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RELAY PROPAGATION OF PARTIALLY COHERENT COSH-GAUSSIAN BEAMS IN NON-KOLMOGOROV TURBULENCE

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Abstract—The analytical formulas for the average intensity and power in the bucket of the relay propagation of partially coherent cosh-Gaussian (ChG) beams in non-Kolmogorov turbulence have been derived based on the extended Huygens-Fresnel principle. The influences of the beam parameters, relay system parameters and the non-Kolmogorov turbulence parameters on relay propagation are investigated by numerical examples. Numerical results reveal that the relay propagation of the beam is different from that in the case of Kolmogorov turbulence. It is shown that the relay propagation has advantages over direct propagation, and the relay propagation of partially coherent ChG beams depends greatly on the beam parameters, relay system and the generalized exponent α. The focusability of the beam at the target in non-Kolmogorov turbulence increases with larger inner scale, larger relay system radius, smaller outer scale, and smaller generalized structure constant. The results are useful for the practical applications of relay propagation, i.e., free-space communication.

1. INTRODUCTION

The optical propagation through the atmosphere is a very important subject for practical application of laser beams, i.e., free-space optical communication, which has attracted considerable theoretical and practical interest in the past decades [1–24]. Many factors limit the propagation of laser beams, such as turbulent atmosphere and transmission. Relay propagation at high elevation angles has been proposed to overcome this problem, which has been proved to be a

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valid method [25–30]. Relay propagation will be affected not only by its travel path but also by the relay system, such as the size of the receiver of relay system and the correction, which is different from the direct propagation. Chu has carried out detailed investigation concerning the influence of turbulence on the relay propagation performance of various laser beams, i.e., flattened Gaussian beams [28], Gaussian Schell-model beams [29], and cosh-Gaussian-Schell beams [30]. In previous study, Kolmogorov's power spectrum of the refractive index fluctuations has been widely used to study the direct and relay optical propagation of laser beam through the atmosphere. However, recent experiments revealed that turbulence in some portions of the practical atmosphere, such as in portions of the troposphere and the stratosphere deviates from Kolmogorov's model, and in the case of laser propagation along the vertical direction, the turbulence also indicates strongly a non-Kolmogorov character [31–34]. Then a non-Kolmogorov model is presented [33], which is more general to describe the practical atmosphere and reduces to the Kolmogorov model in the case of the generalized exponent $\alpha = 11/3$. It has been reported that, based on this non-Kolmogorov spectrum, optical wave will provide a different property when propagating in non-Kolmogorov turbulence [35–41]. Moreover, due to the high elevation angle propagation and the altitude of the relay system, the relay propagation will inevitably propagate in non-Kolmogorov turbulence, and it requires studying the relay propagation in non-Kolmogorov turbulence for more accurate results for practical applications of relay propagation, i.e., free-space communication.

As is well known, various intensity profiles which can be used in some important applications can be obtained by altering the parameters of a cosh-Gaussian (ChG) beam, which can be regarded as the superposition of decentered Gaussian beams [42–45] and can be obtained in practical application based on the method described in [42, 46, 47] and the references therein. The propagation properties of ChG beam, including both completely coherent and partially coherent beams, in turbulent atmosphere have been studied extensively [42–45, 48]. However, the relay propagation of partially coherent ChG beams in non-Kolmogorov turbulence has not been examined until now. In this manuscript, our aims are to investigate the relay propagation of partially coherent ChG laser beams in non-Kolmogorov turbulence. Analytical formulas for averaged intensity distribution and power in the bucket (PIB) are derived and some useful results are found.

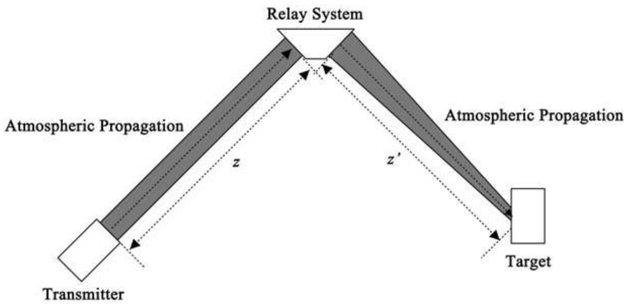

Figure 1. Configuration for free-space communication system.

2. ANALYSIS OF THEORY

Schematic diagram of the configuration for free-space communication system is shown in Fig. 1. The relay system receives the beam and focuses it on a target. z is the distance between the transmitter and the relay system, and z' is the distance between the relay system and the target. The field of a spatially fully coherent two-dimensional ChG beam at the source plane $z = 0$ is expressed as [30]

$$u\left(\vec{r},0\right)=\cosh\left(\Omega_0 x\right)\cosh\left(\Omega_0 y\right)\exp\left(-\frac{x^2+y^2}{w_0^2}\right) \tag{1}$$

where w_0 is the waist width of the Gaussian amplitude distribution, Ω_0 a parameter associated with the cosh part, $\vec{r}=(x,y)$ the transverse coordinate vector, and a amplitude constant is omitted.

Equation (1) can be rewritten as

$$u(\vec{r},0)=\exp\left(\frac{\Omega_0^2 w_0^2}{2}\right)\sum_{r=0}^{1}\sum_{s=0}^{1}\exp\left[-\frac{\left(x-\frac{(-1)^r\Omega_0 w_0^2}{2}\right)^2+\left(y-\frac{(-1)^s\Omega_0 w_0^2}{2}\right)^2}{w_0^2}\right] \tag{2}$$

It shows that ChG can be expressed as a superposition of four decentered Gaussian beams and the Gaussian beam can be created easily in practical applications, such as the output of the fiber laser.

By introducing a Gaussian term of the spectral degree of coherence, the fully coherent ChG beam can be extended to the partially coherent one [48–50], whose cross-spectral density function at the source plane $z = 0$ can be written as

$$W\left(\vec{r_1},\vec{r_2},0\right)=W_x\left(x_1,x_2,0\right)W_y\left(y_1,y_2,0\right), \tag{3}$$

with

$$W_{\mathrm{X}}(\mathrm{X}_1,\mathrm{X}_2,0)=\cosh\left[\Omega_0\left(\mathrm{X}_1\right)\right]\cosh\left[\Omega_0\left(\mathrm{X}_2\right)\right]\exp\left(-\frac{\mathrm{X}_1^2+\mathrm{X}_2^2}{w_0^2}\right)$$
$$\times\exp\left[-\frac{\left(\mathrm{X}_1-\mathrm{X}_2\right)^2}{2\sigma_0^2}\right]\exp\left\{-\frac{ik\left[\mathrm{X}_1^2-\mathrm{X}_2^2\right]}{2F}\right\},\quad (\mathrm{X}=x,y)\qquad(4)$$

where σ_0 is the spatial correlation length of the partially coherent beam and F the phase front radius of curvature. Here, $F > 0$ represents a convergent beam and $F < 0$ represents a divergent beam.

Based on the extended Huygens-Fresnel principle [12–19], the propagation of the cross-spectral density of arbitrary beam can be calculated as follows

$$W\left(\overrightarrow{r_1},\overrightarrow{r_2},z\right)=\left(\frac{k}{2\pi z}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d\overrightarrow{r_1'}d\overrightarrow{r_2'}W\left(\overrightarrow{r_1'},\overrightarrow{r_2'},0\right)$$
$$\exp\left\{\frac{ik}{2z}\left[\left(\overrightarrow{r_1}-\overrightarrow{r_1'}\right)^2-\left(\overrightarrow{r_2}-\overrightarrow{r_2'}\right)^2\right]\right\}$$
$$\times\left\langle\exp\left[\psi\left(\overrightarrow{r_1},\overrightarrow{r_1'}\right)+\psi^*\left(\overrightarrow{r_2},\overrightarrow{r_2'}\right)\right]\right\rangle\qquad(5)$$

where k is the wave number related to the wave length λ by $k = 2\pi/\lambda$. $\psi(\overrightarrow{r_1},\overrightarrow{r_1'})$ is the complex phase function that depends on the properties of the turbulence medium. $\langle\rangle$ denotes average over the ensemble of the turbulent medium, and

$$\left\langle\exp\left[\psi\left(\overrightarrow{r_1},\overrightarrow{r_1'}\right)+\psi^*\left(\overrightarrow{r_2},\overrightarrow{r_2'}\right)\right]\right\rangle$$
$$=\exp\left\{-4\pi^2k^2z\int_0^1\int_0^{\infty}d\kappa d\xi\Phi_n\left(\kappa,\alpha\right)\right.$$
$$\left.\left[1-J_0\left(\kappa\left|\left(1-\xi\right)\left(\overrightarrow{r_2}-\overrightarrow{r_1}\right)+\xi\left(\overrightarrow{r_2'}-\overrightarrow{r_1'}\right)\right|\right)\right]\right\}\qquad(6)$$

where κ is the magnitude of two-dimensional spatial frequency, J_0 the Bessel function of the first kind and zero order, and $\Phi_n(\kappa,\alpha)$ denotes the spatial power spectrum of the refractive-index fluctuations of the atmosphere turbulence. Including both the inner- and outer-scale effects, the non-Kolmogorov spectrum is defined as [35–41]

$$\Phi_n(\kappa,\alpha)=H(\alpha)\tilde{C}_n^2\exp\left(-\kappa^2/\kappa_m^2\right)\left(\kappa^2+\kappa_0^2\right)^{-\alpha/2},\quad 0\le\kappa<\infty,\quad 3<\alpha<4,\quad(7)$$

where $H(\alpha) = \Gamma(\alpha-1)\cdot\cos(\alpha\pi/2)/(4\pi^2)$, $\Gamma(\cdot)$ denotes the Gamma function, $\kappa_0 = 2\pi/L_0$ and $\kappa_m = c(\alpha)/l_0$, in which $c(\alpha) = \{\Gamma\left[(5-\alpha)/2\right]\cdot H(\alpha)\cdot 2\pi/3\}^{1/(\alpha-5)}$, l_0 and L_0 is the inner- and outer-scale, respectively. The term $\widetilde{C}_n^2$ is the generalized structure parameter with units $m^{3-\alpha}$.

Upon substituting Equations (6), (7) into Equation (5) and after careful integration, the cross-spectral density of a partially coherent ChG beam at z-plane can be obtained as

$$W\left(\overrightarrow{r_1},\overrightarrow{r_2},z\right)=W_x\left(x_1,x_2,z\right)W_y\left(y_1,y_2,z\right) \tag{8}$$

with

$$\begin{aligned}&W_{\mathrm{X}}\left(\mathrm{X}_1,\mathrm{X}_2,0\right)\\&=\frac{1}{4\tau}\sum_{p=0}^{1}\sum_{q=0}^{1}\exp\left(A\right)\exp\left\{-\frac{1}{w_0^2\tau^2}\left[\left(\mathrm{X}_1-G_1\right)^2+\left(\mathrm{X}_2-G_2\right)^2\right]\right\}\\&\times\exp\left\{-\frac{\left(\mathrm{X}_1-\mathrm{X}_2\right)^2}{2\sigma^2}+\frac{ik}{2z}\left(\mathrm{X}_1^2-\mathrm{X}_2^2\right)+\frac{z^2i}{k\tau^2}b_1\left[\left(\mathrm{X}_1-\frac{b_2}{2b_1}\right)^2-\left(\mathrm{X}_2-\frac{b_3}{2b_1}\right)^2\right]\right\}\end{aligned} \tag{9}$$

where

$$\tau^2=\tau_1^2+\tau_2^2+\tau_3^2+\tau_4^2 \tag{10a}$$

$$\tau_1=1-\frac{z}{F},\quad \tau_2=\frac{2z}{kw_0^2},\quad \tau_3=\frac{2z}{kw_0\sigma_0},\quad \tau_4=\sqrt{\frac{8z^2T\left(\alpha,z\right)}{k^2w_0^2}}, \tag{10b}$$

$$T(\alpha,z)=\frac{1}{3}\pi^2k^2z\left[\frac{A(\alpha)C_n^2}{2}\frac{\kappa_m^{2-\alpha}\left(2\kappa_0^2-2\kappa_m^2+\alpha\kappa_m^2\right)\exp\left(\frac{\kappa_0^2}{\kappa_m^2}\right)\Gamma\left(2-\frac{\alpha}{2},\frac{\kappa_0^2}{\kappa_m^2}\right)-2\kappa_0^{4-\alpha}}{\alpha-2}\right] \tag{10c}$$

$$A=\frac{w_0^2\Omega_0^2}{4\tau^2}\left\{2\tau^2+\frac{\tau_4^2}{4}\left[1-(-1)^{p+q}\right]\left(\tau_4^2-4\tau_1\right)+\left[(-1)^{p+q}-1\right]\left(\tau_3^2+\tau_4^2\right)\right\} \tag{10d}$$

$$G_1=\frac{w_0^2\Omega_0}{2}\left\{(-1)^p\tau_1-\frac{\tau_4^2}{4}\left[(-1)^p-(-1)^q\right]\right\} \tag{10e}$$

$$G_2=\frac{w_0^2\Omega_0}{2}\left\{(-1)^q\tau_1+\frac{\tau_4^2}{4}\left[(-1)^p-(-1)^q\right]\right\} \tag{10f}$$

$$\sigma=\left\{2T(\alpha,z)\left[1+\frac{1}{\tau^2}\left(1+\tau_1-\frac{\tau_2^2w_0^2T(\alpha,z)}{2}\right)\right]+\frac{1}{\tau^2\sigma_0^2}\right\}^{-1/2} \tag{10g}$$

$$b_1=\frac{2T\left(\alpha,z\right)}{w_0^2z}-\frac{k^2\left(F-z\right)}{2Fz^3}, \tag{10h}$$

$$b_2=\Omega_0\left\{\frac{2\left(-1\right)^p}{w_0^2z}+\left[(-1)^p+(-1)^q\right]\left(\frac{3T(\alpha,z)}{z}-\frac{T(\alpha,z)}{F}+\frac{1}{z\sigma_0^2}\right)\right\}, \tag{10i}$$

$$b_3=\Omega_0\left\{\frac{2\,(-1)^q}{w_0^2 z}+[(-1)^p+(-1)^q]\left(\frac{3T(\alpha,z)}{z}-\frac{T(\alpha,z)}{F}+\frac{1}{z\sigma_0^2}\right)\right\}, \quad (10j)$$

with $\Gamma(\cdot,\cdot)$ denoting the incomplete Gamma function. The parameters τ_1, τ_2, τ_3, and τ_4 describe the influence of the geometrical magnification, diffraction, coherence of initial beams and turbulence, respectively. σ is the spatial correlation length of the beam propagation in non-Kolmogorov turbulence. Let $\overrightarrow{r_2}=\overrightarrow{r_1}$, we can get the average intensity distribution on the relay system

$$\langle I\left(\vec{r},z\right)\rangle=\langle I_x\left(x,z\right)\rangle\langle I_y\left(y,z\right)\rangle \quad (11)$$

with

$$\begin{aligned}&\langle I_{\mathrm{X}}\left(\mathrm{X},0\right)\rangle\\&=\frac{1}{4\tau}\sum_{p=0}^{1}\sum_{q=0}^{1}\exp\left(A-\frac{\left(G_1^2+G_2^2\right)}{w_0^2\tau^2}+\frac{z^2 i}{4b_1 k\tau^2}\left(b_2^2-b_3^2\right)\right)\\&\quad\times\exp\left\{-\frac{1}{w_0^2\tau^2}\left[2\mathrm{X}^2-\left(2\left(G_1+G_2\right)-\frac{z^2w_0^2 i}{k}\left(b_2-b_3\right)\right)\mathrm{X}\right]\right\} \quad (12)\end{aligned}$$

For the applications such as energy transmission, laser power focusability in the far field is a key parameter and the *PIB* is a useful method for characterizing different laser beams, which clearly indicates how much fraction of the total beam power is within a certain area and is defined as

$$PIB=\frac{\int_{-h}^{h}\int_{-h}^{h}\langle I\left(\vec{r},z\right)\rangle dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle I\left(\vec{r},z\right)\rangle dxdy} \quad (13)$$

where h is the radius of the bucket.

Substituting from (11)–(12) into (13), the *PIB* at the relay system is given by

$$PIB_{relay\_system}=\frac{G_1^2}{G_2^2} \quad (14)$$

with

$$\begin{aligned}G_1\;=\;&\frac{w_0}{8}\sqrt{\frac{\pi}{2}}\sum_{p=0}^{1}\sum_{q=0}^{1}\exp\left(A-\frac{\left(G_1^2+G_2^2\right)}{w_0^2\tau^2}+\frac{z^2 i}{4b_1 k\tau^2}\left(b_2^2-b_3^2\right)\right.\\&\left.+\frac{\left(\left(G_1+G_2\right)-\frac{z^2w_0^2 i}{2k}\left(b_2-b_3\right)\right)^2}{2w_0^2\tau^2}\right)\end{aligned}$$

$$\times\left[erf\left(\frac{\sqrt{2}}{w_0\tau}\left[h-\left(\frac{(G_1+G_2)}{2}-\frac{z^2w_0^2i}{4k}(b_2-b_3)\right)\right]\right)\right.$$
$$\left.+erf\left(\frac{\sqrt{2}}{w_0\tau}\left[h+\left(\frac{(G_1+G_2)}{2}-\frac{z^2w_0^2i}{4k}(b_2-b_3)\right)\right]\right)\right] \quad (15a)$$

$$G_2=\frac{w_0}{4}\sqrt{\frac{\pi}{2}}\sum_{p=0}^{1}\sum_{q=0}^{1}\exp\left(A-\frac{(G_1^2+G_2^2)}{w_0^2\tau^2}+\frac{z^2i}{4b_1k\tau^2}\left(b_2^2-b_3^2\right)\right.$$
$$\left.+\frac{\left((G_1+G_2)-\frac{z^2w_0^2i}{2k}(b_2-b_3)\right)^2}{2w_0^2\tau^2}\right) \quad (15b)$$

where $erf(\cdot)$ is the error function.

To study the average intensity distribution at the target, we assume that the relay system focuses the beam on the target, namely, the focal length of relay system is equal to the distance between the relay system and the target, and that the receiver of relay system is the same as the transmitter [28–30]. Based on the above assumptions, we can rewrite Equation (9) in the form of

$$W'(\overrightarrow{r_1},\overrightarrow{r_2},z)=W_x'(x_1,x_2,z)\,W_y'(y_1,y_2,z) \quad (16)$$

with

$$W_{\mathrm{X}}(\mathrm{X}_1,\mathrm{X}_2,0)$$
$$=\frac{1}{4\tau}\sum_{p=0}^{1}\sum_{q=0}^{1}\exp(A)\exp\left\{-\frac{1}{w_0^2\tau^2}\left[(\mathrm{X}_1-G_1)^2+(\mathrm{X}_2-G_2)^2\right]\right\}$$
$$\times\exp\left\{-\frac{(\mathrm{X}_1-\mathrm{X}_2)^2}{2\sigma^2}-\frac{ik}{2z'}\left(\mathrm{X}_1^2+\mathrm{X}_2^2\right)\right\} \quad (17)$$

By using the following hard aperture function

$$U(\vec{r})=\begin{cases}1, & |\vec{r}|\le a\\ 0, & |\vec{r}|>a\end{cases}, \quad (18)$$

with a being the radius of the aperture of the relay system, the cross-spectral density of a partially coherent ChG beam at the target plane can be expressed as

$$W\left(\overrightarrow{r_1},\overrightarrow{r_2},z'\right)$$
$$=\left(\frac{k}{2\pi z'}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dr_1'dr_2'W'\left(\overrightarrow{r_1'},\overrightarrow{r_2'},z\right)U\left(\overrightarrow{r_1'}\right)U^*\left(\overrightarrow{r_2'}\right)$$

$$\times \exp\left\{\frac{ik}{2z'}\left[\left(\overrightarrow{r_1}-\overrightarrow{r_1'}\right)^2-\left(\overrightarrow{r_2}-\overrightarrow{r_2'}\right)^2\right]\right\}$$
$$\times\left\langle\exp\left[\psi\left(\overrightarrow{r_1},\overrightarrow{r_1'}\right)+\psi^*\left(\overrightarrow{r_2},\overrightarrow{r_2'}\right)\right]\right\rangle \tag{19}$$

where

$$\left\langle\exp\left[\psi\left(\overrightarrow{r_1},\overrightarrow{r_1'}\right)+\psi^*\left(\overrightarrow{r_2},\overrightarrow{r_2'}\right)\right]\right\rangle$$
$$=\exp\left\{-4\pi^2k^2z'\int_0^1\int_0^\infty d\kappa d\xi\Phi_n(\kappa,\alpha)\right.$$
$$\left.\left[1-J_0\left(\kappa\left|(1-\xi)\left(\overrightarrow{r_2}-\overrightarrow{r_1}\right)+\xi\left(\overrightarrow{r_2'}-\overrightarrow{r_1'}\right)\right|\right)\right]\right\} \tag{20a}$$

$$U(\vec{r}) = \sum_{t=1}^{M} B_t \exp\left(-\frac{C_t|\vec{r}|^2}{a^2}\right), \tag{20b}$$

where B_t and C_t $(t = j, k)$ are the expansion coefficient, which can be obtained directly by numerical optimization, and a table of B_t and C_t can be found in [51, 52]. For a hard aperture, $M = 10$ assures a very good description of the diffracted beam in the range from < 0.12 times the Fresnel distance to the infinity, and discrepancies exist only in the extreme near field (< 0.12 times the Fresnel distance) [53].

Set $\overrightarrow{r_2} = \overrightarrow{r_1}$, we can get

$$\left\langle I\left(\vec{r},z'\right)\right\rangle$$
$$=\left(\frac{k}{2\pi z'}\right)^2\sum_{j=1}^{M}\sum_{k=1}^{M}B_jB_k^*\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d\overrightarrow{r_1'}d\overrightarrow{r_2'}W'\left(\overrightarrow{r_1'},\overrightarrow{r_2'},z\right)$$
$$\times\exp\left[-\frac{C_j\left(\overrightarrow{r_1'}\right)^2+C_k^*\left(\overrightarrow{r_2'}\right)^2}{a^2}\right]\exp\left\{\frac{ik}{2z'}\left[\left(\vec{r}-\overrightarrow{r_1'}\right)^2-\left(\vec{r}-\overrightarrow{r_2'}\right)^2\right]\right\}$$
$$\times\exp\left[-T\left(\alpha,z'\right)\left(\overrightarrow{r_2'}-\overrightarrow{r_1'}\right)^2\right] \tag{21}$$

Then the average intensity distribution at the target can be expressed as

$$\left\langle I\left(\vec{r},z'\right)\right\rangle=\sum_{j=1}^{M}\sum_{k=1}^{M}B_jB_k^*\left\langle I_x\left(x,z'\right)\right\rangle\left\langle I_y\left(y,z'\right)\right\rangle \tag{22}$$

where

$$\left\langle I_{\mathrm{X}}\left(\mathrm{X},z'\right)\right\rangle=\frac{1}{4\tau}\sum_{p=0}^{1}\sum_{q=0}^{1}\frac{k}{2z'\sqrt{D}}\exp\left(A+\frac{1}{w_0^2\tau^2}\left(\frac{G_1^2}{A_1w_0^2\tau^2}-G_1^2-G_2^2\right)\right.$$
$$\left.+\frac{\left(G_1F+A_1G_2\right)^2}{A_1w_0^4\tau^4D}\right)\times\exp\left(\frac{k^2\left[2F-\left(A_1+A_2\right)\right]}{\left(2z'\right)^2D}\mathrm{X}^2\right.$$
$$\left.+\frac{ik\left[\left(G_1-G_2\right)F-\left(A_2G_1-A_1G_2\right)\right]}{z'w_0^2\tau^2D}\mathrm{X}\right)\tag{23a}$$

$$A_1=\frac{C_j}{a^2}+\frac{1}{w_0^2\tau^2}+\frac{1}{2\sigma^2}+T\left(\alpha,z'\right)\tag{23b}$$

$$A_2=\frac{C_k^*}{a^2}+\frac{1}{w_0^2\tau^2}+\frac{1}{2\sigma^2}+T\left(\alpha,z'\right)\tag{23c}$$

$$D=A_1A_2-\left(\frac{1}{2\sigma^2}+T\left(\alpha,z'\right)\right)^2,\ F=\frac{1}{2\sigma^2}+T\left(\alpha,z'\right)\tag{23d}$$

Substituting from (22), (23) into (13), the PIB at the target plane is given by

$$PIB_{target}=\frac{\sum\limits_{j=1}^{M}\sum\limits_{k=1}^{M}B_jB_k^*V_1^2}{\sum\limits_{j=1}^{M}\sum\limits_{k=1}^{M}B_jB_k^*V_2^2}\tag{24}$$

where

$$V_1=\frac{1}{8i}\sum_{p=0}^{1}\sum_{q=0}^{1}\frac{\sqrt{\pi}}{\tau\sqrt{2F-\left(A_1+A_2\right)}}\exp\left(A+\frac{1}{w_0^2\tau^2}\left(\frac{G_1^2}{A_1w_0^2\tau^2}-G_1^2-G_2^2\right)\right.$$
$$\left.+\frac{\left(G_1F+A_1G_2\right)^2}{A_1w_0^4\tau^4D}+\frac{\left[\left(G_1-G_2\right)F-\left(A_2G_1-A_1G_2\right)\right]^2}{\left[2F-\left(A_1+A_2\right)\right]w_0^4\tau^4D}\right)$$
$$\times\left\{erf\left(\sqrt{\frac{2F-\left(A_1+A_2\right)}{D}}\left(\frac{ikh}{2z'}-\frac{\left[\left(G_1-G_2\right)F-\left(A_2G_1-A_1G_2\right)\right]}{\left[2F-\left(A_1+A_2\right)\right]w_0^2\tau^2}\right)\right)\right.$$
$$\left.+erf\left(\sqrt{\frac{2F-\left(A_1+A_2\right)}{D}}\left(\frac{ikh}{2z'}+\frac{\left[\left(G_1-G_2\right)F-\left(A_2G_1-A_1G_2\right)\right]}{\left[2F-\left(A_1+A_2\right)\right]w_0^2\tau^2}\right)\right)\right\}\tag{25a}$$

$$V_2 = -\frac{1}{4i}\sum_{p=0}^{1}\sum_{q=0}^{1}\frac{\sqrt{\pi}}{\tau\sqrt{2F-(A_1+A_2)}}\exp\left(A+\frac{1}{w_0^2\tau^2}\left(\frac{G_1^2}{A_1 w_0^2\tau^2}-G_1^2-G_2^2\right)\right.$$
$$\left.+\frac{(G_1F+A_1G_2)^2}{A_1 w_0^4\tau^4 D}+\frac{\left[(G_1-G_2)F-(A_2G_1-A_1G_2)\right]^2}{\left[2F-(A_1+A_2)\right]w_0^4\tau^4 D}\right) \tag{25b}$$

3. NUMERICAL CALCULATION AND ANALYSIS

3.1. The Properties of Partially Coherent ChG Beam from Transmitter Plane to the Relay System in Non-Kolmogorov Turbulence

For the sake of simplicity, we assume that the ChG beam emitting from the transmitter is collimated, namely, $F = \infty$, and the travel path is horizontal. The parameters in this paper are taken as $\lambda = 1.064\,\mu\mathrm{m}$, $w_0 = 0.2\,\mathrm{m}$, $\sigma_0 = 0.01\,\mathrm{m}$, $h = 0.1\,\mathrm{m}$. The parameters of the turbulence are set as $\widetilde{C}_n^2 = 1\times10^{-14}m^{3-\alpha}$, $l_0 = 0.001\,\mathrm{m}$, and $L_0 = 5\,\mathrm{m}$. The distance between the transmitter and the relay system is $z = 5\,\mathrm{km}$. The normalized average intensity is defined as

$$I_{Normalized}(\vec{r},z) = \frac{\langle I(\vec{r},z)\rangle}{P_0} \tag{26}$$

where P_0 is the total power at initial plane and is given as

$$P_0 = \frac{\pi w_0^2}{8}\left[1+\exp\left(w_0^2\Omega_0^2\right)+2\exp\left(\frac{w_0^2\Omega_0^2}{2}\right)\right] \tag{27}$$

The normalized average intensity distributions on the relay plane z with different α and the normalized average transversal intensity distributions are shown in Figs. 2–4, respectively. Comparing the results in Figs. 2–4, we can see that the average intensity profile varies with the parameter Ω_0, and with α decreasing, the average intensity profile of the partially coherent ChG beam becomes similar to a Gaussian distribution. Generally, the propagation of the partially coherent ChG beam in non-Kolmogorov turbulence is different from that in the case of Kolmogorov turbulence, that is, $\alpha = 11/3 \approx 3.67$.

To further study the propagation properties of partially coherent ChG beams in non-Kolmogorov turbulence, Fig. 5 plots the dependence of *PIB* on α under different Ω_0 and σ_0. Fig. 5 shows that the focusability of the beam will decrease with α until reaches its minimum value when $\alpha = 3.078$, after which the focusability will increase with α. It also indicates, with Ω_0 increasing and σ_0 decreasing, the focusability decreases, and the difference due to Ω_0 and σ_0 is larger with larger α.

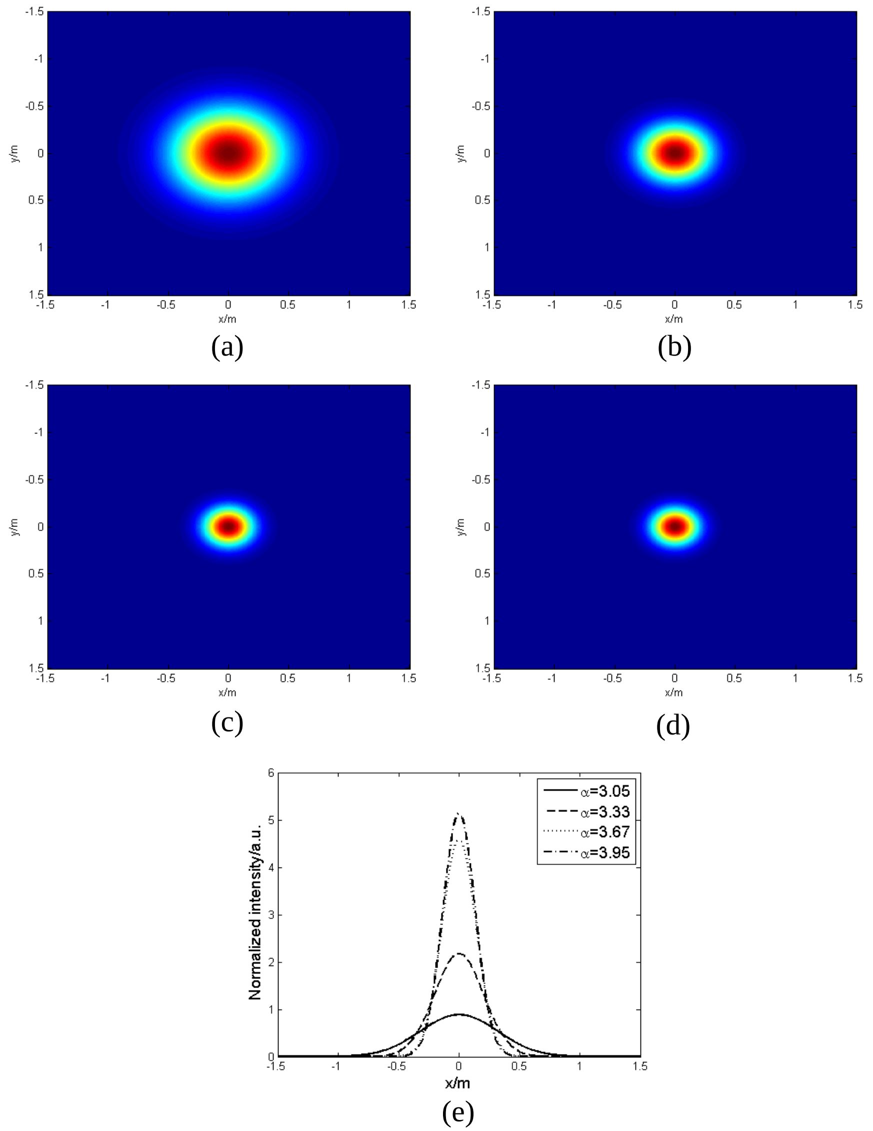

Figure 2. Normalized average intensity distributions on the relay plane z where $\Omega_0 = 1\,\mathrm{m}^{-1}$. (a)–(d) are two-dimensional average intensity distributions and (e) is the transversal intensity distributions. (a) $\alpha = 3.05$. (b) $\alpha = 3.33$. (c) $\alpha = 3.67$. (d) $\alpha = 3.95$.

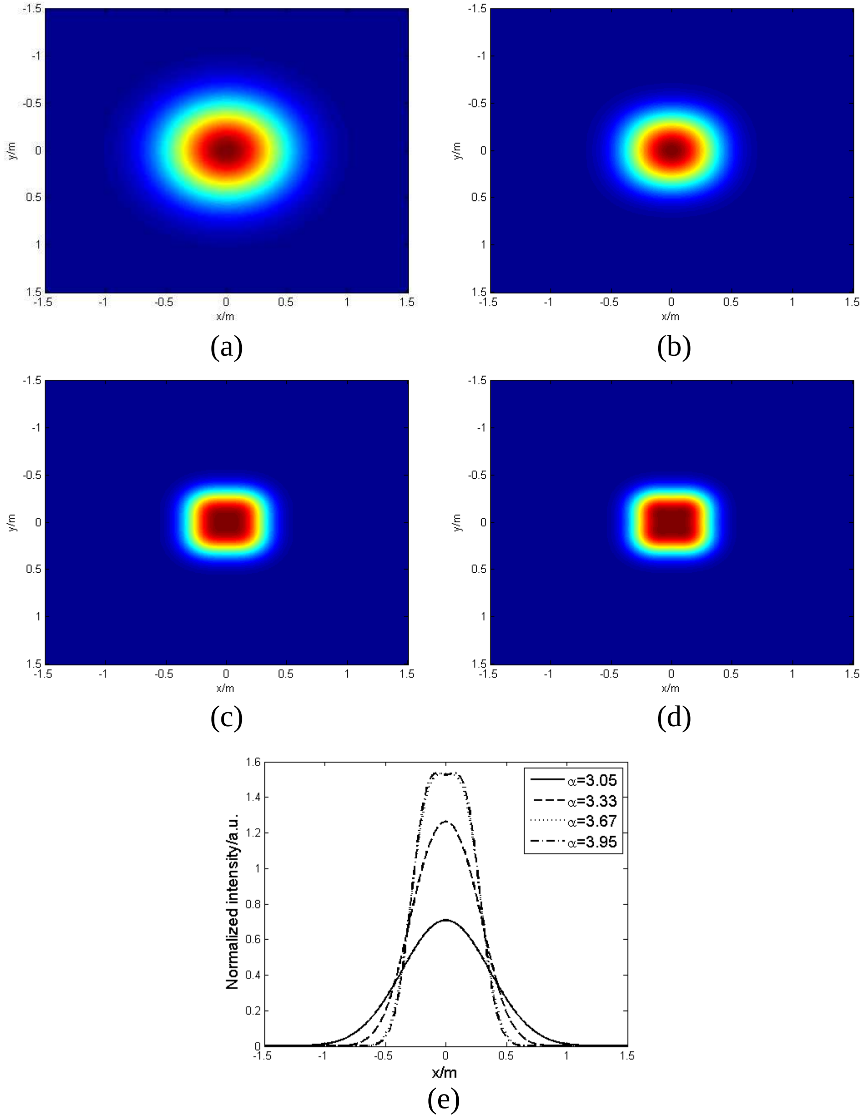

Figure 3. Normalized average intensity distributions on the relay plane z where $\Omega_0 = 8.5\,\mathrm{m}^{-1}$. (a)–(d) are two-dimensional average intensity distributions and (e) is the transversal intensity distributions. (a) $\alpha = 3.05$. (b) $\alpha = 3.33$. (c) $\alpha = 3.67$. (d) $\alpha = 3.95$.

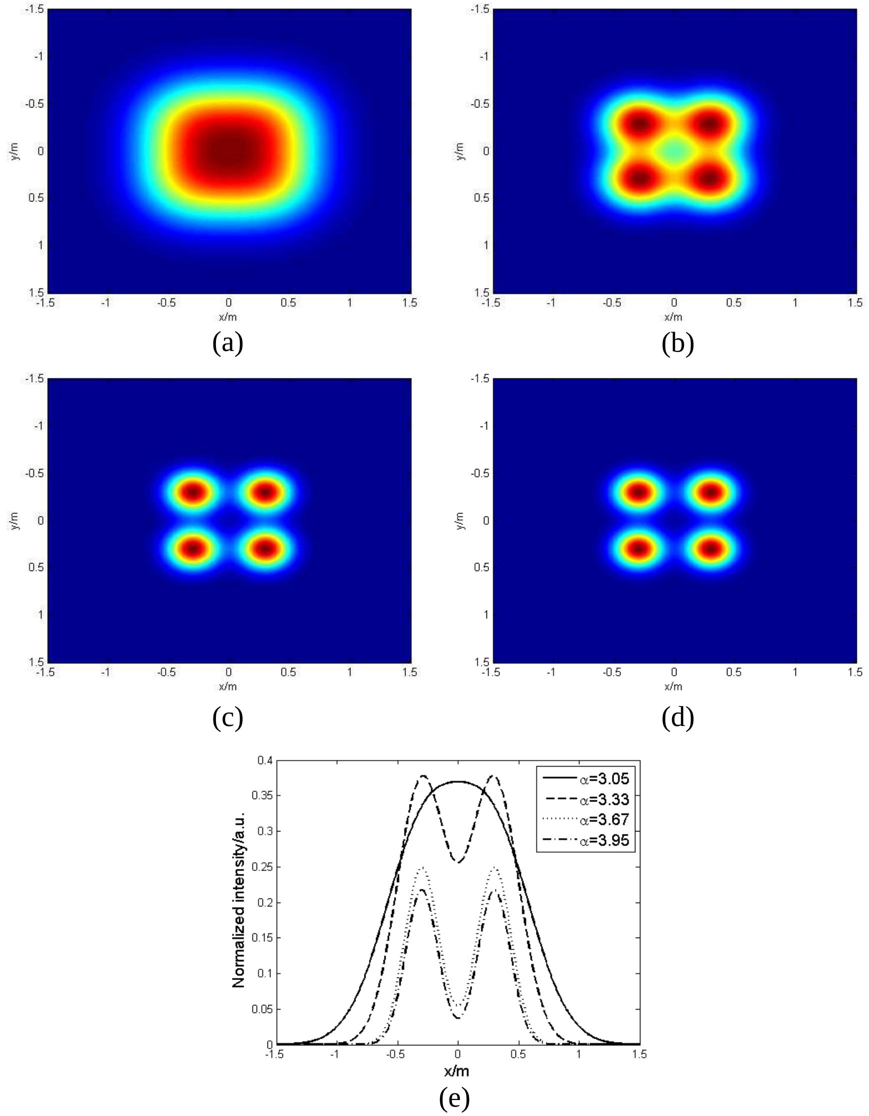

Figure 4. Normalized average intensity distributions on the relay plane z where $\Omega_0 = 15\,\mathrm{m}^{-1}$. (a)–(d) are two-dimensional average intensity distributions and (e) is the transversal intensity distributions. (a) $\alpha = 3.05$. (b) $\alpha = 3.33$. (c) $\alpha = 3.67$. (d) $\alpha = 3.95$.

3.2. The Properties of Partially Coherent Cosh-Gaussian Beam to Target in Non-Kolmogorov Turbulence

The propagation to the target will be affected not only by the beam projected by the relay system, but also by the limitation of the aperture and the turbulent atmosphere. Without any correction, the normalized average transversal intensity distributions are shown in Fig. 6. The parameters are taken as $z = 5\,\mathrm{km}$, $z' = 5\,\mathrm{km}$, $\Omega_0 = 5.76\,\mathrm{m}^{-1}$, $a = 0.5\,\mathrm{m}$, and the other parameters are taken the same as in Fig. 2. The dashed lines are for the case of direct propagation to the target without relay system, and the solid lines are for the case of relay propagation to the target, which employs a relay system to focus the beam to the target. From Fig. 6, we can clearly see the advantage of the relay propagation.

The effects of the relay system and the non-Kolmogorov turbulence on propagation in detail are studied in detail. Fig. 7 presents the dependence of *PIB* on α under different relay system radius without any correction, and the value of α are taken the same in two propagation stages. Fig. 7 shows that the focusability of the beam at the target increases with a increasing, and also decrease with α until reaches its minimum value when $\alpha = 3.078$, after which the focusability will increase with α. Also with larger α, the difference due to relay system radius becomes larger.

For relay propagation, one advantage is that the received beam can be cleaned up in a relay system [24–26], which means the increase of the spatial correction length. To see the effects of the correction performance of the relay system in non-Kolmogorov turbulence, the dependence of *PIB* on α with different σ (which is defined as in

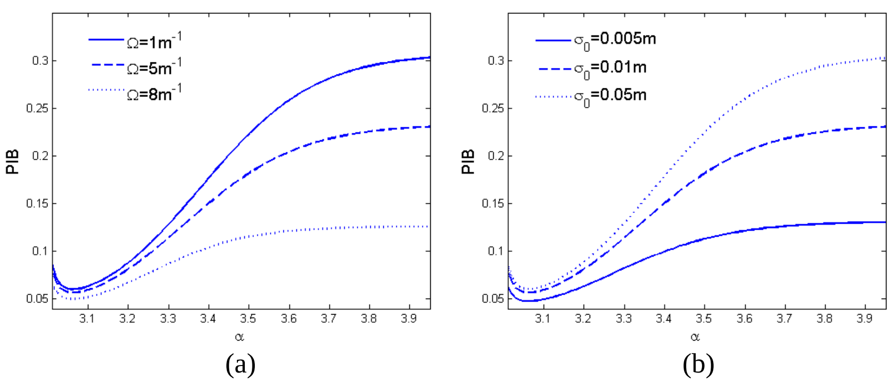

Figure 5. Dependence of *PIB* on α.

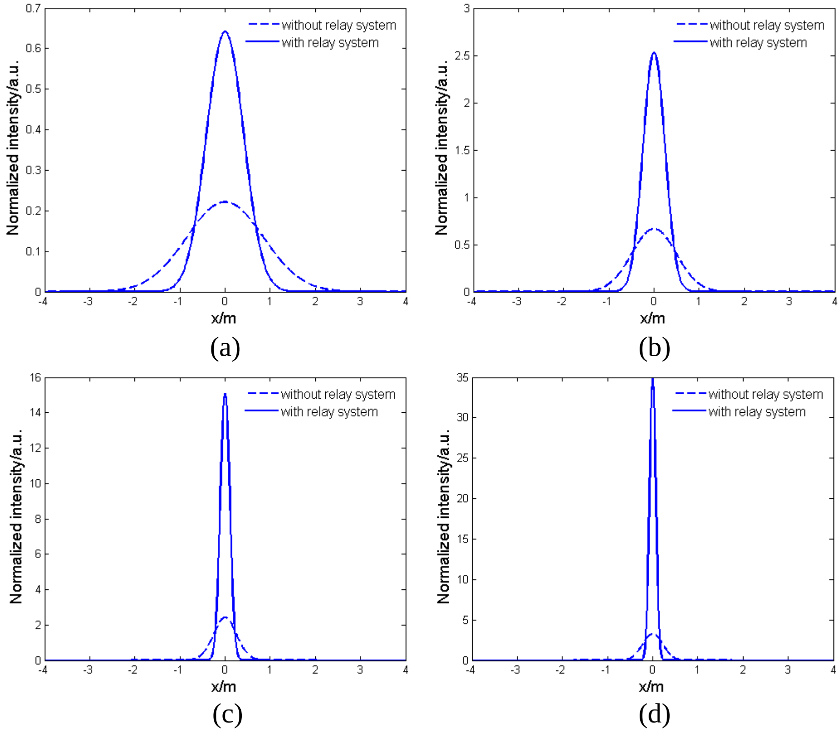

Figure 6. Normalized average transversal intensity distributions for different α. (a) $\alpha = 3.05$. (b) $\alpha = 3.33$. (c) $\alpha = 3.67$. (d) $\alpha = 3.95$.

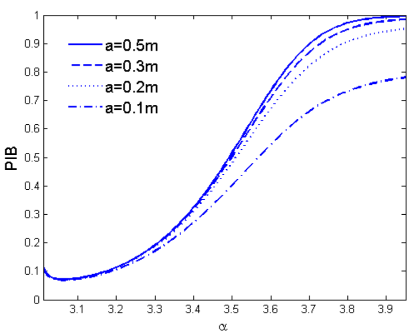

Figure 7. Dependence of *PIB* on α under different relay system radius without any correction.

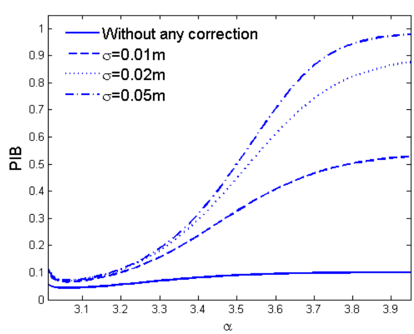

Figure 8. Dependence of *PIB* on α with different correction.

Equation (10g)) is plotted in Fig. 8. For the propagation between transmitter and the relay system, $\alpha = 3.5$, $a = 0.2\,\mathrm{m}$, and the other parameters are taken the same as in Fig. 7. Without any correction, numerical calculation shows that $\sigma = 0.0035\,\mathrm{m}$. From Fig. 8, we can see that with α increasing, the effects of the correction are more significant.

To further study the influence of the non-Kolmogorov turbulence, we plot the variation of the *PIB* at target with different correction for different turbulence in Fig. 9. We can see that the *PIB* at the target plane increases with increasing of α and l_0, and decreases with increasing of $\widetilde{C}_n^2$ and L_0. We can also conclude that if σ is large enough (i.e., $\sigma > 0.1\,\mathrm{m}$), the improvement of correction of relay system is unnecessary. This is because when σ is large enough, the effects of σ are much smaller than that of turbulence between relay system and target, and the variation of the intensity distribution at the target are mainly decided by the turbulence. On the contrary the effect due to the variation of σ is large.

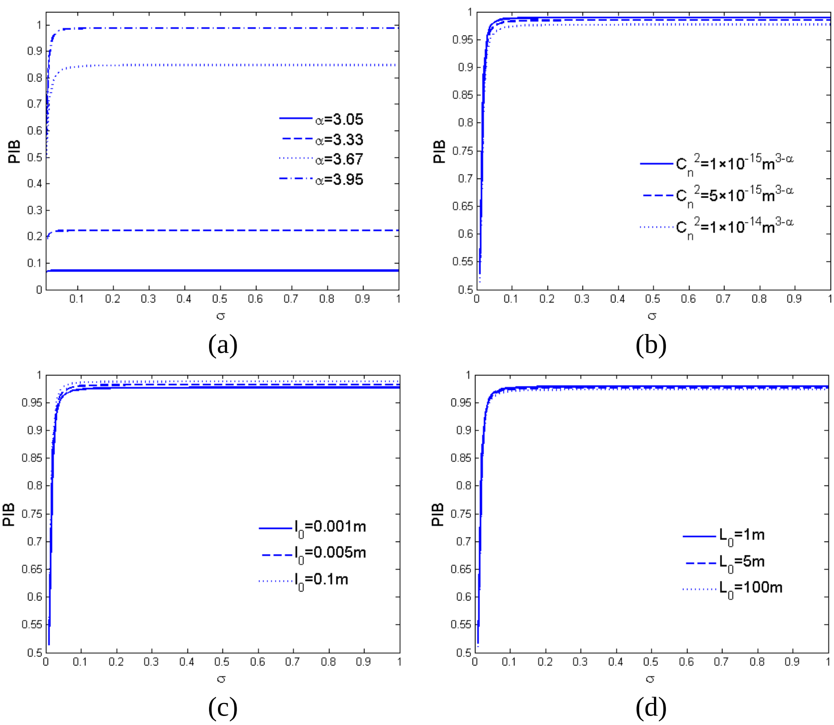

Figure 9. Variation of the *PIB* at target with different correction.

4. CONCLUSIONS

In summary, the analytical formulas for the average intensity and *PIB* of the relay propagation of partially coherent ChG beams in non-Kolmogorov turbulence have been derived based on the extended Huygens-Fresnel principle. The normalized intensity and the *PIB* of partially coherent ChG beams in non-Kolmogorov turbulence have been discussed with numerical examples. Results shows that the relay propagation of partially coherent ChG beams has a close relation with Ω_0, σ_0, $a\alpha$, $\widetilde{C}_n^2$, l_0 and L_0. The relay propagation of the beam has significant advantage compared with direct propagation and is different from that in the case of Kolmogorov turbulence; that is, $\alpha = 3.67$. The focusability of the beam will decrease with α until reaches its minimum value when $\alpha = 3.078$, after which the focusability will increase with α. The average intensity profile of the beams evolves to Gaussian form quicker for smaller exponent α and the focusability of the beam at the target increases with larger a, larger l_0, smaller L_0, and smaller $\widetilde{C}_n^2$. Therefore, for the real relay system optimization, such as the designation of free-space communication system, the non-Kolmogorov spectrum should be taken into consideration. Experimental study will be carried out in the future to further study the properties of the relay propagation in non-Kolmogorov turbulence.

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