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Reliability-Based Multidisciplinary Design Optimization Using Subset Simulation Analysis and Its Application in the Hydraulic Transmission Mechanism Design

The Monte Carlo simulation (MCS) can provide high reliability evaluation accuracy. However, the efficiency of the crude MCS is quite low, in large part because it is computationally expensive to evaluate a very small failure probability. In this paper, a subset simulation-based reliability analysis (SSRA) approach is combined with multidisciplinary design optimization (MDO) to improve the computational efficiency in reliability-based MDO (RBMDO) problems. Furthermore, the sequential optimization and reliability assessment (SORA) approach is utilized to decouple an RBMDO problem into a sequential of deterministic MDO and reliability evaluation problems. The formula of MDO with SSRA within the framework of SORA is proposed to solve a design optimization problem of a hydraulic transmission mechanism. [DOI: 10.1115/1.4029756]

1 Introduction

In multidisciplinary systems, uncertainties will be propagated among coupled disciplines and may cause design solutions to be unsafe [1]. To solve this problem, many methods have been proposed to address the reliability evaluation and optimization issues in MDO, and they are so-called RBMDO [2–20]. In some situations, the extremely high reliability of products is required. To evaluate the reliability accurately, MCS can be employed. MCS is robust and accurate if sufficient samples are used [21,22]. It can solve reliability evaluation problems with different distribution types and high dimensional randomness. However, the crude MCS is not suitable to the case where the failure probability to be evaluated is very small (e.g., $p_f \leq 10^{-7}$). It is because that evaluating failure probability (or reliability) of highly reliable products requires an extremely large number of samples in the crude MCS, leading to the low efficiency.

In this paper, RBMDO problems with rare failure events are investigated. Based on the subset simulation strategy [23], an original reliability evaluation problem of a rare failure event is replaced by a series of reliability evaluation problems of more frequent failure events in conditional probability spaces and initial conditional probability space. The modified Metropolis algorithm is utilized to generate offspring samples of each intermediate failure event. The SORA strategy is also utilized to further improve the computational efficiency.

The paper is organized as follows. In Sec. 2, the RBMDO formulation is given. Existing reliability evaluation methods used in RBMDO are also briefly reviewed. In Sec. 3, the details of SSRA are introduced. In Sec. 4, the procedure of MDO with SSRA within the framework of SORA (called MDO-SSRA-SORA) is proposed. In Sec. 5, the proposed method is implemented to solve a design optimization problem of a hydraulic transmission mechanism. Section 6 concludes the paper.

2 The Simulation and Approximation Methods for the Reliability Evaluation in RBMDO Problems

2.1 The RBMDO Formulation. The mathematical formulation of the RBMDO is given as

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$$\begin{aligned}&\min_{\mathbf{X}_{\mathrm{DV}}} \quad f\left(\mathbf{d}, \boldsymbol{\mu}_{\mathbf{X}}, \boldsymbol{\mu}_{\mathbf{X}_s}, \boldsymbol{\mu}_{\mathbf{Y}}\right)\\ &\text{s.t.} \quad \Pr[g_i(\mathbf{d}, \mathbf{X}, \mathbf{X}_s, \mathbf{Y}) \le 0] \ge [R_i] = 1 - [p_{fi}],\\ &\mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \mathbf{X}^L \le \boldsymbol{\mu}_{\mathbf{X}} \le \mathbf{X}^U, \mathbf{X}_s^L \le \boldsymbol{\mu}_{\mathbf{X}_s} \le \mathbf{X}_s^U,\\ &\mathbf{Y}^L \le \boldsymbol{\mu}_{\mathbf{Y}} \le \mathbf{Y}^U, \mathbf{X}_{\mathrm{DV}} = \left\{\mathbf{d}, \boldsymbol{\mu}_{\mathbf{X}}, \boldsymbol{\mu}_{\mathbf{X}_s}, \boldsymbol{\mu}_{\mathbf{Y}}\right\}, i = 1, 2, \ldots, n\end{aligned} \tag{1}$$

where $f(\bullet)$ denotes a system objective function; $g_i(\bullet) < 0$ denotes the safe region; $g_i(\bullet) > 0$ denotes the failure region, and $g_i(\bullet) = 0$ is defined as the limit state surface which is the boundary between safe and failure regions. $[R_i]$ is the demand reliability for $\Pr[g_i(\bullet) \le 0]$. $[p_{fi}]$ is the acceptable failure probability for $g_i(\bullet) > 0$. $\mathbf{X}_{\mathrm{DV}}$ denotes the vector of design variables. $\mathbf{d}$ denotes the vector of deterministic design variables. $\mathbf{X}$ denotes the vector of the random discipline design variables. $\mathbf{X}_s$ denotes the vector of the random shared design variables. $\mathbf{Y}$ denotes the vector of the linking variables. $\mathbf{Y} = \{\mathbf{Y}_{\bullet i}, \mathbf{Y}_{i\bullet}\}$, $i = 1, 2, \ldots, n$, $\mathbf{Y}_{\bullet i}$ are input linking variables to the i th discipline and $\mathbf{Y}_{i\bullet}$ are output linking variables from the i th discipline. $\boldsymbol{\mu}$ denotes the mean value of random variables. Superscripts L and U denote the lower and upper bounds, respectively. n denotes the total number of disciplines.

2.2 Simulation and Approximation Methods for the Reliability Evaluation. In Eq. (1), the reliability constraint $\Pr(\bullet)$ is used to guarantee the system reliability. We express the functional relationship between performance G and deterministic variables, random variables by $G = g(\mathbf{d}, \mathbf{X}_{\mathrm{R}})$, where $\mathbf{X}_{\mathrm{R}}$ denotes the vector of random design variables, $\mathbf{X}_{\mathrm{R}} = \{\mathbf{X}, \mathbf{X}_s, \mathbf{Y}\}$. Commonly used simulation and approximation methods to evaluate $\Pr(\bullet)$ can be roughly categorized into three types [24]: (1) sampling based methods, (2) moment matching methods, and (3) most probable point (MPP) based methods. Sampling based methods [25–30], such as the crude MCS, Latin hypercube sampling, and importance sampling, are flexible to use and can provide an accurate estimation of probability if sufficient samples are used. However, it is not efficient where the higher reliability is required or performance functions are computationally expensive. For example, 10^{15} samples are required for MCS to estimate a failure probability with 10^{-7} under the precision measure $\delta = 10^{-4}$, where $\delta = \sqrt{\left(1 - [p_f]\right)/\left([p_f]N\right)}$, N is the sample size. The value of δ can be used to reflect the confidence level of numerical results. The less δ we have, the higher confidence level of reliability estimation we obtain. Moment matching methods are usually employed to ease the computational difficulty [31,32], which approximate the distribution of a performance function by fitting its first few moments. Many approaches, such as numerical integrations, point estimate methods [33,34], and Taylor series approximations [35], are proposed to calculate moments. Though moment matching methods are more efficient, it is less accurate than sampling based methods generally [25]. The MPP based method can obtain a good balance between efficiency and accuracy. Typical MPP based methods, such as first order reliability method (FORM) and second order reliability method (SORM) [36–38], approximate a performance function with the Taylor expansion at MPP to ensure the minimal accuracy loss. However, in FORM and SORM, original random variables should be transformed into standard normal variables by Rosenball transformation [39]. The transformation process may increase the nonlinearity of a performance function [40]. To avoid the transformation, first order saddlepoint approximation (FOSA) is proposed [41]. However, an extra optimization problem is introduced to finding most likelihood point in FOSA, which needs more function evaluations [25].

In some extreme working conditions, rare failure events may happen. Thus, accurate reliability estimation for these rare failure events is necessary. However, sometimes it is hard to balance accuracy and efficiency in practical engineering. To improve the efficiency of reliability estimation while sustaining the high accuracy, subset simulation method was proposed [42]. The subset simulation method has been widely used to solve different reliability evaluation problems, such as reliability-based design optimization problems [43,44], reliability benchmark problems [45], and dynamic systems analysis problems [46,47].

3 SSRA Approach

3.1 The Basic Idea of Subset Simulation. The basic idea of subset simulation is that a small probability can be calculated using a product of an initial probability and a series of conditional probabilities with greater values. Then the reliability evaluation problem of rare event can be converted into a series of reliability evaluation problems of an initial event and more frequent conditional events. Use F to denote a target failure event in the random variables space R^n. Denote a decreasing sequence of failure events as $F_1 \supset F_2 \supset \cdots \supset F_m = F$, $F_m = \bigcap_{j=1}^{m} F_j$, where F_j is an intermediate failure event for $j = 2 \sim m$ or an initial failure event for $j = 1$, and m is the number of failure regions. $F_m (= F)$ is the failure event of interest. By the definition of conditional probability, the failure probability can be expressed as a product of a sequence of conditional failure probabilities $\left\{P\left(F_j | F_{j-1}\right) : j = 2, 3, \ldots, m\right\}$ and the initial failure probability $P(F_1)$ as shown in [22]

$$\begin{aligned}P_F = P(F_m) = P\left(\bigcap_{j=1}^{m} F_j\right) = P(F_m | F_{m-1}) P\left(\bigcap_{j=1}^{m-1} F_j\right) = \cdots\\ = P(F_1) \prod_{j=2}^{m} P\left(F_j | F_{j-1}\right)\end{aligned} \tag{2}$$

3.2 Markov Chain Monte Carlo and the Modified Metropolis Algorithm. Markov chain Monte Carlo (MCMC) is a powerful tool to generate random samples and can be used in calculating statistical estimation and marginal and conditional probabilities [48]. MCMC has been used in Bayesian updating of structural models and reliability [49], estimation of small failure probabilities [22], and probabilistic inference [50]. As a specific implementation of MCMC, the Metropolis algorithm can simulate samples as the states of Markov chain which has the target distribution as its limiting stationary distribution under the assumption of ergodicity [22,47]. Use p to denote a probability density function (PDF). The significance of the Metropolis algorithm is that if a sample distributes as the conditional distribution $p\left(\bullet | F_j\right)$, a new offspring sample can be generated as the next state of the Markov chain which will also be distributed as $p\left(\bullet | F_j\right)$. However, the Metropolis algorithm is not applicable in high dimensional space. It is because a zero acceptance ratio for the next candidate state results in extremely repeated samples in Markov Chain [42]. To solve this problem, a modified Metropolis algorithm is proposed [22]. In the modified Metropolis algorithm, a group of one-dimensional proposal PDFs are used, instead of an n-dimensional proposal PDF which is used in Metropolis algorithm. Thus, the acceptance ratio of individual sample can remain nonvanishing in spite of the increasing of dimension [22]. The details of the modified Metropolis algorithm are given as follows.

For every $t = 1, \ldots, N_j$, let $p_j^*\left(\xi | \mathrm{X}_{\mathrm{R},j}(t)\right)$, called the proposal PDF, be a one-dimensional PDF for ξ centered at $\mathrm{X}_{\mathrm{R},j}(t)$ with the symmetry property $p_j^*\left(\xi_{j+1}(t) | \mathrm{X}_{\mathrm{R},j}(t)\right) = p_j^*\left(\mathrm{X}_{\mathrm{R},j}(t) | \xi_{j+1}(t)\right)$. Generate a sequence of samples $\left\{\mathbf{X}_{\mathrm{R},1}, \mathbf{X}_{\mathrm{R},2}, \ldots\right\}$ from a given sample $\mathbf{X}_{\mathrm{R},1}$ by computing $\mathbf{X}_{\mathrm{R},j+1}$ from $\mathbf{X}_{\mathrm{R},j} = \left[\mathbf{X}_{\mathrm{R},j}(1), \mathbf{X}_{\mathrm{R},j}(2), \ldots, \mathbf{X}_{\mathrm{R},j}\left(N_j\right)\right]$, $j = 1, 2, \cdots$. This process includes two steps.

Step 1: Generate a candidate state $\tilde{\mathbf{X}}_{\mathrm{R},j+1}$: For each component $\mathrm{X}_{\mathrm{R},j}(t)$, $t = 1, \ldots, N_j$, simulate ξ_{j+1} from $p_j^*\left(\xi_{j+1}(t) | \mathrm{X}_{\mathrm{R},j}(t)\right)$. Calculate the acceptance ratio

$$r_{j+1}(t) = p\left(\xi_{j+1}(t)\right) / p\left(\mathrm{X}_{\mathrm{R},j}(t)\right) \tag{3}$$

Set the t th component of $\tilde{\mathbf{X}}_{R,j+1}$ according to

$$\tilde{X}_{R,j+1}(t) = \begin{cases} \xi_{j+1}(t) \text{ with probability } \min(1, r_{j+1}(t)) \\ X_{R,j}(t) \text{ with probability } 1 - \min(1, r_{j+1}(t)) \end{cases} \quad (4)$$

Step 2: Accept/reject $\tilde{\mathbf{X}}_{R,j+1}$: Check the location of $\xi_{j+1}(t)$. If $\tilde{\mathbf{X}}_{R,j+1} \in F_j$, accept it as the next sample, $\mathbf{X}_{R,j+1} = \tilde{\mathbf{X}}_{R,j+1}$; otherwise reject it and take the current sample $\mathbf{X}_{R,j}$ as the next sample, $\mathbf{X}_{R,j+1} = \mathbf{X}_{R,j}$.

Based on experience in Ref. [42], the PDF of the uniform distribution, which is centered at the current sample with width equal to two times of the standard deviation of last simulation level, is a good candidate as the proposal PDF.

3.3 Subset Simulation Procedure. By using the modified Metropolis algorithm, the subset simulation proceeds as follows.

Step 1: Evaluate the initial failure probability $P(F_1)$ at the first simulation level by MCS as

$$P(F_1) \approx \hat{P}_1 = \frac{1}{N_1}\sum_{k=1}^{N_1} I_{F_1}(\mathbf{X}_{R,1}) \quad (5)$$

where $\hat{P}_1$ is the estimator; N_1 is the sample size; $I_{F_1}(\bullet)$ is an indicator function, $I_{F_1}(\mathbf{X}_{R,1}) = 1$ if $\mathbf{X}_{R,1} \in F_1$ and $I_{F_1}(\mathbf{X}_{R,1}) = 0$ otherwise; $\mathbf{X}_{R,1}$ is a vector of samples $X_{R,1}(t)$ which are independently and identically distributed according to $p(X_{R,1})$ and present an uncertain state of the system, $\mathbf{X}_{R,1} = \{X_{R,1}(t) : t = 1, \ldots, N_1\}$.

Step 2: Evaluate the conditional failure probabilities at each simulation level by MCMC based on the modified Metropolis algorithm. In the MCMC, the "seeds" samples of subset $j+1$ are from the samples which are in subset j and belong to the failure event F_j. Then the conditional failure probabilities can be calculated by

$$P(F_{j+1}|F_j) \approx \hat{P}_{j+1} = \frac{1}{N_{j+1}}\sum_{k=1}^{N_{j+1}} I_{F_{j+1}}(\mathbf{X}_{R,j+1}) \quad (6)$$

where the conditional PDF of $\mathbf{X}_{R,j+1}$ is $p(X_{R,j+1}|F_j)$.

Step 3: Finally, combining Eqs. (2), (5), and (6), the failure probability estimator is

$$P_F \approx \hat{P}_F = \prod_{j=1}^{m} \hat{P}_j \quad (7)$$

4 MDO Within the Framework of SORA

4.1 The Strategy of SORA. The strategy of SORA is developed to improve the optimization efficiency of RBMDO [51,52]. SORA employs a sequential of cycles of MDO and reliability evaluation. In each cycle, the reliability evaluation is conducted after MDO [51]. The procedure of SORA is illustrated in Fig. 1.

4.2 The Procedure of MDO-SSRA-SORA. In this section, we combine SSRA with MDO within the strategy of SORA. The detailed procedure of MDO-SSRA-SORA is given as follows.

Step 1: Solve an MDO problem with deterministic constraint $G = g(\mathbf{d}, \boldsymbol{\mu}_{\mathbf{X}_R}) \le 0$. In this paper, the engineering design optimization problem is considered as low couplings. So we can use the collaborative optimization (CO) method here. As a hierarchical MDO method, CO has a system optimization problem and discipline optimization problems. The system optimization problem minimizes the system objective f while satisfying the compatibility constraints J_i. The discipline optimization problems use compatibility constraints J_i as discipline objectives and minimize them while satisfying the discipline constraints. The discipline constraints are the original constraints in Eq. (1). The system optimization problem is given as

$$\begin{aligned} &\min_{\mathbf{X}_{DV}} \quad f\left(\mathbf{d}^{sys}, \boldsymbol{\mu}_{\mathbf{X}^{sys}}, \boldsymbol{\mu}_{\mathbf{X}_s^{sys}}, \boldsymbol{\mu}_{\mathbf{Y}^{sys}}\right) \\ &\text{s.t.} \quad J_i = \left(\mathbf{d}^{sys} - \mathbf{d}^{dis,i}\right)^2 + \left(\boldsymbol{\mu}_{\mathbf{X}^{sys}} - \boldsymbol{\mu}_{\mathbf{X}_i^{dis}}\right)^2 \\ &\qquad + \left(\boldsymbol{\mu}_{\mathbf{X}_s^{sys}} - \boldsymbol{\mu}_{\mathbf{X}_s^{dis}}\right)^2 + \left(\boldsymbol{\mu}_{\mathbf{Y}^{sys}} - \boldsymbol{\mu}_{\mathbf{Y}^{dis}}\right)^2 \le \varepsilon, \\ &\mathbf{X}_{DV} = \left\{\mathbf{d}^{sys}, \boldsymbol{\mu}_{\mathbf{X}^{sys}}, \boldsymbol{\mu}_{\mathbf{X}_s^{sys}}, \boldsymbol{\mu}_{\mathbf{Y}^{sys}}\right\}, \quad i = 1, 2, \ldots, n \end{aligned} \quad (8)$$

The discipline optimization problems are given as

$$\begin{aligned} &\min_{\mathbf{X}_{DV}} \quad J_i = \left(\mathbf{d}^{sys} - \mathbf{d}^{dis,i}\right)^2 + \left(\boldsymbol{\mu}_{\mathbf{X}^{sys}} - \boldsymbol{\mu}_{\mathbf{X}_i^{dis}}\right)^2 + \left(\boldsymbol{\mu}_{\mathbf{X}_s^{sys}} - \boldsymbol{\mu}_{\mathbf{X}_s^{dis}}\right)^2 + \left(\boldsymbol{\mu}_{\mathbf{Y}^{sys}} - \boldsymbol{\mu}_{\mathbf{Y}^{dis}}\right)^2 \\ &\text{s.t.} \quad g_i\left(\mathbf{d}_i^{dis}, \boldsymbol{\mu}_{\mathbf{X}_i^{dis}}, \boldsymbol{\mu}_{\mathbf{X}_s^{dis}}, \boldsymbol{\mu}_{\mathbf{Y}^{dis}}\right) \le 0, \mathbf{d}^L \le \mathbf{d}_i^{dis} \le \mathbf{d}^U, \mathbf{X}^L \le \boldsymbol{\mu}_{\mathbf{X}_i^{dis}} \le \mathbf{X}^U, \\ &\quad \mathbf{X}_s^L \le \boldsymbol{\mu}_{\mathbf{X}_s^{dis}} \le \mathbf{X}_s^U, \mathbf{Y}^L \le \boldsymbol{\mu}_{\mathbf{Y}^{dis}} \le \mathbf{Y}^U, \mathbf{X}_{DV} = \left\{\mathbf{d}_i^{dis}, \boldsymbol{\mu}_{\mathbf{X}_i^{dis}}, \boldsymbol{\mu}_{\mathbf{X}_s^{dis}}, \boldsymbol{\mu}_{\mathbf{Y}^{dis}}\right\}, \quad i = 1, 2, \ldots, n \end{aligned} \quad (9)$$

where superscripts "sys" and "dis" denote system and discipline, respectively. The solutions $\mathbf{d}^{(1)}$ and $\boldsymbol{\mu}_{\mathbf{X}_R}^{(1)}$ can be obtained from Eq. (8) when the optimization problems in Eqs. (8) and (9) converge. The superscript (1) denotes the first cycle, $\boldsymbol{\mu}_{\mathbf{X}_R}^{(1)} = \{\boldsymbol{\mu}_{\mathbf{X}}^{(1)}, \boldsymbol{\mu}_{\mathbf{X}_s}^{(1)}, \boldsymbol{\mu}_{\mathbf{Y}}^{(1)}\}$.

Step 2: Run MCS to evaluate the probability of $G = g(\mathbf{d}, \mathbf{X}_R) \le 0$. The initial reliability of $G = g(\mathbf{d}, \mathbf{X}_R) \le 0$ is only around 0.5 because uncertainty is not considered. Thus MCS can be applied directly in the fist cycle. Suppose there are N simulation samples. The failure probability estimator is $P(F) \approx \hat{P}_F = \frac{1}{N}\sum_{t=1}^{N} I_F(X_R(t))$, where $I_F(\bullet)$ is an indicator function, $I_F(X_R) = 1$ if $X_R \in F$ and $I_F(X_R) = 0$ otherwise. And then all simulation samples in an ascending order are listed according to the performance values of $G(\mathbf{d}^{(1)}, \mathbf{X}_R^{(1)})$, i.e., $G(\mathbf{d}^{(1)}, \mathbf{X}_{R1}^{(1)}) < G(\mathbf{d}^{(1)}, \mathbf{X}_{R2}^{(1)}) < \cdots < G(\mathbf{d}^{(1)}, \mathbf{X}_{RN}^{(1)})$.

Find the smallest value $G_s = \min\{G(\mathbf{d}^{(1)}, \mathbf{X}_{Ri}^{(1)}) | i = \text{int}(R \times N)\}$ in the sequence $G(\mathbf{d}^{(1)}, \mathbf{X}_{R1}^{(1)}) < G(\mathbf{d}^{(1)}, \mathbf{X}_{R2}^{(1)}) < \cdots < G(\mathbf{d}^{(1)}, \mathbf{X}_{RN}^{(1)})$ to obtain a simulation MPP (SMPP), $\mathbf{X}_{SMPP} = \{\mathbf{X}_{Ri} | g(\mathbf{d}, \mathbf{X}_{Ri}) = G_s\}$. The notation of $\text{int}(R \times N)$ is the integer part of number $(R \times N)$. Considering the randomness of computer simulation, the simulation can run n times and SMPP can be decided by the average results.

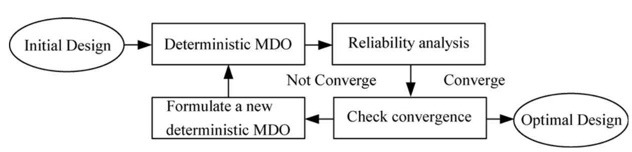

Fig. 1 The procedure of SORA

Step 3: In the first cycle, the failure probability is much larger than the acceptable failure probability. Therefore, a shifting vector $\mathbf{S}^{(1)}$, $\mathbf{S}^{(1)} = \boldsymbol{\mu}_{\mathbf{X}_\mathrm{R}}^{(1)} - \mathbf{X}_\mathrm{SMPP}^{(1)}$, is constructed to move G_s to the safe region, which is shown in Fig. 2. $\mathbf{S} = \{\mathbf{s}_\mathbf{X}, \mathbf{s}_{\mathbf{X}_\mathrm{s}}, \mathbf{s}_\mathbf{Y}\}$.

Step 4: Use CO to solve MDO with shifted constraints. The system optimization problem is shown in Eq. (8), and the discipline optimization problems are shown in Eq. (10). And then we obtain solutions of the k th cycle $\mathbf{d}^{(k)}$, $\boldsymbol{\mu}_{\mathbf{X}_\mathrm{R}}^{(k)}$.

$$\begin{aligned}
\min_{\mathbf{X}_\mathrm{DV}} J_i &= \left(\mathbf{d}_i^{\mathrm{sys},(k)} - \mathbf{d}_i^{\mathrm{dis},(k)}\right)^2 + \left(\boldsymbol{\mu}_{\mathbf{X}_i^\mathrm{sys}}^{(k)} - \boldsymbol{\mu}_{\mathbf{X}_i^\mathrm{dis}}^{(k)}\right)^2 \\
&\quad + \left(\boldsymbol{\mu}_{\mathbf{X}_\mathrm{s}^\mathrm{sys}}^{(k)} - \boldsymbol{\mu}_{\mathbf{X}_\mathrm{s}^\mathrm{dis}}^{(k)}\right)^2 + \left(\boldsymbol{\mu}_{\mathbf{Y}^\mathrm{sys}}^{(k)} - \boldsymbol{\mu}_{\mathbf{Y}^\mathrm{dis}}^{(k)}\right)^2 \\
\text{s.t. } & g_i\left(\mathbf{d}_i^{\mathrm{dis},(k)}, \boldsymbol{\mu}_{\mathbf{X}_i^\mathrm{dis}}^{(k)} - \mathbf{s}_{\mathbf{X}_i^\mathrm{dis}}^{(k)}, \boldsymbol{\mu}_{\mathbf{X}_\mathrm{s}^\mathrm{dis}}^{(k)} - \mathbf{s}_{\mathbf{X}_\mathrm{s}^\mathrm{dis}}^{(k)}, \boldsymbol{\mu}_{\mathbf{Y}^\mathrm{dis}}^{(k)} - \mathbf{s}_{\mathbf{Y}^\mathrm{dis}}^{(k)}\right) \le 0, \\
& \mathbf{d}^L \le \mathbf{d}_i^{\mathrm{dis},(k)} \le \mathbf{d}^U, \\
& \mathbf{X}^L \le \boldsymbol{\mu}_{\mathbf{X}_i^\mathrm{dis}}^{(k)} \le \mathbf{X}^U, \mathbf{X}_\mathrm{s}^L \le \boldsymbol{\mu}_{\mathbf{X}_\mathrm{s}^\mathrm{dis}}^{(k)} \le \mathbf{X}_\mathrm{s}^U, \mathbf{Y}^L \le \boldsymbol{\mu}_{\mathbf{Y}^\mathrm{dis}}^{(k)} \le \mathbf{Y}^U, \\
& \mathbf{X}_\mathrm{DV} = \left\{\mathbf{d}_i^{\mathrm{dis},(k)}, \boldsymbol{\mu}_{\mathbf{X}_i^\mathrm{dis}}^{(k)}, \boldsymbol{\mu}_{\mathbf{X}_\mathrm{s}^\mathrm{dis}}^{(k)}, \boldsymbol{\mu}_{\mathbf{Y}^\mathrm{dis}}^{(k)}\right\}, \quad i = 1, 2, \ldots, n
\end{aligned} \tag{10}$$

Step 5: Define an initial failure event and $m-1$ conditional failure events $F_j = \left\{\mathbf{X}_\mathrm{R}^{(k)} : g\left(\mathbf{d}, \mathbf{X}_\mathrm{R}^{(k)}\right) > G_j, j = 1, 2, \ldots, m-1\right\}$. To ease the simulation, m can be equated to the magnitude index of acceptable failure probabilistic. Denote a failure event F_m as $F_m = \{\mathbf{X}_\mathrm{R} : g(\mathbf{d}, \mathbf{X}_\mathrm{R}) > 0\}$. The probability of $P_F^{(k)} = \Pr\left[g\left(\mathbf{d}^{(k)}, \mathbf{X}_\mathrm{R}^{(k)}\right) > 0\right]$ is calculated by $P_F^{(k)} = P^{(k)}(F_1)\prod_{j=1}^{m-1} P^{(k)}\left(F_{j+1}|F_j\right)$; The conditional failure probabilities $P^{(k)}\left(F_{j+1}|F_j\right)$ are calculated by $P\left(F_{j+1}|F_j\right) \approx \hat{P}_{j+1} = (1/N_{j+1})\sum_{k=1}^{N_{j+1}} I_{F_{j+1}}\left(\mathbf{X}_{\mathrm{R},j+1}\right)$; and the first failure probability $P^{(k)}(F_1)$ is calculated by MCS directly.

Step 6: Once $P_F^{(k)}$ is obtained, it is compared with $\left[p_f\right]$. If $P_F^{(k)} < \left[p_f\right]$ and the value of objective is stable, the algorithm converges; if $P_F^{(k)} \ge \left[p_f\right]$, define the desired final conditional failure probability as

$$\begin{aligned}
P_\mathrm{desire}^{(k)}(F_m|F_{m-1}) &= \frac{\left[p_f\right]}{P^{(k)}(F_{m-1})} = \frac{\left[p_f\right]}{P^{(k)}(F_1)\prod\limits_{j=1}^{m-1} P^{(k)}\left(F_{j+1}|F_j\right)} \\
&= P(G \le G_s | G \le G_{m-1})
\end{aligned} \tag{11}$$

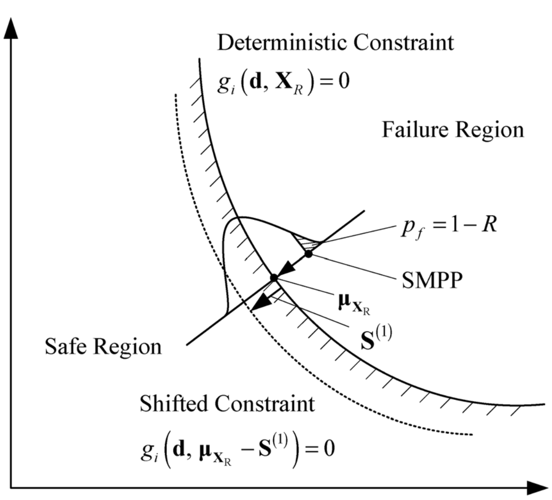

Fig. 2 The schematic diagram of shifting constraint boundary

where $P_\mathrm{desire}^{(k)}(F_m|F_{m-1})$ is the expected probability of the failure event of interest and satisfies $P^{(k)}(F_{m-1}) \times P_\mathrm{desire}^{(k)}(F_m|F_{m-1}) = \left[p_f\right]$. Like at step 2, we find SMPPs again by listing all MCMC samples in an ascending order according to their performance values and construct the shifting vector $\mathbf{S}^{(k)} = \boldsymbol{\mu}_{\mathbf{X}_\mathrm{R}}^{(k)} - \mathbf{X}_\mathrm{SMPP}^{(k)}$. Then go to step 1.

The MDO-SSRA-SORA process is performed until $P_F^{(k)} < \left[p_f\right]$ and the value of objective is stable. The flowchart of MDO-SSRA-SORA is illustrated in Fig. 3.

4.3 Numerical Example. In this section, a numerical example is given to show the proposed method in detail. We also combine mean value first order second moment (MVFOSM), FORM, SORM, and MCS with CO to solve this problem. The proposed method is compared with them. The solutions obtained by MCS based RBMDO (MDO-MCS) are used as reference.

The formulation of mathematical example is given in Eq. (12).

$$\begin{aligned}
& \text{Find } \mu_{x_1}, \mu_{x_2}, \mu_{x_3}, y_{12}, y_{21} \\
& \min f = (y_{12} - 1)^2 + \mu_{x_1}^2 + \mu_{x_2}^2 + (y_{21} - 2)^2 + \mu_{x_3}^2 \\
& \text{s.t. } \Pr_1\left[g_1 = x_1 x_2^2 + y_{12} - 0.4 \le 0\right] \ge 0.9985 = 1 - \left[p_f\right], \\
& \quad \Pr_2\left[g_2 = x_3^2 + y_{12}^2 + y_{21} - 1.75 \le 0\right] \ge 0.9985 = 1 - \left[p_f\right], \\
& \quad y_{12} = \mu_{x_1} - \mu_{x_2} + 2y_{21}, \; y_{21} = \mu_{x_3} - y_{12}, \; -5 \le \mu_{x_1} \le 0, \\
& \quad 0 \le \mu_{x_2} \le 1, 0 \le \mu_{x_3} \le 5, 0 \le y_{12} \le 10, 0 \le y_{21} \le 10
\end{aligned} \tag{12}$$

There are two disciplines in this problem, which is shown in Fig. 4. f is the system objective; x_1, x_2 and x_3 are discipline design variables in discipline 1 and discipline 2, respectively; y_{12} and y_{21} are linking design variables. The detailed uncertainty information is given in Table 1.

The acceptable failure probability $\left[p_f\right]$ of each reliability constraint is 1.5×10^{-3}. In SORA, the MDO problem includes the system optimization problem in Eq. (13) and the discipline optimization problems in Eqs. (14) and (15).

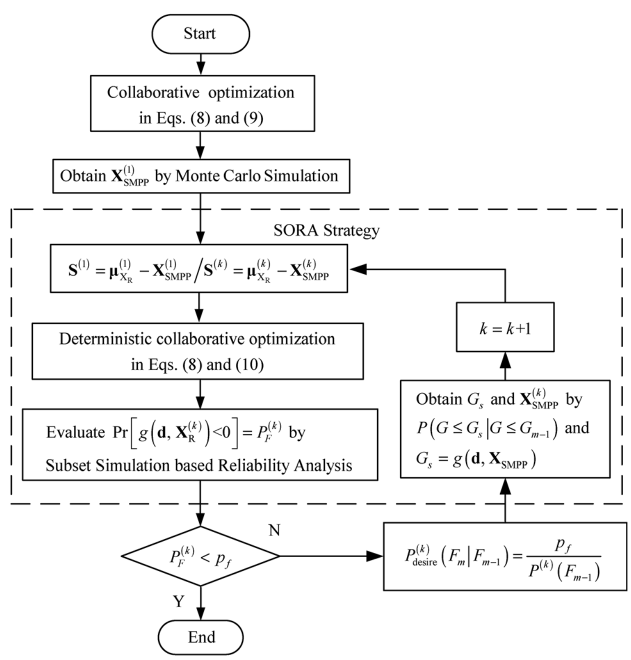

Fig. 3 The flowchart of MDO-SSRA-SORA

(1) System optimization problem

$$
\begin{aligned}
&\text{Find} \quad \mu_{x_1^{\text{sys}}}, \mu_{x_2^{\text{sys}}}, \mu_{x_3^{\text{sys}}}, y_{12}^{\text{sys}}, y_{21}^{\text{sys}} \\
&\min \; f = \left(y_{12}^{\text{sys}} - 1\right)^2 + \left(\mu_{x_1^{\text{sys}}}\right)^2 + \left(\mu_{x_2^{\text{sys}}}\right)^2 + \left(y_{21}^{\text{sys}} - 2\right)^2 + \left(\mu_{x_3^{\text{sys}}}\right)^2 \\
&\text{s.t.} \quad J_1 \le \varepsilon, \quad J_2 \le \varepsilon
\end{aligned}
\tag{13}
$$

(2) Optimization problem for discipline 1

$$
\begin{aligned}
&\text{Find} \quad \mu_{x_1^{\text{dis1}}}, \mu_{x_2^{\text{dis1}}}, y_{12}^{\text{dis1}}, y_{21}^{\text{dis1}} \\
&\min \quad J_1 = \left(\mu_{x_1^{\text{sys}}} - \mu_{x_1^{\text{dis1}}}\right)^2 + \left(\mu_{x_2^{\text{sys}}} - \mu_{x_2^{\text{dis1}}}\right)^2 + \left(y_{12}^{\text{sys}} - y_{12}^{\text{dis1}}\right)^2 + \left(y_{21}^{\text{sys}} - y_{21}^{\text{dis1}}\right)^2 \\
&\text{s.t.} \quad \left(\mu_{x_1^{\text{dis1}}} - s_{x_1^{\text{dis1}}}^{(k)}\right)\left(\mu_{x_2^{\text{dis1}}} - s_{x_2^{\text{dis1}}}^{(k)}\right)^2 + y_{12}^{\text{dis1}} - 0.4 \le 0, \; -5 \le \mu_{x_1^{\text{dis1}}} \le 0, \\
&\qquad 0 \le \mu_{x_2^{\text{dis1}}} \le 1, 0 \le y_{21}^{\text{dis1}} \le 10, 0 \le y_{12}^{\text{dis1}} \le 10, y_{12}^{\text{dis1}} = \mu_{x_1^{\text{dis1}}} - \mu_{x_2^{\text{dis1}}} + 2y_{21}^{\text{dis1}}
\end{aligned}
\tag{14}
$$

(3) Optimization problem for discipline 2

$$
\begin{aligned}
&\text{Find} \quad \mu_{x_3^{\text{dis2}}}, y_{12}^{\text{dis2}}, y_{21}^{\text{dis2}} \\
&\min \quad J_2 = \left(\mu_{x_3^{\text{sys}}} - \mu_{x_3^{\text{dis2}}}\right)^2 + \left(y_{12}^{\text{sys}} - y_{12}^{\text{dis2}}\right)^2 + \left(y_{21}^{\text{sys}} - y_{21}^{\text{dis2}}\right)^2 \\
&\text{s.t.} \quad \left(\mu_{x_3^{\text{dis2}}} - s_{x_3^{\text{dis2}}}^{(k)}\right)^2 + \left(y_{12}^{\text{dis2}}\right)^2 + y_{21}^{\text{dis2}} - 1.75 \le 0, \\
&\qquad 0 \le \mu_{x_3^{\text{dis2}}} \le 5, 0 \le y_{12}^{\text{dis2}} \le 10, 0 \le y_{21}^{\text{dis2}} \le 10, y_{21}^{\text{dis2}} = \mu_{x_3^{\text{dis2}}} - y_{12}^{\text{dis2}}
\end{aligned}
\tag{15}
$$

The compatibility constraint accuracy ε is 0.001. Because the acceptable failure probability $[p_f] = 1.5 \times 10^{-3} = 1.5 \times (10^{-1})^3$, the magnitude index is three. We subdivide the failure event into an initial failure event and two conditional failure events, which are predefined as $P_1 = 1.5 \times 10^{-1}$, $P_2(F_1) = 10^{-1}$ and $P_3(F_2) = 10^{-1}$. After $k = 3$ cycles, the optimal solutions are obtained, shown in Table 2.

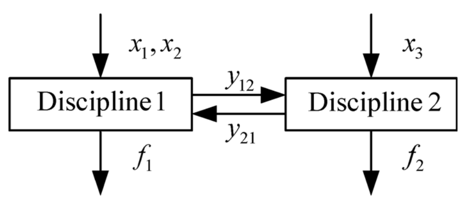

Fig. 4 The MDO problem of the numerical example

Table 1 Distribution details of random design variables in the numerical example

Variables	Mean	Standard deviation	Distribution
x_1	μ_{x_1}	$0.01\mu_{x_1}$	Normal
x_2	μ_{x_2}	$0.01\mu_{x_2}$	Normal
x_3	μ_{x_3}	$0.01\mu_{x_3}$	Normal

We can see that MDO-MVFOSM and MDO-FORM enjoy the higher computational efficiency, however less accuracy. MDO-SSRA and MDO-SORM need almost the same computational time. However, solutions from MDO-SSRA are closer to accurate solutions, compared with solutions from MDO-SORM. MDO-MCS needs longer computational time than MDO-SSRA. It is because more simulation samples are needed using MDO-MCS. To evaluate the failure probability under $\delta = 0.1$, MCS needs 2×10^5 samples. However, SSRA only needs $2 \times 3 \times 10^3$ samples.

5 The Hydraulic Transmission Mechanism Design

The hydraulic transmission mechanism comprised rails, sliders, and a connecting rod, which is shown in Fig. 5. The rotation process is as follows: the connecting rod rotates around the axle B; the sliders move on the rails; the moving section rotates around the axle A; the hydraulic transmission mechanism completes the whole moving cycle from the initial position E to the final position E″.

Table 2 Solutions of the numerical example

	MDO-SSRA	MDO-MCS	MDO-MVFOSM	MDO-FORM	MDO-SORM
μ_{x_1}	−0.2932	−0.2909	−0.3312	−0.3205	−0.3012
μ_{x_2}	0.2920	0.2898	0.2957	0.3005	0.2991
μ_{x_3}	0.7961	0.7760	0.8601	0.8523	0.8054
y_{12}	0.3357	0.3238	0.3644	0.3612	0.3368
y_{21}	0.4604	0.4522	0.4957	0.4911	0.4686
f	3.6166	3.6236	3.6038	3.6043	3.6139
time (s)	31	119	17	21	30

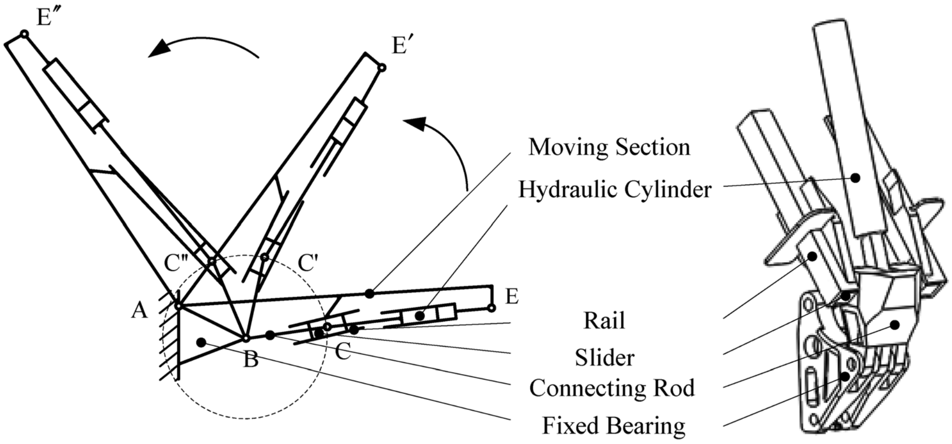

Fig. 5 The structure sketch of a hydraulic transmission mechanism

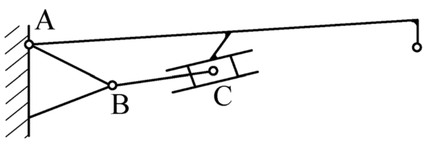

Fig. 6 The power transmission discipline

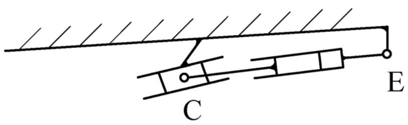

Fig. 7 The power input discipline

The objective is maximizing the folding torque M. There are two disciplines, the power transmission discipline and the power input discipline, as shown in Figs. 6 and 7. There are eight design variables $\mathbf{X} = [x_B, y_B, l, \alpha, x_E, y_E, D, \beta]$, three design parameters $\mathbf{P} = [P, E, d]$, and three linking variables $\mathbf{Y} = [x_C, y_C, F]$. The detailed design information is given in Table 3 and also shown in Fig. 8. The coupled information is shown in Fig. 9. Here, all of random variables are assumed to be normally distributed, $\mathbf{X} \sim N(\mu_{\mathbf{X}}, 0.01\mu_{\mathbf{X}})$, where $\mu_{\mathbf{X}}$ is the mean of design variables, and the standard deviation is $0.01\mu_{\mathbf{X}}$. Then the RBMDO model of this problem is given as follows:

Table 3 Design variables and design parameters of a hydraulic transmission mechanism

		Description	Initial value	Lower bound	Upper bound
Design variables of the power input discipline	x_B (mm)	The abscissa of B	125	0	954
	y_B (mm)	The ordinate of B	−104	−208	0
	l (mm)	The length of connecting rod	146	132	160
	α (deg)	Initial angle of connecting rod	5.5	4.5	6.5
Design variables of the power transmission discipline	x_E (mm)	The abscissa of E	948	0	954
	y_E (mm)	The ordinate of E	−114	−208	0
	D (mm)	The diameter of hydraulic cylinder	100	80	120
Shared design variables	β(deg)	Initial angle of rail	5	3.5	6.5
Linking variables	x_C	The abscissa of C	270	0	954
	y_C(mm)	The ordinate of C	−90	−208	0
	F(N)	The axial force of hydraulic rod	8.48×10^4	—	12.2×10^4
Design parameters	P(Pa)	Hydraulic Pressure	1080	—	—
	E(GPa)	Elastic modulus of carbon steel AISI 1045 for fixed bearing, connecting rod, rail and hydraulic rod	209	—	—
	d(mm)	The diameter of hydraulic rod	56	—	—

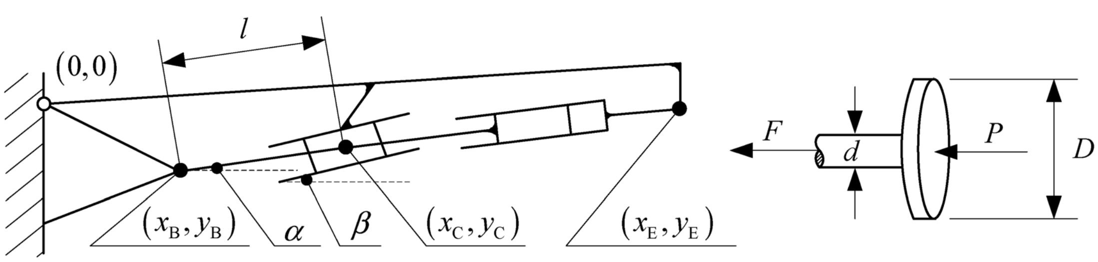

Fig. 8 The design variables and design parameters shown in the sketch

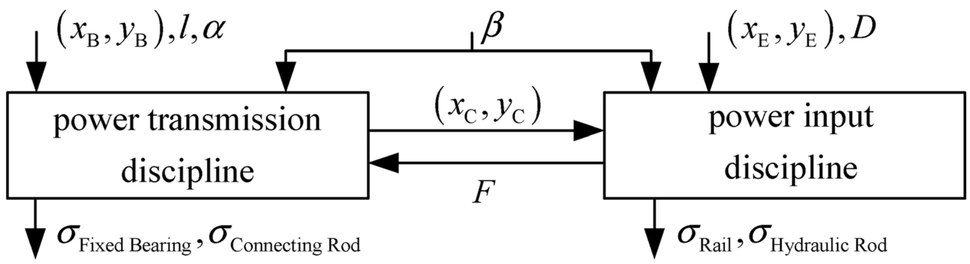

Fig. 9 The coupled information of a hydraulic transmission mechanism MDO problem

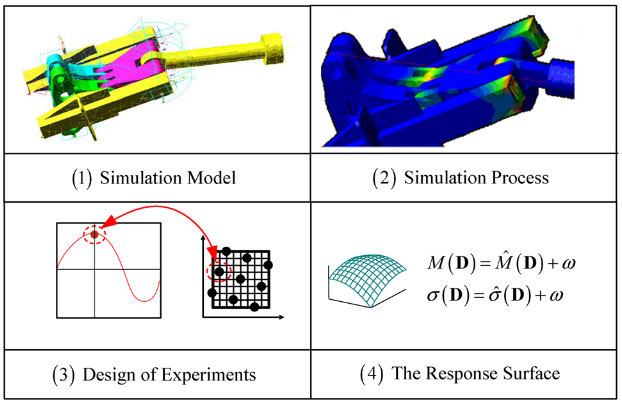

Fig. 10 The application of response surface model modeling technique

$$
\begin{aligned}
&\text{Max } M = M\left(\mu_{x_B}, \mu_{y_B}, \mu_l, \mu_\alpha, \mu_\beta, \mu_{x_E}, \mu_{y_E}, \mu_D\right)\\
&\text{s.t.} \quad \Pr_1\left[\sigma_{\text{Fixed Bearing}}(x_B, y_B, l, \alpha, \beta, F) \le [\sigma]\right] \ge 1 - [p_f],\\
&\qquad \Pr_2\left[\sigma_{\text{Connecting Rod}}(x_B, y_B, l, \alpha, \beta, F) \le [\sigma]\right] \ge 1 - [p_f],\\
&\qquad \Pr_3\left[\sigma_{\text{Rail}}(x_E, y_E, D, \beta, x_C, y_C)\right] \le [\sigma] \ge 1 - [p_f],\\
&\qquad \Pr_4\left[\sigma_{\text{Hydraulic Rod}}(x_E, y_E, D, \beta, x_C, y_C) \le [\sigma]\right] \ge 1 - [p_f],\\
&\qquad 0 \le \mu_{x_B}, x_C, \mu_{x_E} \le 954, \; -208 \le \mu_{y_B}, y_C, \mu_{y_E} \le 0,\\
&\qquad 132 \le \mu_l \le 160, 4.5 \le \mu_\alpha, \mu_\beta \le 6.5,\\
&\qquad 80 \le \mu_D \le 120, F \le 12.4 \times 10^5
\end{aligned} \tag{16}
$$

where $x_C = l \times \cos\alpha + x_B$; $y_C = l \times \sin\alpha + y_B$; $F = P \times 10^6 \times (\pi/4) \times (D \times 10^{-4})^2$; $\Pr(\bullet)$ is the reliability constraints, $[p_f] = 2 \times 10^{-6}$; $\sigma(\bullet)$ is the stress constraint of component; $[\sigma]$ is the yield strength. Here, the material is carbon steel AISI 1045, thus $[\sigma] = 505$ MPa.

As shown in Fig. 10, we use the response surface method to construct the functions $M(\bullet)$ and $\sigma(\bullet)$. The response surface modeling postulates models

$$
\begin{aligned}
M(\mathbf{D}) &= \hat{M}(\mathbf{D}) + \omega\\
\sigma(\mathbf{D}) &= \hat{\sigma}(\mathbf{D}) + \omega
\end{aligned} \tag{17}
$$

where ω is random error and assumed to be independent and identically normally distributed at each observation; $\hat{M}(\mathbf{D})$ and $\hat{\sigma}(\mathbf{D})$ are quadratic polynomial functions of design variables $\mathbf{D}$

$$
\begin{aligned}
\hat{M}(\mathbf{D}) &= \varphi_0 + \sum_{i=1}^{8} \varphi_i \mathrm{D}_i + \sum_{i=1}^{8} \varphi_{ii} \mathrm{D}_i^2 + \sum_i \sum_{j>i} \varphi_{ij} \mathrm{D}_i \mathrm{D}_j\\
\hat{\sigma}(\mathbf{D}) &= \lambda_0 + \sum_{i=1}^{6} \lambda_i \mathrm{D}_i + \sum_{i=1}^{6} \lambda_{ii} \mathrm{D}_i^2 + \sum_i \sum_{j>i} \lambda_{ij} \mathrm{D}_i \mathrm{D}_j
\end{aligned} \tag{18}
$$

The parameters of polynomials in Eq. (18) can be determined by the least-squares regression as

$$
\varphi = \left[D'_{\text{design}} D_{\text{design}}\right]^{-1} D'_{\text{design}} M \quad \text{and} \quad \delta = \left[D'_{\text{design}} D_{\text{design}}\right]^{-1} D'_{\text{design}} \sigma \tag{19}
$$

where D_{design} is the design matrix of samples; D'_{design} is its transpose; M and σ can be obtained by simulation.

Table 4 Solutions of structure optimization of a hydraulic transmission mechanism

	MDO-SSRA	MDO-MCS	MDO-MVFOSM	MDO-FORM	MDO-SORM
μ_{x_B} (mm)	126.56	126.01	126.85	126.73	126.59
μ_{y_B} (mm)	−109.24	−109.32	−110.02	−109.75	−109.44
μ_l (mm)	148.42	149.17	152.57	151.28	150.31
μ_α (deg)	5.64	5.67	5.71	5.70	5.69
μ_{x_E} (mm)	946.98	947.25	945.38	945.91	946.32
μ_{y_E} (mm)	−113.41	−113.69	−113.01	−113.24	−113.34
μ_D (mm)	110	108	116	116	112
μ_β (deg)	3.72	3.74	3.77	3.77	3.75
x_C (mm)	274.26	274.45	274.69	274.61	274.52
y_C (mm)	−94.65	−94.58	−94.37	−94.40	−94.68
F (N)	10.26×10^4	9.89×10^4	11.41×10^4	11.41×10^4	10.64×10^4
$\sigma_{\text{Fixed Bearing}}$ (MPa)	221.45	222.66	228.76	227.35	222.68
$\Pr_1$	99.9999%	99.9998%	99.9999%	99.9999%	99.9999%
$\sigma_{\text{Connecting Rod}}$ (MPa)	170.55	179.01	210.11	201.95	185.24
$\Pr_2$	99.9998%	99.9998%	99.9998%	99.9998%	99.9998%
σ_{Rail} (MPa)	264.87	265.40	275.32	274.28	270.24
$\Pr_3$	99.9998%	99.9999%	99.9998%	99.9998%	99.9998%
$\sigma_{\text{Hydraulic Rod}}$ (MPa)	375.33	377.53	384.12	383.94	379.27
$\Pr_4$	99.9998%	99.9999%	99.9999%	99.9999%	99.9998%
M (N × m)	36593.592	36502.547	37112.541	36967.258	36709.325
time (s)	123	585	66	87	121

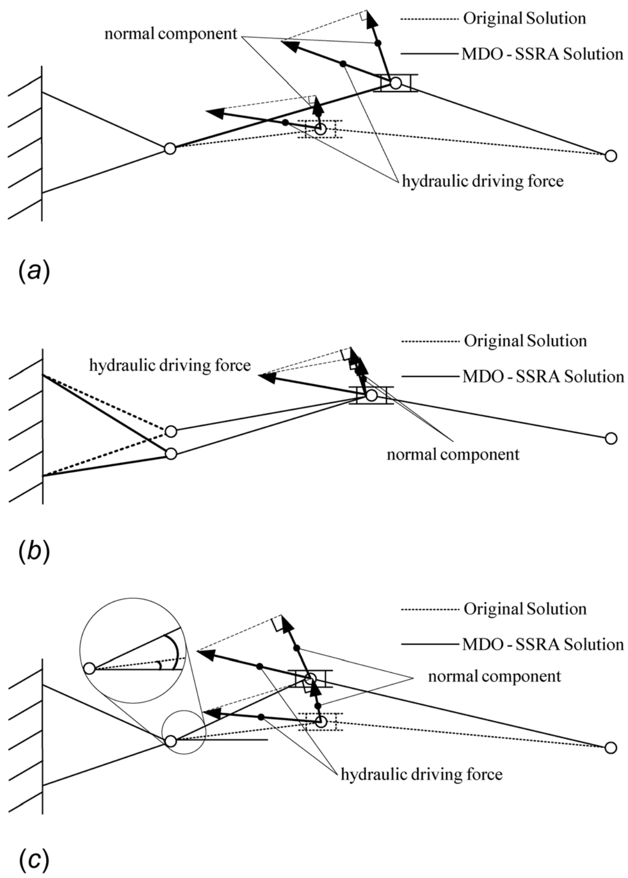

Fig. 11 (*a*) The structure comparisons of MDO solution and MDO-SSRA solution: increase the length of connecting rod, (*b*) the structure comparisons of MDO solution and MDO-SSRA solution: reduce the ordinate of point B, and (*c*) the structure comparisons of MDO solution and MDO-SSRA solution: increase the value of angle α

In this example, we also use MDO-MVFOSM, MDO-FORM and MDO-SORM for comparison. After $k = 3$ cycles, the solutions are obtained and listed in Table 4, respectively. Compared with the folding torque $M = 37112.541\,(\mathrm{N}\times\mathrm{m})$ from MDO-MVFOSM and $M = 36967.258\,(\mathrm{N}\times\mathrm{m})$ from MDO-FORM, the solutions $M = 36593.592\,(\mathrm{N}\times\mathrm{m})$ from MDO-SSRA and $M = 36709.325\,(\mathrm{N}\times\mathrm{m})$ from MDO-SORM are closer to the reference $M = 36502.547\,(\mathrm{N}\times\mathrm{m})$ from MDO-MCS, although using greater computational expense. MDO-SSRA and MDO-SORM have almost the same computational efficiency. However, more conservative solutions can be obtained by MDO-SSRA. Furthermore, the failure event is subdivided into an initial failure event and five sequential partial failure events using MDO-SSRA. Each conditional failure probability is predefined to $P_i(F_j) = 0.1$, $i = 1 \sim 4$, $j = 1 \sim 6$. $4 \times 6 \times 10^3$ simulation samples are needed in MDO-SSRA compared with 4×10^8 simulation samples in MDO-MCS under $\delta = 0.1$. Thus MDO-SSRA is more efficient than MDO-MCS. Compared with the initial solutions, there are three improvements in the solutions from MDO-SSRA, shown in Figs. 11(*a*)–11(*c*). They are increasing the length of the connecting rod, reducing the ordinate of point B and increasing the value of angle α, respectively. All improvements increase the normal component of the hydraulic driving force, which can enhance the folding torque M.

6 Conclusions

In this paper, a subset simulation-based decouple-loop RBMDO approach, MDO-SSRA-SORA, is proposed to improve the computational efficiency. SSRA and CO are combined within the framework of the SORA strategy. In SSRA, an initial probability and a sequence of larger conditional probabilities are calculated instead of calculating the probability of failure event of interest directly. We use the modified Metropolis algorithm to obtain offsprings of simulation samples. A hydraulic transmission mechanism optimization problem is taken as an example to show the engineering application of the proposed method. Compared with MDO-MCS, MDO-SSRA can enjoy higher efficiency when calculating a very small failure probability. However, using CO in MDO-SSRA may result in low computational efficiency or induce the divergence issue. This is because CO is more suitable to solve MDO problems with low couplings. More compatibility constraints are needed when there are many linking variables in RBMDO problems. In this situation, a MDO problem will become more complex. To solve this problem, many other MDO methods can be introduced into MDO-SSRA. In future works, we will introduce different MDO methods to solve RBMDO problems with high couplings.

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