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Reliability-based design optimization of axial compressor using uncertainty model for stall margin †

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Abstract

Reliability-based design optimization (RBDO) of the NASA stage 37 axial compressor is performed using an uncertainty model for stall margin in order to guarantee stable operation of the compressor. The main characteristics of RBDO for the axial compressor are summarized as follows: First, the values of mass flow rate and pressure ratio in stall margin calculation are defined as statistical models with normal distribution for consideration of the uncertainty in stall margin. Second, Monte Carlo Simulation is used in the RBDO process to calculate failure probability of stall margin accurately. Third, an approximation model that is constructed by an artificial neural network is adopted to reduce the time cost of RBDO. The present method is applied to the NASA stage 37 compressor to improve the reliability of stall margin with both maximized efficiency and minimized weight. The RBDO result is compared with the deterministic optimization (DO) result which does not include an uncertainty model. In the DO case, stall margin is slightly higher than the reference value of the required constraint, but the probability of stall is 43%. This is unacceptable risk for an aircraft engine, which requires absolutely stable operation in flight. However, stall margin obtained in RBDO is 2.7% higher than the reference value, and the probability of success increases to 95% with the improved efficiency and weight. Therefore, RBDO of the axial compressor for aircraft engine can be a reliable design optimization method through consideration of unexpected disturbance of the flow conditions.

Keywords: Axial compressor; Reliability-based design optimization; Stall margin; Uncertainty model; Multidisciplinary design optimization

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1. Introduction

For many years, various compressor design optimization technologies have been researched to improve performance of the compressors. In the early stages, these researches focused on a specific discipline and were performed by adopting objective functions limited to aerodynamic design [1-7], structural design [8], etc. However, a compressor is a very complex system, and various design factors of several disciplines need to be satisfied in compressor design at the same time. Therefore, multidisciplinary design optimization (MDO), which can allow different disciplines of entire system to be considered simultaneously, has been introduced to the compressor design optimization. The MDO technologies have been applied to the compressor design optimizations while mostly combining two disciplines; aerodynamics and structure [9-13]. These researches have successfully combined disciplines and improve each factor of the performance simultaneously. Nevertheless, there are many factors to consider in the compressor MDO, such as efficiency (η), weight (W), pressure ratio (PR), stall margin (SM), and safety factor (SF) of the compressor.

Stall margin is an especially important factor for stable operation of the aircraft compressor when the aircraft is in flight, but it has complex characteristics. Keskin and Bestle [6], Choi [7] and Chen et al. [11] adopted stall margin as a factor of the multiobjective function. And they assumed that the pressure ratio and the mass flow rate at the stall point could be calculated by the computational fluid dynamics analysis or via commercial programs. However, stall margin is influenced by both the flow conditions at the stall point and the initial point. And it is difficult to detect these conditions accurately by the unstable properties of the stall state according to compressor operation. The accurate flow conditions at the stall point, such as mass flow rate and pressure ratio, should be determined by experiments in comparison with numerical analyses. However, the results of the numerical analysis include margin of error, which is an uncertainty; thus it is necessary to consider the

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uncertainty in numerical analysis. In previous researches, the uncertainty for stall margin of the compressor has not been considered, while the uncertainty for safety factor of the compressor was considered by Lian and Kim [10]. Therefore, uncertainties are considered in the flow conditions of stall margin calculation for a realistic and stable design result, while reliability-based design optimization (RBDO) is adopted in this study.

RBDO has been adapted in various areas of aerospace engineering to consider the uncertainty of design optimization, including compressor design optimization [14] and aircraft wing MDO problems [15, 16]. In this study, RBDO has been used to investigate the uncertainty in the compressor design optimization with an uncertainty model of the flow conditions in the stall margin calculation. RBDO is carried out based on the NASA stage 37 compressor [17], which has been used in the various compressor design researches. Monte Carlo Simulation [14, 16] is used to estimate accurately the reliability of stall margin in this study. For an efficient design optimization process, an approximation model in the RBDO process is required to represent the given design space. And the artificial neural network (ANN) [16, 18-20] is adopted to formulate the approximation model because ANN is efficient to simulate nonlinear problems like to the compressor design. Design points which are required to compose the approximation model are extracted using the D-optimal design of experiment (DOE) method [13, 21]. These design optimization techniques are applied to the compressor RBDO process, and the results are compared with the results of conventional deterministic optimization without consideration of the stall margin uncertainty. Finally, improvements of efficiency and weight by RBDO with uncertainty model are analyzed.

2. Numerical approaches

2.1 Reliability- based design optimization

RBDO has been carried out by assessment of the limit-state function with a mathematically integrated form of the joint probability density function. To do this, design methods such as the reliability index approach (RIA), performance measure approach (PMA), approximate moment approach (AMA), and Monte Carlo simulation (MCS) have been introduced and researched. In the RBDO procedure, the safety index or most probable point (MPP) was explored. This exploration enables design optimization to be done while preventing violations of constraints through the introduction of uncertainty. Generally, the fundamental formula of RBDO is defined as follows [16]:

$$\begin{aligned} &\textit{minimize} \;\; F(x) \\ &\textit{subject to} \;\; P_f = P\left[G_j(X) \le 0\right] \le p_j , \end{aligned} \tag{1}$$

where $F(X)$ is an objective function to minimize and $G(X)$ is a limit-state function which is transformed from the constraints of the design requirements. Failure of the problem may occur when the value of $G(X)$ is below zero, and p_j is a target failure possibility. The reliability or the failure probability (P_f) is the probability that the designed result satisfies or does not satisfy the design requirements of the problem. P_f is an important factor of RBDO as it takes into account the uncertainty of the given design problem. The mathematical formula of P_f is given like to Eq. (2) [14, 16].

$$P_f = P\left(G_j(X) \le 0\right) = \int_{G(X)\le 0} f(x)\,dx . \tag{2}$$

In Eq. (2), $f(x)$ is a joint probability density function and P_f is calculated by an integral form of the $f(x)$. Unfortunately, $f(x)$ is not usually defined clearly, and even when it is defined, it typically takes a very complex form of the direct analysis. In addition, if there are large numbers of cycles in complicated RBDO problems, it may be difficult to calculate the reliability numerically. Therefore, P_f is calculated by alternative methods such as an analytical method and a directive method. The integral form of $f(x)$ in Eq. (2) may be approximated to first- or second-order equation using the Taylor approximation as the analytical method. This method can improve the efficiency of calculation for a complicated integral equation, but it cannot guarantee the accuracy of the solution. The directive method (e.g., MCS) can obtain a feasible solution by calculation of the mean value or the degree of deviation from many samples without a limit on the complexity of the problem. Therefore, MCS is adapted as the sampling method of RBDO in this study and ANN which will be explained in next section, is chosen to calculate P_f via MCS.

2.2 Artificial neural network

The approximation model has been usually applied to design optimization problems using numerical analysis data, and it is similar to a correlation formula from experimental results. There are many approximation models such as response surface model (RSM) [13, 21-24], ANN, or Kriging model [16]. RSM has a good advantage in that it can construct an approximation model for the design optimization efficiently using second-order polynomial. However, by the same token, it may have a drawback in complex design optimization problems with strong nonlinear characteristics in the relationship between the design variables and the objective function. The relationships between the design variables and the objective functions are sometimes nonlinear in compressor design optimization problems [10, 13]. Hence, RSM has limited success in constructing the approximation model in compressor design optimization problems, and ANN is adopted in this study to replace RSM in the effort to overcome this limitation.

Jun et al. [16] researched the characteristics of three approximation models in order to choose a most suitable metamodel for the RBDO of the aircraft wing. They were polynomial regression (*e.g.,* RSM), Kriging model and ANN. The advantages and disadvantages were determined by a comparative analysis of three properties: rapid response, nonlinearity

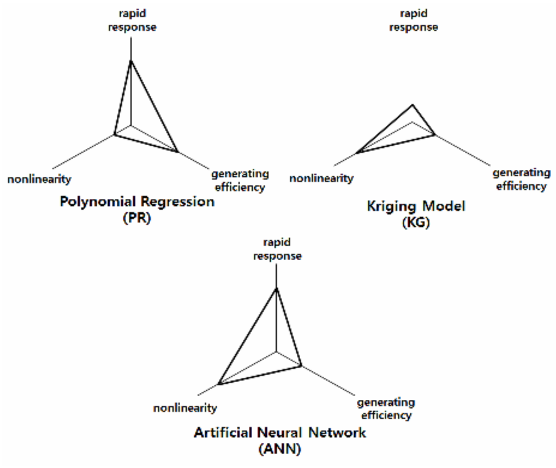

Fig. 1. Comparison of approximation models [16].

and the generating efficiency. The rapid response property refers to how quickly responses can be gained with examples of the constructed approximation model, and the nonlinearity refers to the ability to represent the nonlinearity of design space.

The last property, the generating efficiency, considers the required time to construct the approximation model. These properties of each approximation model are compared and displayed in Fig. 1. ANN shows excellent capability of capturing the nonlinearity of performances, providing the response rapidly. Thus, the use of ANN is feasible to compute the failure probability of the stall margin of a compressor via MCS.

2.3 Monte Carlo simulation

MCS has been used in many RBDO researches to calculate reliability or failure probability when it is impossible to obtain an analytical solution numerically or when the failure domain cannot be approximated in analytical form [14, 16]. In other words, RBDO problems which need to adopt MCS are usually complicated design problems with large numbers of design variables to apply other RBDO analysis methods. The mathematical formulation method of MCS is simple, and it can handle very complicated RBDO problems. However, as the number of samples for MCS is increased to acquire an improved result, the calculation cost of MCS may also increase exponentially. Therefore, as mentioned, many sampling methods, known as variance reduction techniques, have been introduced for efficient calculations with a reduced number of samples. In this manner, Eq. (2) can be simplified in according with a nonbiased assumption as expressed in Eq. (3) [24].

$$P_f = P\left(G_j\left(X\right) \le 0\right) = \int_{G(X)\le 0} f\left(x\right) dx \cong N_f / N . \tag{3}$$

In Eq. (3), N is the total number of simulation cases and N_f is the number of cases where failure occurs. The ANN approximation model can improve the efficiency of MCS because it simplifies the relationships between the design variables and the objective function not defined from the entire set of data at all design points but by the approximation model.

Table 1. Design specifications of the NASA stage 37 compressor [17].

Specification	Value
Mass flow rate, kg/s	20.1
Rotational speed, rpm	17185.7
Stage pressure ratio	2.05
Rotor aspect ratio	1.19
Stator aspect ratio	1.28
Number of rotor blades	36
Number of stator blades	46

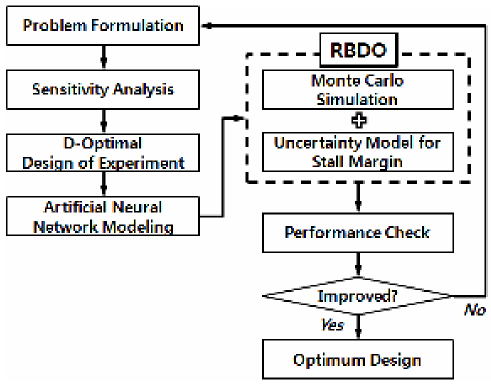

Fig. 2. Flow chart of the RBDO process.

3. RBDO of the NASA stage 37 compressor

The NASA stage 37 compressor is used as the baseline for the RBDO with an uncertainty model of stall margin. This is a transonic axial compressor that was introduced in a technical paper by Reid and Moore [17]. It has been used in various researches of the compressor design optimization, such as Benini [3], Samad and Kim [4]. The design specifications of the NASA stage 37 compressor are listed in Table 1, and this chapter presents the RBDO procedure using these specifications. The flow chart of RBDO for the NASA stage 37 transonic axial compressor is represented in Fig. 2. A brief outline of the RBDO procedure is listed as follows:

1. Sample design points based on the selected design variables with the D-optimal method.
2. Evaluate the objectives at the design points with analysis tools of each discipline.
3. Construct an ANN approximation model of the objective function.
4. Carry out MCS to generate the probability density function.
5. Reconstruct an uncertainty model for the stall margin while varying the parameters.
6. Perform multiobjective RBDO by Pareto-optimal analysis.

7. Obtain an optimal compressor design shape.
8. Compare the objectives of RBDO with the results of the initial shape.

3.1 Objective function and constraints

To construct an objective function for compressor RBDO considering its aerodynamics and structure, the features of each discipline are required to be combined. Therefore, the objective function is made up of multiobjective function that consists of two factors from the aerodynamics and the structure. The factor from the aerodynamic discipline is η of the entire compressor system, and the factor from the structural discipline is W of the rotors and the stators. This multiobjective function is set to maximize the efficiency and minimize the weight. It is defined as Eq. (4).

$$f = f(\eta, W) = w_1 \cdot \frac{\eta}{\eta_0} + w_2 \cdot \frac{W_0}{W} , \tag{4}$$

where $w_1 + w_2 = 1$.

In this equation, W takes a contrasting reciprocal form to maximize the objective function and w_1 and w_2 are weighting factors of η and W.

There are three constraints in this problem (Table 2). Safety factor is defined as the ratio of a maximum yield stress (σ_Y) and a calculated maximum stress (σ) as shown in Eq. (5) [13-14] and has to be more than 1.50 [27].

$$SF = \frac{\sigma_Y}{\sigma} . \tag{5}$$

In Eq. (5), minimization of the stress is equivalent to maximization of the safety factor. Stall margin is defined by calculation of the mass flow rate and the pressure ratio at the stall point and the initial point [17, 28]. The stall margin must exceed 1.10 and it is defined as [17]:

$$SM = \left[\frac{PR_{stall} \times \dot{m}_{ref}}{PR_{ref} \times \dot{m}_{stall}} - 1 \right] \times 100 , \tag{6}$$

where the subscript, *ref,* means the value of the initial point. Finally, the pressure ratio of the operation condition is set to be more than 1.82 [17]. These multiobjective functions and the three constraints are used in RBDO of the given compressor.

Aerodynamic analysis is performed using a commercial turbomachinery design and analysis tool Axial™ [29, 30]. Axial™ is a part of the agile engineering design systems from Concepts NREC. Developed for nearly 20 years, it can be applied to calculate efficiency, pressure ratio and flow conditions of the operation point and the stall point.

Structural analyses of the compressor such as calculation of weight and safety factor are performed using the commercial tool ANSYS.

Table 2. Constraints of the compressor MDO.

Constraint	Safety Factor (*SF*) ≥ 1.50 [27]
	Stall Margin (*SM*) ≥ 1.10 [17]
	Pressure Ratio (PR) ≥ 1.82 [17]

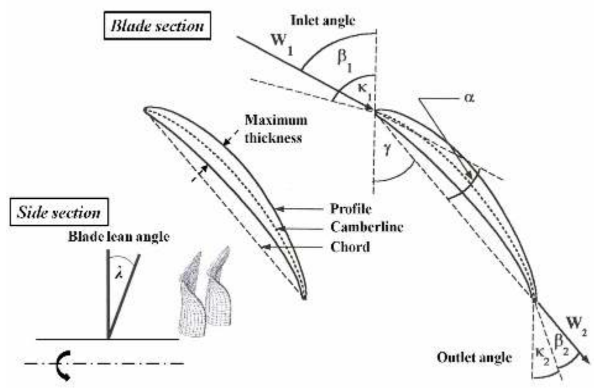

Fig. 3. Design variables of the rotor and stator.

3.2 Definition of design variables and sensitivity analysis

Generally, the blade shape of the rotor and stator has a decisive effect on the performance and the weight of the compressor. Therefore, the majority of adopted design variables are universally concerned with the geometry of the blade of the rotor and stator. The geometry variables of the rotor and stator itself (e.g., the lean angle) are also important in the design of the compressor. Fig. 3 shows the design variables applied in this study: the inlet angle, the outlet angle, the chord length, the maximum thickness and the blade lean angle of the rotor and stator. The chord length and the maximum thickness ratio are divided into three sections: the tip, the hub and the meanline. Additionally, the solidity of the rotor and stator is adopted in the design optimization process. As a result, there are 22 design variables. The initial value of the design variables originated from the NASA stage 37 compressor.

In this study, experiment points for DOE are extracted in ANN process. However, the number of experiment points can increase with the number of design variables, and it requires additional calculation time. Therefore, it is necessary to select some major design variables that have a great effect on the objective function for the efficient RBDO of compressor. This process is known as sensitivity analysis of the design variables. Sensitivity analysis is done by confirming the tendency of the objective function according to the changes of each design variable. The initial values of the design variables are set from the NASA stage 37 compressor. The gaps between the minimum and the maximum values of the design variables are termed the design space. It is essential to verify the influence of each design variable on the objective function in the design space for the sensitivity analysis. This is done for 22 design variables as suggested in the early stage of this paper using the commercial process integration and design optimization (PIDO) tool PIAnO [31].

Table 3. Design space of the selected design variable.

Design variable	Lower Boundary	Baseline (Initial value)	Upper Boundary
β_{1R}	-58.3361	-57.6361	-56.2361
β_{2R}	-44.8185	-43.3185	-41.8185
t_R	0.0342	0.0518	0.0647
β_{1S}	40.2020	44.6689	48.2424
t_S	0.0400	0.0606	0.0758

According to the result of the sensitivity analysis, five major design variables are verified, compiled in Table 3, to affect the multiobjective function more than 2% with more than 1% influence of each factor of the objective function at the same time. They are the inlet angle of the rotor/stator, the outlet angle of the rotor and the maximum thickness ratio of the rotor/stator. Table 3 represents details of these variables.

3.3 Design of experiment and approximation model

The approximation model needs data that can reproduce the behaviors of the objective functions and the constraints in the entire given design space. It is necessary to calculate the objective function at properly distributed experiment points in the design space to acquire the data for a reproduction of the design space. This process is known as the abstraction of the experiment points in design optimization, as this is similar to the correcting process with data from real experiments in what is known as DOE method. The simplest method of DOE is a full factorial method that extracts a large number of experiment points to make the reproduction of the actual design space more likely. If the full factorial method is adapted in the design optimization with five design variables, the number of experiment points required for DOE is 2^5 (2k full factorial) or 3^5 (3k full factorial). It may be difficult to obtain data of hundreds of experiment points due to the increased computational cost. Thus, the D-Optimal method is applied to the approximation model of ANN with fewer experiment points [13].

The D-Optimal method requires experiment points that exceed $(n+1)(n+2)/2$ to construct an approximation model for n design variables. Therefore, the number of experiment points required for five design variables must exceed only 21. In accordance with this requirement of the D-Optimal method, 33 experiment points are chosen for the construction of the more accurate and efficient approximation model. The approximate model of ANN for RBDO is formulated by applying experiment points which are extruded by the D-Optimal method. The approximation model of ANN consists of an input layer with five chosen design variables, a hidden layer with 20 nodes and an output layer with seven responses (efficiency, weight, pressure ratio, safety factor, and pressure ratio at the operating point, mass flow rate, and pressure ratio at the stall point). The accuracy of the approximation model can be determined by calculating of the coefficient of determination (R^2) and the root mean square error (RMSE) [16].

These results are presented in Table 4. As shown, all values

Table 4. Accuracy of the ANN approximation model.

	η	W	SF	SM	PR
R^2	0.9999	0.9999	0.9999	0.9998	0.9999
RMSE	0.004028	0.001498	0.004405	0.005437	0.001234

Table 5. Propagated values of variations for the uncertainty model.

Propagated variables	$PR_{initial}$	PR_{stall}	$\dot{m}_{stall}$
Variation value	0.0249	0.0109	0.0136

of R^2 exceed 0.999; therefore, the approximation model in this study accurately represents the actual design space of the compressor, highlighting its reliability.

3.4 RBDO procedure with uncertainty model

An uncertainty model for the stall margin calculation is applied to RBDO. It is necessary to detect the stall point and determine the flow conditions at the stall point to calculate stall margin as shown in Eq. (6). The flow conditions at the stall point for the stall margin calculation are computed by Axial™. The mass flow rate and the pressure ratio at the stall point are calculated using an iterative method with the mass flow rate decreasing from the design point to the stall point in Axial™. As a result, the value of the mass flow rate and the pressure ratio at the stall point in Axial™ is not a deterministic solution but is a highly adjacent value of the exact solution. By the same token, the pressure ratio at the initial design point is also computed using the iterative method in Axial™. Moreover, as in most commercial tools, Axial™ supplies a conservative stable design or analysis result, and the margin values tend to be calculated as smaller value than the experiment result or the analytic solution. This property enables more stable design of the compressor, but it may have some limitations if it is used to determine the exact performance metrics of the compressor. Therefore, it is possible to overdesign. For this reason, the uncertainty model for stall margin is adopted for the mass flow rate and pressure ratio at the stall point and the mass flow rate at the operating point to minimize difference between the experimental result and the numerical result.

To construct the uncertainty model for stall margin, it is assumed that the stall margin can be influenced by changing the variables in Eq. (6). It is assumed that values of the pressure ratio and the mass flow rate in Eq. (6) are statistical models with a normal distribution and that variations of these values are calculated by deviations of them. The propagated values of each variable for the uncertainty model are listed in Table 5. The P_f of stall margin can be calculated by the integral of the joint probability functions in Eq. (2), like Eq. (7).

$$P_f = P\left(SM\left(X\right) \le 1.1\right) = \int_{SM\left(X\right)\le 1.1} f\left(x\right)dx\,. \tag{7}$$

The requirement of p_j in Eq. (1) is set to 5%, implying reliability of 2σ for the stall margin calculation. The results of P_f

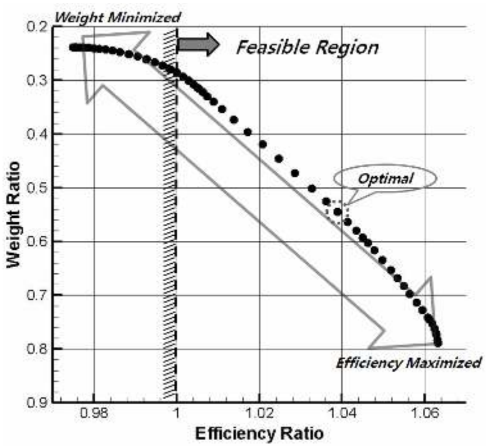

Fig. 4. Pareto front of DO.

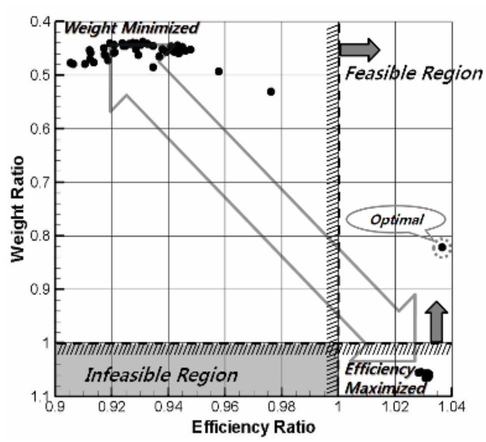

Fig. 5. Pareto front of RBDO.

in DO and RBDO are compared in the next chapter.

4. Results and dscussion

First, the Pareto fronts of deterministic optimization (DO) and RBDO are analyzed and compared to confirm the tendencies of the factors objective function. The Pareto fronts of the factors which provide variety to the weighting factors in the DO process are presented in Fig. 4. The weighting factor w_1 is oriented to η, and the weighting factor of W, which is w_2, is 1-w_1. The Pareto solutions change according to the change of w_1 from 0 to 1 and w_2 from 1 to 0 at the same time. The values of the axis are non-dimensional values of the each factor of the objective function about the initial value. As shown in Fig. 4, there is an inverse proportion between η and W. The weighting factor of W, w_2, increases toward the left side of the graph and there is a weight-minimized region. In contrast, the weighting factor of η, w_1, increases toward the right side and there is efficiency-maximized region.

The feasible region of this Pareto front is comprised of the regions where the non-dimensional value of η is higher than 1.0 and where those of W are lower than 1.0. However, all values of W in this figure do not exceed 1.0; therefore, the constraint is limited to η. When selecting the optimal point, bias of the improvement to one side of the objective function cannot be allowed, and the improvement in the factors of the objective function can be achieved simultaneously. The optimal point of DO is selected at the point where the weighting factors of η and W are 0.9 and 0.1, respectively. The efficiency of this point is set as close to the optimal result of RBDO.

The Pareto front of RBDO is represented in Fig. 5. Similar to the Pareto front of DO, the left side is the weight-minimized region, and the right side is the efficiency-maximized region. The values of axis are also non-dimensional values about the initial values. The infeasible regions are those that are less than 1.0 for efficiency and over 1.0 for weight and they are drawn in the graph. The detected tendency of the Pareto front is that efficiency and weight are in inverse proportion to each other and that the optimal point is located in the feasible region of both factors. The weighting factors of the multiobjective function are 0.88 for efficiency and 0.12 for weight, which are the similar values noted with the DO results. This indicates that weight is a more oscillatory factor than efficiency. Therefore, as suggested by Fig. 4 and Fig. 5, the results of DO and RBDO show a good tendency for MDO with interdependence in the compressor design optimization.

Table 6. Comparison of design optimization results.

Variable	Initial	DO	RBDO
β_{1R}	-57.6361	-57.8737	-57.5404
β_{2R}	-43.3185	-44.1446	-44.6944
t_R	0.0518	0.0390	0.0508
β_{1S}	44.6689	48.1345	45.5895
t_S	0.0606	0.0729	0.0664
η	0.8403	0.8730	0.8711
W	2.0003	1.0907	1.6428
PR	2.0504	2.0713	1.9958
SM	1.0792	1.1001	1.1272
P_f *of SM*	n/a	43.03%	5.00%

The approximation model of RBDO by ANN is constructed to define the relationship between the design variables and the objective function. This approximation model is applied to DO that excludes consideration of the uncertainty in the compressor design process. These results are then compared with the results of RBDO, which includes the uncertainty of the stall calculation. The comparison of the initial values and each optimized design results is listed in Table 6. As shown in Table 6, the values of η, W, f and pressure ratio by DO are improved in comparison with the results of the initial shape. However, stall margin is calculated as 1.1001, indicating a 10.01% margin of the stall point by DO.

This is slightly higher than the reference value of 1.10 in Table 2. In addition, P_f for stall margin by DO is surprisingly at 43.03%; this is too high to apply to the design of compressor. Therefore, some supplementations are necessary to improve the reliability of the DO result. According to this re-

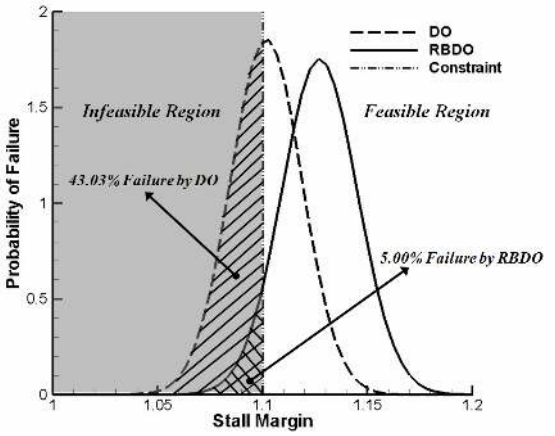

Fig. 6. Probability of the failure distribution of stall margin.

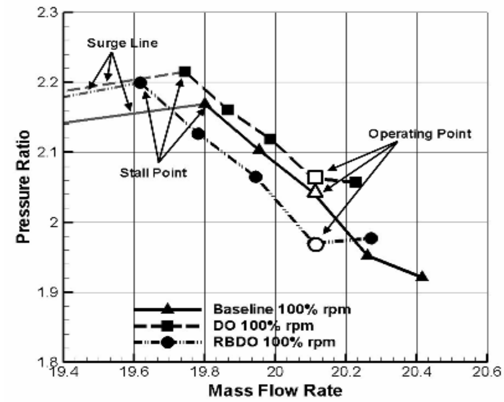

Fig. 7. Performance of the three compressors.

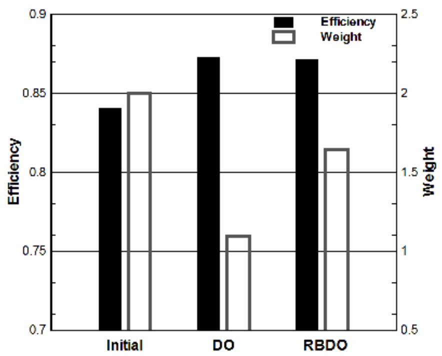

Fig. 8. Comparison of the objective functions of the three compressors.

quirement, RBDO, which enables reliable design optimization, is conducted in this study. It is carried out with an uncertainty model with flow condition variables which appear as random variables in the stall margin calculation.

RBDO of the compressor is done up to the point of the stall margin convergence by MCS for data gathering using the approximation model in DO process. RBDO is carried out while applying the uncertainty model with three random variables: the pressure ratio and the mass flow rate at the stall point and the pressure ratio at the initial point. The reliability of RBDO is set to k=2, indicating 2σ (95%) reliability. In other words, the P_f value of stall margin is set to be lower than 5% by RBDO. As a result of the RBDO process, stall margin is 1.1272, signifying a 12.72% margin with 5% P_f , as shown in Table 6. This value is sufficiently higher than the reference value of 10%. The P_f distributions of stall margin by DO and RBDO are presented in Fig. 6. The horizontal axis is the calculated stall margin and the vertical axis is the P_f values. The graphs of P_f show normal distributions with a constraint at 1.10, and the feasible region of RBDO is much larger than that of DO. This figure shows that the reliability of stall margin is improved by RBDO in comparison with DO due to the decrease of P_f from 43.03% in DO to 5.00% in RBDO. This is considerable improvement, and stall margin of RBDO can satisfy the constraint value with more than 95% success. Therefore, RBDO with the uncertainty model using random variables for the stall margin calculation can provide a more reliable result in the design optimization of the compressor by improving both the value of stall margin and the probability of failure for stall margin. This may be a proper supplementation of the commercial compressor design program Axial™.

Performances of the NASA stage 37 compressor and the optimally designed compressors are compared in a performance map according to the mass flow rate in Fig. 7. In this case, the rotational speed is set to 100% rpm of the operating rotational speed in Table 1, and the pressure ratios (PR) determined by Axial^{TM} according to the mass flow rates are presented. As shown in Fig. 7, the DO and RBDO results show a similar tendency to the initial shape. It is verified that the surge lines of both DO and RBDO rise from the initial value as shown in Fig. 7. The operating point is set to the mass flow rate at 20.1 kg/s, and the pressure ratios of the results at this point are located in the order of size: DO, the initial values and RBDO, representing an identical result to that shown in Table 6. Moreover, in the RBDO case, the length of the performance curve from the operating point to the stall point is the longest. This is related to the value of stall margin in RBDO and proved by the stall margin results in Table 6.

As shown in Table 6, the stall margin of RBDO is very good compared with not only the initial point but also DO. However, η, W and pressure ratio values of RBDO show deteriorating results in comparison with the result of DO. Nevertheless, the decrease of η is only 0.22%, which may be manageable for the compressor design optimization with a significant improvement in its stall margin reliability.

And W of RBDO is much improved in comparison with the initial value, though it is worse than DO. Additionally, the pressure ratio of RBDO is higher than 1.5, the reference value in Table 2, though it is lower than DO. The factors of objective function, η and W of the initial shape, DO, and RBDO are compared in Fig. 8. The result of DO shows a considerable improvement of both η and W. However, the probability of failure for stall margin in DO is over 40%, which is much greater than that in RBDO. It means that these results can be certain under only 60% of the operating conditions. Therefore,

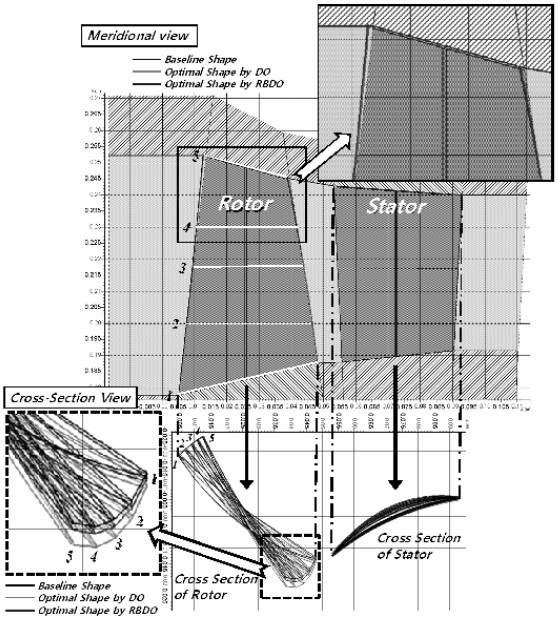

Fig. 9. Configurations of the three compressors.

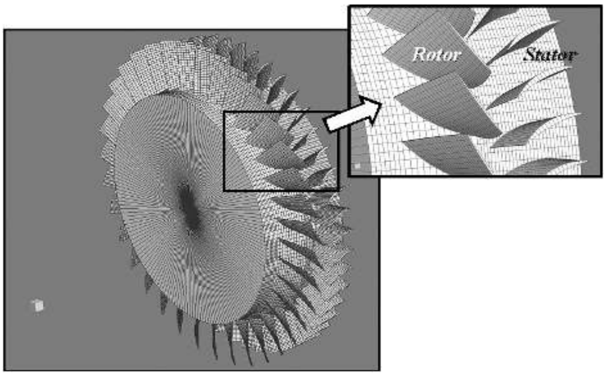

Fig. 10. 3D view of RBDO result.

RBDO is applied to construct more realistic and efficient design optimization method of the compressor.

Finally, the optimal design shapes of DO and RBDO are presented in Fig. 9 with the baseline shape of the NASA stage 37 compressor. These three shapes of the compressor show little differences in terms of their meridional view, but the shapes at the tip show a notable difference, as shown in the enlarged section in the upper-right corner of Fig. 9. Five cross-sections along the radial direction of the rotor and stator are drawn at the bottom of Fig. 9. The numbers (1~5) are given from the hub to the tip of the rotor, showing the twist of the rotor. As shown in Table 6 and Fig. 9, the rotor exit angle of DO is the smallest in the results. The efficiency of DO is greater than that of the baseline and RBDO because the shape of DO shows considerable change from the initial baseline shape that exceeds that of the RBDO result. The maximum thickness ratio of the rotor is related to the weight of compressor; the DO and RBDO results have a lower value than the initial value, as shown in the expanded in the lower left area of the figure. In particular, the weight of the DO process shows the smallest value in Table 6, while the maximum thickness ratio of the rotor in the DO result is also the smallest.

The full 3D shape of the compressor stage by RBDO is presented at the left side in Fig. 10, and blown-up images of the rotors and stators are drawn on the right using the meridional and cross-section views.

5. Conclusions

Reliability-based design optimization (RBDO) of the NASA stage 37 transonic axial compressor was performed by applying an uncertainty model to enhance the stall characteristics. It was assumed that uncertainty models of the mass flow rate and the pressure ratio for the stall margin calculation are statistical models with normal distributions. Monte Carlo Simulation (MCS) was carried out to calculate the probability of failure (P_f) of stall margin. Artificial neural network was adopted to determine the efficient performance levels of MCS and RBDO. The result of RBDO for the NASA stage 37 compressor was compared with the result of deterministic optimization (DO) and the initial shape. As shown in the results of this study, RBDO can serve as a good approach for nonlinear and uncertain design problems like a compressor design optimization. Moreover, RBDO can improve the result from the initial shape and make up for the weak points of DO by considering the uncertainty property of the compressor design. The results can be summarized as follows:

(1) The result of DO may cause stall in operation of the compressor with 43.03% probability by disturbance, but the probability of RBDO is only 5%. In other words, RBDO can provide 95% reliability as regards the stall characteristic.

(2) The value of stall margin by RBDO using the uncertainty model is 12.72%, which is greater than both the initial value and DO. Additionally, this is enough marginal to the given constraint value, 10%.

(3) RBDO for the compressor provides 3.67% increased efficiency and 17.87% decreased weight in comparison with the baseline compressor. This is advantageous for an aircraft engine.

In this manner, RBDO with an uncertainty model can be an one of efficient and realistic method of stall margin calculation for an aircraft compressor design.

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Nomenclature

k	:	Sigma level
P_f	:	Probability of failure
PR	:	Pressure ratio
SF	:	Safety factor
SM	:	Stall margin
t_R	:	Maximum thickness ratio of a rotor
t_S	:	Maximum thickness ratio of a stator
W	:	Weight
W_0	:	Weight of the initial shape (baseline)
w_1	:	Weighting factor of the efficiency
w_2	:	Weighting factor of the weight
β_{1R}	:	Inlet angle of a rotor
β_{1S}	:	Inlet angle of a stator
β_{2R}	:	Outlet angle of a stator
η	:	Efficiency
η_0	:	Efficiency of the initial shape (baseline)
λ	:	Blade lean angle
σ	:	Calculated maximum stress of the compressor
σ_Y	:	Yield stress of the compressor

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