K_u. As well-known from the describing function analysis [36], the first apparent harmonic oscillation will correspond to the point where the negative inverse describing function $-1/N(a)$ crosses the Nyquist curve of the process $P(j\omega)$. The describing function of relay with hysteresis is given by

$$N(a) = \frac{4k}{\pi a}\left(\sqrt{\frac{a^2-\epsilon^2}{a^2}} - j\frac{\epsilon}{a}\right), \quad\cdots\cdots (5)$$

where a is the amplitude of the relay input, correspondingly process output which is steady-state oscillating. The relay amplitude and hysteresis width parameters are denoted by k and ϵ correspondingly, cf. with Section 1. Obviously, an ideal relay, for which $\epsilon \to 0$, yields

$$-\frac{1}{\bar{N}(a)} = -\frac{\pi a}{4k}, \quad\cdots\cdots (6)$$

that lies on the negative real axis, therefore requiring the

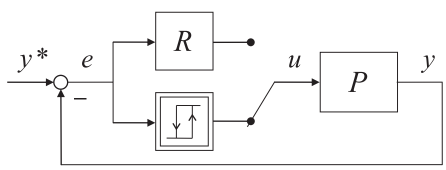

Fig. 2. Closed-loop configuration for relay auto-tuning

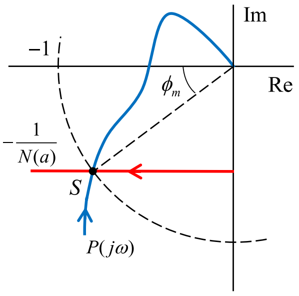

Fig. 3. Nyquist plot of the process P and negative reciprocal of the describing function N of the relay with hysteresis; if the intersection point S lies on the unit circle, the closed-loop system has the phase margin ϕ_m

$P(j\omega)$ to have a phase lag of at least $-\pi$ at higher frequencies. Otherwise, the Nyquist curve of $P(j\omega)$ is not intercepting the negative real axis, and no appearance of the limit cycles can be expected. Quite on the contrary, a hysteresis relay allows shifting the straight of the negative inverse describing function along the imaginary axis, by simply varying the ϵ parameter, cf. Fig. 3. This way, various M points can be determined in the complex plain, therefore providing the tuning process with a second degree-of-freedom, in addition to the relay gain k. Note that in Fig. 3, the harmonic balance intersection of both curves is shown already in the desired point S on the unit circle. This case, the closed-loop system has the phase margin ϕ_m, which can be seen as a design reference. Since $N(a)$ is given, and the amplitude and frequency of the process output oscillations are derived from the measurements, the open-loop transfer function can be identified from the harmonic balance $N(a)P(j\omega_c) + 1 = 0$.

For instance, when assuming the open-loop to be $KG(s) = R(s)P(s)$ one perform an auto-tuning of the loop gain K, for the required phase margin ϕ_m. For the intersection of $KG(j\omega)$ Nyquist curve and negative reciprocal of the describing function lies on the unit circle, cf. with Fig. 3, the relay parameters should be set as

$$k = \frac{\pi a^*}{4}, \quad \epsilon = a^* \sin(\phi_m), \cdots\cdots\cdots (7)$$

cf. with [12]. Note that a^* is the desired amplitude of oscillations and, therefore, an auxiliary design parameter for experiments, that respects particularities of the operation range of the process under control. While the desired intersection point S can be determined by the amplitude of oscillations, the auto-tuning strategy, proposed in [12], is to reach this point by adapting the K-gain in dependency of the instantaneous amplitude error $\alpha = a(n) - a^*$ at the n-th discrete step. The loop gain adaption law [12]

$$K(n+1) = K(n) - \alpha \frac{K(n) - K(n-1)}{a(n) - a(n-1)} \cdots\cdots\cdots (8)$$

provides a quadratic convergence rate close to the solution.

Even without an explicit auto-tuning algorithm, the use of the limit cycles generated by a relay in feedback has certain advantages in comparison to the classical Ziegler-Nichols closed-loop tuning strategy. First, the process output quantity becomes inherently limited and can be adjusted (by the relay parameters) to a desired level. This is also significant

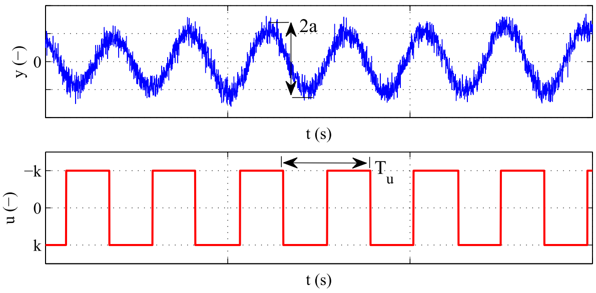

Fig. 4. Oscillating harmonics of the process output (above) and relay (below) connected into the closed-loop

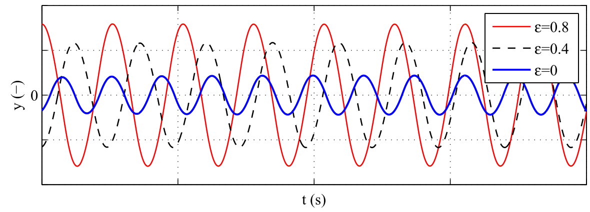

Fig. 5. Process output oscillations for variable ϵ

when the output noise should be taken into account. Furthermore, an accurate read-out of the T_u-value from the measured process output can turn out as awkward when applying a standard Ziegler-Nichols closed-loop tuning procedure. The relay-based tuning, on the contrary, provides an exact T_u-value, since that one remains the same for the square-wave relay output. The latter is, indeed, a controller-internal quantity, without possible distortions by an external measurement or process noise. The numerically simulated time series of the process and relay outputs, connected in a closed-loop so as to induce the steady limit cycles, are exemplary shown in Fig. 4. For better highlighting the measurement issues mentioned above, the process output in feedback is subject to an additional band-limited white noise.

Considering the first, i.e. dominant, harmonic of the relay output oscillations, it follows from a Fourier series expansion that the ultimate gain is approximately given by

$$K_u \approx \frac{1}{\pi a^2} \left| 4ak \sqrt{\frac{a^2 - \epsilon^2}{a^2}} - j\,4k\epsilon \right|, \cdots\cdots\cdots (9)$$

when the amplitude of process output is a. This also follows directly from (5) when evaluating the absolute value $|N(a)|$. Recall that the relay describing function should fulfil the harmonic balance condition $-1/N(a) = \mathrm{Re}(P) + j\mathrm{Im}(P)$ of the limit cycles. One can recognize that for an ideal relay, the ultimate gain (9) reduces to $4k/(\pi a)$, cf. with [12]. An interesting fact is that increasing the hysteresis width, i.e. $\epsilon \uparrow$ while keeping the same k-value, can lead to an increased a, cf. Fig. 5, while the ultimate gain will reduce according to (9). Also the period T_u of the limit cycles, correspondingly oscillation frequency, changes so that the relay with hysteresis provides generally more flexibility when designing a feedback tuning loop. Once the ultimate gain and period are determined, the parameters of a feedback controller $R(s)$, for instance PID one, can be assigned by using various reference tables and plots like e.g. Ziegler-Nichols tuning rules, see e.g. in [37][38].

3. Backlash Identification with Relay Feedback

One of the application-specific identification problems, to benefit from the relay feedback systems, is the estimation of unknown (hidden) backlash in the drive chains – in general terms. The backlash related issues are very common in the system and control engineering, see e.g. [(39)] for survey, and have been addressed already in the early works [(40)], and that from a viewpoint of system dynamics and effect on the control behavior. Backlash (also known as mechanical play) can arise nearly everywhere, once two adjacent movable parts of a mechanism are connected via joint and/or gearing, in order to transmit, correspondingly transform, relative motions and forces. Applications area is broad and the related studies, just to mention some of them, can be found for servo drive systems [(41)–(43)], industrial and medical robotics [(44)–(46)], automotive power trains [(47)(48)], and others. Exact information about the presence and extent of backlash in an actuated mechanism is rarely known in advance, and is rather conditional upon the factors such as wear, fatigue and incipient failures in components. All the more, a detection and estimation of backlash during regular operations, without special disassembling and sensing measures, can be required for monitoring purposes, equally as for a control-based attenuation of performance degradation caused by the backlash. When both sides of a backlash pair can be measured, for instance equipped with position encoders, the identification of backlash becomes a trivial task to be accomplished under quasi-static, to say low-excitation, conditions. If only a one-side sensing of the relative motion is available, which is a common case for numerous applications, the backlash identification become a non-trivial task, requiring dedicated algorithms and analysis of the system dynamics. Some previous works [(47)–(49)] proposed different approaches for the backlash identification correspondingly observation of its dynamic state in applications by using, however, more than one sensing element in the drive chain. Once a mechanical drive chain can be only one-side measured, the backlash becomes enclosed between, at least, two inertial terms and acts as a (hidden) internal nonlinearity with only piecewise continuous, to say switching, dynamics. In what follows, we will address such case of a two-mass, correspondingly two-inertia, system with backlash, while using only a one-side measurement for estimation of the backlash size via a relay feedback approach. The main principles will be discussed, together with some strengthening experimental results, while for more detailed developments of the method an interested reader is referred to [(32)].

The general structure of a two-inertia system with backlash is shown in Fig. 6. Note that independently whether a translational or rotational motion is meant, the generalized coordinates x_1 and x_2 of both rigid bodies, with the total masses (inertias) m_1 and m_2, can be assumed. The bodies are connected via a link (or shaft) which has a mechanical play stipulating appearance of the backlash. The total backlash size, also denoted as a backlash gap, is 2β. The first inertial body is actuated by the controllable (generalized) force u. Both linked bodies are on the common ground, with a normal contact interface that induces tangentially counteracting forces, generally known as friction. An experimental evaluation, shown below, has been accomplished on the

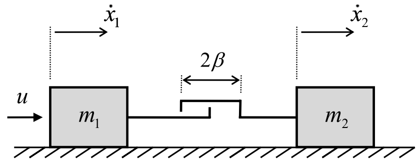

Fig. 6. Structure of two-inertia system with backlash

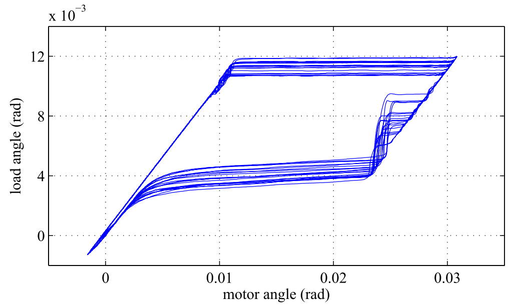

Fig. 7. Reference measured backlash

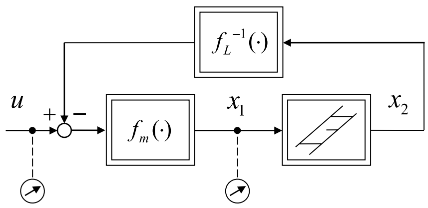

Fig. 8. Transformed structure of two-mass system with kinematic backlash as play-type hysteresis operator

laboratory setup consisting of two identical motors, with 20-bit high-resolution encoder each. The first one is actuated and low-level torque-controlled and, therefore, denoted further as *motor*. The second one serves as a passive rotary *load*. We stress that the angular measurement of the (first) motor only is used; the available load encoder serves solely for the sake of reference measurements. The utilized geared coupling between the motor and load contains a backlash. For further details on the experimental setup we refer to [(32)]. The reference measured input-output backlash map is shown in Fig. 7. It is worth noting that due to non-zero input frequency the resulted backlash trajectories are not entirely static and contain certain transients during the impact phases. Moreover, it should be underlined that the trajectories developing in positive direction are not entirely symmetrical to those in negative direction. The increasing trajectories diverge from a proper kinematical backlash (i.e. play-type hysteresis map), due to some additional adhesive by-effects in the geared coupling. Notwithstanding, a well-reproducible backlash gap of $2\beta \approx 0.019$ rad can be detected and assumed as reference.

A two-mass system with the backlash inside can be seen as a closed-loop dynamics with a piecewise linear coupling via a play-type hysteresis operator, cf. Fig. 8. Note that this transformed structure differs from a more common modeling approach, where a dead-zone nonlinearity is feedback to both sub-dynamics, i.e. of the motor and load, and coupled with a high but finite joint stiffness, see e.g. in [(39)]. The $f_m : (u - d) \mapsto x_1$ mapping of the motor dynamics incorporates the inertial and damping terms, including possible nonlinearities like e.g. friction. Important to recall is that the generalized input force and output displacement x_1 are

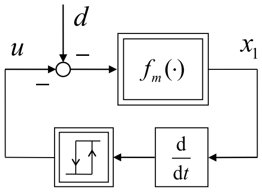

Fig. 9. Relay feedback system with motor dynamics

the single measurable values available in the system, as indicated in Fig. 8. The kinematic backlash, as piecewise differentiable nonlinear mapping between the motor and load position states, can be captured by a play-type hysteresis operator, also known as Prandtl-Ishlinskii model, for more details see e.g. [50]. The operator in the differential form is given by

$$\dot{x}_2 = \begin{cases} \dot{x}_1 & \text{if } x_2 = x_1 - \beta,\ \dot{x}_1 \geq 0\,, \\ \dot{x}_1 & \text{if } x_2 = x_1 + \beta,\ \dot{x}_1 \leq 0\,, \\ 0 & \text{if } x_1 - \beta \leq x_2 \leq x_1 + \beta\,, \end{cases} \quad \cdots\cdots\cdots\cdots (10)$$

cf. with [2]. Considering the kinematic backlash map, the input reversal points capture decoupling between the motor and load inertias. Consequently, the merging points in the backlash graph represent the colliding impact between moving bodies, both assumed as absolutely stiff and with the unity restitution coefficient. It should be noted that this is an ideal case without structural damping at impact, while the actual damping can be associated with internal friction, both linear viscous and nonlinear as well. Another important feature of the transformed structure of dynamics, as in Fig. 8, is that $\dot{x}_2 = \dot{x}_1 \equiv v$ during the backlash engagement mode, while nothing can be said about $\dot{x}_2$ within the gap mode, i.e. during decoupled motor and load displacements. However, this transient loss of the load state observability is irrelevant, from the motor dynamics viewpoint, since no back propagation of the load force occurs within the gap. With assumptions explained above, the backlash acts as a structure-switching nonlinearity, so that the inverse mapping $f_L^{-1} : x_2 \mapsto d$ of the load dynamics becomes case varying and can be described as

$$d = \begin{cases} \dot{v}\,(m_1 + m_2) + g(v) & \text{in engagement mode,} \\ 0 & \text{in gap mode,} \\ \dot{\gamma} & \text{at impact.} \end{cases}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots (11)$$

Here the aggregated nonlinearity, mostly damping, of the motor and load in engagement, is summarized in g, and $\gamma = m_1 \dot{x}_1 - m_2 \dot{x}_2$ is the total momentum of system with backlash before and after the impact.

The delayed relay $k \cdot h(\dot{x}_1)$, cf. with (2), in feedback of the motor velocity can be used as shown in Fig. 9, while being gained by k and, hence, providing two degrees-of-freedom for the control parameterization. The k-gain determines the level of dynamics excitation required for inducing the limit cycles. In the gap mode, the motor and load are decoupled, i.e. $d = 0$, and a stable limit cycle arises from the harmonic balance $1 + k\,h(\dot{x}_1)\,f_m(u) = 0$, cf. Section 2. A symmetric unimodal limit cycle, with the characteristic points x1–x4, is shown in Fig. 10 on the left, while the experimentally measured limit cycles under the relay feedback control are depicted on the right. Important to note is that the corresponding displacement amplitude of the induced limit cycle is by

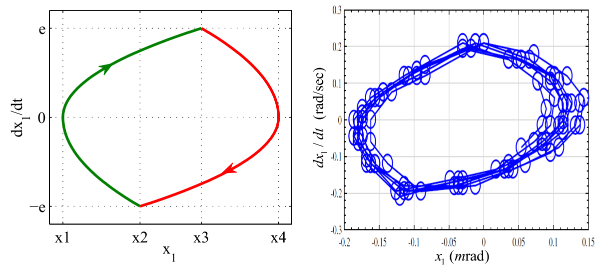

Fig. 10. Principal shape of symmetric unimodal limit cycle (left), experimentally measured limit cycles (right)

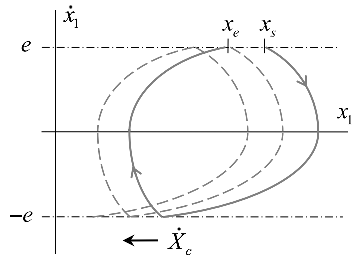

Fig. 11. Drifting limit cycle by asymmetric relay

two orders of magnitude smaller than the backlash gap, cf. Fig. 7 and Fig. 10 (on the right). That means the steady limit cycles can be operated, in robust manner, within the backlash gap without hitting the boundaries, i.e. switching to the engagement mode. Details on the relay parameterization and analytic solution of the limit cycle, in terms of the displacement amplitude and period of oscillations, can be found in [32].

A steady limit cycle, shown above, can be converted to the drifting one, when assigning different k-values for the positive and negative relays' state. An asymmetric relay gain leads to the motor dynamics is differently forced in both directions, so that the limit cycle trajectory does not close after one period (as in Fig. 10) and becomes continuously drifting as exemplified in Fig. 11. Since the analytic solution for an average velocity $\dot{X}_c$ of the limit cycles' drift is available, cf. [32], one can design a trajectory with flipping k-gain asymmetry. In this way, the limit cycle becomes forced to drift alternately in both direction, while the assumed traveling distance in one direction is supposed to be larger than the backlash gap. After the impact, following by the engagement mode, the drifting limit cycle changes qualitatively its shape, since a slow periodic propulsion of the load appears during each of the following impacts. Note that due to conspicuously changing initial conditions at each impact, and time- as well as state-varying damping, correspondingly stiction, in the mechanical backlash pair, the shape of the drifting limit cycle can yield highly irregular during the engagement mode. All the more, will be then a pattern of the drifting limit cycles differs between the gap and engagement modes. This can provide an unique signature of the backlash gap in the recorded time series that allows identifying the backlash size; also in a robust manner by averaging over several drifting periods. A controlled and measured drifting limit cycle, with the backlash signature used for identification as explained above, is exemplary shown in Fig. 12.

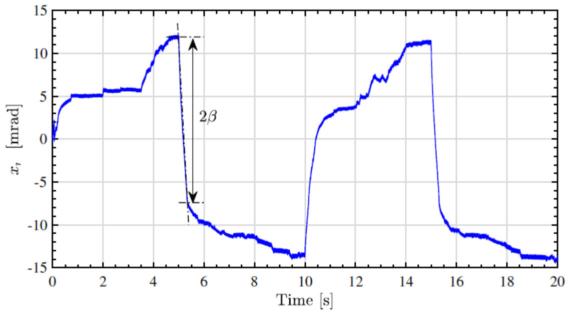

Fig. 12. Drifted motor position with backlash signature

4. Conclusions

In this paper, a brief tutorial on the relay feedback systems is provided. First, the relay feedback systems, consisting of the LTI dynamics (in a general state-space notation) in loop with the non-ideal (hysteresis) relays without cross-couplings, have been introduced. We gave a short historical retrospect on the former developments of the relay feedback systems, as been driven by emergence of the applications and practical engineering problems that required a solution. A well-known and rather 'classical' field of using the relay feedback methods for tuning the simple linear controllers has been summarized with some explanative examples. We also discussed the appearance of the limit cycles, as a most principal behavior associated with the relay feedback systems. Finally, the open-end potential of the relay feedback systems for identification purposes, especially for case-specific nonlinearities, has been brought into the light. We demonstrated a recently developed approach for estimating the unknown (hidden) backlash in the drive chains. While combining certain advantages of both, time- and frequency-domain analysis, the relay feedback systems seem to be promising for further developments in the area of system identification and control design, correspondingly tuning.

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