Relay Strategies for Interference-Forwarding

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***Abstract*—We consider relaying strategies in networks with multiple source-destination pairs and possibly additional outside sources of interference. We study these networks in the discrete, memoryless setup, and focus on relaying strategies based on forwarding the interference. In particular, the relay encodes the interference signal so as to make it easier for the receiver to remove it. The objective is to help receivers with weak interference by making the interference strong enough so that these receivers are able to cancel it completely. Our proposed approach is a combination of ideas from decode-and-forward (DF) and/or estimate-and-forward (EF) but applied to the interfering signal rather than the desired signal. When based only on DF, the relay first decodes (part of) the interfering signal it wants to enhance. It then encodes the interference in such a way as to increase the interference at the assisted receiver. The rate of the relayed interference is not limited by the rate from the relay to the original destination of the forwarded message, thus, interference cancellation is not a by-product of enhancing the desired information at its intended destination, but a goal in itself. We call this method interference-forwarding (IF). IF can also be based on EF where, instead of forwarding the exact interfering signal, the relay simply sends a compressed version of it to the assisted receiver. Rate increase can thus be obtained even if the signal received at the relay is independent of the desired message and consists only of interference and noise.**

I. Introduction

In discrete, memoryless channels there are two basic strategies a relay can use to assist a receiver: estimate-and-forward and decode-and-forward [1]. These strategies were originally developed for the single-relay scenario, where there is one source-destination pair as well as a helper relay to aid the destination in decoding the source's messages. In EF the relay compresses its channel output and sends the compressed version to the destination. Since in EF the relay does not decode the source message, the signal transmitted by the relay contains both the desired information signal as well as noise. DF is fundamentally different than EF as the relay removes all the noise from its received signal. This allows the relay to achieve full coordination with the source. DF thus achieves maximum enhancement of the *desired information* at the *intended destination*. Note that in both schemes the relay tries to enhance reception of the desired information at the destination receiver.

A fundamental property of the single-relay scenario is that there is no interference. Here, interference is defined as a signal selected from a codebook known to all receivers in the network, that carries information only to its intended destination receiver. It is worth nothing that also in the multiple-relay channel with a single source-destination pair there is no interference.

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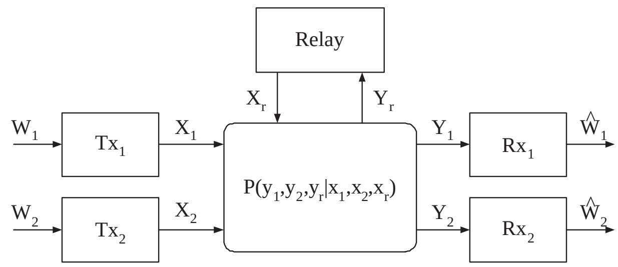

Fig. 1. The interference channel with a relay.

When considering *a network with multiple source-destination pairs* the situation is different. In this scenario, in addition to noise, there is another source of impairment – at any given receiver the signals intended for the other receivers create interference. This interference can make it more difficult for the receiver to decode its desired message.

Consider for example the classic interference channel (IC), where two independent transmitters, Tx_1 and Tx_2, send data to two independent receivers, Rx_1 and Rx_2. The component of the signal received at Rx_1 originating from the transmission of Tx_2 does not carry any desired information for Rx_1, and the same applies to the signal from Tx_1 received at Rx_2. The class of ICs is generally divided into two: weak interference and strong interference. In the weak interference regime, there is no one optimal strategy. In the strong interference regime, the optimal strategy was derived in [2]: when the interference is strong, jointly decoding the desired information and the interference at both receivers is the optimal strategy.

Adding a relay to the IC gives rise to the interference channel with relay (ICR) scenario, depicted in Figure 1. In the ICR, following current approaches, the relay has two options (and combinations thereof): one option is to operate in an oblivious manner, hence, compress (possibly with a different compression for each destination receiver) its received signal and forward it to the receivers using a broadcast code (if two compressions are used then a broadcast code with private messages is utilized). Alternatively, it can use the DF philosophy: decode the messages an re-encode them into a broadcast codeword whose purpose is to enhance the *desired information* at each receiver.

We now introduce a third option for the ICR: the relay can also generate a signal whose purpose is to *increase the interference already present at the destination receiver*. If the relay can make the interference strong enough, then the assisted receiver can cancel the effect of this interference on its received signal. Thus, the relay can help a receiver *without operating on the information desired at that receiver*. This method has the basic component of DF except that the relay decodes the signal of the interferer rather than the source. Thus, the relay does not try to enhance the desired information at its destination receiver. Instead, it enhances

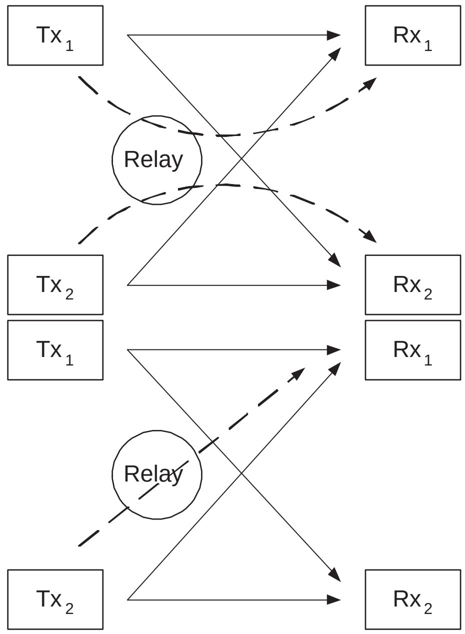

Fig. 2. The conceptual flow of information for DF (top) and IF (bottom) in the interference channel with relay. The dashed lines represent the flow of information in the path that goes through the relay.

the interference at the other receiver. We therefore refer to this method as *interference-forwarding* (IF). The conceptual difference between IF and DF is illustrated in Figure 2. Put in mathematical terms, if we denote the cooperation information at the relay with a random variable (RV) U, the source signal with a RV X and the signal received at the destination with a RV Y, then DF results in a rate expression of the form $I(X, U; Y)$ while IF results in a rate expression of the form $I(X; Y|U)$. Moreover, the rate of the forwarded message *is not limited by the rate from the relay to its original destination* but by the rate from the relay to the interfered receiver. Thus *interference cancellation is not a by-product of signal enhancement at the other receiver, but a goal in itself.*

If the interference is made strong enough, then the assisted receiver can use interference cancellation to improve its rate. Furthermore, if the relay can drive a receiver into the "very strong" interference regime then the maximum possible rate to that receiver is achieved. When based on DF this method is abbreviated as DIF. This work will focus on the DIF scheme, but as noted earlier an EIF variant is also possible, so as not to restrict the rate due to decoding the interference at the relay. The downside of EIF is that the compressed signal contains both noise as well as information about the interference.

In the general ICR scenario the relay can receive signals from both Tx_1 and Tx_2 and can transmit to both Rx_1 and Rx_2 (see Figure 3). Here, the transmission of the relay is actually a broadcast transmission, and encoding is implemented via a broadcast code. The relay has to decide how to split its information into private and common messages. For example, if the relay uses a single EF compression it should encode the compressed signal as a common message. But, the relay can also apply two different compressions each aiming at helping a different receiver. In this case we believe that the best strategy is to encode the compressed information as private messages to each receiver. The same reasoning applies to DF and IF. For

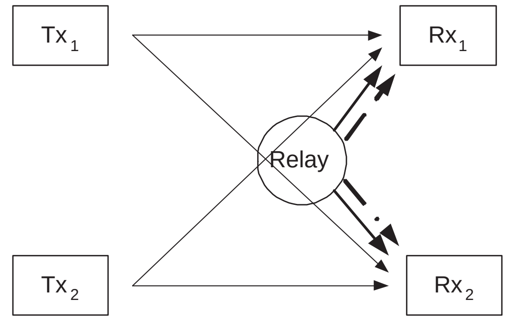

Fig. 3. The general ICR scenario. The relay can receive information from both transmitters and transmit to both receivers. The bold solid lines indicate common information, the dashed line indicates private information to Rx_1 and the dash-dot line indicates private information to Rx_2.

example, decoded information can be encoded as a private message to the desired receiver to enhance reception of the desired signal, as a common message to both receivers to facilitate simultaneous signal enhancement and interference cancellation, or *as a private message to the other receiver to facilitate interference cancellation at the other receiver*, following the IF approach. Thus interference cancellation is not limited by the rate available for signal enhancement. The notion of interference-forwarding opens up a new class of relaying strategies for channels with multiple source-destination pairs. In this paper we will consider strategies for enhancing only the interference at one receiver and compare them with sending a signal that improves channel conditions for both transmit-receive pairs (i.e. "opens the channel", see below). These two strategies and the network conditions under which they are investigated are described in more detail below.

Main Contributions and Organization

In this work we introduce a new approach to relaying in networks with multiple source-destination pairs. In this approach, instead of enhancing the desired information at the destination receiver, we enhance the interference at the other receiver(s). We investigate the implications of such an approach in the ICR model of Figure 1, where Rx_2 has strong interference and Rx_1 has weak interference. Furthermore we specialize the channel such that the relay cannot receive information from Tx_1 and cannot deliver new information to Rx_2. This implies that the relay cannot help $\mathrm{Tx}_1 - \mathrm{Rx}_1$ or $\mathrm{Tx}_2 - \mathrm{Rx}_2$ by enhancing their desired information, and it can do only one of the two: either transmit a signal that "opens" the channel for the IC communication (i.e. transmit a (possibly fixed) signal, generated independently of its received signal, that facilitates the communication between the two pairs, as in [1, Theorem 2]), or try to increase the interference at Rx_1. We find the conditions under which it is better to enhance the interference. This shows that when considering relaying in networks with multiple source-destination pairs, IF maybe a better alternative than existing relaying strategies. *In particular, if the relay cannot receive information from some transmitters, it can still help their corresponding receivers.* This demonstrates most clearly the decoupling of interference cancellation from signal enhancement.

The rest of this paper is organized as follows: Section II formally introduces the model, Section III presents the main results followed by a discussion in Section IV. Lastly, Section V concludes the paper.

II. Model

First a word about notations: we denote random variables (RVs) with upper case letters, e.g. X, Y, and their realizations with lower case letters x, y. A RV X takes values in a set $\mathcal{X}$. We use $p_X(x)$ to denote the probability mass function (p.m.f.) of a discrete RV X on $\mathcal{X}$. For brevity we may omit the subscript X when it is obvious from the context. We use $p_{X|Y}(x|y)$ to denote the conditional p.m.f. of X given Y. We denote vectors with boldface letters, e.g. $\mathbf{x}$, $\mathbf{y}$; the i'th element of a vector $\mathbf{x}$ is denoted with x_i and we use x_i^j where $i < j$ to denote the vector $(x_i, x_{i+1}, ..., x_{j-1}, x_j)$; x^j is short form notation for x_1^j, and $\mathbf{x} \equiv x^n$. We use $H(\cdot)$ to denote the entropy of a RV and $I(\cdot;\cdot)$ to denote the mutual information between two RVs.

A. A General Model for Interference Channels with Relays

We now define the ICR scenario formally.

Definition 1: The *discrete interference channel with a relay* (ICR) consists of three discrete input alphabets, $\mathcal{X}_1$, $\mathcal{X}_2$ and $\mathcal{X}_r$, three discrete output alphabets $\mathcal{Y}_1$, $\mathcal{Y}_2$ and $\mathcal{Y}_r$ and a set of conditional p.m.fs $p(y_1, y_2, y_r|x_1, x_2, x_r)$. The discrete ICR is *memoryless* if

$$p(w_1, w_2, \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_r, \mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_r) = p(w_1)p(w_2)p(\mathbf{x}_1|w_1)p(\mathbf{x}_2|w_2) \times \prod_{i=1}^{n} p(x_{r,i}|y_{r,1}^{i-1})p(y_{1,i}, y_{2,i}, y_{r,i}|x_{1,i}, x_{2,i}, x_{r,i}),$$

where we used the fact that the relay is allowed to operate only on its past received channel outputs.

Let R_1 and R_2 be the information rates to Rx_1 and Rx_2 respectively. Rates are non-negative real numbers.

Definition 2: An (R_1, R_2, n) *code* for the ICR consists of two message sets, $\mathcal{W}_1 = \{1, 2, ..., 2^{nR_1}\}$ and $\mathcal{W}_2 = \{1, 2, ..., 2^{nR_2}\}$, and mapping functions $f_1, f_2, \{f_{r,i}\}_{i=2}^{n}, g_1, g_2$, where for $t = 1, 2$, $f_t : \mathcal{W}_t \mapsto \mathcal{X}_t^n$ are the mappings at the encoders, $x_{r,i} = f_{r,i}(y_{r,1}^{i-1})$, $i = 2, 3, ..., n$, is the set of mappings at the relay with $x_{r,1}$ being an arbitrary symbol from $\mathcal{X}_r$. The decoders at Rx_1 and Rx_2 are defined by $g_t : \mathcal{Y}_t^n \mapsto \mathcal{W}_t$ for $t = 1, 2$.

Definition 3: The *average probability of error* of an (R_1, R_2, n) code for the ICR when the messages W_1 and W_2 are selected independently and uniformly over their respective message sets is $P_e^{(n)} \triangleq \Pr(g_1(Y_1^n) \neq W_1 \text{ or } g_2(Y_2^n) \neq W_2)$.

Definition 4: A rate pair (R_1, R_2) is called *achievable* if for every $\epsilon > 0$ and $\delta > 0$ there exists a block length $n(\epsilon, \delta)$ such that an $(R_1 - \delta, R_2 - \delta, n)$ code with $P_e^{(n)} \leq \epsilon$ can be constructed for all $n > n(\epsilon, \delta)$.

B. A Special Case of the General ICR Model

In this work we study a special case of the general ICR scenario to demonstrate the interference-forwarding relay strategy. This special case is characterized by the following assumptions:

A1. Y_r, the channel output at the relay, is independent of X_1, given X_2 and X_r. This can happen in the wireless channel, for example, when due to heavy shadowing the channel input X_1 is not observed at the relay:

$$p(y_r|x_1, x_2, x_r) = p(y_r|x_2, x_r). \quad (1)$$

A2. Y_2, the channel output at Rx_2, is independent of X_r when X_1 and X_2 are given:

$$p(y_2|x_1, x_2, x_r) = p(y_2|x_1, x_2). \quad (2)$$

A3. Rx_2 observes strong interference. This means that the channel $\mathrm{Tx}_1 - \mathrm{Rx}_2$ is better than the channel $\mathrm{Tx}_1 - \mathrm{Rx}_1$:

$$I(X_1; Y_2|X_2, X_r) \geq I(X_1; Y_1|X_2, X_r) \quad (3)$$

for all $p(x_1)p(x_2, x_r)$. This implies that there is no rate loss when forcing Rx_2 to decode W_1, hence, rate-splitting on W_1 is not necessary. Subject to A2, $I(X_1; Y_2|X_2, X_r)$ becomes:

$$\begin{aligned} I(X_1; Y_2|X_2, X_r) &= H(Y_2|X_2, X_r) - H(Y_2|X_2, X_1) \\ &= H(Y_2|X_2) - H(Y_2|X_2, X_1) \\ &= I(X_1; Y_2|X_2), \end{aligned}$$

thus A3 becomes

$$I(X_1; Y_2|X_2) \geq I(X_1; Y_1|X_2, X_r). \quad (4)$$

A4. Rx_1 observes weak interference. This means that the channel $\mathrm{Tx}_2 - \mathrm{Rx}_1$ is worse than the channel $\mathrm{Tx}_2 - \mathrm{Rx}_2$:

$$I(X_2; Y_1|X_1, X_r) \leq I(X_2; Y_2|X_1, X_r) \quad (5)$$

for all $p(x_1)p(x_2, x_r)$. Note that due to A2,

$$\begin{aligned} I(X_2; Y_2|X_1, X_r) &= H(Y_2|X_1, X_r) - H(Y_2|X_1, X_r, X_2) \\ &= H(Y_2|X_1, X_r) - H(Y_2|X_1, X_2) \\ &\leq I(X_2; Y_2|X_1). \end{aligned}$$

Thus (5) implies

$$I(X_2; Y_1|X_1, X_r) \leq I(X_2; Y_2|X_1). \quad (6)$$

This scenario is depicted in Figure 4.

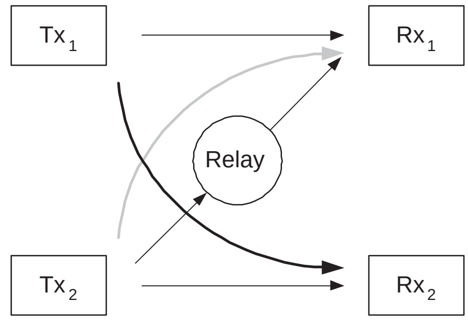

Fig. 4. The interference channel with relay, where the relay cannot receive information from Tx_1, and Rx_2 cannot receive information from the relay. The arrows in the figure indicate which node can receive information from which node. The light gray line $\mathrm{Tx}_2 - \mathrm{Rx}_1$ indicates weak interference and the bold black line $\mathrm{Tx}_1 - \mathrm{Rx}_2$ indicates strong interference.

When the relay cannot receive information from Tx_1 then the EF and DF relaying strategies cannot help Rx_1. Similarly, when the relay cannot transmit information to Rx_2 then again, EF and DF cannot help Rx_2. Therefore, the relay has two ways to assist Rx_1: either to open the channel or increase the interference. We use this scenario to characterize the conditions under which IF is useful. In the more general scenario a combination of all strategies should be considered.

III. Main Results

Our main objective is to characterize the situation where by enhancing the interference, the relay can assist the communication better than any other relaying strategy. Since there is strong interference at Rx_2 (assumption A3), the focus is on assisting only Rx_1. Note that A3 implies that rate-splitting on W_1 is not necessary. We compare two regions: the first region $\mathcal{R}_{\text{open}}$ is obtained by letting the relay open the channel. Here we also use rate-splitting of W_2 at Tx_2 (see, e.g. [3]) as this results in the largest known achievable rate region. For the second region, $\mathcal{R}_{\text{IF}}$, the relay uses IF but Tx_2 does not rate-split W_2. We note that IF combined with rate-splitting at Tx_2 will give a more general result for which $\mathcal{R}_{\text{open}}$ is a special case, trivially giving $\mathcal{R}_{\text{IF with rate-splitting}} \supseteq \mathcal{R}_{\text{open}}$. However, a "$\supseteq$" relationship does not prove that increasing the interference strictly increases the region. The reason is that in the weak interference regime, the achievable region does not necessarily increase as the interference increases. The purpose of this paper is to characterize the situations in which IF is strictly the best option based only on the parameters of the channel. This is possible if we find an achievable region for IF without auxiliary variables (e.g. $\mathcal{R}_{\text{IF}}$), and compare it with the largest $\mathcal{R}_{\text{open}}$.

The rate region when the relay simply facilitates communication by opening the channel is given in Theorem 1:

Theorem 1: (Han-Kobayashi transmission [3] without rate-splitting of W_1 and with relay opening the channel) For the ICR of Definition 1 subject to assumptions A1 and A2, any rate pair (R_1, R_2) satisfying

$$R_1 \le I(X_1; Y_1 | U_2, X_r) \quad (7a)$$
$$R_2 \le I(X_2; Y_2 | X_1) \quad (7b)$$
$$R_1 + R_2 \le I(X_1, U_2; Y_1 | X_r) + I(X_2; Y_2 | U_2, X_1) \quad (7c)$$
$$R_1 + R_2 \le I(X_1, X_2; Y_2) \quad (7d)$$
$$2R_1 + R_2 \le I(X_1, U_2; Y_1 | X_r) + I(X_1, X_2; Y_2 | U_2) \quad (7e)$$

for some probability distribution $p(x_1, u_2, x_2, x_r, y_1, y_2, y_r) = p(x_1)p(u_2, x_2)p(x_r)p(y_1, y_2, y_r | x_1, x_2, x_r)$, is achievable. $\mathcal{R}_{open}$ is the convex hull of all rate pairs (R_1, R_2) that satisfy (7).

Proof outline: This rate region is achieved with the rate-splitting scheme where due to the strong interference at Rx_2 (assumption A3), there is no need to rate-split at Tx_1. The message W_2 is split into common and private parts, where the common part is encoded into a codeword $\mathbf{U}_2$. The relay generates a codebook whose rate satisfies $R_r \le \min\{I(X_r; Y_1), I(X_r; Y_2)\}$. This implies that the relay codeword can be decoded at both receivers and then canceled. Rx_1 now decodes W_1 from $\mathbf{Y}_1$ using $\mathbf{U}_2$, $\mathbf{X}_1$ and $\mathbf{X}_r$, and Rx_2 decodes W_2 from $\mathbf{Y}_2$ using $\mathbf{U}_2$, $\mathbf{X}_1$, $\mathbf{X}_2$ and $\mathbf{X}_r$. As the relay signal $\mathbf{X}_r$ can always be recovered, decoding proceeds in the standard way, see for example [3].

When the relay uses IF, the resulting region is given in Theorem 2:

Theorem 2: For the ICR of Definition 1 subject to assumptions A1 and A2, any rate pair (R_1, R_2) satisfying

$$R_1 \le I(X_1; Y_1 | X_2, X_r) \quad (8a)$$
$$R_2 \le I(X_2; Y_2 | X_1) \quad (8b)$$
$$R_1 + R_2 \le I(X_1, X_2, X_r; Y_1) \quad (8c)$$
$$R_1 + R_2 \le I(X_1, X_2; Y_2) \quad (8d)$$
$$R_2 \le I(X_2; Y_r | X_r) \quad (8e)$$

for some probability distribution $p(x_1, x_2, x_r, y_1, y_2, y_r) = p(x_1)p(x_2, x_r)p(y_1, y_2, y_r | x_1, x_2, x_r)$, is achievable. $\mathcal{R}_{IF}$ is the convex hull of all rate pairs (R_1, R_2) that satisfy (8).

Proof outline: Tx_1 generates a random codebook $\mathbf{X}_1$. Tx_2 and the relay use the block-Markov construction of Cover & El-Gamal in [1, Theorem 1] to generate correlated codebooks $(\mathbf{X}_2, \mathbf{X}_r)$. At block b, Rx_1 decodes first the bin index s_b and then uses it together with $(\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_r, \mathbf{Y}_1(b), \mathbf{Y}_1(b-1))$ to decide that $(\hat{W}_{1,b}(1), \hat{W}_{2,b-1}(1))$ was sent. Rx_2 uses $(\mathbf{X}_1, \mathbf{X}_2, \mathbf{Y}_2(b))$ to decide that $(\hat{W}_{1,b}(2), \hat{W}_{2,b}(2))$ was sent. Equivalently, the block-Markov encoding and backward decoding [4, Section 7] at decoder 1 can be used; decoder 1 waits until block time B to decide on $(\hat{W}_{1,B}(1), \hat{W}_{2,B-1}(1))$ and then uses this information to proceed with decoding backwards.

Remark: Assumption A1 (Equation (1)) implies that $X_1 - X_2, X_r - Y_r$ is a Markov chain so X_1 does not affect the rate constraint at the relay. When the relay is required to recover X_2 this implies that the relay cannot coordinate its transmission with Tx_1 and therefore $p(x_1, x_2, x_r) = p(x_1)p(x_2, x_r)$ in Theorem 2.

IV. Discussion

Comparing Theorems 1 and 2 we note that (7b) = (8b) and (7d) = (8d), and all four bound expressions are evaluated under the same p.m.f. $p(x_1, x_2) = p(x_1)p(x_2)$. This is expected as the relay does not have any impact on the signal Y_2 when X_1 and X_2 are known, and X_1 can be decoded completely at Rx_2 due to the strong interference assumption A3.

Comparing the rate constraints on R_1 we see that (7a) $\le$ (8a). To see this rewrite (7a) as:

$$\begin{aligned} I(X_1; Y_1 | U_2, X_r) &= H(X_1 | U_2, X_r) - H(X_1 | Y_1, U_2, X_r) \\ &\overset{(a)}{=} H(X_1 | X_2, X_r) - H(X_1 | Y_1, U_2, X_r) \\ &\le H(X_1 | X_2, X_r) - H(X_1 | Y_1, X_2, U_2, X_r) \\ &\overset{(b)}{=} I(X_1; Y_1 | X_2, X_r), \end{aligned} \quad (9)$$

where (a) is because X_1 is independent of U_2, X_2 and X_r under $p(u_2, x_2)p(x_1)p(x_r)$ and (b) is because $p(x_1 | y_1, x_2, u_2, x_r) = p(x_1 | y_1, x_2, x_r)$. Moreover, the underlaying chain for Theorem 2 allows dependence between X_2 and X_r strengthening the inequality.

Finally, comparing the sum-rate constraints (7c) and (8c) we note that

$$\begin{aligned} &I(X_1, U_2; Y_1 | X_r) + I(X_2; Y_2 | U_2, X_1) \\ &\quad \le I(X_1, X_2; Y_1 | X_r) + I(X_2; Y_2 | X_1) \\ &\quad = I(X_1, X_2, X_r; Y_1) + I(X_2; Y_2 | X_1) - I(X_r; Y_1) \\ &\quad \overset{(a)}{<} I(X_1, X_2, X_r; Y_1), \end{aligned}$$

where (a) holds if $I(X_2;Y_2|X_1) < I(X_r;Y_1)$.

We therefore obtained the following proposition stating the conditions under which Theorem 2 is strictly better than Theorem 1:

Proposition 1: If for all distributions $p(x_1)p(x_2,x_r)$ it hold that

C1. $I(X_2;Y_2|X_1) < I(X_r;Y_1)$ *(strong relay – Rx$_1$ link),*

C2. $I(X_2;Y_r|X_r) > I(X_2;Y_2|X_1)$ *(strong Tx$_2$ – relay link),*

then $\mathcal{R}_{IF} \supset \mathcal{R}_{open}$.

Comparing Theorems 1 and 2 when C2 holds (i.e. decoding at the relay does not incur rate loss on R_2) we observe that the rate constraints for decoding at Rx$_2$ are the same in both theorems (due to A2), but comparing (7a) with (8a) we see that enhancing the interference may allow a higher rate to Rx$_1$:

$$\begin{aligned}\max R_1^{\text{open}} &= I(X_1;Y_1|U_2,X_r)\\ &< I(X_1;Y_1|X_2,X_r) = \max R_1^{\text{IF}},\end{aligned}$$

as long as U_2 is only partial information on X_2. Setting $U_2 \overset{\text{a.s.}}{=} X_2$, both expressions become $I(X_1;Y_1|X_2,X_r)$. But since for (7a) the underlying distribution becomes $p(x_1)p(x_2)p(x_r)$, while for (8a) the underlying distribution is $p(x_1)p(x_2,x_r)$, then the inequality relationship "$\leq$" still holds.

Next, let us compare the regions of Theorems 1 and 2, subject to C2, by considering one point of the achievable regions: $P = (I(X_1;Y_1|X_2,X_r), I(X_2;Y_2)) \equiv (R_{1,P}, R_{2,P})$. For the moment ignore the difference in the distribution chains. We observe the following:

- *The region $\mathcal{R}_{open}$:* Achieving $R_{1,P}$ requires setting $U_2 \overset{\text{a.s.}}{=} X_2$. This assignment implies that P satisfies (7a) and (7b). Now examine the sum-rate bound at Rx$_1$: when $U_2 = X_2$ (7c) becomes $I(X_1,X_2;Y_1|X_r)$, and can be expanded as

$$I(X_1,X_2;Y_1|X_r) = I(X_2;Y_1|X_r) + I(X_1;Y_1|X_2,X_r).$$

Hence, for P to be admissible by (7c) we need

$$I(X_2;Y_2) \leq I(X_2;Y_1|X_r). \tag{10}$$

Examine next the sum-rate at Rx$_2$ given in (7d): $R_1 + R_2 \leq I(X_1,X_2;Y_2)$. This can be expanded as

$$\begin{aligned}R_1 + R_2 &\leq I(X_2;Y_2) + I(X_1;Y_2|X_2)\\ &\geq I(X_2;Y_2) + I(X_1;Y_1|X_2,X_r),\end{aligned}$$

due to A3. This means that P is admissible by (7d). Finally examine (7e) with $U_2 \overset{\text{a.s.}}{=} X_2$:

$$\begin{aligned}2R_1 + R_2 &\leq I(X_1,X_2;Y_1|X_r) + I(X_1;Y_2|X_2)\\ &= I(X_2;Y_1|X_r) + I(X_1;Y_1|X_2,X_r) + I(X_1;Y_2|X_2).\end{aligned}$$

Thus A3 and (10) imply that P satisfies (7e) as well. In conclusion, $P \in \mathcal{R}_{open}$ *only if* (10) *holds.*

- *The region $\mathcal{R}_{IF}$:* The above point demonstrates most clearly the benefit of increasing the interference. The point P satisfies (8a) and (8b). The sum-rate at Rx$_1$, given in (8c) is now

$$I(X_1,X_2,X_r;Y_1) = I(X_2,X_r;Y_1) + I(X_1;Y_1|X_2,X_r).$$

Therefore, P is admissible by (8c) if

$$I(X_2;Y_2) \leq I(X_2,X_r;Y_1). \tag{11}$$

Similarly to (7d) it follows that P satisfies (8d). Finally, (8e) is satisfied by C2. In conclusion, $P \in \mathcal{R}_{IF}$ *only if* (11) *holds.*

Comparing (10) and (11) we observe that as long as $I(X_r;Y_1) > 0$, then $I(X_2,X_r;Y_1) > I(X_2;Y_1|X_r)$. Therefore, we conclude that there are ICR scenarios for which the point P can be achieved only with IF. We note that the situation is even more in favor of IF as when $U_2 \overset{\text{a.s.}}{=} X_2$ the underlying distribution for $\mathcal{R}_{\text{open}}$ is more restrictive than for $\mathcal{R}_{\text{IF}}$, allowing possibly higher rates for IF though the expressions are the same. Also note that (11) is weaker than C1 since we consider only the specific point P rather than the entire region. We see that the relay in fact turned the weak interference into strong interference for Rx$_1$.

In summary, in the scenario of section II-B when conditions C1 and C2 hold, IF is better than simply opening the channel. Admittedly, in this scenario the relay does not have any alternative to opening the channel but to apply IF, however, the concept remains correct also when the paths Tx$_1$-relay and relay-Rx$_2$ exist. The difference is that now IF will have to be evaluated against DF and EF (and probably combined with them). The best relay strategy vs. the channel conditions is summarized in Table I. The entries in the table indicate the *strategy for generating the messages* to be transmitted from the relay to each receiver and not the encoding scheme. We point out that when the relay is received at both Rx$_1$ and Rx$_2$, the messages to be transmitted should be encoded via Marton's broadcast code. It can be seen from the table that there are situations in which IF is the only possible strategy.

TABLE I
BEST RELAYING STRATEGIES FOR DIFFERENT CHANNEL CONDITIONS.

Relay can receive from	Relay is received at			
	Rx$_1$ only	Rx$_2$ only	Rx$_1$ & Rx$_2$	neither
Tx$_1$ only	CR(1)	***IF(2)***	CR(1) & ***IF(2)***	∅
Tx$_2$ only	***IF(1)***	CR(2)	***IF(1)*** & CR(2)	∅
Tx$_1$ & Tx$_2$	***COM(1)***	***COM(2)***	***COM(1) & COM(2)***	∅
neither	OC(1)	OC(2)	OC(1) & OC(2)	∅

IF = interference-forwarding (DIF/EIF, if better than OC), CR = classic relaying (DF/EF), OC = opening the channel, COM = combination of IF and CR to the same receiver. The number in brackets indicates the receiver for which the relay applies the specified strategy (for example, IF(1) means IF is used to assist Rx$_1$).

V. Conclusions

We introduced a new relaying approach for networks with multiple source-destination pairs, and showed that the relay may be of more benefit by increasing the interference at a given receiver rather than enhancing the desired signal. In particular, some nodes may benefit more from enhancing the *undesired* signal, thus allowing them to cancel its effect on their received signal. Therefore, alongside with DF and EF, the interference-forwarding scheme should be considered as well.

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