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Reliability-based design methodology of multi-pile composite foundation

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ABSTRACT: This paper presents an approach based on reliability to design the replacement ratio (percent coverage) of piles in a multi-pile composite foundation. The spatial variability of the soil is analyzed by using the stochastic theory and the model uncertainty is estimated by introducing a model uncertainty factor which describes the stochastic functional dependence between the true and the predicted value. In this study, the reliability of both bearing capacity and settlement of multi-pile composite foundation are analyzed to obtain the relationship between the reliability index and the replacement ratio of piles. The indices of the reliability are calculated using the Monte-Carlo simulation which can be readily applied to a nonlinear and complex performance function. A target reliability index of 3.2 is selected based on the analysis of previous studies to estimate the replacement ratios of multi-pile composite foundation. In order to facilitate the reliability-based design methodology of multi-pile composite foundation, the results of the analysis are presented graphically suitable for use by practicing engineers.

1 INTRODUCTION

In geotechnical engineering, the bearing capacity and settlement of composite foundation, especially multi-pile foundation, were traditionally evaluated by using a deterministic approach. The factor of safety used in the deterministic approach accounts for natural variability, statistical uncertainty, measurement errors, and limitations of analytical models and is an indirect way of limiting deformation. A factor of safety of 2.5-3.0 is generally adopted to account for this variability (Bowles 1996). Over the last two decades, there has been a slow but worldwide move toward the increased use of risk-based on design methodologies for geotechnical engineering. Load and resistance factor design (LRFD) codes have been adopted by the American Association of State Highway and Transportation Officials (1996); Ministry of Transportation of Ontario (1991); and American Petroleum Institute (1993). Partial factors of safety codes are implemented in the Canadian Foundation Engineering Manual (Technical Committee on Foundation 1992) and Eurocode 7 (European Committee for Standardization 1994). The impetus for LRFD codes originates from the structural engineering community. The same applies to transmission line structures, where structural reliability-based design (RBD) initiatives (Task Committee on Structural Loadings 1991) took place ahead of similar geotechnical initiatives (Phoon et al. 1995, 2003; Low 2005; Sivakumar Babu et al. 2006).

The objective of this paper is to present a simple methodology for (ⅰ) analyzing the major components of uncertainties associated with the prediction of properties for a given soil material and combining them to obtain the statistics of soil variables that will govern the performance of a geotechnical system, (ⅱ) utilizing a random model uncertainty factor to deal with the difference between the true and the predicted value of the capacity and settlement of multi-pile composite foundation and (ⅲ) analyzing the relationship between the reliability index, the bearing capacity, and the replacement ratio of piles based on the RBD methodology and illustrating this relationship in the form of charts.

2 SOIL PROPERTIES CHARACTERIZATION

There are three major sources of uncertainty associated with geotechnical engineering practice: inherent variability, measurement, and transformation uncertainty (Phoon 1999a). Analysis of the sources of uncertainty in soil properties and its influence on design decisions and implications has been studied extensively (Vanmarcke 1977, 1983; Meyerhof 1982; Phoon 1999a, 1999b; Cherubini 2000). Due to the variability of the properties of soils, along with disturbance caused by sampling, it is necessary to evaluate the spatial average of the design properties over some depth interval, rather than use the value of the design property at a point. Therefore, spatial variability is an important factor affecting design involving foundation problems. The spatial averaging of soil properties reduces its point variance. A variance reduction factor is derived in terms of scale of fluctuation (δ); and averaging distance (L), the distance over which the geotechnical properties are averaged.

The variability of soil property u_i from point to point is measured by the standard deviation σ_i, and the standard deviation of the spatially averaged property u_L is given by σ_L. With an increase in the averaging distance, more fluctuations in the soil property u_i get cancelled out, and subsequently the variance in the soil property value is reduced in the process of spatial averaging. The σ_L/σ_i ratio was defined as the variance reduction factor $\Gamma_u(L)$ by Vanmarcke(1977, 1983):

$$\Gamma_u(L) = \sigma_L/\sigma_i \tag{1}$$

The approximate relationship between the variance reduction function in terms of the averaging distance and the scale of fluctuation is as follows (Vanmarcke 1983):

$$\Gamma^2(L) = \begin{cases} \left[\dfrac{\delta}{L}\left(1-\dfrac{\delta}{4L}\right)\right]; \dfrac{L}{\delta} > 0.5 \\ 1.0; \dfrac{L}{\delta} \le 0.5 \end{cases} \tag{2}$$

Taking into account of the inherent soil variability (w), measurement error (e) and transformation uncertainty (ε), the design property (ξ_d) can be assumed to be predicted as:

$$\xi_d = (t + w + e)\varepsilon \tag{3}$$

in which t is the deterministic trend function. Note that the mean of $w, e,$ and ε is zero.

A second-moment probabilistic approach to evaluate the coefficient of variation (COV) of the spatial average described below (Phoon 1999b):

$$\mathrm{COV}_{\xi_a}^2 \approx \Gamma^2(L)\mathrm{COV}_w^2 + \mathrm{COV}_e^2 + \mathrm{COV}_\varepsilon^2 \tag{4}$$

in which $\Gamma^2(\bullet)$ is the variance reduction function; COV_w is the COV of inherent variability; COV_e is the COV of measurement error; COV_ε is the COV of transformation uncertainty. The values of COV_e and COV_ε can be taken as 0.15 and 0.29 (Sivakumar Babu et al 2006), respectively.

3 MODELLING UNCERTAINTY

Conducting a limit state-based reliability analysis for a geotechnical engineering problem involves specification of a limit state function. This function is usually based on a standard deterministic model for analysis of the problem in question. The analysis model is associated with a model uncertainty because of its imperfect representation of reality, e.g., due to simplifications and idealizations that have been made, purposely or due to lack of knowledge (Ditlevsen 1982). If the model is conservative, it is obvious that the probabilities of failure calculated subsequently will be biased, because those design situations that belong to the safe domain could be assigned incorrectly to the failure domain, as a result of built-in conservatism. Therefore, even a simple estimate of the average model bias is crucial for index of reliability analysis. A random model uncertainty factor I was introduced by former researchers (e.g. Ronold 1992) to describe the stochastic functional dependence between the true and the predicted value in the following form:

$$Z = I \cdot M \tag{5}$$

in which Z is the true capacity; M is the predicted capacity.

Take the bearing capacity of piles for example. Full-scale measurements of pile capacities form a possible way to obtain information about the true, but unknown, capacities Z. After one test, a realization of Z can be obtained, notated as z_1, and a realization of M can be calculated as well, notated as m_1. When the z_1 and m_1 are available, a realization notated as i_1 of I can be predicted by

$$i_1 = z_1 / m_1 \tag{6}$$

Assume now that test data are available from n full-scale tests, i.e., n outcomes z_i of measured capacities, and n corresponding outcomes m_i of calculated value can be obtained. This will give n realizations of the model uncertainty factor I. A statistical analysis of these n realizations will give information about the probability distribution of I. For different calculation models, the computed capacity M may either larger or smaller than the true/measured capacity Z. And it is reasonable to expect that the each realization i_i of I fluctuates within a limited interval.

The Beta distribution (He 1991) which can cover all kinds of distribution form such as rectangular to normal distribution as well as asymmetrical distributions and has the necessary flexibility to closely represent the distribution of I that is likely to result from calculation models. The four parameters of Beta distribution $B(a,b,\gamma,\eta)$: a,b =lower and upper limits of the distribution; γ,η , shape exponents are determined by a method given by He (1991) , and a simplified method given by Liu Yong et al (2006). Detailed discussions on the Beta distribution are given elsewhere (He 1991 ; Liu Yong et al 2006).

4 CALCULATION MODEL OF BEARING CAPACITY

In vertical reinforcement of composite foundation, the three types of pile, including discrete material pile, flexible pile and rigid pile, of which the bearing capacity and distortion characteristic are different from each other, have their own applicable scopes and deficiencies. Taking the advantages and disadvantages of each type of pile into account, engineers employ two or more types of piles comprehensively to improve soft ground. Consequently, this technique is defined as multi-pile composite foundation by which can not only enhance the bearing capacity but also reduce the settlement of a composite foundation considerably. Among the piles in a specific multi-pile composite foundation, the kind of pile with relatively high strength is considered as primary pile and that with relatively low strength as secondary pile. There are two main categories of multi-pile foundation used for the purpose of enhancing the bearing capacity:

1) the secondary-pile comprises flexible piles with relatively high strength (e.g., soil-cement deep mixing pile). In this situation the bearing capacity is expressed as (Zheng Jun-jie,2004)

$$f_{sp,k} = m_1 \frac{R_{k1}^d}{A_{p1}} + \beta_2 m_2 \frac{R_{k2}^d}{A_{p2}} + \beta_1 \left(1 - m_1 - m_2\right) f_{sk} \tag{7}$$

2) the secondary-pile comprises flexible piles with relatively low strength(e.g., compacted lime-soil pile). In this situation the bearing capacity is expressed as (Zheng Jun-jie,2004)

$$f_{sp,k} = m_1 \frac{R_{k1}^d}{A_{p1}} + \beta_2 m_2 f_{pk2} + \beta_1 \left(1 - m_1 - m_2\right) f_{sk} \tag{8}$$

in which m_1, m_2 =area replacement ratio of primary-pile and secondary-pile, respectively; R_{k1}^d, R_{k2}^d = standard value of single-pile bearing capacity of primary-pile and secondary-pile, respectively; A_{p1}, A_{p2} = cross-sectional area of single-pile of primary-pile and secondary-pile, respectively; $f_{sp,k}$ =standard value of the composite foundation bearing capacity; f_{pk2} = strength of secondary-pile; f_{sk} = standard value of bearing capacity of soil between piles; β_1, β_2 = efficiency factor of soil and secondary-pile, respectively.

In this study, the latter situation is selected as a representative equation to estimate the bearing capacity of multi-pile composite foundation. The former situation could be analyzed similarly. In Eq.(8), m_1 , m_2 , A_{p1} , β_1 , β_2 are taken as deterministic variables. And f_{sk} , f_{pk2} are given by the empirical value locally.

The total vertical bearing capacity of a pile is a function of the side resistance (Q_s), and tip resistance (Q_t). The bearing capacity R_{k1}^d is given by

$$R_{k1}^d = Q_s + Q_t \tag{9}$$

The side and tip resistances are analyzed below, respectively.

4.1 Side resistance

For drained loading, the side resistance is given as follows:

$$Q_s = \pi B \sum_i^n q_{si} l_i \quad \text{(Zheng Jun-jie et al 2002)} \tag{10a}$$

$$Q_s = \pi B \alpha \int_0^D s_u(z) dz \quad \text{(Phoon et al 2000)} \tag{10b}$$

in which q_s =unit side resistance; q_{si} = unit side resistance of layer i; n=the number of layers; l_i = thickness of layer i; α =adhesion factor; s_u =undrained shear strength. B , l_i can be taken as deterministic. The unit side resistance q_s and s_u are considered to be spatially random for their values varying from point to point. As far as the spatial variability of soil properties are concerned, q_s and s_u can be assumed to be two random fields. In order to reduce the errors of estimation, q_s and s_u in each layer are considered as random fields. Therefore, its COV of the spatial average can be given by Eq.(4). Both q_s and s_u which are characterized by their means and their reduced variations are assumed to have a log-normal distribution, primarily because of its simple relationship with the normal distribution, and because it is non-negative.

4.2 Tip resistance

The tip resistance in compression is provided by the bearing capacity of the soil beneath the tip, as given by:

$$Q_t = q_p \pi B^2 / 4 \tag{11}$$

in which B =diameter of pile; q_p =unit tip resistance. The value of q_p can be given by locally empirical value.

Incorporating Eq.(10a) and Eq.(11), the bearing capacity of the pile can be obtained:

$$R_{k1}^d = \pi B \sum_i^n q_{si} l_i + q_p \pi B^2 / 4 \tag{12}$$

5 CALCULATION MODEL OF SETTLEMENT

The settlement of composite foundation under a vertical working load usually includes two components:

$$s = s_1 + s_2 \tag{13}$$

in which s = total settlement, s_1 = settlement of the reinforced area, s_2 = settlement of underlying stratum in the reinforced area. As for s_1 , the piles and soil can be viewed as a composite material, of which the settlement can be valuated by compound modulus method,

$$s_1 = \sum_{i=1}^n \frac{\Delta p_i}{E_{csi}} l_i \tag{14}$$

in which Δp_i =stress increase at the middle of layer i , l_i =thickness of layer i , E_{csi} = compound modulus of layer i , which can be calculated as:

$$E_{csi} = m_1 E_{p1} + m_2 E_{p2} + (1 - m_1 - m_2) E_s \tag{15}$$

in which E_{p1} = modulus of the main-pile, E_{p2} = modulus of the secondary pile, m_1, m_2 =area displacement ratio of mail-pile and secondary-pile, respectively, E_s = modulus of soil.

The settlement of underlying stratum in the reinforced area (s_2) can be valuated as:

$$s_2 = \sum_{i=1}^n \frac{\Delta e_i}{1 + e_0(i)} l_i \tag{16}$$

in which Δe_i = change of void ratio caused by the stress increase in layer i , $e_0(i)$ =initial void ratio of layer i , l_i =thickness of layer i .

In the Eq.(15), the E_{p1}, E_{p2} are taken as deterministic simply because the piles are relatively isotropic materials , and the E_s can be assumed to be a random field. The spatial characteristic of E_s in each layer can be analyzed as both q_s and s_u in a similar manner.

6 LIMIT STATE EQUATION OF COMPOSITE FOUNDATION

As for the bearing capacity of composite foundation, the strength limit state at the failure point can be expresses as

$$g_b = m_1 I_b \cdot R_{k1}^d / A_{p1} + \beta_2 m_2 f_{pk2} + \beta(1 - m_1 - m_2) f_{sk} - Q_0 \tag{17}$$

in which g_b = safety margin of bearing capacity; I_b = model uncertainty factor of bearing capacity; Q_0 =total load.

As for the settlement of composite foundation, the limit state equation can be expressed as

$$g_s = I_s \cdot s - s_0 \tag{18}$$

in which g_s = safety margin of settlement; I_s = model uncertainty factor of settlement, s =predicted settlement given by Eq.(13), s_0 = allowable settlement.

For a given type of foundation or for a given soil, when other factors are invariable, we can adjust the values of m_1, m_2 to meet the requirements of both bearing capacity and settlement. Monte-Carlo simulation can be employed to calculate the reliability index under different values of m_1, m_2 in Eqs.(17,18). For reliability-based design (RBD), a target reliability index of 3.2 is adopted based on analysis of previous studies (Phoon1995, 2003) for ultimate limit state design. Based on RBD, an optimum design for m_1, m_2 can be obtained.

7 EXAMPLE

The following is an example of the design for m_1, m_2 referring to a bearing capacity and settlement problem by utilizing Eqs.(17)and (18). The basic numerical characteristics of variables are shown in Table 1. In this problem, $\beta_2 = 0.8$, $\beta_1 = 1.0$, q_p =1.3 MN/m^2 , and standard value of bearing capacity of soil (f_{sk}) is taken as 180kPa . The treatment process is expected to use two types of piles, soil-cement deep mixing pile ($B = 500$mm, total length=11.0m, in this situation) and lime pile ($B = 300$mm, f_{pk2} =1587kN, total length=5.5m, in this situation) as for primary and secondary piles, respectively. The total load Q_0 in Eq.(17) and allowable settlement s_0 in Eq.(18) are given as 121.5 kPa and 13.5 cm, respectively.

As for the model uncertainty factors of bearing capacity and settlement, the authors have collected 128 data (Liu Yong et al 2006) on the bearing capacity of piles, in which the data is represented a bias factors λ

$$\lambda = R_m / R_n \tag{19}$$

in which R_m =measured value of bearing capacity, R_n = predicted bearing capacity by given prediction method.

By utilizing the method to determine the parameters of Beta distribution, we can get the Beta distribution as $B(0.0305, 1.6381, 8.9051, 3.7192)$. Owing to lack of data on settlement of composite foundation, it is acceptable to take the model uncertainty factor of settlement as a constant, 1. The scale of fluctuation (δ) in Eq.(2) can be taken as 0.95m as illustrated by Gao Da-zhao (1996).

Table 1 Numerical characteristic of variables

Layer	Depth (m)	E_s		q_s		s_u	
		Mean (MPa)	COV (%)	Mean (MN/m^2)	COV (%)	Mean (kN/m^2)	COV (%)
1 (miscellaneous fill)	0-4.5	5.0	35	2.5	15	118.0	19
2 (silty clay)	4.5-8.0	4.5	31	1.9	27	110.5	20
3 (silty sand)	8.0-11.0	6.8	25	3.2	25	125.5	15

The relationship between the reliability index and the replacement ratio of piles are illustrated in Figs.(1,2).

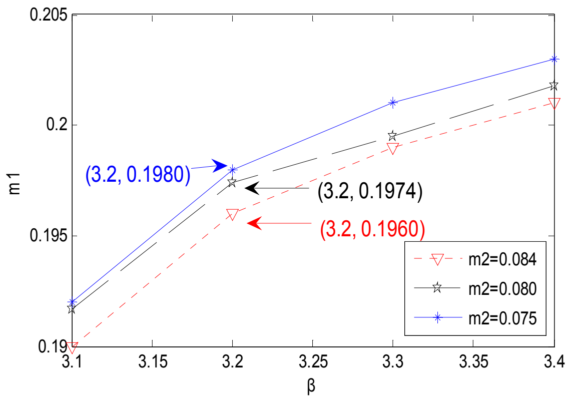

Fig.1 Relationship of $m_1 : \beta$ (bearing capacity)

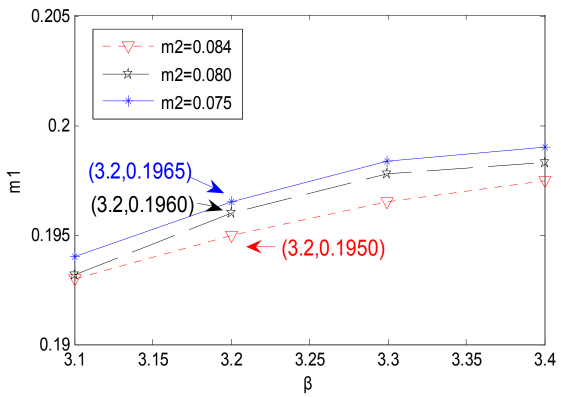

Fig.2 Relationship of $m_1 : \beta$ (settlement)

With regard to the bearing capacity of multi-pile composite foundation, the RBD method goes as follows. Fig.1 shows that when the area replacement ratio of secondary-pile (i.e., m_2) is kept constant, the value of the area replacement ratio of primary-pile (i.e., m_1) increases with an increase in the desired target reliability index (i.e., β), and decreases with an increase in the value of m_2 if β is kept constant. When a target reliability index of 3.2 is selected as an acceptable value to assure safety of a practical engineering, the value of m_1 can be determined under a specific value of m_2. For example, if the value of m_2 and β are determined as 0.084 and 3.2 respectively in advance, the value of m_1 illustrated in Fig.1 can be designed as 0.1960. The same situation applies to design the value of m_1 by analyzing the settlement of multi-pile composite foundation which is illustrated in Fig.2. Furthermore, figs.(1,2) also show that under an equal increment of β, for a certain value of m_2, the increment of m_1 in Fig.1 is greater than that in Fig.2. This phenomenon demonstrates that the bearing capacity of multi-pile composite foundation is more sensitive to the reliability index than the settlement dose in this practical engineering.

8 CONCLUSIONS

This paper presents a reliability-based design methodology for multi-pile composite foundation. On the basis of this study, the following conclusions can be drawn.

(1) Geotechnical performances are often governed by spatial average soil properties. Since soil properties are not exactly measured at every point of a soil stratum, a variance reduction factor is employed to deal with the problem of space characteristic of soil properties. A second-moment probabilistic approach is put forward to evaluate the variance of the spatial average.

(2) Proper modelling of each component of uncertainties requires first an understanding of its characteristics, whether it would give rise to biased or unbiased estimators, random or systematic error, or subject to spatial variations or averaging over the soil stratum. Such biased or unbiased estimators can be handled well by introducing a random model uncertainty factor, which describe the stochastic functional dependence between the true and the predicted value. And the Beta distribution which can cover all kinds of distribution form rectangular to normal distribution as well as asymmetrical distributions has the necessary flexibility to closely represent the distribution of the model uncertainty factor.

(3) Compared with the settlement, bearing capacity of multi-pile composite foundation is more sensitive to the reliability index as illustrated in this paper. By utilizing a specified target reliability index of 3.2, the replacement ratios of multi-pile composite foundation can be evaluated in a rational manner.

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