RELIABILITY BASED ANALYSIS OF SLOPE, FOUNDATION AND RETAINING WALL USING FINITE ELEMENT METHOD

A Project Report

Submitted by

SUBRAMANIAM P

In partial fulfillment of the requirements for the award of the Degree of

MASTER OF TECHNOLOGY

in

GEOTECHNICAL ENGINEERING

DEPARTMENT OF CIVIL ENGINEERING
NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA
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ROURKELA-769008.
JUNE 2011

Department of Civil Engineering
National Institute of Technology Rourkela
Rourkela – 769008, India www.nitrkl.ac.in

CERTIFICATE

This is to certify that the project entitled ***Reliability based analysis of Slope, Foundation and Retaining Wall using Finite Element Method*** submitted by Mr. Subramaniam.P (Roll No. 209CE1035) in partial fulfilment of the requirements for the award of Master of Technology Degree in Civil Engineering at NIT Rourkela is an authentic work carried out by him under my supervision and guidance.

Date:

Prof. Sarat Kumar Das

Associate Professor

Department of Civil Engineering

National Institute of Technology Rourkela

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Subramaniam P

Table of Contents

List of Figures

List of Tables

ABSTRACT

Uncertainties in geotechnical Engineering is inevitable. The soil properties may disperse within a significant range over a domain. Thus the factor of safety is used in the deterministic approach which account for the uncertainty associated with the soil properties. This does not consider the sources and amount of uncertainty associated with the system. Limit state design of the structure is difficult to estimate using deterministic methods. So it is reasonable to study the probability of failure of the structure. In the present study reliability analysis has been made for slope stability, geogrid reinforced footing and the sheet pile wall using finite element method (FEM). The limit state function is developed using response surface methods based on finite element models using commercial software PLAXIS 9.0. The FEM model is validated by analysing case studies, wherever available. Full factorial design is used for development of response surface models. The reliability analysis is performed using first order reliability method in the present study. The need for reliability analysis and the corresponding factor of safety is discussed. Parametric study has been done by considering the variability in soil parameter.

CHAPTER-1

INTRODUCTION

1.1 INTRODUCTION

Uncertainties in the geotechnical engineering are unavoidable. The geotechnical engineering deals mostly with natural materials. So the variability of the material is inevitable. This is termed as spacial variability. The soil properties are obtained from field or from laboratory testing and the properties vary depending upon borehole location, number of samples, borehole methods etc. Method of sampling (undisturbed or disturbed), method of laboratory testing, interpretation of statistical results from testing data, the method of analysis for the particular problem such as Meyerhof, Terzaghi, Vesic bearing capacity methods, instrumental error, human error are also considered as uncertainty associated with the performance of the system.

More over in some cases though the probability of failure is high but system shows high factor of safety in deterministic analysis. Factor of safety is chosen based on past experience and the outcome of failure. The factor of safety is used in the deterministic approach which account for natural soil variability, measurement errors, statistical approximations, model transformation and limitation in analytical models. This does not consider the sources and amount of uncertainty associated with the system. A factor of safety of 2.5–3.0 is adopted to account this variability in various geotechnical bearing capacity problems. Serviceability of the structure is difficult to estimate using deterministic methods.

Reliability:

Reliability of the system is the relationship between loads the system must carry and its ability to carry. Reliability of the system is expressed in the form of reliability index (β). This reliability index is related to the probability of failure of the system (p_f). Risk and reliability are complementary terms. Risk is unsatisfactory performance or probability of failure. On the other hand reliability is satisfactory performance or probability of success.

Benefits of reliability method in concurrence with conventional design:

1. All sources of uncertainties involved in the project are taken into account.
2. Support in decision making regarding risk – cost analysis.
3. Probability of failure can be known for each design methods.
4. The structure can be designed according to serviceability conditions.
5. The overall risk involved in the project is clearly identified.

1.2 SCOPE AND ORGANIZATION OF THE THESIS

After the brief introduction (Chapter 1), the recent trend in reliability analysis in geotechnical engineering problems is described in Chapter 2. The literature pertaining to reliability analysis, finite element analysis and response surface methods in geotechnical engineering are critically evaluated in this chapter. For the present finite element study commercial software PLAXIS is used.

Chapter 3 describes the use of reliability analysis for slope stability analysis of unreinforced slopes. A case study of failed slope is analyzed using finite element method. Considering the variability in cohesion, angle of internal friction of the soil different FEM models are developed as per full factorial design as per response surface method to formulate the performance function for reliability analysis. The reliability index and probability of failure is calculated using first order reliability method (FORM).

While describing the reliability analysis of slope, effect of various soil parameters like cohesion and angle of internal friction on reliability index is also discussed. A comparative study of reliability analysis and the corresponding factor of safety are also discussed.

Safe bearing capacity of foundation is one of the important stability problems in geotechnical engineering, which depends upon the bearing capacity and the allowable settlement of foundation. In Chapter 4, using PLAXIS, actual and predicted settlement of footing are compared. The problem has been taken from the Briaud & Gibbons (1994) prediction symposium. The settlement is predicted through FE software PLAXIS. The young's modulus is obtained from the different correlations and the results are discussed. The variability in soil properties of the layered soil is discussed. Then after reliability analysis is performed for settlement of Geogrid reinforced footing using FEM. Variability in soil parameters and soil-geogrid interface is taken into account for reliability analysis. Full factorial design is used in the design of experiments. Response surface model is generated using this input variables and output response. Using this limit state function, reliability Index and probability of failure of the system are calculated.

In chapter 5, anchored sheet pile wall is analysed using the FEM. The horizontal displacement is found out. The reliability study is carried out for the horizontal displacement of the sheet pile wall and the failure of anchorage considering variability in soil parameter. The failure of the cantilever sheet pile wall is also studied based on reliability analysis.

In Chapter 6, generalized conclusions made from various studies made in this thesis are presented and the scope for the future work is indicated. The general layout and different problems analyzed in the present thesis using different computational techniques in each chapter (from Chapter 3 to 5) is shown in a flow diagram (Figure 1.1) for ready reference.

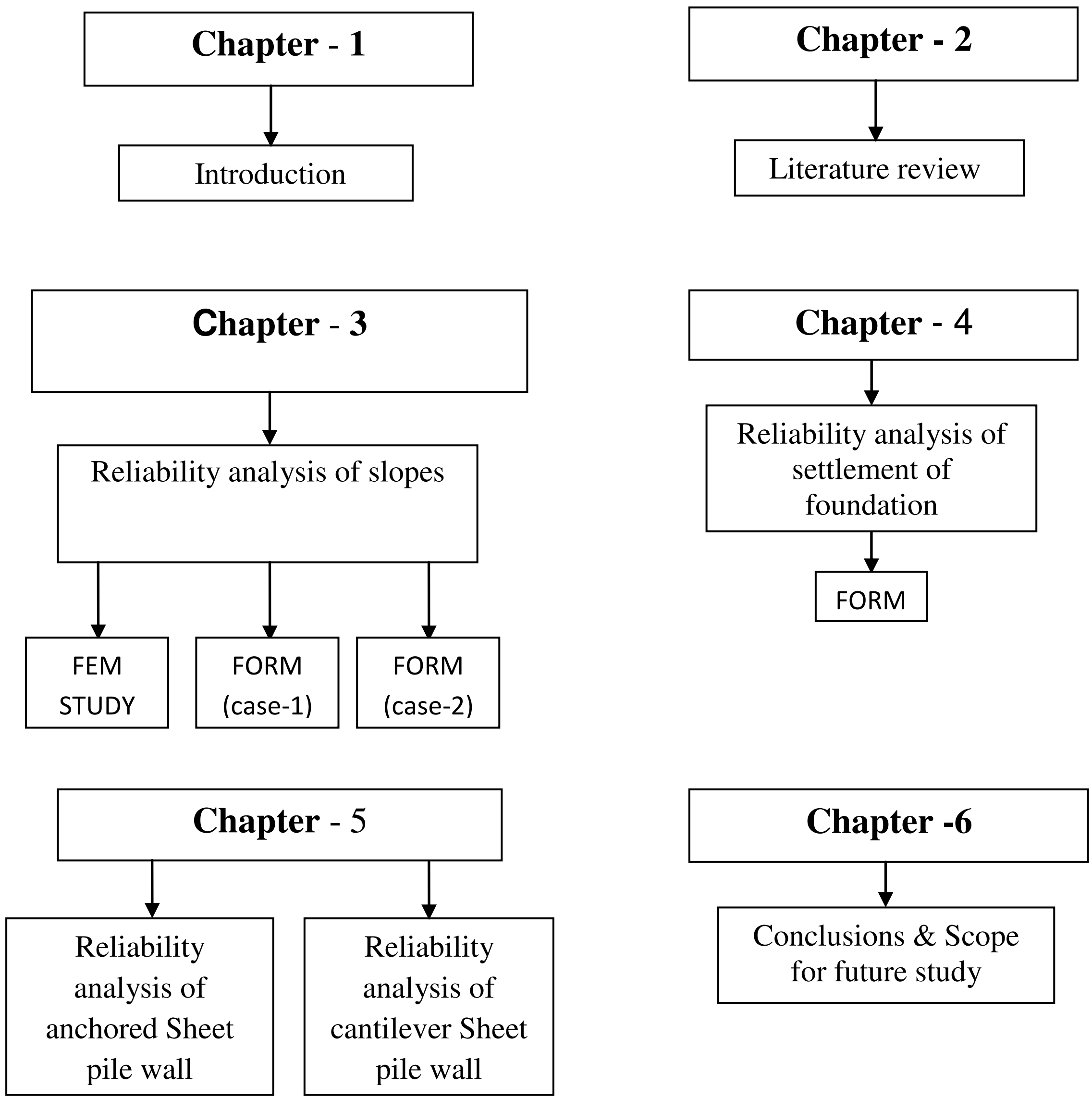


Figure 1.1 Flow diagrams showing the organization of the thesis

CHAPTER-2

REVIEW OF LITERATURE
&
METHODOLOGY

2.1 REVIEW OF LITERATURE:

The reliability analysis in geotechnical engineering developed over the years starting from the probabilistic methods, and some of the studies are discussed as follows. Fardis and Veneziano (1981) developed a probabilistic model based on statistical analysis of liquefaction potential of sands using the results of 192 published cyclic simple shear tests taking into account the uncertainties caused by the effect of sample preparation, effect of system compliance and stress non uniformities. Chowdhury and Grivas (1982) have developed a probabilistic model for progressive failure of slopes. Harr (1987) conducted the extensive study in the application and methods of reliability analysis in civil engineering. Hwang and Lee (1991) considered uncertainties in both site par meters and seismic parameters to calculate probability of liquefaction index, P_L, based on *SPT N-value* which measures the severity of liquefaction. Low and Tang (1997) have proposed the procedure to calculate the Hasofer Lind second moment reliability index using spread sheet. Low (2003) explained the practical probabilistic slope study with case studies. Low (2005) compared the expanding ellipsoid, Hasofer-Lind method and FORM. Low (2005) analyzed the retaining walls for overturning and sliding. Correlated normal variables have used in the study. Monte Carlo simulation method is a probabilistic method which uses random number generators. Greco (1996) and Malkawi et al (2001) have analyzed the slopes using Monte Carlo simulation methods. But it involves high computational expenses. Phoon and Kulawy (1999) have explained the variation in geotechnical property. He explained about measurement error, transformation uncertainty and soil variability. Coefficient of variation has been evidently explained by him. Babu et al. (2007) have analysed the stability of earthen dams by Monte Carlo simulations and conducted the reliability analysis. Babu and Srivastava (2010) have conducted the reliability study on earth dams by developing response surface models by Finite difference method. Babu and Basha (2008) have analyzed the sheet pile walls by target reliability approach. Inverse first order reliability method has used to analyze the anchored cantilever sheet pile wall. Christian et al. 1994, Low 2003, Low and Tang 1997 have proposed reliability-based approaches to slope stability problems. Xue and Cavin (2007) considered the variables in polar coordinates and the reliability index defined with the Hasofer-Lind method is formulated as a function of the soil properties and the slip surface. With genetic algorithm, the nonlinear programming problem has solved. In this method, the reliability index and critical slip surface are found concurrently.

2.2 METHODOLOGY:

The parameters involved in the particular problem are studied. The random variables are chosen which affect the required output. The variability of the random variables is inspected. Then using Full Factorial design, experimental design is developed. For each set of input variables required output is developed using Finite Element Method. These set of input variables and its corresponding output is used to develop the linear response models. These linear response models are used to develop the limit state function. First order reliability method is used to find out the reliability index. The reliability index is minimized using Excel solver with the constraint as performance function. From this reliability index probability of failure is obtained. The flow chart for the above considered for the present study is presented in Figure 2.1 as shown below.

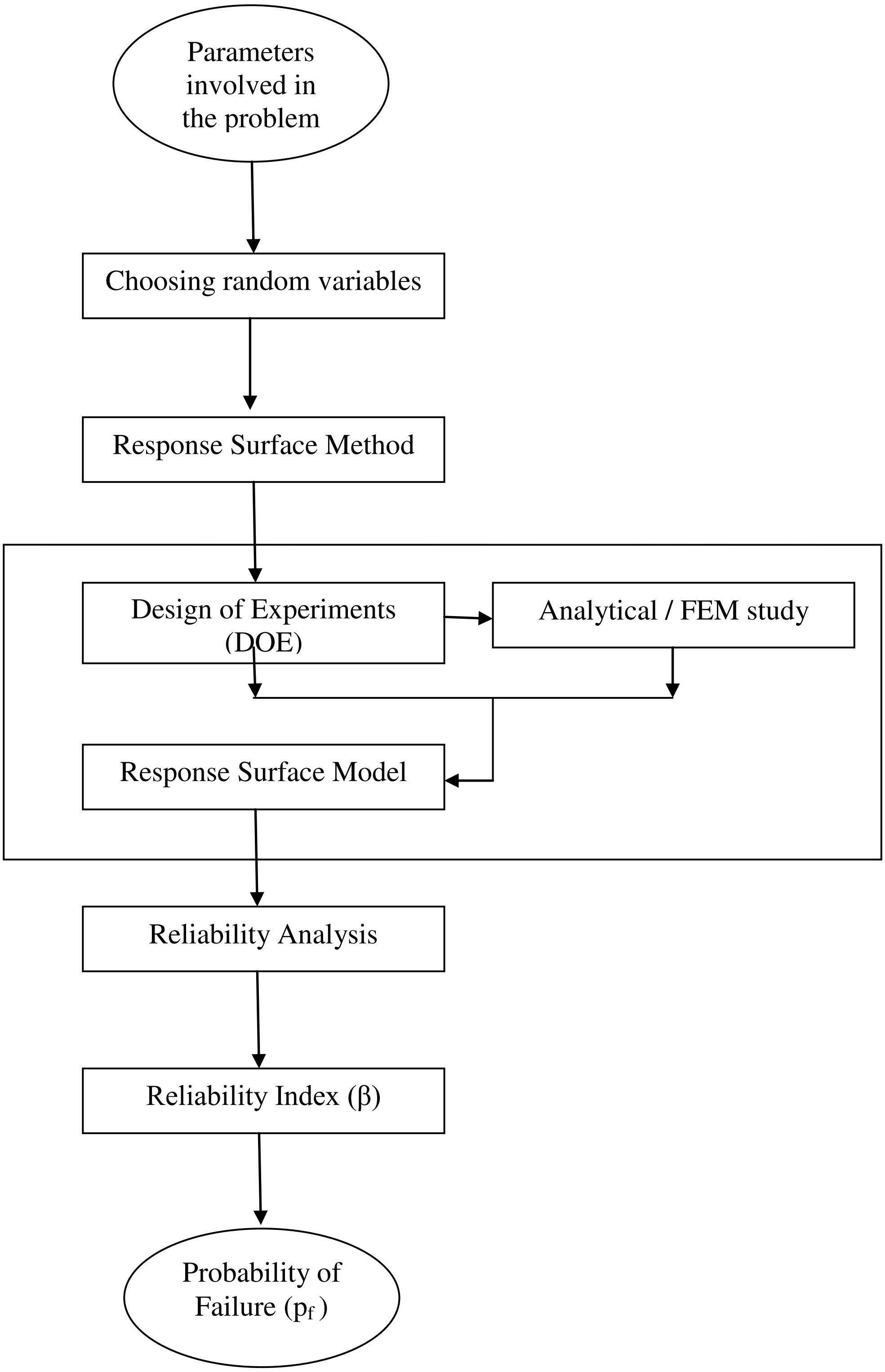


Fig: 2.1 Flow chart for the reliability analysis

2.2.1 Finite Element Method:

It allows modelling complicated non linear soil behaviour through constitutive model, various geometrics with different boundary conditions & interfaces. It can predict the stresses, deformations and pore pressures of a specified soil profile.

PLAXIS*:*

According to Burd (1999), the initiation of this Finite Element Program was held at Delft University of Technology Netherland by Pieter Vermeer in 1974. PLAXIS name was derived from PLasticity AXISymmetry, a computer program developed to solve the cone pentrometer problem by Pieter Vermeer and De borst. The commercial version of PLAXIS was released in 1987. Earlier version of PLAXIS was in DOS interface. PLAXIS V-7 was released in windows with automated mesh generation. Advanced soil models were also incorporated.

2D Finite Element Model in PLAXIS:

Axisymmetric and Plane strain conditions with two translation degrees of freedom along x-axis and y-axis are available in PLAXIS. However, axisymmetric models are applied only for circular structures with a uniform radial cross section. The loads are also assumed as circular symmetric around the central axis. In the plane strain model the displacements and strains in z-direction are assumed to be zero. But normal stresses in z-direction are considered.

Elements

PLAXIS 2D uses 2^{nd} order 6-node with 3 gauss point & 4^{th} order 15-node with 12 gauss point triangular elements to model the soil. 3 node & 5 node beam elements are available to model shell, retaining wall and other slender members. 3-node element has 2 pair of Gaussian stress points and 5-node element has 4 pair of Gaussian stress points. Bending moments and axial forces of these Plates are calculated from the stresses at the Gaussian stress points.

Constitutive models

Mohr-Coulomb Model

This is the simple model to represent the soil behaviour. This is an elastic perfectly plastic soil model. The model engages with five parameters: Cohesion (c), Angle of Friction (ϕ), Dilatancy angle (Ψ), Young's modulus (E) and Poisson's ratio (ν).

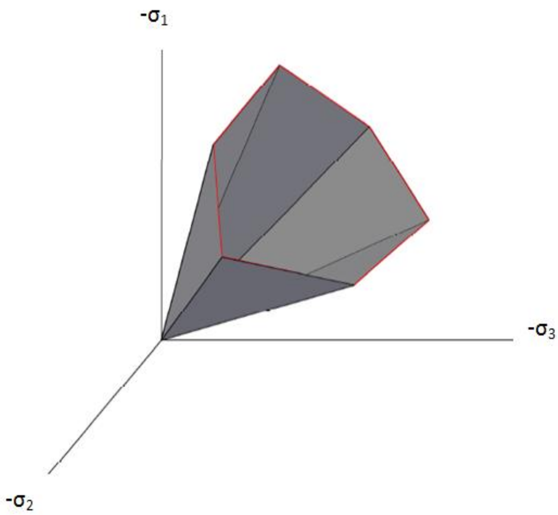


Fig: 2.2 Yielding surface at principle stress space (c=0) for M-C model

Linear Elastic Model

This model is based on Hooke's law. The model involves with two parameters: Young's modulus (E) and Poisson's ratio (ν). The model is used to simulate the structural elements in soil such as footing, Pile or Rock.

Mesh Properties

PLAXIS involves automatic mesh generation. PLAXIS produces unstructured mesh generation. The mesh generation is based on robust triangulation procedure. Global refinement (to increase the number of elements globally), Local refinement (to increase the number of elements in particular cluster), Line refinement (to increase the element numbers at the cluster boundaries), Point refinement (increasing the element coarseness around the point) are available to obtain the better results. The number of mesh elements considerably affects the results. So sensitivity study on mesh elements for each analysis should be investigated.

Model Simulation:

In the present study PLAXIS 9.0 is used to simulate the Settlement of footing, Slope stability and Retaining wall.

Strength reduction technique:

Sudden increase in the dimensionless displacement of soil mass and the algorithm unable to converge within the iteration limit can be considered as the failure of the slope. In PLAXIS arc length procedure provides the strength displacement curves. Arc length control composes the procedure strong since the procedure need not be associated with a non-converging iterative procedure. The method avoids the strength parameters decreases beyond critical value. When further reduction in the shear strength parameters is not possible, the construction has collapsed and at that point the safety factor is obtained. During the calculation phase, Arc length control should be activated. In the calculation of Factor of safety Young's modulus (E) of the soil has no influence and Poisson's ratio (ν) has negligible influence.

$$FOS = \frac{Available\ shear\ strength}{Shear\ strength\ at\ failure}$$

In PLAXIS this FOS is indicated in terms of sum of incremental multiplier (∑Msf). The displacement of the soil during failure has no practical meaning. When the Phi-c reduction method is applied to advanced soil models, it follows Mohr-Coulomb failure criteria.

In the slope Stability analysis, initial stresses are developed in the calculation stage according to gravity loading method because this always results in equilibrium stress state. But K0 procedure does not applicable for sloping ground. During the gravity loading self weight of the soil and generated pore pressure are activated. If the gravity loading is used, it causes displacements. So in the next calculation phase the displacement should be reset to zero. The initial stress condition in K_0 procedure is generated by Jaky's formula (Jaky 1944)

$$k0 = 1 - sin\varphi'$$

φ' = effective friction angle.

Convergence criteria:

Convergence study is conducted for mesh coarseness. By increasing the number of elements the variation in the output parameter is inspected. The number of mesh element is varied and inspected until the output parameter for the two successive meshing is negligible. If the system fails before it reaches the maximum number of step then the calculation is controlled by allowing tolerated error. The mesh size can be inspected if it does not converge in the calculation stage.

2.2.2 Response Surface Method

The response surface method (RSM) originated by Box and Wilson (1951) is a collection of statistical and mathematical techniques helpful for developing, improving and optimizing processes through empirical model building. Response surface methodology is the practice of adjusting predictor variables to move the response in a desired direction to an optimum by iteration. The method generally engages a combination of both computation and visualization. The use of quadratic response surface models makes the method simpler than standard nonlinear techniques for determining optimal designs. The Response surface method consists of design of experiments and response surface analysis. Response surface models are multivariate polynomial models. They typically arise in the design of experiments, where they are used to determine a set of design variables that optimize a response.

In a designed experiment, the data-generating process is manipulated to improve the quality of information and to eliminate unused data. An experiment is a series of tests, called runs, in which changes are made in the input variables in order to identify the causes for changes in the output response. A common goal of all experimental designs is to collect data as cheaply as possible while providing sufficient information to precisely estimate model parameters.

Response surface analysis aims to interpolate the available data in order to predict the correlation locally or globally between variables and objectives. If the data follows a flat surface, a first order model is usually sufficient.

A simple model of a response y in an experiment with two controlled factors x_1 and x_2 might look like this:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \varepsilon$$

The response can be characterized graphically, either in the three-dimensional space or as contour plots that aid visualize the shape of the response surface.

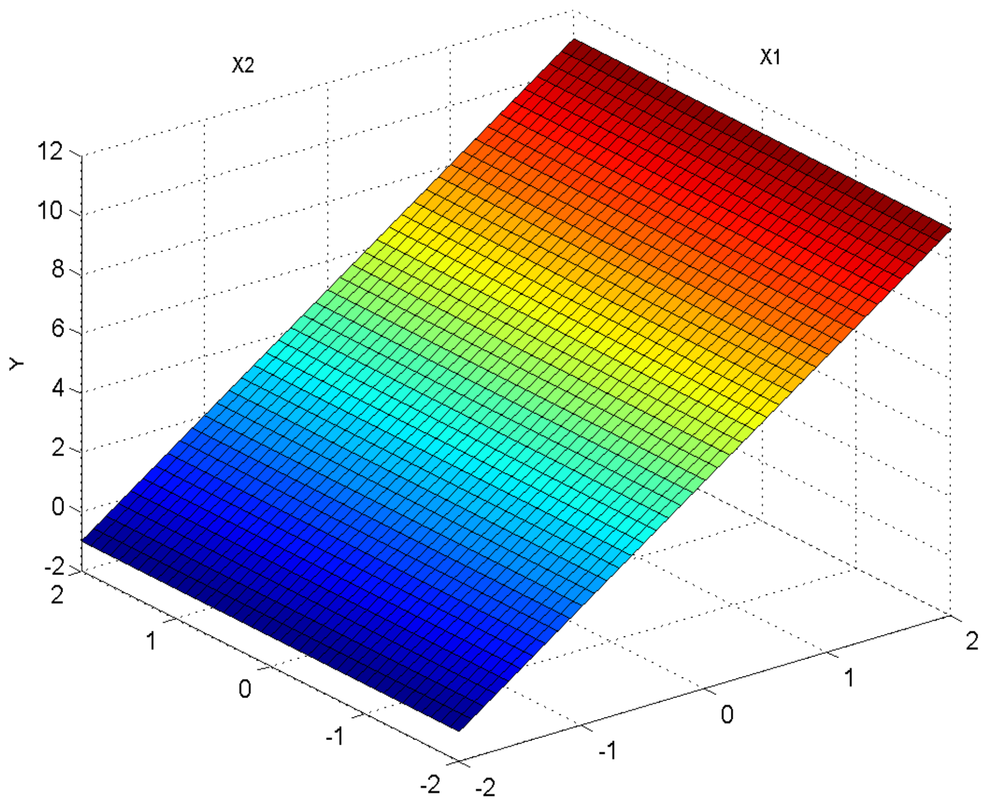


Fig 2.3 Linear Response surface

Here ε includes both experimental error and the effects of any uncontrolled factors in the experiment. The terms $\beta_1 x_1$ and $\beta_2 x_2$ are main effects and the term $\beta_{12} x_1 x_2$ is a two-way interaction effect. A designed experiment would systematically manipulate x_1 and x_2 while measuring y, with the objective of accurately estimatingβ_0,β_1,β_2, and β_{12}.

If there is curvature in the data, a first order model would show a significant lack of fit. A higher order model must be used to "mold" to the curvature. Polynomial models are generalized to any number of predictor variables x_i (i = 1, N) as follows:

$$y = \beta_0 + \sum_{j=1}^{k} \beta_j x_j + \sum_{j=1}^{k} \beta_{jj} x_j^2 + \sum_{i<} \, {}_j\sum_{=2}^{k} \beta_{ij} x_i x_j + \varepsilon$$

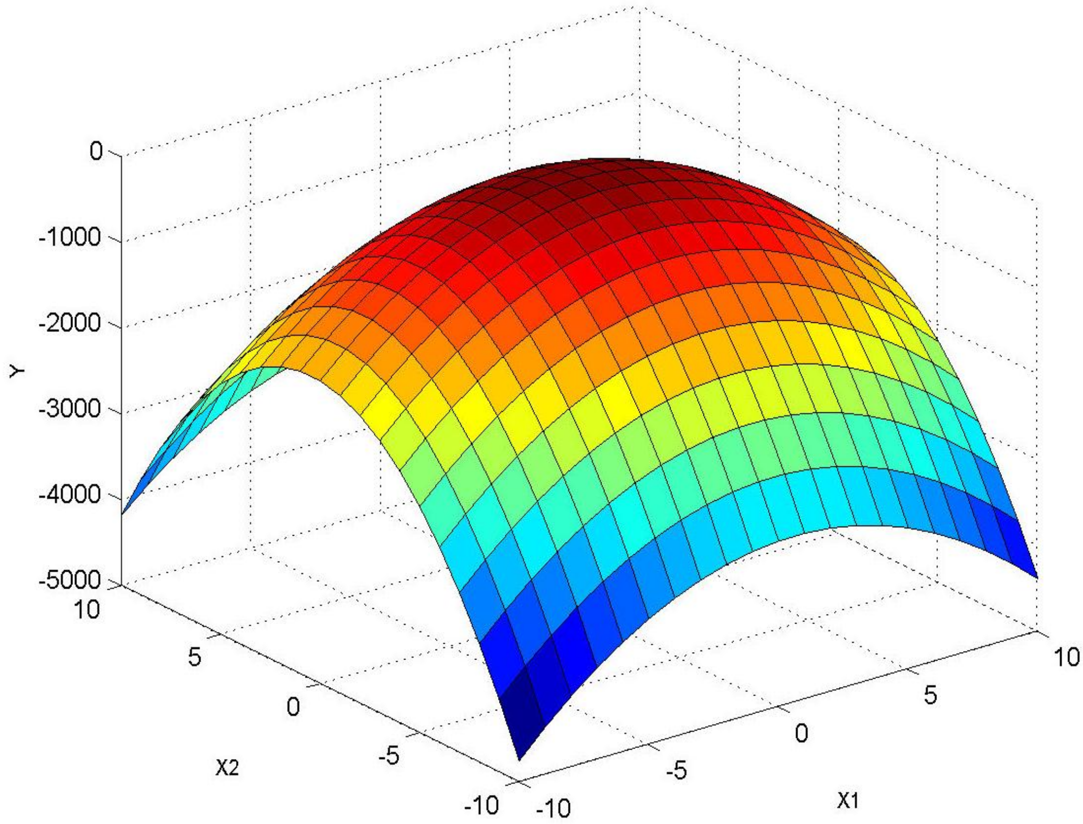


Fig 2.4 Non-linear response surface

Design of Experiments: (DOE)

Factorial Designs

A factorial experiment is an experimental tactic in which design variables are varied together, instead of one at a time. In experiments, factorial designs are used to investigate the joint effects of the factors on a response variable. The important special case of the factorial design is two level factors in which each of the k factors of interest has only two levels. In this, each design has 2^k experimental trials. These designs are known as 2^k factorial designs. The 2^k design is the basic building block. So this is used to create other response surface designs. A 2^k design is useful at the start of a response surface study. Screening experiments should be performed to identify the important system variables. This design is also used to fit the first order response surface model.

Two-level full factorial design:

2^2 Factorial design:

The simplest design in 2^k series is with two factors x_1 and x_2 and this run in two levels.

Matlab code for design of experiments:

dFF2 = ff2n(n)

dFF2 is R-by-C, where R is the number of treatments in the full-factorial design. Each row of dFF2 corresponds to a single treatment. Each column contains the settings for a single factor, with values of 0 and 1 for the two levels.

If the number of parameters involved in the design is 3, then the design can be generated in Matlab as follows. These binary set don't have any meaning and simply considered as design set.

```
>> dFF2 = ff2n(3)
dFF2 =
    0    0    0

    0    0    1

    0    1    0

    0    1    1

    1    0    0

    1    0    1

    1    1    0

    1    1    1
```

In this experimental design eight set of data has generated for 3 input parameter. 0 and 1 are then estimated as $\mu + 1.65\sigma$ and$\mu - 1.65\sigma$. μ is the mean of the variable. σ is standard deviation of the corresponding variable.

σ =μ * cov

Cov is the coefficient of variation of the particular parameter of the soil. The decoded design sets (x_1, x_2, *and* $x3$) are used to conduct experiments and output response (y_1) is obtained. Using this eight set of input-output parameters linear or nonlinear regression model is developed using MS Excel.

2.2.3 Reliability Analysis

Reliability:

Reliability of a geotechnical structure is its capacity to fulfill its design purposes for specific time. It is the probability that the structure will not attain the specified limit state during a specified time.

Methods of reliability:

1. First Order Reliability Method (FORM)
2. Second Order Reliability Method (SORM)
3. Monte Carlo Sampling (MCS)
4. Numerical Integration (NI)
5. Increased Variance Sampling (IVS)

The above Reliability methods can be classified into four groups:

Level-1 methods:

Load & Resistance factor design (LRFD)

Level-2 methods:

These methods are based on analytical approximation to solve the complicated integral

1. First order reliability methods (FORM)
2. Second order reliability methods (SORM)

Level-3 methods:

These are advanced methods known as full distribution approach. These methods cover the complete probability analysis. This entails integration of the multidimensional joint PDF of the random variable over the safety domain. It is extremely difficult to calculate the multidimensional integration. Directional Adaptive Response Surface Sampling (DARS), Directional Sampling (DS) are level-3 methods. These methods can be used to validate the applicability of FORM. These are effective if the system is non-linear and include system effects.

Level-4 methods:

These methods are suitable for most sensible structures such as foundations for power plant, Highway bridges. Economics, maintenance, repair are also included in the sources of uncertainty.

Terminology

Mean

It is average or expected value of data set. It measures the central tendency of data. It is known as first central moment.

Variance

It is the measure of spread in the data about the mean or average of the sample. It is known as second central moment.

Coefficient of Variation: (CoV)

It is the measure of dispersion of data. If the CoV is higher than dispersion will be higher about its mean.

Covariance:

Covariance indicates the degree of linear relationship between two random variables (x, y).

$$Cov(x, y) = E[(X - \mu_X)(Y - \mu_Y)] = E[XY - \mu_X \mu_Y] = E(XY) - E(X)E(Y)$$

Correlation coefficient: ρ_{xy}

It is a non dimensional parameter. It is obtained by dividing the covariance of two random variables Cov(X, Y) with the product of standard deviation of individual variables (σ_x , σ_y)

$$\rho_{xy} = \frac{Cov(x, y)}{\sigma_x \sigma_y}$$

The correlation coefficient varies between -1 to +1. If the ρ_{xy} is high then the two random variables have high correlation. These are mostly linear dependent variable.

Continuous random variable:

The mathematical model satisfying the properties of Probability density function (PDF), Probability mass function (PMF), Cumulative distribution function (CDF) can be used to quantify the uncertainties in a random variable. Continuous random variable may follow normal distribution, lognormal distribution or beta distribution.

Properties of Normal distribution:

1. The parameter varies between $-\infty \; to + \infty$
2. It is perfectly symmetric about mean
3. Mean, median and mode values are same.

Normal distribution $\quad f_x(x) = \frac{1}{\sigma_x\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x}\right)^2\right], -\infty \leq x \leq +\infty$

Reliability can be taken as the probability of endurance and is equal to one minus the probability of failure $(1 - p_f)$. Let the resistance of the structure be R and the load on the structure be Q. The structure is supposed to fail when R is less than Q and its probability of failure is specified as,

$$p_f = P[R \leq Q] = P[(R - Q) \leq 0]$$

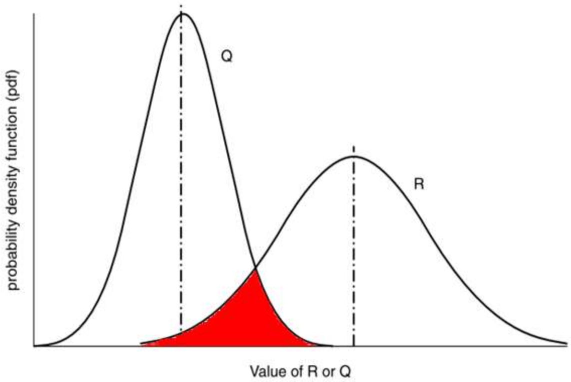


Fig 2.5 the overlapped area is the probability of failure of random variable R and Q

If $G_R(r)$ is the cumulative distribution function (CDF) of resistance R and $G_Q(q)$ is the probability density function (PDF) of Q, then the probability of failure is the shaded area of overlapping as shown in the Figure 2.5 & it can be mathematically denoted as

$$p_f = \int_{-\infty}^{\infty} G_R(q) G_Q(q) dq$$

Reliability, $$R_0 = 1 - \int_{-\infty}^{\infty} G_R(q) G_Q(q) dq$$

This is the basic case which involves two random variables R and Q only. But in many geotechnical problems these R and Q are the functions of several random variables.

Limit state function:

A mathematical model is derived to relate variables such as load and resistance, for the limit state of interest. Then the limit state equation is represented as

$$Z = (R\text{-}Q) = g(R,Q) = g(X_1,X_2,X_3, \ldots,X_n).$$

Z = margin of safety.

When this limit state function is equal to zero, it is called as the failure surface equation or the limit state equation.

i.e., $g(X_1,X_2,X_3, \ldots,X_n)=0$, this defines the safe and unsafe which may be linear or non linear.

This provides the basis for the quantitative measure of reliability,

i.e. $R_0=1-p_f$

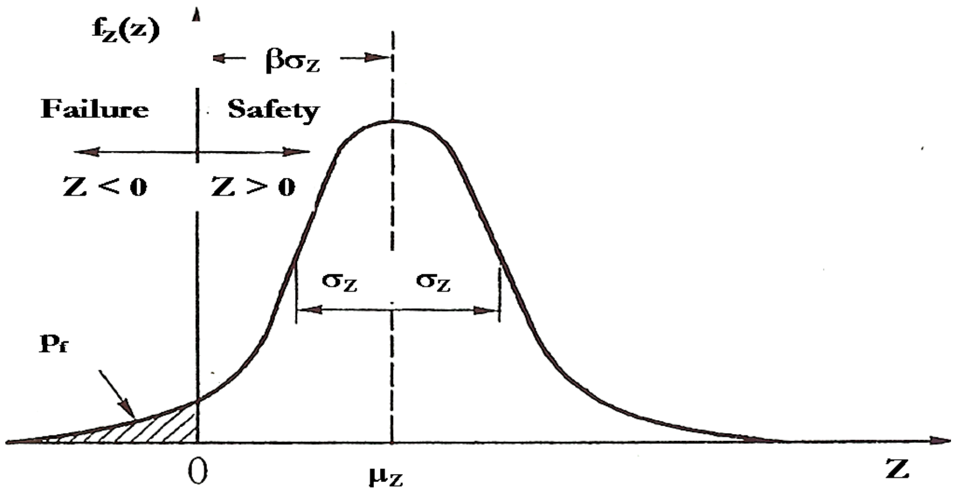


Fig 2.6 shows ddistribution of safety margin, $Z = R\text{-}Q$ *(Melchers 2002)*

If the failure function Z tracks *normal* distribution, the reliability of the system can be measured by reliability index, β. It was first pronounced by Cornell.

$$\beta = \frac{\mu_z}{\sigma_z}$$

and $p_f = \Phi(-\beta)$

μ_z and σ_z are the mean and the standard deviation of the random variable Z.

Φ is the CDF of standard normal variable.

First order reliability methods (FORM):

This method uses the first terms of a Taylor series expansion to estimate the mean value and variance of performance function. This method is often called as first order second moment method (FOSM) because the variance is a form of second moment. The methodology of the FOSM in detail is described in Baecher G.B and Christian J.T. (2003), but for sake of completeness it is described briefly as below.

FOSM:

Let the loading of an engineering system is Q and the available resistance is R. The values of both R and Q are uncertain. Then these variables have mean or expected values E(x), variances and covariances. The performance function of the system can be expressed in terms of margin of safety i.e. M=g(R,Q) and the failure surface equation can be written as

M = R-Q = 0

Then the probability of failure is

$$p_f = P[(R-Q) \leq 0]$$

when R and Q are normal variables and independent, the reliability index β as given by Cornell is

$$\beta = \frac{(\mu_R - \mu_Q)}{\sqrt{\sigma_R^2 - \sigma_Q^2}}$$

If M is a linear function of basic variables, say

$$M = b_0 + b_1X_1 + b_2X_2 + \cdots + b_nX_1$$

Then

$\mu_M = B_0 + \sum_{I=1}^{n} b_i \, \mu_i$ &

$$\sigma_M^2 = \sum_{i=1}^{n} b_i^2 \sigma_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \rho_{ij} \, b_i b_j \sigma_i \sigma_j$$

When the variables are uncorrelated equation reduces to,

$$\sigma_M^2 = \sum b_i^2 \sigma_i^2$$

β can be found from eqn.

If M is a non linear function of variables (X_1, X_2,............,X_n), then Taylor's series expansion is made about the mean value and the method is called mean value first order second moment method (MVFOSM).

$$\mu_M = g(\mu_1, \mu_2, \ldots., \mu_n)$$

$$\sigma_M^2 = \sum_{i=1}^{n} \sum_{j=i}^{n} (\frac{\partial g}{\partial X_i})$$

MVFOSM method has some short comings. In this method, due to limit state function the failure surface is linearized at the mean values. So there are unacceptable errors in approximating non linear failure failure surface. For the different limit state function which is mechanically equivalent for the particular problem, it shows variation in the results. this variance occurs due to the linear expansions which are taken about the mean value.

Advanced FOSM method: (*Hasofer-Lind reliability method):*

Hasofer and Lind (1974) proposed the development on the FOSM method based on a geometric analysis. It is used to find the first order approximation of the probability of failure. Hasofer-lind reliability index is only applicable for normal variables. So it is compulsory to convert the non normal variable into alike normal variables. The equivalent normal μ & σ of all the non normal random variables at the design point should be calculated approximately. It uses the transformed or reduced coordinate system instead of original coordinate to indicate the reliability index. The reliability index is the measure of the distance in dimensionless space between the peak of the multivariate distribution of the input variables and a limit state function defining the failure surface. Nowadays the Hasofer – Lind method is known as FORM. This method overcomes the lack of invariance of the problem.

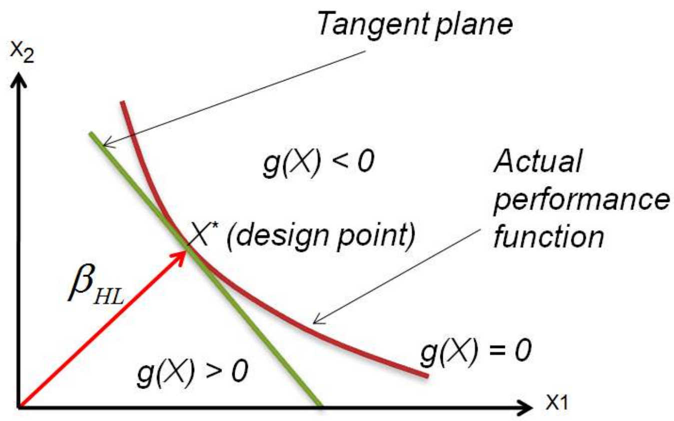


Fig: 2.7 Hasofer- Lind reliability index for nonlinear performance function

In this method the basic variables are normal and uncorrelated. The basic variables are first transformed to standard normal variates with zero mean and unit standard deviation.

$$x_i = \frac{X_i - \mu_i}{\sigma_i} \quad , i=1,2,.....,n$$

The limit state equation in reduced space coordinates

$$g_1(x_1, x_2, ..., x_n) = 0$$

The x* on g(x) =0 is referred as design point.

X* is known as most probable point of failure. So the reliability index is the minimum distance between the origin and X*. For the non linear limit state function, this minimum distance computation acts as constrained optimization problem.

$$\beta_{HL} = \min_{G(z^*)=0} \sqrt{(X')^t(X')}$$

$$p_f = \varphi(-\beta_{HL})$$

Fig shows the relationship between reliability index (β) and probability of failure of the system (p_f).

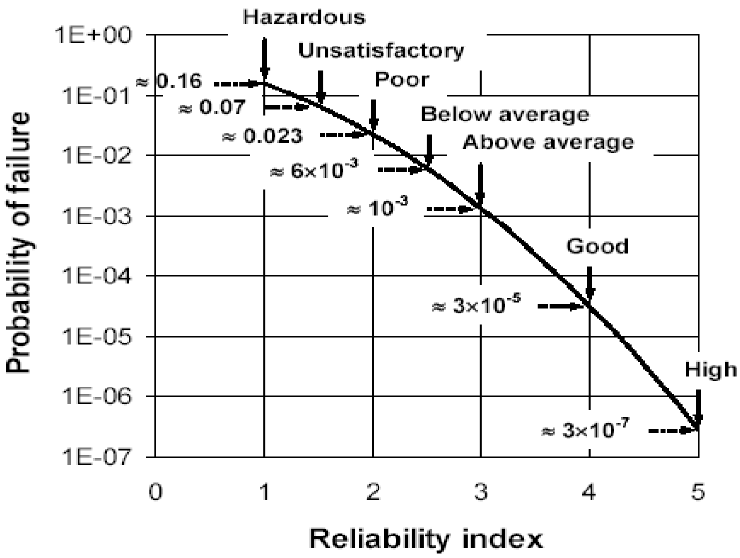


Fig 2.8 Relationship between reliability index (β) and probability of failure (p_f)

USACE (1997)

CHAPTER-3

RELIABILITY ANALYSIS OF SLOPE

3.1 DETERMINISTIC ANALYSIS OF SLOPE

Slopes involving compacted soils include highway and railway embankments, earth dams, and levees. In general, embankment slopes are designed using shear strength parameters obtained from tests on samples of the proposed material compacted to the design density. The stability analyses of embankments and fills do not usually involve the same difficulties and uncertainties as natural slopes and cuts because fill materials are preselected and processed. In this section the embankment slope has been selected from the literature, analysed by PLAXIS and compared with the literature.

3.1.1 Analysis of failed Oklahoma Birch Dam

This example is drawn from the slope analysis of Birch Dam in Oklahoma, as described by Nguyen (1985). Oklahoma birch dam is a zoned earthen Embankment. Dam consists of Low plasticity clay at the core and compacted sandy silt at the shell. The geometrical features of the dam are shown in Fig.3.1. Nguyen used simplex reflection technique and grid search method to analyze the dam. The PLAXIS model is shown in Fig. 3.2.The displacement of the soil is shown in Fig.3.3 in terms of deformed mesh. The critical slip surface is shown in Fig.3.4 as the shear shadings of incremental strains.

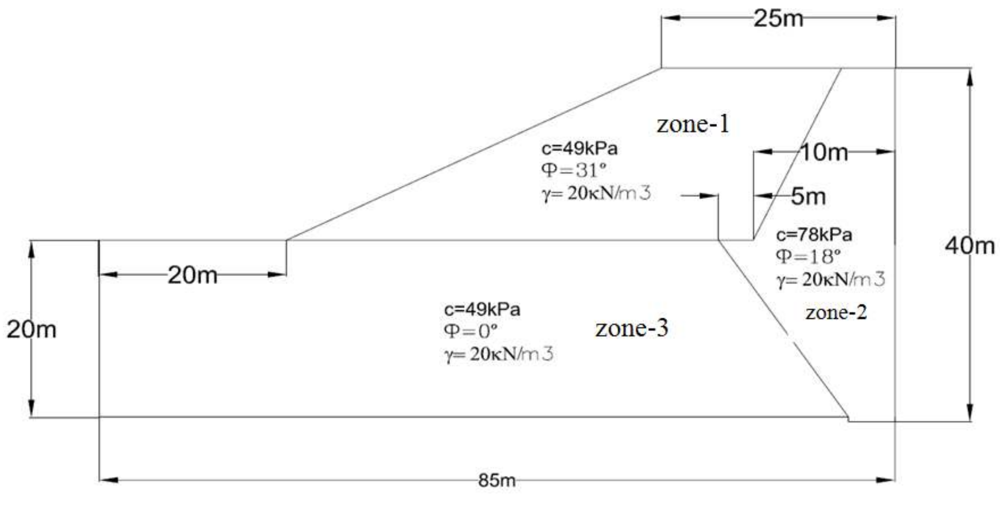


Fig: 3.1 Slope model of Birch Dam in Oklahoma

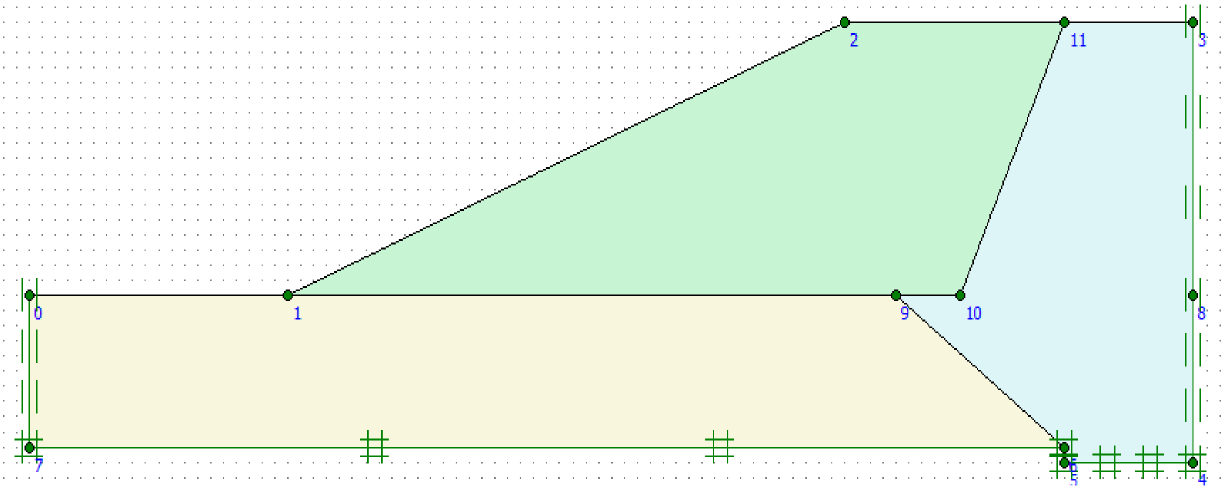


Fig: 3.2 PLAXIS Model

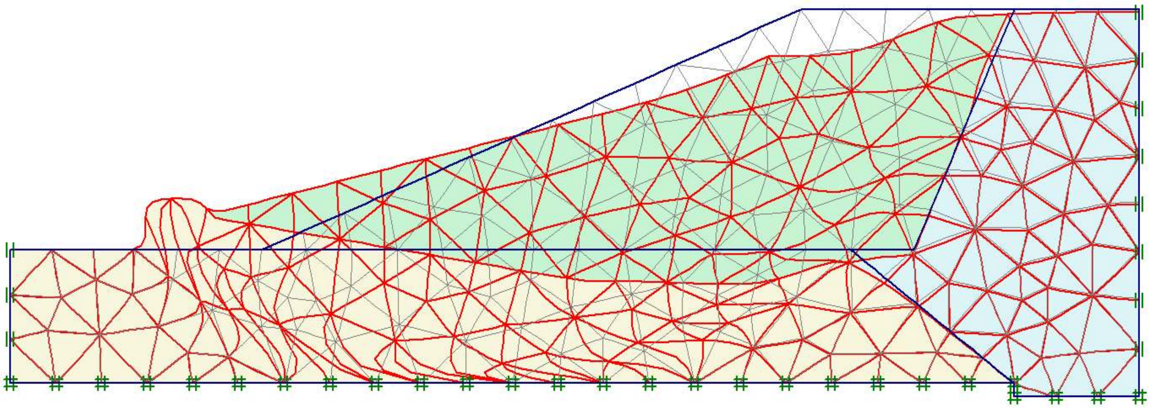


Fig: 3.3 deformed mesh

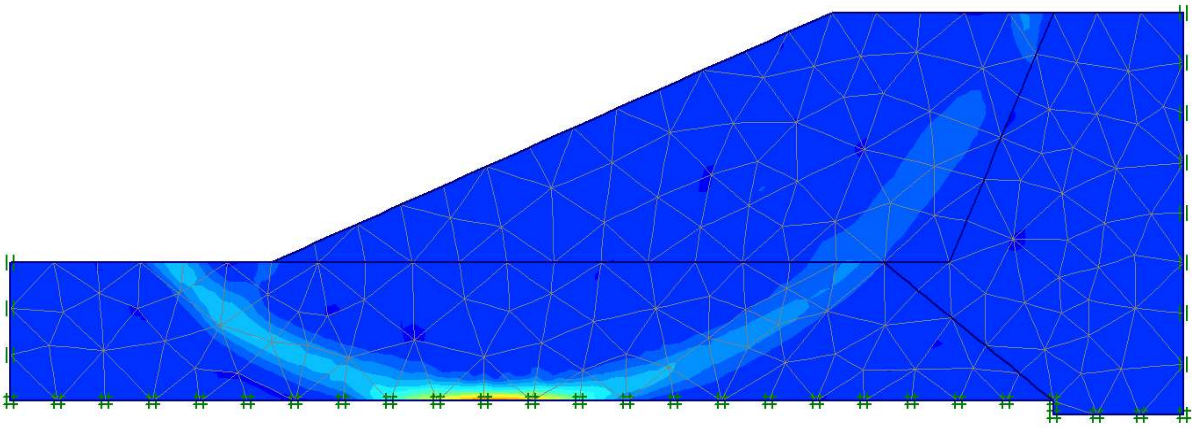


Fig: 3.4 Shear shading of incremental strains

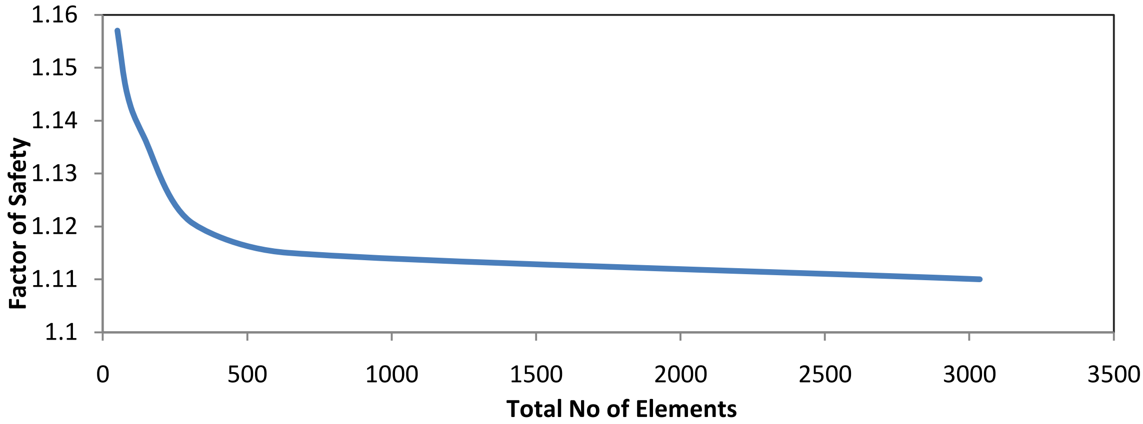


Fig. 3.5 Variation of factor of safety with respect to mesh refinements

Comparison of Results:

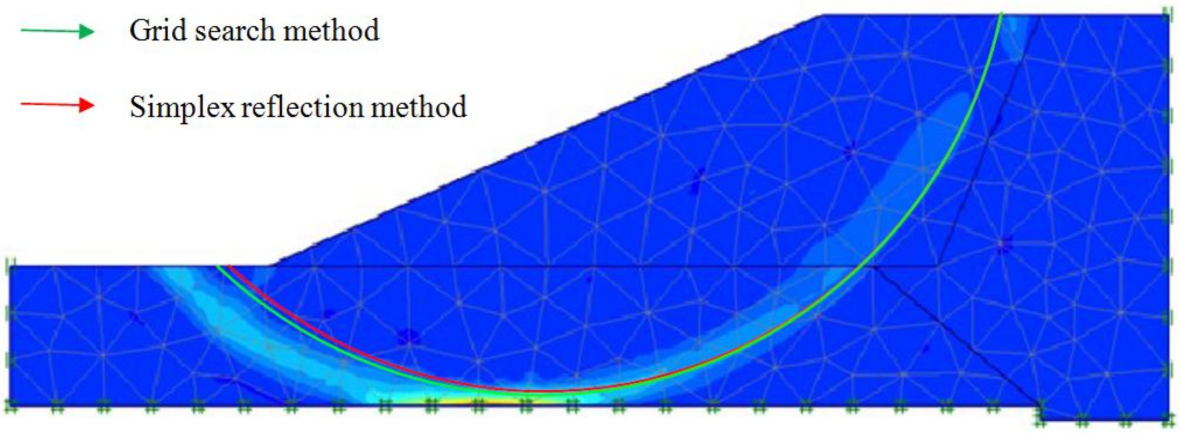


Fig: 3.6 Comparison of slip surfaces

The critical slip surfaces predicted by various methods and PLAXIS are plotted together in Fig.3.5. The red and green colored lines indicate the critical slip surface predicted using Simplex Reflection Technique and Grid Search Method respectively by Nguyen. The critical slip surface predicted by PLAXIS is indicated in terms of shear shadings as shown in figure.

Table: 3.1 Comparison of FOS for the Example problem as shown in Figure – 3.5

Method		Factor of Safety
Simplex Reflection Technique(Nguyen- 1985)		1.121
Grid Search Method (Nguyen)		1.117
Present Study (FEM)		1.116
Celestino and Duncan 1981 (Bishop)		1.121
De Natale 1991		1.093
Husein et al 2001	(Bishop – Random walking method)	1.028
	(Bishop – Random jumping method)	1.052
	(Bishop- Random walking - jumping)	1.027

3.1.2 Reliability analysis of Oklahoma birch dam

Uncertainty associated with spacial variability of soil is considered. The shear strength parameters of the soil are considered as random variables.

Table: 3.2 Mean and Coefficient of variation of the soil parameters

	Mean (μ)		
	ɣ (kN/m^3)	C (kPa)	ϕ °
zone-1	20	49	31
zone-2	20	78	18
zone-3	20	49	0
COV(%)	7	20	13

As explained in the Chapter – 1, regression analysis has performed based on least square error approach. The developed response surface model (RSM) is used to find out the reliability index (β).

The parameters are considered as uncorrelated normally distributed and correlated normally distributed. The lower limit (μ+1.65σ) and upper limit (μ-1.65σ) of the normal distribution of soil parameters are considered to quantify each point in design sets.

Standard deviation (σ) = μ * Cov

Full Factorial Design

The design has done using Matlab (MathWorks 2005)

```
>> dFF2 = ff2n(5)

dFF2 =

     0     0     0     0     0
     0     0     0     0     1
     0     0     0     1     0
     0     0     0     1     1
     0     0     1     0     0
     0     0     1     0     1
     0     0     1     1     0
     0     0     1     1     1
     0     1     0     0     0
     0     1     0     0     1
     0     1     0     1     0
     0     1     0     1     1
     0     1     1     0     0
     0     1     1     0     1
     0     1     1     1     0
     0     1     1     1     1
     1     0     0     0     0
```

1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Table: 3.3 FOS of Oklahoma slopes corresponding to thirty two sample points in RSM analysis using PLAXIS

	Zone- 1 C_{Z_1}	Zone-2 C_{Z_2}	Zone-3 C_{Z_3}	Zone-1 ϕ_{Z_1}	Zone-2 ϕ_{Z_2}	FOS
(0) =μ+1.65σ	65.17	103.74	65.17	37.65	21.86	
(1)= μ-1.65σ	32.83	52.26	32.83	24.35	14.14	
1	65.17	103.74	65.17	37.65	21.86	1.46
2	65.17	103.74	65.17	37.65	14.14	1.45
3	65.17	103.74	65.17	24.35	21.86	1.37
4	65.17	103.74	65.17	24.35	14.14	1.37
5	65.17	103.74	32.83	37.65	21.86	0.86
6	65.17	103.74	32.83	37.65	14.14	0.85
7	65.17	103.74	32.83	24.35	21.86	0.83
8	65.17	103.74	32.83	24.35	14.14	0.83
9	65.17	52.26	65.17	37.65	21.86	1.36
10	65.17	52.26	65.17	37.65	14.14	1.27
11	65.17	52.26	65.17	24.35	21.86	1.36
12	65.17	52.26	65.17	24.35	14.14	1.27
13	65.17	52.26	32.83	37.65	21.86	0.83
14	65.17	52.26	32.83	37.65	14.14	0.80
15	65.17	52.26	32.83	24.35	21.86	0.82
16	65.17	52.26	32.83	24.35	14.14	0.80
17	32.83	103.74	65.17	37.65	21.86	1.35
18	32.83	103.74	65.17	37.65	14.14	1.35
19	32.83	103.74	65.17	24.35	21.86	1.26
20	32.83	103.74	65.17	24.35	14.14	1.26
21	32.83	103.74	32.83	37.65	21.86	0.80
22	32.83	103.74	32.83	37.65	14.14	0.80
23	32.83	103.74	32.83	24.35	21.86	0.75
24	32.83	103.74	32.83	24.35	14.14	0.75
25	32.83	52.26	65.17	37.65	21.86	1.35
26	32.83	52.26	65.17	37.65	14.14	1.28
27	32.83	52.26	65.17	24.35	21.86	1.26
28	32.83	52.26	65.17	24.35	14.14	1.25
29	32.83	52.26	32.83	37.65	21.86	0.79
30	32.83	52.26	32.83	37.65	14.14	0.79
31	32.83	52.26	32.83	24.35	21.86	0.75
32	32.83	52.26	32.83	24.35	14.14	0.75

The preliminary analysis shows that for the mean value the slip surface does not pass through the Zone-2. But due to the consideration in the variability of the soil in the Zone-1 and Zone-3, slip surface may pass through the Zone-2. The statistical design results show the soil variability in the Zone-2 does not affect the result considerably. Sensitivity study may be conducted to eliminate the insignificant parameters to minimize the computational cost. Using the data mentioned in Table-3.3

Regression analysis is carried out to get a linear response surface model as

$$FOS = -0.0448 + \left(0.001991 * C_{Z_1}\right) + \left(0.00072 * C_{Z_2}\right) + \left(0.016396 * C_{Z_3}\right)$$
$$+ \left(0.003318 * \varphi_{Z_1}\right) + (0.002655 * \varphi_{Z_1})$$

$(R^2 = 0.988;\ R^2_{adj} = 0.986)$

Case: 1 (The Parameters c, φ are considered as uncorrelated normally distributed)

The correlation coefficients are found from the field data by conversion. In the first case, correlation coefficients are set to zero to be uncorrelated parameters for conservative deformation behavior, and a linear correlation model between the geotechnical parameters is assumed.

The Performance function can be defined as

$g(x) = fos - 1$

Minimize $\beta_{HL} = \underset{G(z^*)=0}{min} \sqrt{(X')^t (X')}$

$$X = \frac{x - \mu}{\sigma}$$

X' is a matrix which contains the value of X. Initially the value of x is assumed nearer to the mean value of the input parameter.

The minimum distance from the origin to the design point (reliability index) is obtained using MS-Excel solver.

$\beta = 0.417$

The probability of failure of the slope

$p_f = \varphi(-\beta_{HL})$

From the excel, =normdist(-0.417) gives $p_f = 0.338$.

Case: 2a (The Parameters c, φ are considered as correlated normally distributed)

The parameters c, φ are considered as linearly correlated. The correlation coefficient is -0.35.

The correlation is negative.

Correlation matrix: c

	c_1	c_2	c_3	φ_1	φ_2
c_1	1	0	0	-0.25	-0.25
c_2	0	1	0	-0.25	-0.25
c_3	0	0	1	-0.25	-0.25
φ_1	-0.25	-0.25	-0.25	1	0
φ_2	-0.25	-0.25	-0.25	0	1

Minimize $\beta = \underset{G(z^*)=0}{min}\sqrt{(X')^t c^{-1}(X')}$

$\beta = 0.436$

$p_f = 0.331.$

Case: 2b the correlation coefficient is -0.4.

Correlation matrix: c

	c1	c2	c3	φ1	φ2
c1	1	0	0	-0.4	-0.4
c2	0	1	0	-0.4	-0.4
c3	0	0	1	-0.4	-0.4
φ1	-0.4	-0.4	-0.4	1	0
φ2	-0.4	-0.4	-0.4	0	1

Minimize $\beta = \underset{G(z^*)=0}{min} \sqrt{(X')^t c^{-1} (X')}$

$\beta = 0.444$

$p_f = 0.328.$

The results of reliability analysis show the dam is under most hazardous region as per USACE chart. At this point it may be mentioned as per the deterministic approach, the FOS was more than 1.0, but in reality it is a failed slope. In present reliability analysis for both correlated and un-correlated cases, show probability of failure as most hazardous. This study highlights the importance of reliability analysis in slope stability.

CHAPTER-4

RELIABILITY ANALYSIS OF GEOGRID REINFORCED FOOTING

4.1 INTRODUCTION

The analysis of shallow foundation pertains to load bearing capacity and corresponding settlement. This has drawn attention of various researchers and the analysis has closely followed the development in computation and experimental geomechanics. In this study an attempt has been made to analyze the above problem using finite element method using elastic perfectly plastic Mohr Coulomb soil model for the layered soil. The settlement of footing has been analysed using field test data from standard penetration test (SPT), cone penetration test (CPT) and cross hole wave test (CHT) available in literature. The load corresponding to 25 mm and 150 mm settlement has been compared with the actual settlement. The elastic parameters of the soils are estimated based on the in-situ tests using different available correlations. It was observed that the prediction based on CHT is better compared to other in-situ tests. The predicted settlement corresponding to CHT is found to well within 10% of the observed value.

The designs of shallow foundation used in buildings, bridges are based on two criteria: bearing capacity criterion and settlement criterion. The bearing capacity of soil is determined based on laboratory and in-situ tests as per relevant standard. Various empirical methods have been developed to calculate the settlement of footing based on various field tests such as Standard penetration test (SPT), Cone penetration test (CPT), Cross-Hole wave tests (CHT), Pressure meter Tests (PMT). Indian standard IS 8009:1976 (part-1) illustrates about settlement of shallow foundation. These empirical methods consider only the elastic deformation of the soil. Due to the advances in numerical methods and constitutive soil models plastic strains can also be considered for the settlement criteria.

In 1994 Federal Highway Administration (FHWA) conducted a prediction symposium to examine various design methodologies for the settlement of spread footings based upon field tests (Briaud and Gibbens 1994). The participants were asked to predict the load corresponding to 25 mm and 150 mm settlement of five spread square footing where numerous field tests were conducted. Several methods were used to predict the settlement of shallow footing for different field tests. It was observed that the predicted settlements are very much different from the observed settlement. The percentage variation was more than 25%.

With above in view, in the present study, 3mx3m (North footing) and 1.5mx1.5m (West footing) have been analyzed using finite element method. The finite element package PLAXIS has been used for the simulation. Different correlation for soil modulus has been used to predict the settlement of footing. The finite element results based on field tests; SPT, CPT and CHT have been compared with the actual settlement of footing.

4.2 SOIL PROFILE

Figure 1 shows the soil profile at the site. SPT-N values and CPT-q_c values over the depth have listed in the Table 1 and Table 2. From the wave energy measurements it was assumed that the blow counts reported were measured with an energy efficiency averaging 53±5%. The in situ relative density was estimated from SPT blow counts and CPT values as being 55%. The angle of internal friction is 34.2° at 0.6 m and 36.4° at 3.0 m. The water table was reported at the level of 4.9m.

Table. 4.1 SPT-N and CPT-q_c for North footing.

Depth(m)	Blow count (N)	q_c (kPa)
0.3	13	2875
0.9	18	5500
1.5	25	11000
2.1	16	7500
2.7	17	4900
3.6	18	8000

5.1	26	12454
6.0	21	2874
7.5	17	5650
9.0	10	4790
10.5	44	23950
12.0	97	20000
13.5	45	15000
15	38	9101

Table. 4.2 SPT-N and CPT-q_c for West footing.

Depth(m)	Blow count (N)	q_c (kPa)
0.3	11	3353
0.9	15	4598
1.5	17	4790
2.1	16	4790
2.7	15	4790
3.6	14	6035
5.1	11	9580
6.0	15	9580
7.5	14	9580
9.0	20	9580
10.5	60	9580
12.0	70	7185
13.5	53	7185
15	60	7185

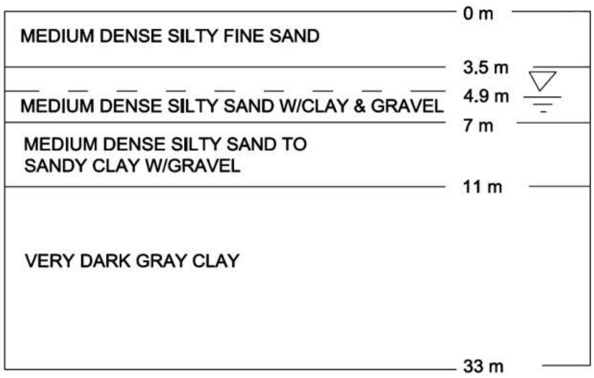


Fig. 4.1 Soil layer at the FHWA testing site.

4.3 Numerical Simulation

The FEM analysis of footing has been modelled in FEM based package PLAXIS 2D. The square strip footing has modelled in terms of equivalent radius of circular footing. The axi-symmetric circular footing has modelled as linear elastic element. The layered soil has modelled for elastic perfectly plastic Mohr coulomb failure criteria with 15-noded triangular element. Sensitivity study was carried out for the convergence of all models. The elastic modulus of soil has been considered based on the available correlation with the in-situ test results e.g. SPT, CPT, CHT and PMT.

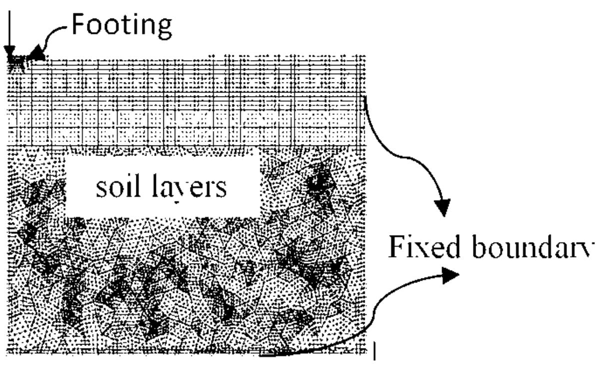


Fig. 4.2 Axi-symmetric model footing in FEM

Sensitivity study was conducted for all PLAXIS models by varying the number of mesh elements. To avoid the collapse of soil body and to obtain the settlement of footing up to 150 mm calculation was controlled manually by allowing tolerated error up to 0.4. This affects the load settlement curve path beyond 30 mm settlement in all the cases. Figure 3 shows the typical sensitivity study.

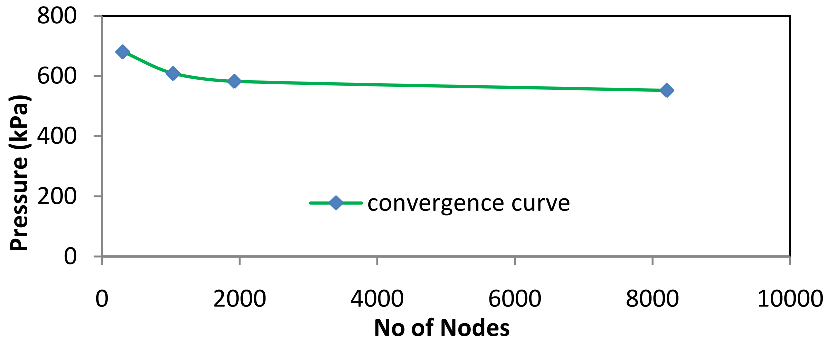


Fig. 4.3 Sensitivity plot corresponding to 25mm.

4.4 Settlement analysis using SPT

The SPT values have corrected for overburden pressure based on Mayerhof (1976) and presented in Table 1 and Table 2.

Normalised $N = N_{measured} \times 0.77\log(1920/\sigma_v')$ (1)

According to the type of soil profile shown in (Fig 1), Different correlations for the soil modulus considered for the present PLAXIS modelling is presented as follows.

Over consolidated sand $E_s = 40000 + 1050N$ (2)

Clayey sand $E_s = 320\,(N+15)$ (3)

Sandy silt $E_s = 300\,(N+6)$ (4)

$$E_s\,(MPa) = 8 * \left(\frac{N_{60}}{10}\right) \qquad (5)$$

The Eq. 2-4 have been considered as per Bowles (2002) and Eq. 5 as per Das (1999).

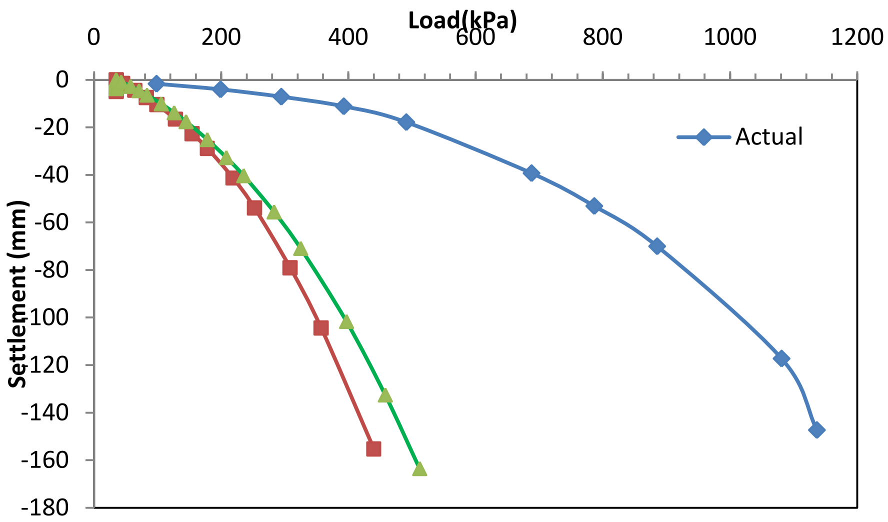


Fig. 4.4 Load settlement curve for 3m x 3m footing using SPT data

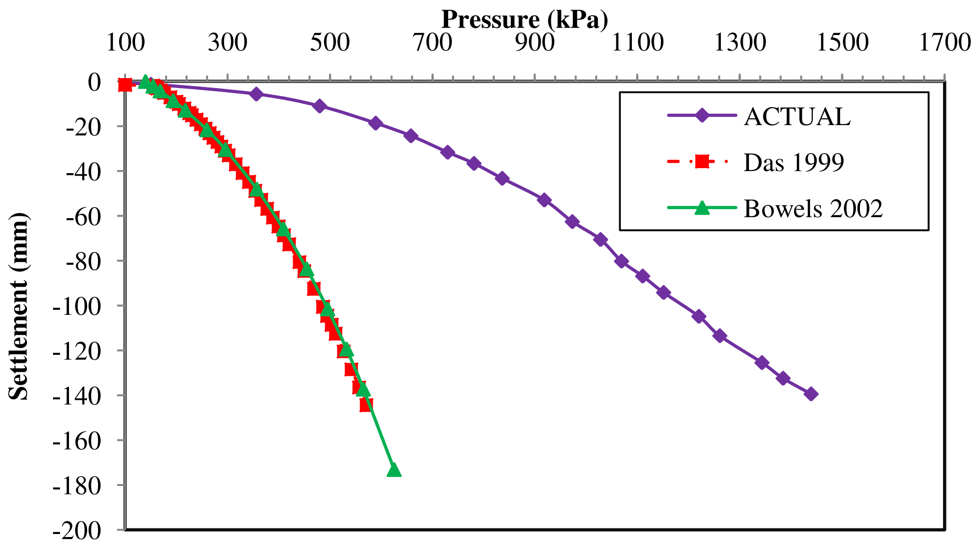


Fig. 4.5 Load settlement curve for 1.5m x 1.5m Footing using SPT data

Using the above correlations the load-displacement curves for the 3m x 3m North footing and 1.5m x 1.5m west footing are presented in Figures 4 and 5 receptively. It can be seen that for the North footing the predicted settlement are more than the observed values. The predicted settlement values as per Eq.5 are found to be better than as predicted based on Eq. 2-4. However, both the predicted values are overestimating the observed values. The predicted settlement values for the 1.5m x 1.5 west footing are found to almost matching irrespective of the soil modulus correlation used for the SPT value.

4.5 Settlement using CPT

Cone tip resistance (q_c) from cone penetration test has been used to obtain the elastic modulus (E). The tip resistance value has been tabulated for 3m x 3m and 1.5m x 1.5m footings in Table 1 and Table 2 respectively.

The correlation for elastic modulus as per Vesic (1970) is considered as follows

$$E = 2(1 + D_r^2)q_c \qquad (6)$$

D_r = Relative density (%)

q_c = Tip resistance (kPa)

Similarly the correlations as per Bogdanovi (1973) are present in Eq. 7-10.

$E = 1.5q_c$ Sand and sandy gravel (7)

$E = 1.5 to 1.8q_c$ Silty saturated sand (8)

$E = 1.8 to 2.5q_c$ Clayey silt with Silty sand (9)

$E = 2.5 to 3q_c$ Silty saturated sand with silt (10)

Vesic (1970) and Bogdanovi (1973) correlations were used to find out soil modulus from cone tip resistance. Based on the results of present analysis the load-displacement curves for the North footing is presented in Figure 6. Similar to SPT correlations the predicted values are found to overestimate the settlement values. Figure 7 shows the load settlement curve for 1.5m x 1.5m (East) footing. The predicted values are found to very close irrespective of the correlations used and are very much on higher side compared to the observed settlement.

4.6 Settlement from Cross- Hole Wave Test:

The tests were performed at the Texas site in accordance with ASTM D4428, using an impulse down-hole energy source (Briaud & Gibbens). The tests conducted between Cht-2 and Cht-1 (North-South direction) was used to find the shear modulus of the soil below North footing. The tests conducted between Cht-2 and Cht-5 (East-West direction) was used to analyse the settlement of West footing. The shear modulus values are listed in Table 3. Using the correlation Eq. 13 and 14 soil elastic modulus values has been obtained for different soil layers and used to analyse in PLAXIS. The load settlement curve for North footing and East footing was compared with actual settlement curve and are presented in Figures 8 and Figures 9 respectively.

$$V_s = \sqrt{\frac{G}{\rho}} \qquad (13)$$

$$G = \frac{E}{2(1+\nu)} \qquad (14)$$

V_s = Shear wave velocity of the soil (m/s);

G = shear modulus (kPa);

E=stiffness moduli(kPa);

ν =Poisson's ratio;

ρ (kN/m^3) = density of the soil

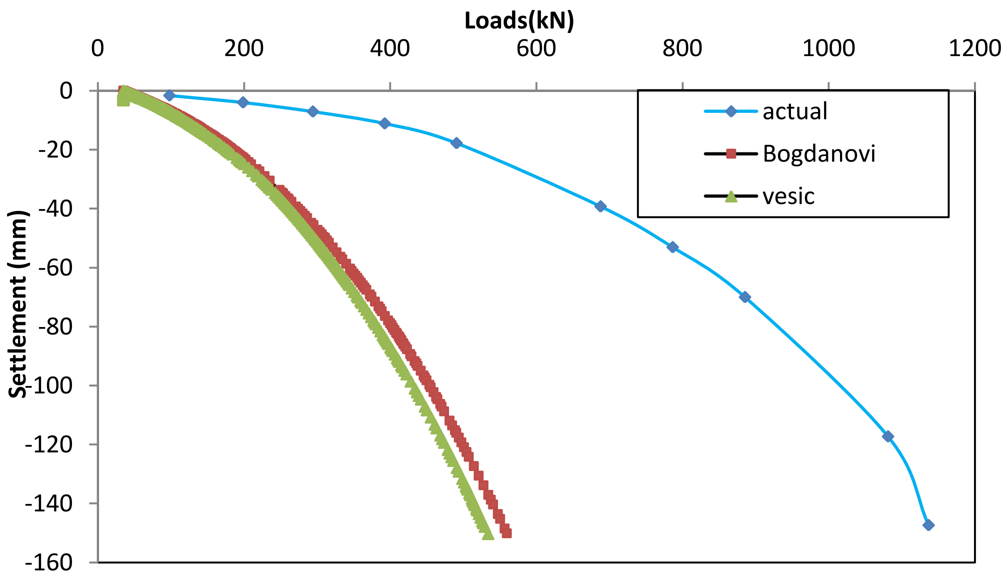


Fig. 4.6 Load-settlement curve for 3m x 3m footing using CPT

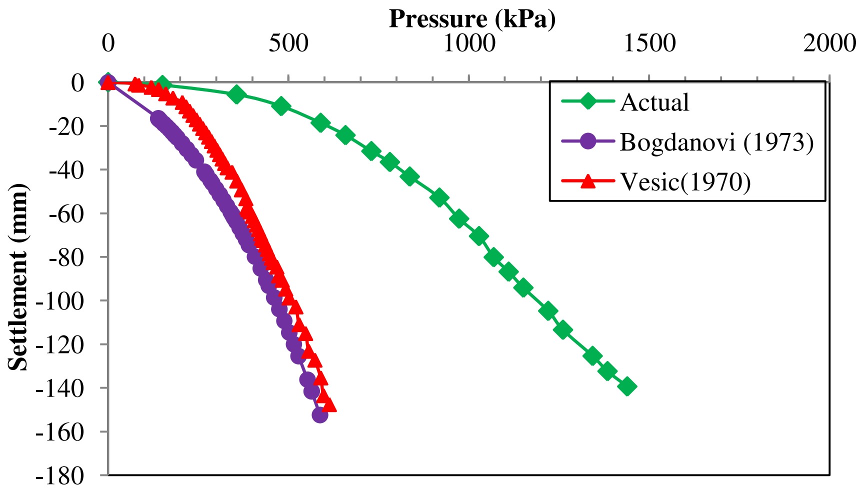


Fig. 4.7 Load-settlement curve for 1.5m x 1.5m Footing using CPT data

Table. 4.3 Shear modulus for North and East footing

Depth (m)	Shear modulus (MPa)	
	North footing (3m x 3m)	West footing (1.5m x 1.5m)
2	104	73
4	162	80
6	142	79
8	71	52
10	102	95

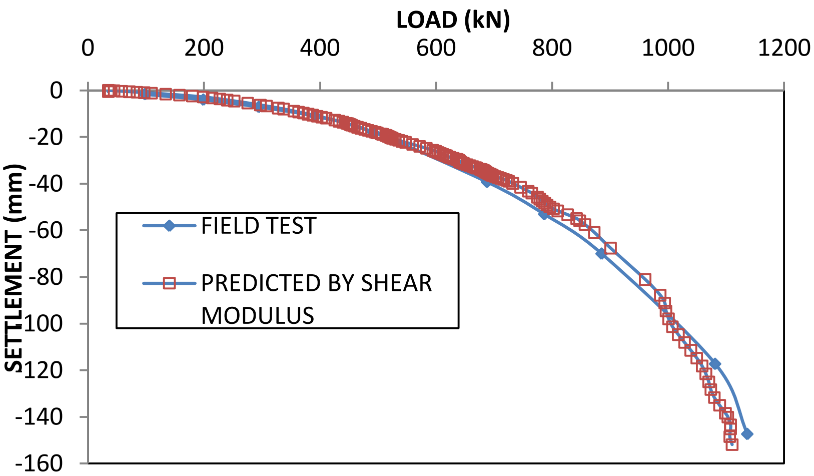


Fig. 4.8 Load-settlement curve for 3m x 3m footing using CHT data

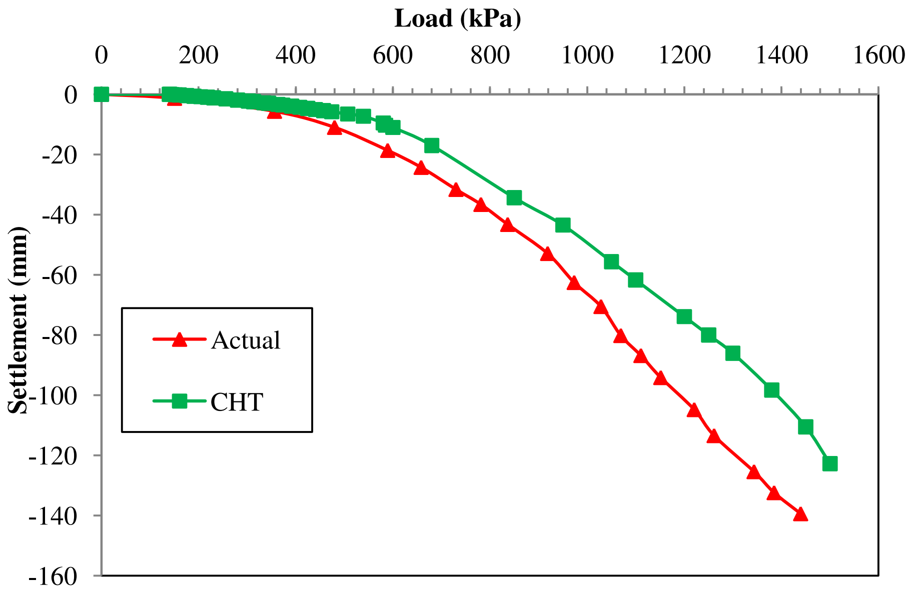


Fig. 4.9 Load-settlement curve for 1.5m x 1.5mm footing using CHT

It can be seen that compared to SPT and CPT data the load settlement curve based on CHT data is close to observed values and the error was within 10%. However, it is also important to note that at certain load point the predicted settlement is less than the observed settlement. This has implication terms of the reliability of the method. As it is well that reliability and accuracy are two different concepts and it needs judicious decision regarding the preference of reliability over accuracy and vice versa. This particular study shows the importance of reliability analysis. The reliability analysis is carried out on the settlement of the geogrid reinforced footing.

4.7 RELIABILITY ANALYSIS OF SETTLEMENT OF GEOGRID REINFORCED FOOTING

The settlement of a shallow foundation can be estimated by using a deterministic approach in combination with a probabilistic approach. In the deterministic approach, correlations based on field tests are used to find the settlement of the shallow foundation. In the serviceability limit state design, the footing should fulfil the settlement criteria. Response surface method is adopted to generate the limit state function for the settlement of shallow foundation resting on cohesionless soil for a range of expected variation in the parameters. Variability in the soil parameters & the interfacial strength between soil and geogrid is taken into account to generate response surface models. The problem has taken from Patra et al. (2005). In the deterministic analysis the FEM results are compared with experimental results presented in the literature. The response surface model has used to perform the reliability analysis. In the cohesionless soil, Unit eight (ɣ), Angle of internal friction (φ), modulus of elasticity (E), Poisson's ratio (ν) are the parameters which influence the settlement of footing. In the present study geogrid reinforced footing has examined.

Finite element model:

Footing:

The footing has modelled for plane strain conditions. Surface Strip footing has considered for the present study. The footing is modelled using linear elastic element. Material type is considered as non-porous. Concrete properties have given to represent the linear elastic element. The unit weight of concrete is considered as 24 kN/m^3. The Youngs modulus of concrete is obtained by $E = 5000\sqrt{f_{ck}}$. f_{ck}= Characteristic compressive strength of concrete. The poisson's ratio is 0.35. No interface between the base of the footing and soil is considered.

Soil:

The simple elastic-perfectly plastic Mohr Coulomb model is considered to represent the soil. Material is considered as drained.

Table: 4.4 Properties of the soil

Parameter	Mean value
Unit weight (kN/m^3)	19
Angle of internal friction (φº)	38
Cohesion (kPa)	0.01
Young's modulus (E)	45000
Poisson's ratio(ν)	0.2
R_{inter}	0.8

Geogrid:

It is a geosynthetic material. These are slender members with a normal stiffness but no bending stiffness. It carries tensile forces rather than compressive forces. The geogrid material type is considered as elastic. Axial stiffness is considered as $1*10^5$ kN/m. 5-node geogrid element is chosen automatically for 15 node soil element. Two geogrid layers is provided. The width of the geogrid is 5 times the width of the footing.

Interfacial strength:

The purpose of interface element is to simulate the relative friction between geogrid and soil. The roughness of the interaction is modelled by selecting an appropriate value for the strength reduction factor (R_{inter}) in the interface. The strength of interface element is equal to the strength of surrounding soil multiplied by the coefficient $R_{inter.}$ Interfacial strength between the soil and geogrid is modelled using Mohr Coulomb model. The same soil properties has assigned with reduced strength. The transfer of stresses between the reinforcement and soil is decided by this interface model. The factor R_{inter} indicates the scale of interaction between the soil and geogrid.

$$c_{inter} = R_{inter} C_{soil}$$

$$tan\varphi_{inter} = R_{inter} tan\varphi_{soil}$$

The interfacial strength between the soil and geogrid is based on both type of soil and geogrid. The aperture type of geogrid and friction of soil chiefly describe the interfacial strength. If there is no relative sliding between the soil and geogrid then $R_{inter} = 1$. If the deformation or sliding of geogrid is more than the soil body $R_{inter} < 1$. This interfacial strength can be found from the laboratory tests. Direct sliding or pull-out test may be conducted to find out R_{inter}.

Meshing:

Type of meshing and number of elements considerably affect the performance of footing. Converge study has been conducted for each footing model. Very fine element coarseness was used. Line refinement has done for geogrid elements. The close view of meshing along the soil- geogrid layer has been shown in fig.5.1

Table: 4.5 Properties of meshing

Total no of nodes	5221
Total number of elements	42987
Number of stress points	62652
Average element size	$401.23*10^{-3}$m

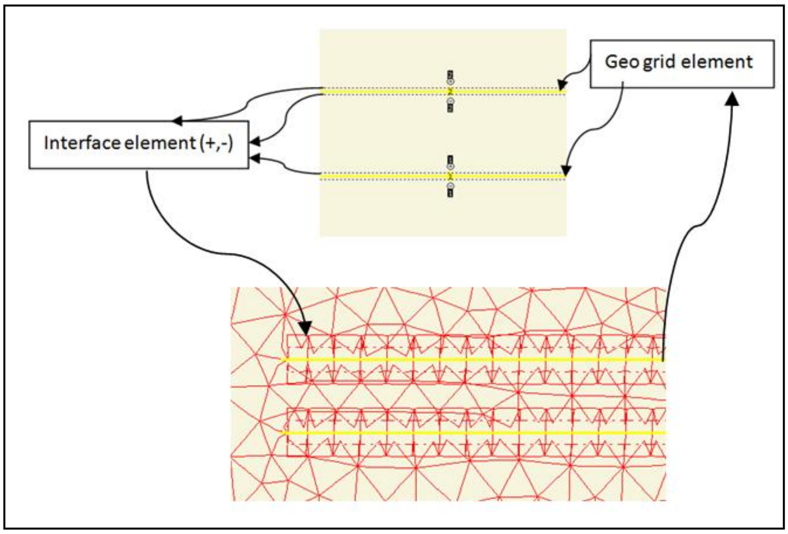


Fig: 4.10 Interfacial elements

Loading condition:

Static point load of 200 kN is applied on the centre of the footing without any eccentricity. Initial stresses are developed in the input stage by deactivating the footing. It is assumed that the self weight of the footing has added to the point load. Two stage calculations are performed. In the first stage geogrid layers and footing is activated. In the second stage point load is applied. The footing model is shown in Fig.4.2

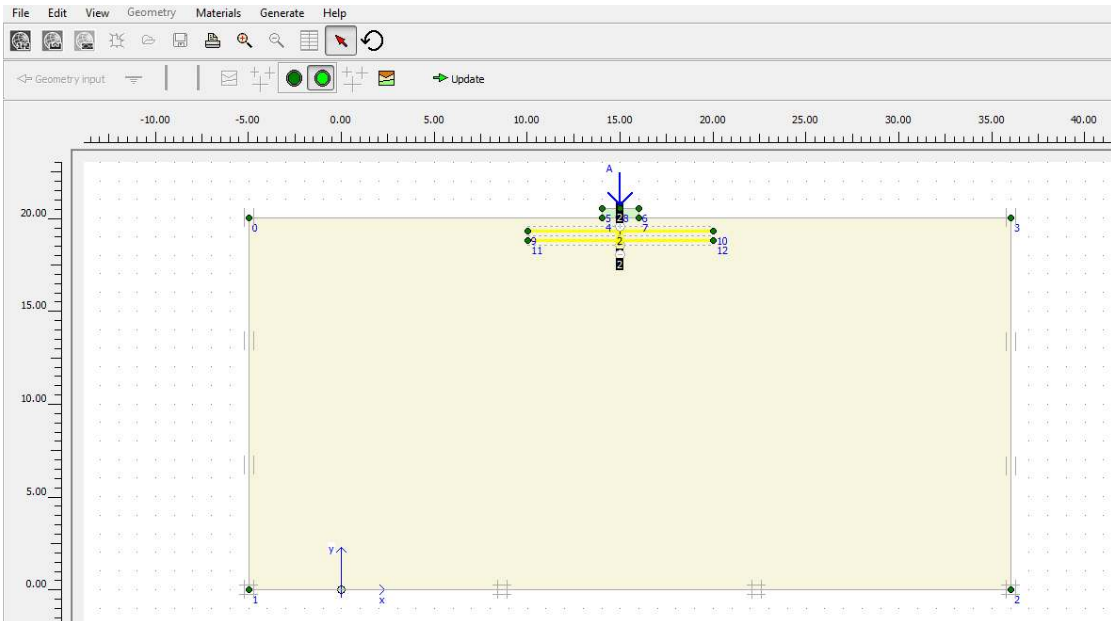


Fig: 4.11 Reinforced footing model in plaxis

4.9 DETERMINISTIC RESULTS:

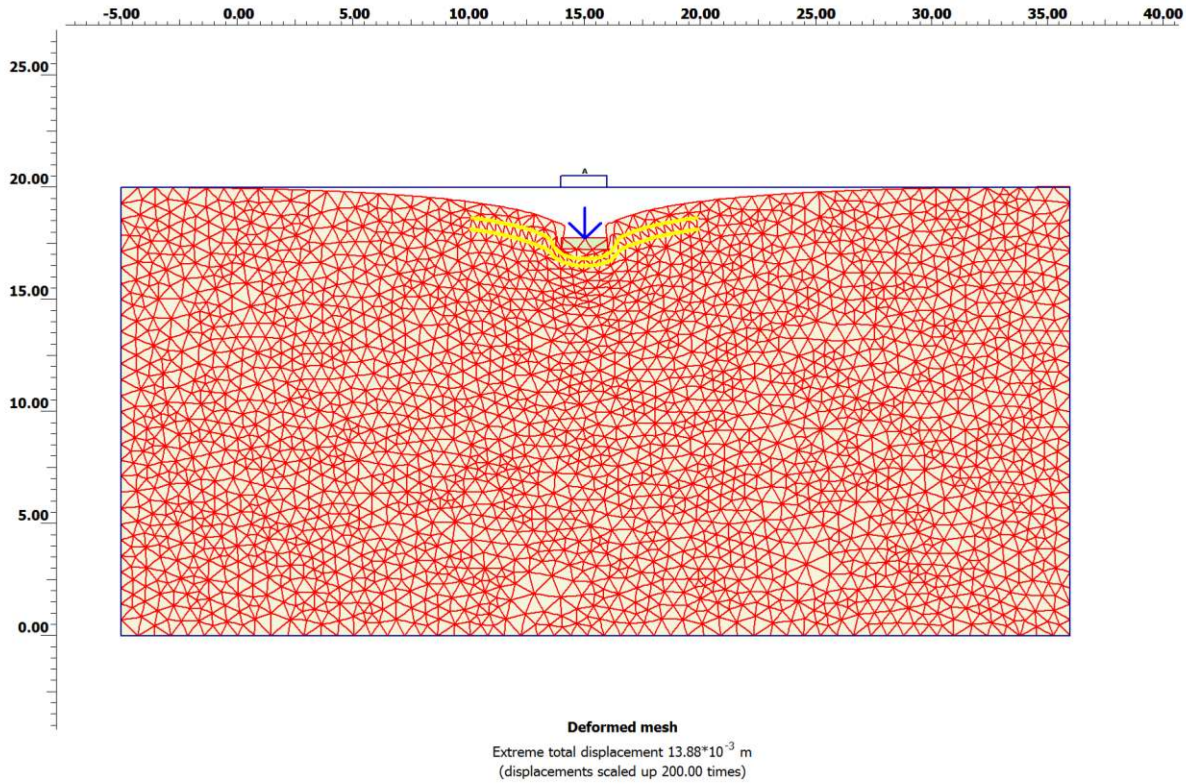


Fig. 4.12 Deformed mesh of geogrid reinforced footing

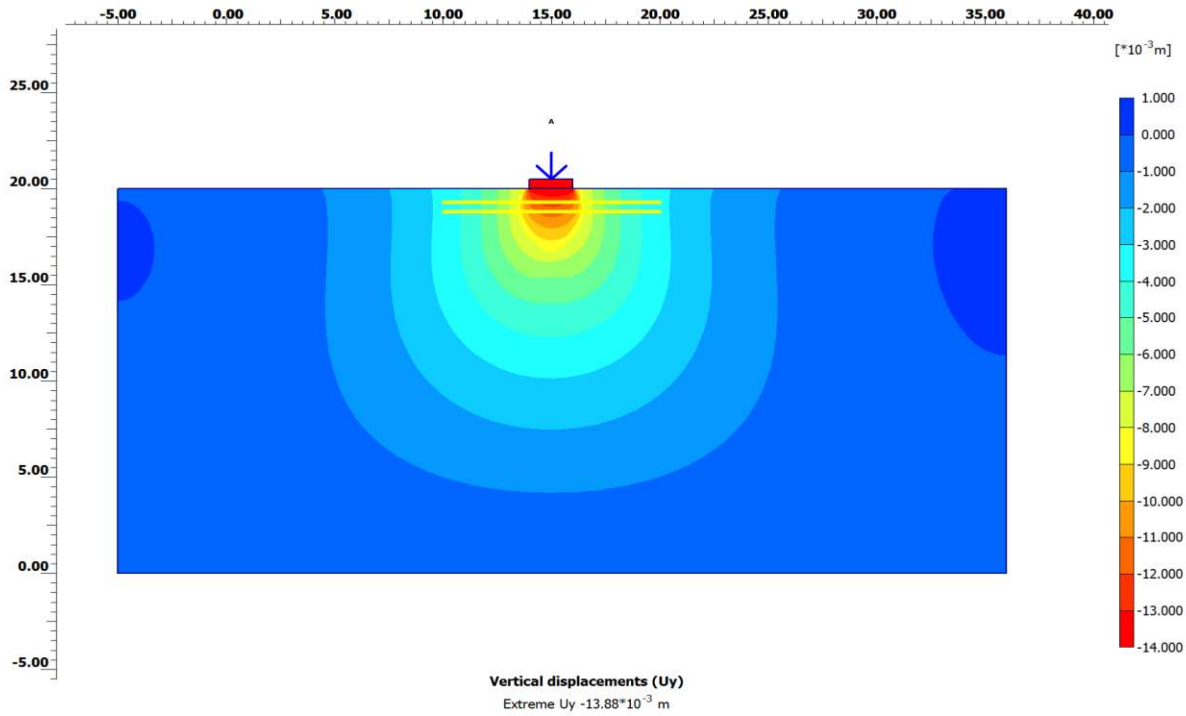


Fig. 4.13 Vertical displacement in terms of relative shadings

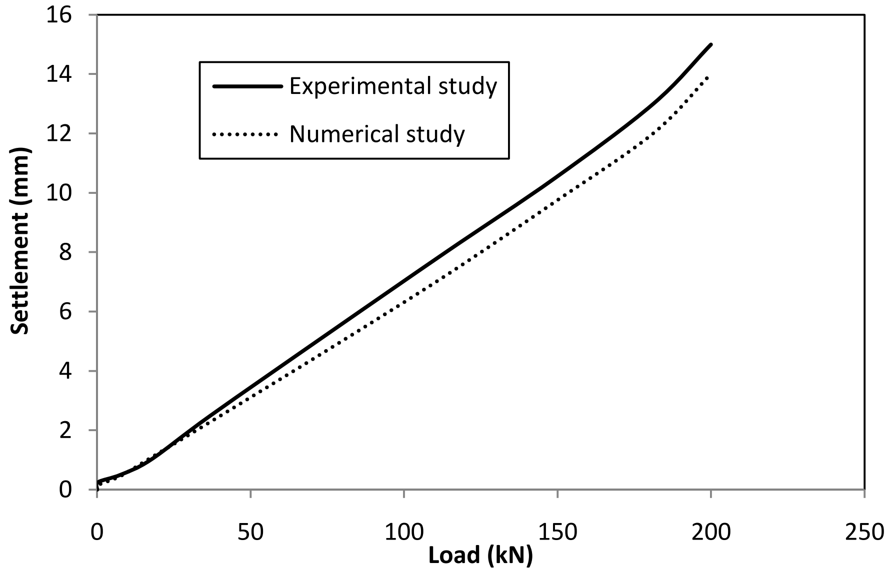


Fig. 4.14 Load settlement curve for geogrid reinforced footing

4.8 RELIABILITY ANALYSIS:

Random variables for reliability analysis:

Unit weight (ɣ), Angle of internal friction (ф), Young's modulus (E) and Interfacial strength coefficient between soil and geogrid (R_{inter}) are considered to develop response surface models.

Table: 4.6 Coefficient of variation:

	μ	Cov	Sd (σ)
ф°	41	0.13	5.33
E(kN/m^2)	30000	0.34	10200
ɣ (kN/m^3)	14.81	0.07	1.0367
R_{inter}	0.7	0.15	0.105

Design of Experiments:

Full factorial design model has been developed in Matlab .

```
>> dFF2 = ff2n(4)
```

$\phi°$	E	γ	R_{inter}
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

Table: 4.7 Settlement of reinforced footing corresponding to sixteen sample points in RSM analysis using PLAXIS

	$\phi°$	E(kN/m2)	γ(kN/m3)	R-inter	Settlement(δ) mm
(0) =μ+1.65σ	44.11	3174.73	21.01	0.87	
(1)= μ-1.65σ	28.53	892.83	16.66	0.53	
1	49.79	46830.00	16.52	0.87	8.05
2	49.79	46830.00	16.52	0.53	8.04
3	49.79	46830.00	13.10	0.87	8.28
4	49.79	46830.00	13.10	0.53	8.24
5	49.79	13170.00	16.52	0.87	28.6
6	49.79	13170.00	16.52	0.53	28.52
7	49.79	13170.00	13.10	0.87	29.31
8	49.79	13170.00	13.10	0.53	29.20
9	32.21	46830.00	16.52	0.87	9.66
10	32.21	46830.00	16.52	0.53	9.60

11	32.21	46830.00	13.10	0.87	10.72
12	32.21	46830.00	13.10	0.53	10.56
13	32.21	13170.00	16.52	0.87	34.65
14	32.21	13170.00	16.52	0.53	34.50
15	32.21	13170.00	13.10	0.87	37.46
16	32.21	13170.00	13.10	0.53	37.67

Linear response surface model:

$$\delta = 57.49419 - 0.25996\varphi - 0.00069355E - 0.358801\gamma + 0.144300144R_{inter}$$

R^2 =0.9863; Adjusted R^2 = 0.9814

The Performance function can be defined as $g(x) = x - \delta$

x is the permissible settlement of footing.

Minimize $\beta_{HL} = \min_{G(z^*)=0} \sqrt{(X')^t(X')}$

$$X = \frac{x-\mu}{\sigma}$$

X' is a matrix which contains the value of X. Initially the value of x is assumed nearer to the mean value of the input parameter.

The minimum distance from the origin to the design point (reliability index) is obtained using MS-Excel solver.

If permissible settlement $x = 40mm$.

$\beta = 2.831$

The probability of failure of the slope

$p_f = \varphi(-\beta_{HL})$

From the excel, =normdist(-2.831) gives $p_f = 0.0023$

The results of reliability analysis show the settlement of footing is under average region as per USACE chart. At this point it may be mentioned as per the deterministic approach, the settlement for the mean value is 13 mm. But it should be noted that in the controlled experiments like laboratory based methods the soil variability is less as compared with the field. This study highlights the importance of reliability analysis in the reinforced footing.

CHAPTER-5

RELIABILITY ANALYSIS OF ANCHORED SHEET PILE WALL

5.1 INTRODUCTION

Sheet pile wall is one of the most common types of flexible earth retaining structure and is being used since early civilization, particularly for continuous waterfront wall structures, river protection walls, excavation and temporary supports in foundation with high ground water table. There are two primary types of sheet pile walls; cantilevered sheet pile and anchored sheet pile. The design of sheet pile wall is based on the overall stability of the retaining system and the moment of resistance of the sheet pile. The limit equilibrium method, with earth pressures on either side of the wall as the disturbing and restoring forces is still more popular due to its simplicity (Basha and Babu 2008), but it does not consider the flexibility of the sheet pile. In this study anchored sheet pile is considered and FEM is used to analyse the same.

5.1.1 Deterministic analysis of an anchored sheet pile

A 15 m high sheet pile wall has penetrated on the dense sand layer. The total depth of penetration is 5m. The sheet pile is anchored in the sand layer and grouted with the sand layer. The total depth of excavation is 14m. In the first stage 1m excavation is done. In the next phase 13 m excavation is done. Water table is located below the considerable depth.

FEM Model:

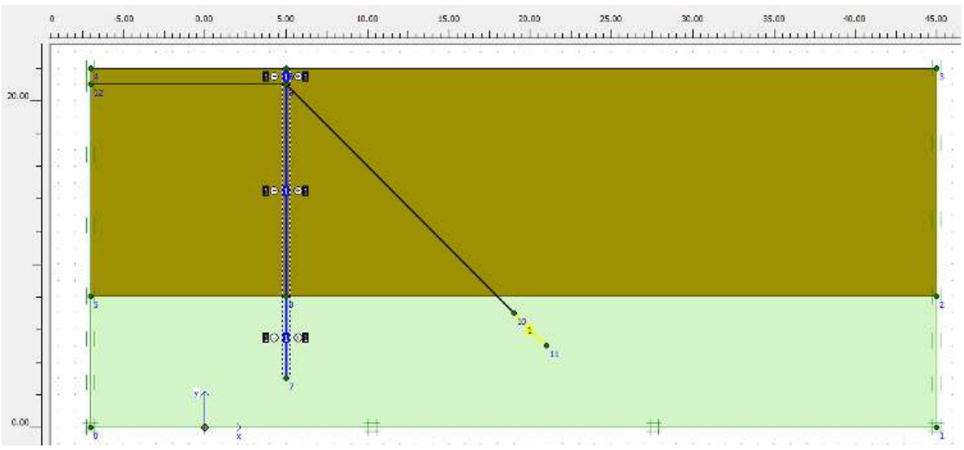


Fig.5.1 Anchored sheet pile wall model in PLAXIS

Soil:

Mohr-Coulomb model is used to simulate the soil body.

The mean values of the soil is given in Table 5.1

Table : 5.1 Properties of the soil

Parameter	Layer-1 (sandy clay)	Foundation layer – sand
Unit weight (ɣ) (kN/m^3)	18	19
Angle of internal friction (φ°)	15	1
Cohesion (c) (kPa)	30	38
Young's modulus (E) (kPa)	4000	200000
Poisson's ratio(ν)	0.3	0.3
R_{inter}	0.7	0.8

Sheet pile wall:

Sheet pile wall is modelled as Elastic plate with the thickness of 0.346 m.

Anchor:

Node to node anchor is used to model the anchor. 100kN/m prestressing force is applied on the Anchor element. Spacing between the anchors is 2.5m.

Grout body:

Elastic geogrid element is used to model the grout body with the normal stiffness of $1*10^5$ kN/m.

Meshing:

General options

Model : Plane Strain

Elements : 15-Noded

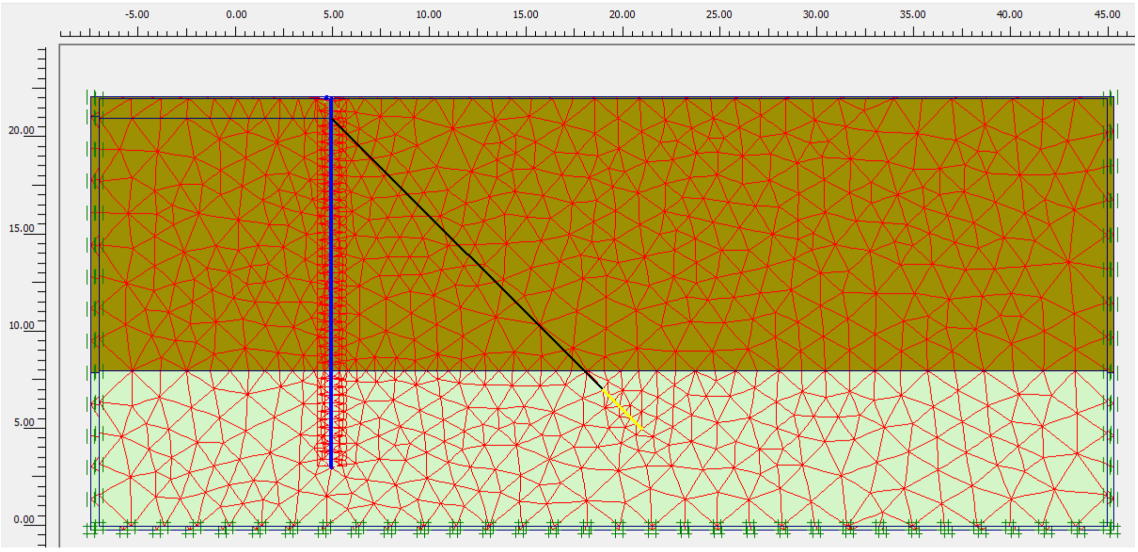
Mesh

Number of elements : 1571

Number of nodes : 13131

Number of stress points : 18852

Average element size : $853.35*10^{-3}$ m



Meshing details

Very fine global coarseness is used for meshing. Line refinement has done on sheet pile and grout body.

Construction Stages in PLAXIS:

Stage-1: Generation of initial stresses (gravity loading)

Stage-2: Excavation depth 1m (deactivating the cluster)

Activating the sheet pile wall, Anchor, Grout body

Stage-3: Excavation depth up to 14m

Deformed mesh:

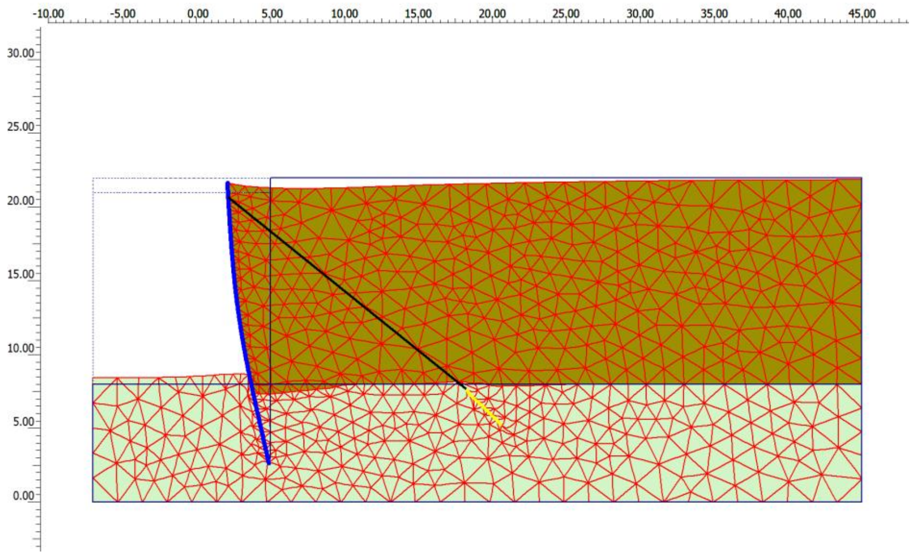


Fig. 5.2 Deformed mesh

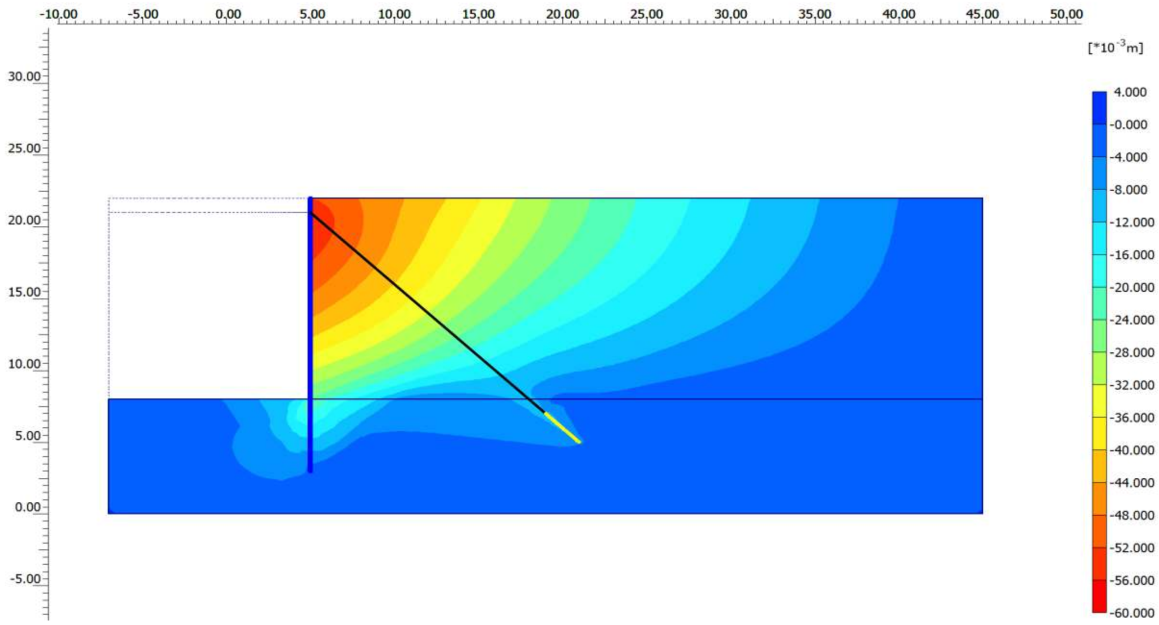


Fig 5.3 Horizontal displacement of sheet pile wall

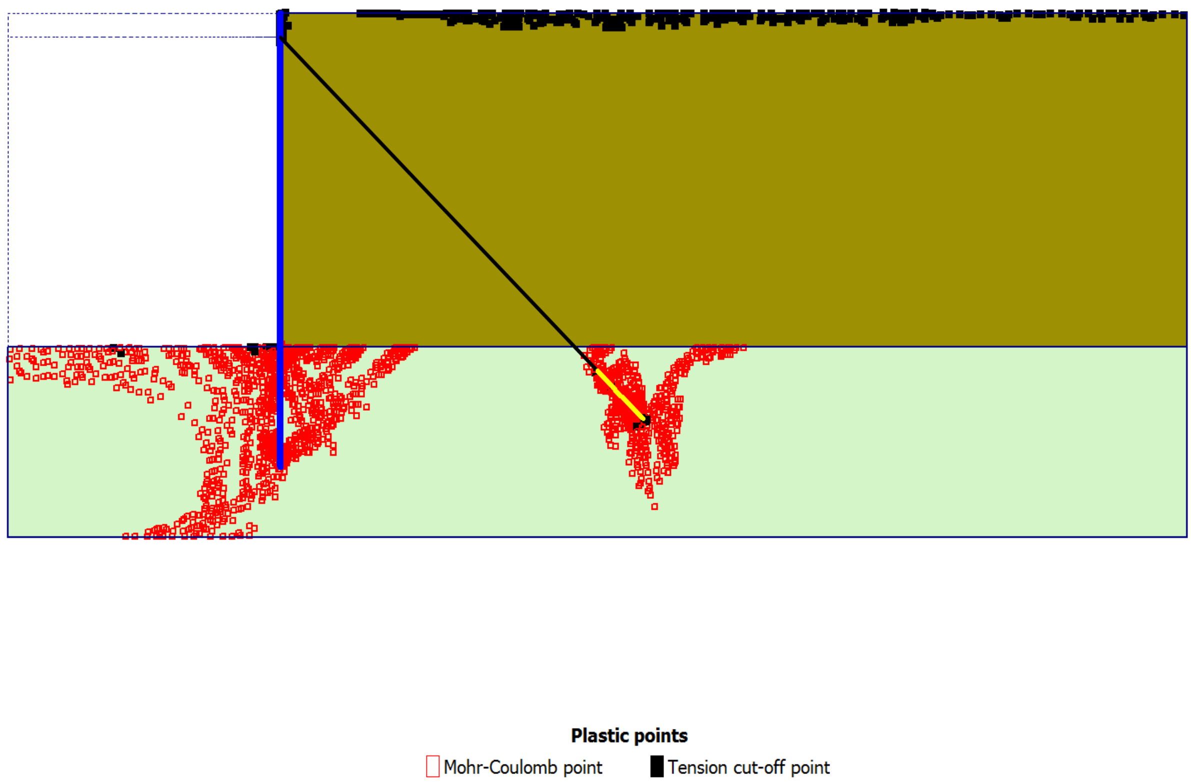


Fig 5.4 Plastic points in the sheet pile wall

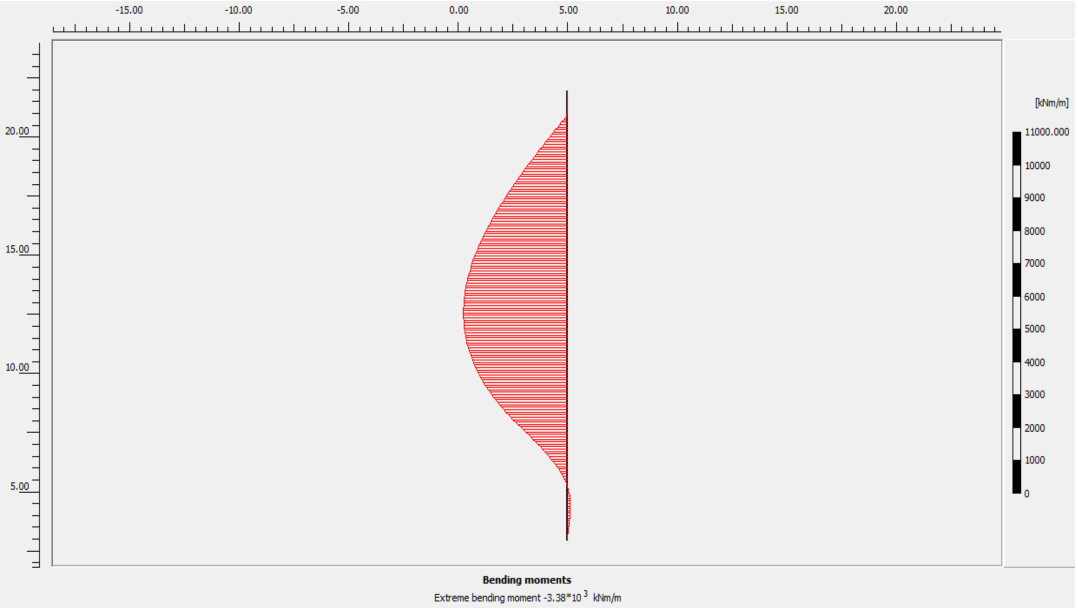


Fig. 5.5 bending moment diagram of sheet pile wall

Fig. 5.6 Axial force diagram of sheet pile wall

5.2 RELIABILITY ANALYSIS:

Random variables for reliability analysis:

Unit weight (ɣ), Angle of internal friction (ϕ), Young's modulus (E) and Interfacial strength coefficient between soil and geogrid (R_{inter}) are considered to develope response surface models.

Table 5.2 Coefficient of variation of soil around the sheet pile wall.

	μ	Cov	Sd (σ)
ɣ (kN/m^3)	18	0.07	1.526474
C (kPa)	30	0.2	6.000262
E(kN/m^2)	4000	0.34	1359.999

Table 5.3 Design of Experiments for the horizontal displacement of the sheet pile wall.

	X			Y
	ɣ	c	E	Horizontal displacement
(+) =μ+1.65σ	20.07	39.90	6244	(mm)
(-)= μ-1.65σ	15.92	20.10	1756	
1	20.07	39.90	6244	40.55
2	20.07	39.90	1756	49.19
3	20.07	20.10	6244	40.63
4	20.07	20.10	1756	49.24
5	15.92	39.90	6244	30.31
6	15.92	39.90	1756	35.15
7	15.92	20.10	6244	30.29
8	15.92	20.10	1756	35.03

Case:1 (Horizontal displacement)

The Parameters c, φ *and* E are considered as uncorrelated normally distributed.

$$\delta_H = -8.1823 + \gamma * 2.9401 + 0.00101 * c - E * 0.0014906$$

R^2 =0.981; Adjusted R^2 = 0.968

The Performance function can be defined as $g_1(x) = \delta_H - 50$

50 mm is considered as the permissible horizontal displacement of the sheet pile.

Minimize $\beta_{HL} = \min_{G(z^*)=0} \sqrt{(X')^t(X')}$

$$X = \frac{x - \mu}{\sigma}$$

X' is a matrix which contains the value of X. Initially the value of x is assumed nearer to the mean value of the input parameter.

$\beta = 2.744$

$p_f = \varphi(-\beta_{HL})$

From the excel, =normdist(-2.744) gives $p_f = 0.003$.

Case: 2 (Anchor rod)

In this case failure of anchor rod is considered for reliability analysis. The exceed of yield stress is considered as failure criterion.

The stress acting on the anchor rod

$$\sigma = \frac{Axial\ force\ in\ the\ anchor\ rod}{cross\ sectional\ area\ of\ the\ anchor} = \frac{F_A}{A_A}$$

F_A in kN/m^2; A_A in m^2

Limit state equation can be established as:

$$g_2(x) = f_y - \sigma$$

Table 5.4 Design of Experiments for the moment of the sheet pile wall.

	X			Y
	γ	c	E	Axial force
(+) =μ+1.65σ	20.07	39.90	6244	(kN)
(-)= μ-1.65σ	15.92	20.10	1756	
1	20.07	39.90	6244	738.9
2	20.07	39.90	1756	859.6
3	20.07	20.10	6244	739.9
4	20.07	20.10	1756	860.0
5	15.92	39.90	6244	601.8
6	15.92	39.90	1756	689.0
7	15.92	20.10	6244	602.2
8	15.92	20.10	1756	689.1

$$F_A = 148.737379 + 37.05507475 * \gamma - 0.0239898 * c - 0.023111631 * E$$

It is assumed that 50 mm anchor rod is used.

The yield strength of steel is considered as 515 N/mm^2

Minimize $\beta_{HL} = \underset{G(z^*)=0}{min} \sqrt{(X')^t(X')}$

$$X = \frac{x - \mu}{\sigma}$$

X' is a matrix which contains the value of X. Initially the value of x is assumed nearer to the mean value of the input parameter.

$\beta = 4.65$

$p_f = 1.65\text{E} - 06$.

5.3 Sheet pile wall in cohesive soil:

The **Example** problem 20.1 Murthy. (2003) has been considered for the present study**.** An allowable flexural stress = 415 MN/m^2. The soil has an effective unit weight of 17 kN/m^3 and angle of internal friction of 30°. The problem has shown in Fig. 5.7.

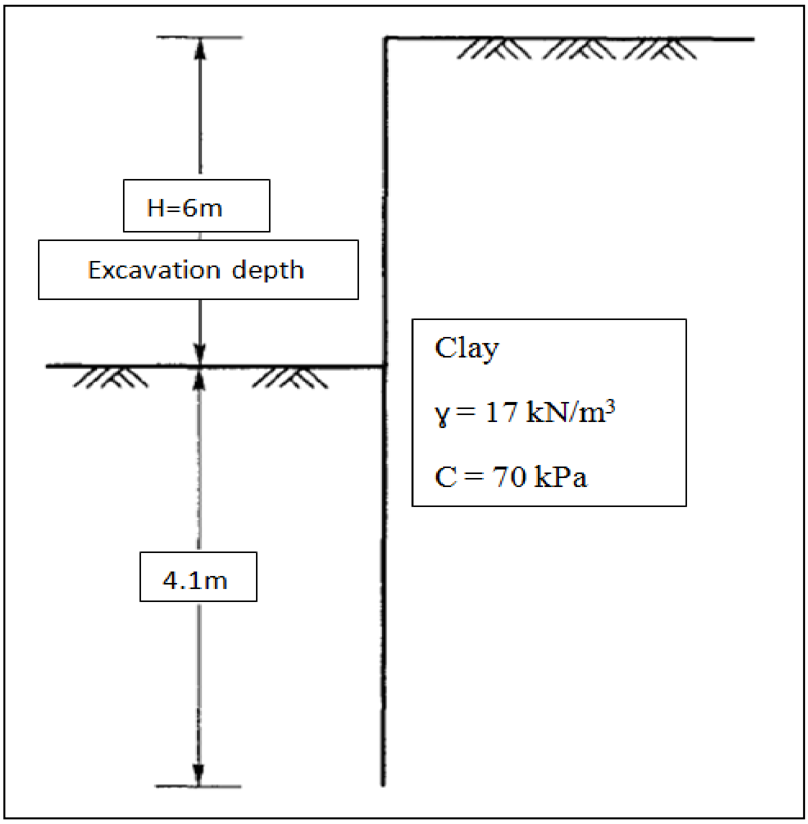


Fig. 5.7 Sheet pile wall in cohesive soil (Moorthy)

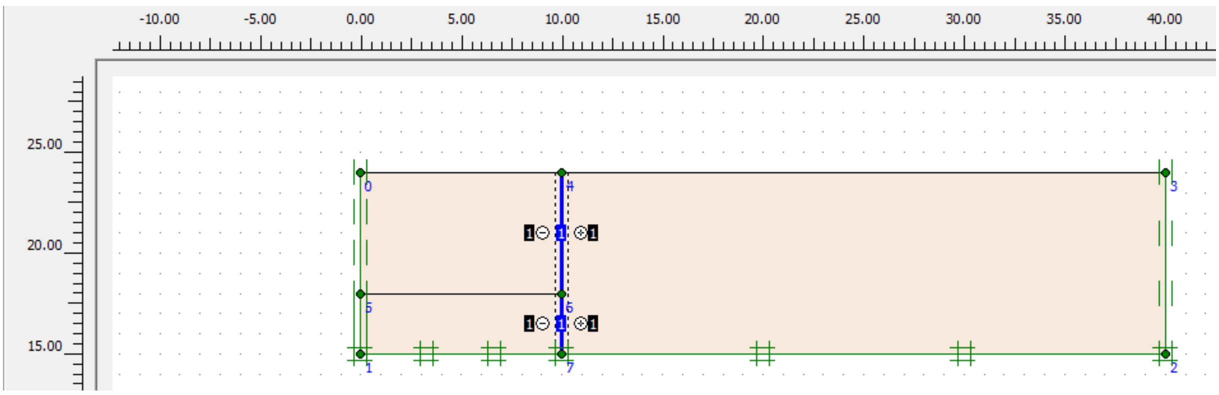


Fig. 5.8 PLAXIS model

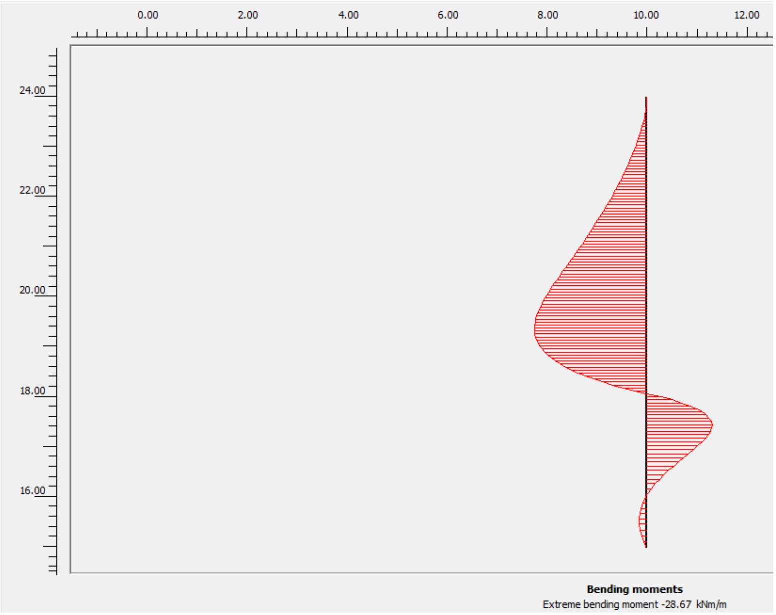


Fig. 5.9 Bending Moment of the sheet pile wall

Maximum moment: 28.67 kNm/m

5.4 RELIABILITY ANALYSIS:

Random variables for reliability analysis:

Unit weight (ɣ), Angle of internal friction (ɸ), Young's modulus (E) and Interfacial strength coefficient between soil and geogrid (R_{inter}) are considered to develope response surface models.

Coefficient of variation:

	μ	Cov	Sd (σ)
ɣ (kN/m^3)	17	0.07	1.19
C (kPa)	70	0.2	14
E(kN/m^2)	4000	0.34	1360

Case: Failure of the sheet pile wall

In this case failure of sheet pile wall is considered for reliability analysis. The exceed of yield stress is considered as failure criterion.

The stress acting on the anchor rod

$$\sigma = \frac{Moment\ developed\ in\ the\ sheet\ pile\ wall}{Section\ modulus\ of\ the\ sheet\ pile\ wall} = \frac{\mathrm{M}}{\mathrm{Z}}$$

M in kNm/m; Z in m^3/m

Limit state equation can be established as:

$$g_2(x) = f_y - \sigma$$

Table 5.5 Design of Experiments for the moment of the sheet pile wall.

	X			Y
	ɣ	c	E	Sheet pile moment
(+) =μ+1.65σ	18.9635	93.1	6244	(kNm/m)
(-)= μ-1.65σ	15.0365	46.9	1756	
1	18.9635	93.1	6244	22.93
2	18.9635	93.1	1756	46.76
3	18.9635	46.9	6244	21.70
4	18.9635	46.9	1756	38.41
5	15.0365	93.1	6244	19.74
6	15.0365	93.1	1756	41.00
7	15.0365	46.9	6244	19.41
8	15.0365	46.9	1756	37.91

$$F_A = 31.24478 + 0.74738965 * \gamma - 0.07034632 * c - 0.004473039 * E$$

The yield strength of steel is considered as 515 N/mm^2

Minimize $\beta_{HL} = \underset{G(z^*)=0}{min} \sqrt{(X')^t(X')}$

$$X = \frac{x - \mu}{\sigma}$$

X' is a matrix which contains the value of X. Initially the value of x is assumed nearer to the mean value of the input parameter.

$\beta = 5.32$

p_f =5.1884E-08

In the case of anchored sheet pile wall, failure of the anchor and the deflection of the sheet pile wall are considered. The results of reliability analysis show the performance of anchor is under high region as per USACE chart. But the horizontal deflection of the sheet pile wall is in the average region. In the case of cantilever sheet pile wall, failure of the sheet pile wall is studied through reliability analysis. The results reveal that the performance of the sheet pile wall is in the high.

CHAPTER-6

CONCLUSIONS AND SCOPE FOR FURTHER STUDY

6.1 Conclusions

In the present study reliability analysis of slope, footing and sheet pile wall has been done using first order reliability method (FORM) Hasofer-Lind reliability index and probability of failure was obtained for these cases. Response surface method was used to develop limit state function. The input data obtained from the design of experiments is analysed using Finite Element Method (FEM) PLAXIS 9.0.

In the Chapter-2 basics of Finite Element Method (FEM) in PLAXIS, Response Surface Method (RSM) and Reliability analysis have been discussed. The variables are considered as uncorrelated normally distributed and correlated normally distributed. In the Chapter-3 Oklahoma slope has been analysed by deterministic method and reliability study was conducted. In the Chapter-4 settlement of geogrid reinforced footing was found by FEM method and the probability of exceeding 50 mm settlement was studied. In the Chapter-5 horizontal displacement and the probability of failure of sheet pile, anchor was studied. . Based on the present study following conclusions are made.

1. The application of reliability analysis in geotechnical engineering is limited compared to the deterministic methods used. But, considering the uncertainty associated in geotechnical engineering, now reliability analysis is becoming more acceptable.
2. Based on deterministic FEM analysis of failed slope for Oklahoma Birch dam, the factor of safety is found to 1.116 which varies from 1.027 to 1.121 according to the limit equilibrium methods available in literature. But based on the reliability analysis the probability of failure is 0.338 for uncorrelated and 0.331 for correlated for soil parameters (c and ϕ). This corresponds to most hazardous as per USACE standards.
3. The FEM model for two unreinforced footing is discussed based on a high quality field data available in literature. The predicted settlement based on SPT values for

both the footings overestimating the observed values. Based on the CPT tests the predicted settlements are found to overestimating the values. The difference in predicted settlement values are found to marginal irrespective of the correlation.

4. It was observed that compared to SPT and CPT data the load settlement curve based on CHT data is close to observed values and the error was within 10%. However, it is also observed that at certain load point the predicted settlement is less than the observed settlement. This has implication in terms of the reliability of the method.
5. In the absence of in-situ filed data for the geogrid reinforced footing, laboratory field data available in literature is considered for comparison of FEM results using deterministic approach. Variability in unit weight (γ), Young's modulus of soil (E), angle of internal friction (φ), interfacial strength between geogrid and soil are considered. The results of reliability analysis show the settlement of footing is under average region as per USACE chart. But it should be noted that in the controlled experiments like laboratory based methods the soil variability is less as compared with the field. This study highlights the importance of reliability analysis in the reinforced footing.
6. In the case of anchored sheet pile wall, failure of the anchor and the deflection of the sheet pile wall are considered, taking a theoretical example, due to absence of laboratory or field data. The results of reliability analysis show the performance of anchor is under high region as per USACE chart. But the horizontal deflection of the sheet pile wall is in the average region. In the case of cantilever sheet pile wall, failure of the sheet pile wall is studied through reliability analysis. The results reveal that the performance of the sheet pile wall is in high.

6.2 Scope for further study

Based on the present work it is observed that further intensive study is required in the area of Finite element study and the reliability analysis.

(1) Advanced computational tools to find out the non linear limit state function.

(2) The effect of variation in coefficient of variation of input random variables should be studied.

(3) Reliability study may be conducted for the variables follow beta distribution or lognormal distribution.

(4) System reliability may be investigated for the complex geotechnical engineering problems.

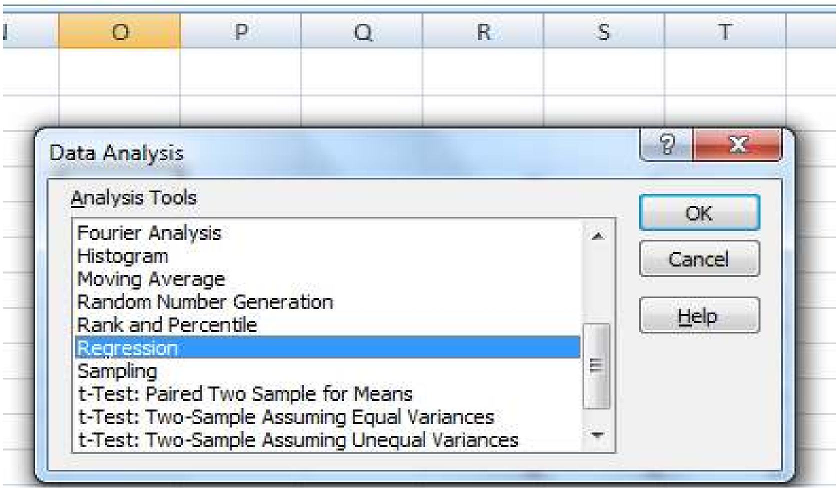
(5) In the present study only the settlement of geogrid reinforced footing has been evaluated. System reliability analysis can be conducted by considering the bearing capacity criteria.

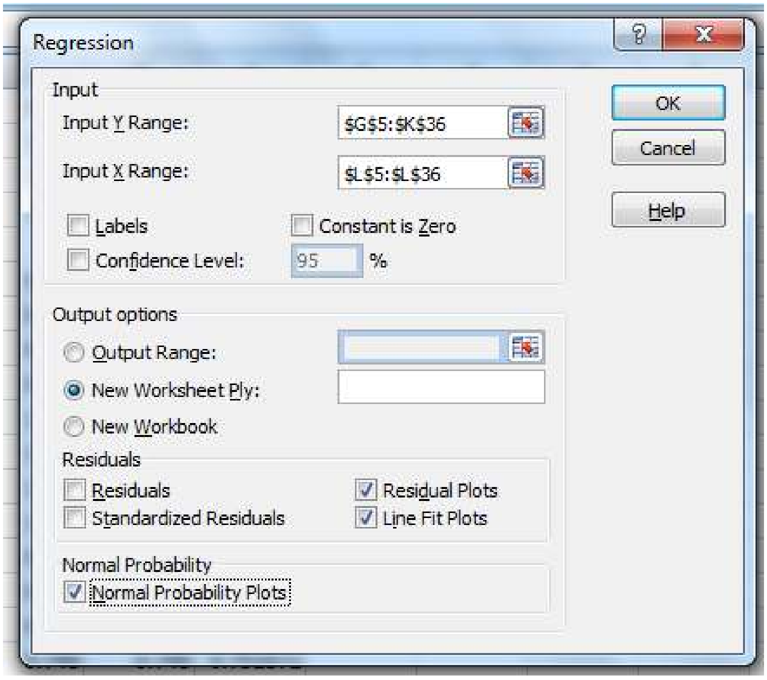
APPENDIX

Regression analysis in MS-Excel:

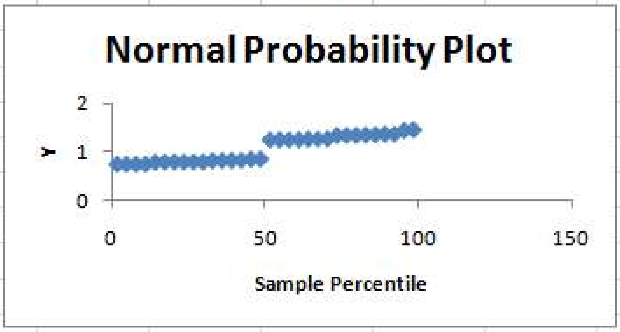
Data set for slope model:

	A	B	C	D	E	F	G	H	I	J
2						ϕ°	E(kN/m2)	γ(kN/m3)	R-inter	Settlement(δ) mm
3					(+) =μ+1.65σ	44.00225	45419.67	21.05401	0.895284	
4		x*	cov	sd	(-)= μ-1.65σ	28.45926	12773.37	16.6941	0.68649	
5	phi	36.23076	0.13	4.709998	1	46.151	70245	21.1945	0.9056	14.91
6	E	29096.52	0.34	9892.817	2	46.151	70245	21.1945	0.6944	15.32
7	Gamma	18.87406	0.07	1.321184	3	46.151	70245	16.8055	0.9056	15.07
8	geogrid	0.790887	0.08	0.063271	4	46.151	70245	16.8055	0.6944	15.48
9					5	46.151	19755	21.1945	0.9056	52.69
10		mu			6	46.151	19755	21.1945	0.6944	54.13
11		38		4.94	7	46.151	19755	16.8055	0.9056	53.29
12		45000		15300	8	46.151	19755	16.8055	0.6944	54.72
13		19		1.33	9	29.849	70245	21.1945	0.9056	19.87
14		0.8		0.064	10	29.849	70245	21.1945	0.6944	23.95
15					11	29.849	70245	16.8055	0.9056	20.77
16					12	29.849	70245	16.8055	0.6944	27.7
17					13	29.849	19755	21.1945	0.9056	65.76
18		x*			14	29.849	19755	21.1945	0.6944	79.4
19		-0.35815			15	29.849	19755	16.8055	0.9056	73.73
20		-1.03944			16	29.849	19755	16.8055	0.6944	98.63
21		-0.0947								
22		-0.14239								



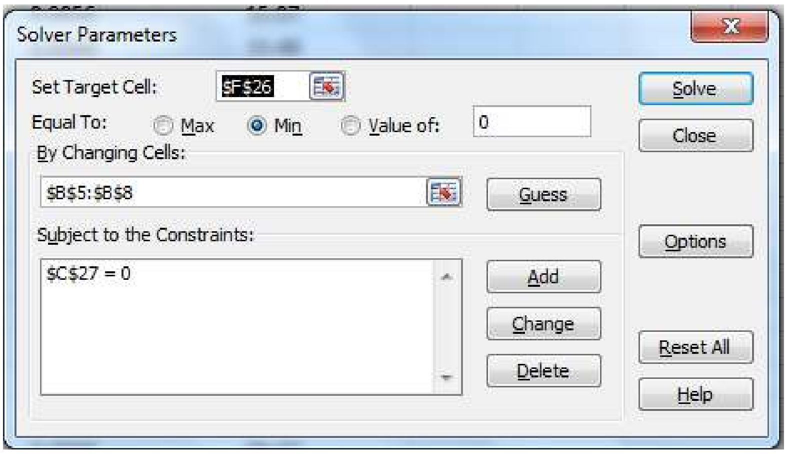


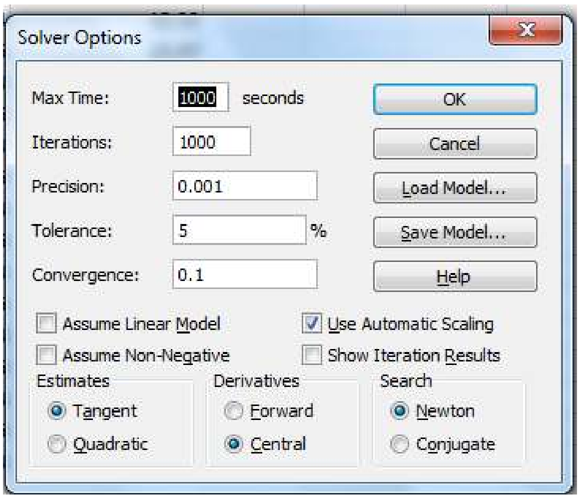
	A	B	C	D	E	F	G	H	I
1	SUMMARY OUTPUT								
2									
3	*Regression Statistics*								
4	Multiple R	0.994235							
5	R Square	0.988504							
6	Adjusted R Squa	0.986293							
7	Standard Error	0.032162							
8	Observations	32							
9									
10	ANOVA								
11		*df*	*SS*	*MS*	*F*	*gnificance F*			
12	Regression	5	2.312438	0.462488	447.1117	2.44E-24			
13	Residual	26	0.026894	0.001034					
14	Total	31	2.339332						
15									
16		*Coefficient*	*andard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*pper 95.0%*
17	Intercept	-0.04484	0.048249	-0.92931	0.361275	-0.14401	0.054338	-0.14401	0.054338
18	X Variable 1	0.001991	0.000352	5.661353	5.92E-06	0.001268	0.002713	0.001268	0.002713
19	X Variable 2	0.000721	0.000221	3.264897	0.003066	0.000267	0.001175	0.000267	0.001175
20	X Variable 3	0.016396	0.000352	46.63196	1.36E-26	0.015673	0.017119	0.015673	0.017119
21	X Variable 4	0.003318	0.000855	3.8805	0.000638	0.00156	0.005075	0.00156	0.005075
22	X Variable 5	0.002655	0.001473	1.802838	0.083017	-0.00037	0.005682	-0.00037	0.005682



Optimization of reliability index in MS-Excel:

	A	B	C	D	E	F	G	H	I	J
2						ϕ°	E(kN/m2)	γ(kN/m3)	R-inter	Settlement(δ) mm
3					(+) =μ+1.65σ	44.00225	45419.67	21.05401	0.895284	
4		x*	cov	sd	(-)= μ-1.65σ	28.45926	12773.37	16.6941	0.68649	
5	phi	36.23076	0.13	4.709998	1	46.151	70245	21.1945	0.9056	14.91
6	E	29096.52	0.34	9892.817	2	46.151	70245	21.1945	0.6944	15.32
7	Gamma	18.87406	0.07	1.321184	3	46.151	70245	16.8055	0.9056	15.07
8	geogrid	0.790887	0.08	0.063271	4	46.151	70245	16.8055	0.6944	15.48
9					5	46.151	19755	21.1945	0.9056	52.69
10		mu			6	46.151	19755	21.1945	0.6944	54.13
11		38		4.94	7	46.151	19755	16.8055	0.9056	53.29
12		45000		15300	8	46.151	19755	16.8055	0.6944	54.72
13		19		1.33	9	29.849	70245	21.1945	0.9056	19.87
14		0.8		0.064	10	29.849	70245	21.1945	0.6944	23.95
15					11	29.849	70245	16.8055	0.9056	20.77
16					12	29.849	70245	16.8055	0.6944	27.7
17					13	29.849	19755	21.1945	0.9056	65.76
18		x*			14	29.849	19755	21.1945	0.6944	79.4
19		-0.35815			15	29.849	19755	16.8055	0.9056	73.73
20		-1.03944			16	29.849	19755	16.8055	0.6944	98.63
21		-0.0947								
22		-0.14239								
23				-0.35815	-1.039443006	-0.0947	-0.14239			
24										
25										
26		settlemer	59.9999998		beta, d	1.112634				
27		g(x)	1.96896E-07		probability of	0.132933				





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