Reliability, Maintainability, and Availability Analysis of a Computerized Numerical Control Machine Tool Using Markov Chains

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Abstract: Reliability, maintainability, and availability analysis of Computerized Numerical Control Machine Tools (CNCMT) is vital as they are widely used in manufacturing industries for mass production. This paper proposes a generalized framework for Time-Between-Failure (TBF) and Time-To-Repair (TTR) data analysis, integrated with Markov chains for estimating the system's Steady State Availability (SSA). A case study of a typical CNCMT illustrates the applicability and the effectiveness of the proposed framework. The effect of variation of sub-systems' failure and repair rates on the availability of the CNCMT is studied. The critical sub-systems from reliability, maintainability, and availability point of view are identified. The analysis reveals that the CNCMT's failure and repair rates are nearly constant and the CNCMT fails four times per year. The Lubrication Sub-system (LS) is the utmost severe sub-system as far as maintainability aspect is concerned and Turret Sub-system (TS) is the utmost severe sub-system from a reliability perspective.

Keywords: steady-state availability; reliability; maintainability; computerized numerical control machine tool; Markov chains

1 Introduction

Computerized Numerical Control Machine Tools (CNCMT) are the sinews of the modern manufacturing industry and are used for manufacturing various components with high precisions [1, 2, 3, 4]. They have become the heart of the machining industry due to their accuracy, flexibility, and productivity. Moreover, machining processes are highly optimized [5, 6]. A typical CNCMT consists of many components, and the failure of a single component can hamper the production of an entire workshop or manufacturing system [7, 8]. The cost of maintenance is high when an unexpected failure takes place [9]. Considering these aspects, the manufacturers of the CNCMT should give topmost priority for reliable and maintainable CNCMTs with a desired level of availability.

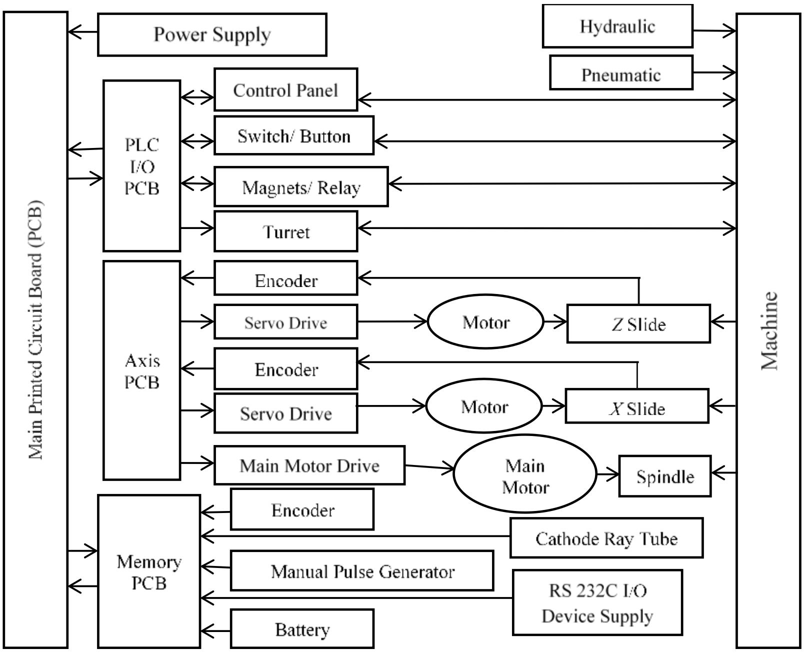

Figure 1

Configuration of a typical CNCMT [2, 9]

The configuration of the typical CNCMT lathes that have *Z* and *X* axes driven by AC or DC motors through ball lead screws simultaneously is shown in Figure 1. The turrent may exchange tools automatically. The CNCMT is made up of several sub-systems such as mechanical, hydraulic, pneumatic, electronic, electric, and software. Chuck mounted on the spindle is a mechanical sub-system on which the workpiece to be machined is mounted. The servomotor through the main transmission sub-system rotates the chuck-spindle assembly at the required machining speed. The hydraulic sub-system regulates the clamping and de-clamping of the workpiece. The cutting tool mounted on the turret moves along

X- and *Z*- axes and carry out machining operations. The cutting tool's simultaneous movement along the *X*- and *Z*-axes is precisely done by the servomotors through lead screws. The turret has an indexing mechanism and is capable of changing the tool automatically as per the machining operations [31]. Cooling, lightening, tail-stock, and pneumatic sub-systems are also incorporated for the ease of machining operations. The motion and operations of different sub-systems are controlled with the help of a centralized Computerized Numerical Control (CNC) sub-system that is also called the heart of the CNCMT. The CNC sub-system consists of thousands of electronic components such as Printed Circuit Board (PCB), Programmable Logic Control (PLC), Cathode Ray Tube (CRT) or Medium Dependent Interface (MDI) encoders (for manual date input), limit switches, relays, Manual Pulse Generator (MPG), RS-232 serial communication device, and contactor switches.

Table 1

Sub-systems of CNCMT

Sub-system	**Code**	**Sub-system**	**Code**
Main Transmission	MT	Spindle Sub-system	SS
Chuck Sub-system	ChS	*X* and *Z* Axis Sub-system	XZAS
Turret Sub-system	TS	Cooling Sub-system	CS
Lubrication Sub-system	LS	Hydraulic Sub-system	HS
CNC Sub-system	CNCS	Electrical and Electronic Sub-system	EES
Swarf Conveyor	SC	Pneumatic Sub-system	PS
Tail-Stock Sub-system	TSS	Other Sub-system	OS

The CNCMT consists of several sub-systems, assemblies, and components. The sub-systems of the CNCMT are categorized into 14 sub-systems according to their functionality and dependency and reported in Table 1. The failure and repair data collected from the service engineers, maintenance registers and experts in the field are considered for the analysis.

This paper presents a generalized framework for the reliability, maintainability, and availability analysis of the CNCMT. In particular, the Time-Between-Failure (TBF) and Time-To-Repair (TTR) data are analyzed and the proposed framework is integrated with Markov chains to investigate the Steady-State Availability (SSA). The rest of the paper is divided into four sections. Section 2 reviews the main published TBF and TTR data analysis frameworks and reliability analysis of CNCMT over the years. Section 3 presents the proposed new generalized framework developed for the analysis of TBF and TTR data. The selection of sample size for reliability, maintainability, and availability analysis of the CNCMT is presented in Section 4, and the analysis of the results is presented in Section 5. Finally, Section 6 concludes the paper.

2 Literature Review

In 1976, Ferris-Prabhu and Lubart [10] proposed an analytical method for the reliability assessment of a system. The method extended and developed a detailed reliability analysis framework in 1984 conducted by Ascher and Feingold [11]. These modified and extended frameworks were then developed for different applications as per the availability of the data. Abdel-Ghaly et al. [12] presented various statistical tools for predicting software reliability. Kumar et al. [13] investigate Load Hauual Dump (LHD) machines' operational reliability using trend test and goodness-of-fit test. Weibull-Poisson process was developed by Crow [14] and applied to a complex repairable system to predict its reliability. Further, several simplified frameworks were developed and evaluated for the reliability analysis of various systems. Few to mention are: A comprehensive model based on the Bayesian approach was proposed by Pulido et al. [15] and required time-to-failure data. Kim and Yum [16] performed a simulative study to select appropriate distribution between Weibull and lognormal distributions for censored and complete data. Barabady and Kumar [17] applied a reliability data analysis framework to analyze the failure and repair data of a mining plant, present, and predict the reliability and maintainability using best-fit distribution. Louit et al. [18] studied numerous methods used to assess data trends and developed a simplified framework for TBF and TTR data analysis. Several other reliability data analysis and modeling frameworks considering different parameters are also published in the literature [19-24, 36-48]. Table 2 summarizes the review of TBF and TTR data analysis frameworks and models.

Table 2

Review of frameworks and models developed for TBF and TTR data analysis

Parameters/ techniques **Authors**	**Sample size selection**	**Pareto analysis**	**Analytic hierarchy process**	**Binary state system analysis**	**MSS analysis**	**Bayesian method**	**Non-parametric methods**	**Parametric methods**	**Trend analysis**	**Goodness-of-fit**	**Reliability analysis**	**Maintainability analysis**	**Availability analysis**
Ferris-prabhu and Lubart, 1976, [10]				✔							✔		
Ascher and Feingold, 1984, [11]				✔					✔		✔		
Abdel-Ghaly et al., 1986, [12]						✔				✔	✔		
Kumar et al., 1989, [13]								✔	✔	✔	✔		

Parameters/ techniques Authors	Sample size selection	Pareto analysis	Analytic hierarchy process	Binary state system analysis	MSS analysis	Bayesian method	Non-parametric methods	Parametric methods	Trend analysis	Goodness-of-fit	Reliability analysis	Maintainability analysis	Availability analysis
Crow, 1990, [14]								✓	✓	✓	✓		
Ansell and Phillips, 1990, [36]								✓	✓		✓		
Kumar and Klefsjo, 1992, [37]								✓	✓	✓	✓		
Kumar and Huang, 1993, [38]											✓	✓	✓
Kamps, 1995 [39]	✓	✓							✓				
Lawless, and Thiagarajah, 1996, [40]								✓	✓	✓	✓		
Coetzee, 1997, [41]								✓	✓	✓	✓	✓	
Kvaloy, and Lindqvist, 1998, [42]								✓	✓				
Ziegel, 2001, [43]								✓	✓	✓	✓	✓	
Yanez et al., 2002, [44]									✓				
Lindqvist et al., 2003, [45]								✓	✓				
Samanta et al., 2004, [46]											✓	✓	✓
Wang, 2005, [47]									✓				
Lindqvist, 2006, [48]									✓				
Kim and Yum, 2008, [16]								✓					
Barabady and Kumar, 2008, [17]		✓		✓				✓	✓	✓	✓	✓	✓
Louit et al., 2009, [18]				✓		✓		✓	✓	✓	✓		
Regattieri et al., 2010, [19]								✓			✓		
Lad and Kulkarni, 2010, [20]								✓			✓		
Castet and Saleh, 2010, [21]					✓		✓	✓			✓		
Barabadi, 2013, [22]								✓			✓		
Barabadi et al., 2014, [23]								✓			✓		

Parameters/ techniques **Authors**	**Sample size selection**	**Pareto analysis**	**Analytic hierarchy process**	**Binary state system analysis**	**MSS analysis**	**Bayesian method**	**Non-parametric methods**	**Parametric methods**	**Trend analysis**	**Goodness-of-fit**	**Reliability analysis**	**Maintainability analysis**	**Availability analysis**
Bobrowski et al., 2015, [24]							✓	✓			✓		

Reliability analysis CNCMTs are being conducted since 1982. The first study on reliability, maintainability, and availability analysis of CNCMTs was presented by Keller et al. [9]. Table 3 summarizes the reliability studies on the CNCMTs, such as the machining center, lathe, and milling center. It can be observed that most of these studies have been conducted based on some particular distributions for data analysis, and few of them used goodness of fit tests. Therefore, there is a need to apply statistical tests to assess the data trends and estimate appropriate distribution to accurately estimate the reliability metric. Statistical tests help to identify anomalies present in the data, trends in the data, and the amount of samples required for the analysis. The field failure data often consists of outliers entered due to human errors and needs to be removed from the sample. Furthermore, over the period of time maintenance policies may be changing that if any needs to tranced to minimize variations in the data. Statistical tests and trend analysis methods are, therefore, useful for refining the field failure data.

Table 3
Review of reliability analysis of CNC assisted machine tools

Parameters/ techniques **Authors**	**Pareto analysis**	**Bayesian method**	**Analytic hierarchy process**	**MSS analysis**	**Failure modes**	**Human factors**	**Organizational factors**	**Software reliability**	**Trend analysis**	**Goodness-of-fit**	**Reliability**	**Maintainability**	**Availability**
Keller et al., 1982, [9]	✓	✓									✓	✓	✓
McGoldrick and Kulluk, 1986, [49]	✓										✓		
Gupta and Somers, 1989, [50]	✓										✓		✓

Parameters/ techniques **Authors**	**Pareto analysis**	**Bayesian method**	**Analytic hierarchy process**	**MSS analysis**	**Failure modes**	**Human factors**	**Organizational factors**	**Software reliability**	**Trend analysis**	**Goodness-of-fit**	**Reliability**	**Maintainability**	**Availability**
Yazhou et al., 1995, [51]											✓		
Yazhou et al., 1995, [52]										✓	✓		
Karyagina et al., 1998, [53]												✓	
Wang et al., 1999, [25]	✓				✓						✓		
Wang et al., 1999, [26]	✓				✓					✓	✓		
Sehgal et al., 2000, [54]					✓						✓		
Dasic, P., 2001, [55]										✓	✓		
Wang et al., 2001, [33]	✓		✓								✓		
Wang et al., 2001, [32]					✓						✓		
Dai and Jia, 2001, [56]					✓						✓		
Dai et al., 2003, [28]										✓	✓		
Wang et al., 2003, [57]											✓		
Jolly and Wadhwa, 2004, [58]											✓	✓	✓
Zhou et al., 2005, [29]	✓	✓			✓						✓		
Zhang et al., 2007, [30]	✓									✓	✓		
Lad and Kulkarni, 2008, [59]											✓	✓	
Lad and Kulkarni, 2010, [20]											✓	✓	
Sung and Lee, 2011, [66]					✓						✓		
Yang et al., 2013, [27]	✓				✓					✓	✓	✓	
Yang et al., 2015, [60]											✓		
Chen et al., 2015, [61]	✓				✓					✓	✓	✓	
Yang et al., 2016, [62]		✓									✓		
Li et al., 2016, [63]											✓		
Peng et al., 2016, [64]		✓									✓		
Patil and Kothavale, 2018, [7]	✓				✓						✓	✓	
Patil et al., 2018, [2]	✓	✓	✓	✓					✓	✓	✓	✓	✓

The literature survey shows that the existing data analysis frameworks need several tests to assess the trend in the data and obtain the best fit reliability distribution. It can also be seen that several studies and frameworks analyze the data using a specific reliability distribution such as exponential, Weibull, normal, and lognormal. There is a need to develop a generalized framework for modeling the failure and repair data. Most of the frameworks are applicable for the system's binary state and not Multi-State System (MSS). Several reliability analyses assume that the system has binary states (either working or failed). It is also essential to consider the system degraded states whenever necessary to get detailed failure characteristics and their associated impact on the system. The accuracy of the predicted reliability depends on the sample size, i.e., the amount of data available. There are very few papers describing the sample size selection for a known or unknown population. Therefore, it is necessary to include a sample size selection approach in the framework. The reliability of any system influenced by its four key elements: hardware, software, organizational, and human. Reliability and maintainability studies are often carried out using one or two elements, particularly hardware and software. Studies have shown that several incidents occur due to the mistake made by the human or policies implemented by the organization. Human and Organizational Factors (HOFs) significantly affect system reliability. Therefore, it is critical to identify and eliminate the failure and repair data affected by HOFs, often called anomalies.

This paper proposes a generalized framework for failure and repair data analysis addressing the key concerns of the existing frameworks. It is then integrated with the Markov chains, and an availability model is developed for the analysis of CNCMT. The reliability, maintainability, and availability characteristics of the CNCMT are estimated. The Steady-State Availability (SSA) of the CNCMT is estimated, and the sub-systems that are critical from a reliability and maintainability point of view are identified.

3 Framework for TBF and TTR Data Analysis of CNCMTs

Several TBF and TTR data analysis frameworks were developed for modeling and analysis of failure and repair data [11, 17, 18]. However, these data analysis frameworks are complex in nature and it is suggested to conduct a large number of tests for trend assessment and verifying goodness-of-fit. In this context, a simplified data analysis framework is developed by modifying the existing one to make it flexible, so that it can be applied for the analysis of the selected CNCMT with sufficient accuracy and with reduction of analysis duration.

The first step in reliability, maintainability, and availability modeling is system selection. Reliability modeling is a time-consuming and critical process and therefore, it is essential to understand and define the necessity. After defining the system, increase the understanding level of the system and divide the system into different sub-systems and components based on their functionality and dependency. Several systems have multi-state components or sub-systems. In this case, the Multi-State System (MSS) approach can be used for modeling otherwise binary state system analysis is preferred. In the MSS approach, a clear distinction between various degraded states is essential. In this view, the critical information can be collected from the available failure and repair data, and judgments of the experts.

The next step is to decide the appropriate methodology or technique for analyzing the data. Baye's technique can be used for reliability modeling if the available data is insufficient or incomplete, or there is no data. Data of other systems, sub-systems, or components can be used for modeling the same. Furthermore, if sufficient data is not available, non-parametric methods can be applied for early reliability prediction of the system. However, reliability prediction with non-parametric methods is not accurate. It is used only at the preliminary stage of analysis. However, in critical systems, the required accuracy of the modeling and analysis is to be very high. In this situation, parametric methods are widely used. The present framework considers two stochastic processes 'as good as new (perfect repair)' and 'as bad as old (minimal repair)'. The framework uses only specific and required tests for trend analysis and estimation of goodness-of-fit.

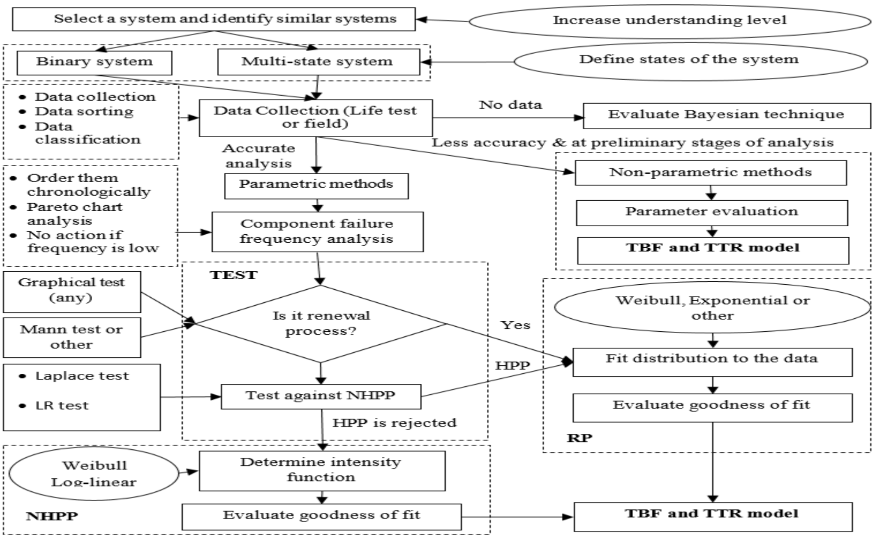

Figure 2

Generalized framework for the selection of TBF and TTR model [2, 11, 34]

Figure 2 shows a generalized framework used for the TBF and TTR data analysis of the CNCMT. The proposed framework is a simple way for reliability, maintainability, and availability predictors to appropriately evaluate the failure mechanisms and distinguish whether a renewable process or minimal repair process needs to be used. Graphical tests such as cumulative failure versus time, scatter plots of successive service lives, and analytical methods such as the Mann test are used for tests against RP. Methods such as Laplace, Lewis-Robinson, and military handbook are the most suitable for test against NHPP.

4 Sample Size Selection for Reliability, Maintainability and Availability Analysis of CNCMTs

This section attempts to select an appropriate sample (TBF and TTR data) of the CNCMT under consideration to predict the reliability and maintainability characteristics precisely. In this context, various terms such as the universe, population, and sample are defined.

The manufacturer, SPM Toold, Ichalkaranji, India, produces three models of CNCMT: $CNCMT_1$, $CNCMT_2$, and $CNCMT_3$ with different production capacities. The group of all the CNCMTs ($CNCMT_1$, $CNCMT_2$, and $CNCMT_3$) is considered as the universe. The manufacturer told us that the $CNCMT_2$ is the most popular and salable model, and they recommended we analyze the reliability of that model. Therefore, this paper uses the TBF and TTR data of $CNCMT_2$. In this view, the group of all the $CNCMT_2$ models produced is considered as population. Furthermore, the appropriate sample size can be defined as the number of $CNCMT_2$ models and the amount of TBF and TTR data required for predicting the reliability, maintainability, and availability characteristics accurately. As the most significant step is to estimate the appropriate sample size (number of machines and the amount of TBF and TTR data) from the population. An attempt is made to select an appropriate sample size from the population and is as given below.

Table 4

Summary of sample size and data collection period for CNC assisted machine tools

Authors	Number of machine tools	Data collection period
Keller et al., (1982), [1]	35	3 years
McGoldrick and Kullukt, (1986), [2]	Lathe 69 + NC 14	1 year
Gupta and Somers, (1989), [3]	05 types of CNC machines	3 years
Karyagina et al., (1995), [4]	09	---
Yazhou et al., (1995), [5]	24	1 year
Wang et al., (1999), [6]	80	2 years
Wang et al., (2001), [7]	09	---
Dai and Jia, (2001), [8]	14	2 years

Authors	Number of machine tools	Data collection period
Jolly and Wadhwa, (2004), [9]	04	3 years
Wang et al., (2013), [10]	12	5 years
Yang et al., (2015), [10]	---	3000 hours
Waghmode and Patil, (2016), [65]	10	2 years
Present work	50	5 years

The central limit theorem is widely used in statistical inference. It explains the relationship between the shape of the population distribution and the sampling distribution. It reveals that if the sample size (n) is greater than 30, the shape of the sampling distribution takes a shape like a normal distribution [35]. Therefore, the central limit theorem reveals that a sample size greater than 30 could be used. However, the validity of the central limit theorem is verified by using the sample size used in the literature for CNCMT's reliability analysis. The summary of sample size (number of CNCMTs) and the data collection period taken for the reliability analysis of CNC assisted machine tools by various researchers is shown in Table 4. It shows that the required TBF and TTR data of nearly 25 CNC-assisted machine tools over almost 2 years has to be collected. The last row of Table 4 gives the number of CNCMT_2 models and the TBF and TTR data collection period for the present work carried out. Furthermore, the reliability characteristics of the CNCMT_2 are estimated using the methodology presented in Figure 2 for different sample sizes and given in Table 5. It is observed that Weibull 3P is the best-fit distribution for the CNCMT_2. The distribution parameters such as shape parameter (β), scale parameter (θ), and location parameter (γ) are converging as the sample size increases.

Table 5

Variation in reliability characteristics for Weibull 3P distribution

Sample size	No of Machines	β	θ	γ	MTBF	Change in MTBF	% Deviation
112	5	1.0388	2177	10.41	2155		
235	10	1.0087	2067	0.42	2060	-95	-4.61
338	15	0.9046	2062	11.54	2175	115	5.29
466	20	0.8717	1937	12.93	2089	-86	-4.12
601	25	0.9082	1902	11.05	2003	-86	-4.29
693	30	0.8993	1908	12.18	2021	18	0.89
771	35	0.8993	1880	13.08	1992	-29	-1.46
846	40	0.8966	1903	14.16	2020	28	1.39
928	45	0.906	1898	13.75	2004	-16	-0.80
959	50	0.909	1895	13.23	1996	-8	-0.40

Table 5 (continued)

Variation in reliability characteristics for Weibull 3P distribution

σ	Change in σ	% Deviation
2172		
2067	-105	-5.07
2062	-5	-0.25
1933	-129	-6.65
1908	-25	-1.33
1914	6	0.32
1886	-28	-1.49
1902	16	0.83
1906	5	0.24
1900	-6	-0.34

Similarly, reliability characteristics such as Mean-Time-Between-Failure (MTBF) and standard deviation (σ) are also converging with an increase in sample size. The percentage deviation in MTBF and standard deviation (σ) is less than 1% for 959 TBF data. It clearly shows that a sample size (959 TBF data) collected from 50 $CNCMT_2$ models is sufficient for reliability analysis. The shape parameter of the $CNCMT_2$ is nearly equal to 1 and reveals that the failure rate of the $CNCMT_2$ is almost constant. Therefore, exponential distribution can be used to estimate and predict reliability characteristics precisely. The value of the location parameter (assured life) is very small, i.e., 13.23 hours, which is very small, and therefore, the Weibull 2P distribution can also be used for predicting reliability characteristics instead of the Weibull 3P distribution. The MTBF of the $CNCMT_2$ is nearly 2000 hrs. It shows that almost four to five failures of the $CNCMT_2$ will occur per year.

Furthermore, one more attempt is made to estimate the required sample size when the population's size is unknown. Equation (1) is used for calculating the sample size when the population is unknown [35]. The analysis is required to be carried out very accurately. Therefore, the standard variate (z) is taken as 1.96 for a 95% confidence level. The standard deviation (σ) of the population and sample is assumed to be the same and is taken as 1900 hrs (see Table 5). The acceptable error (e), i.e., precision is taken as ±190 hrs (10% of the population standard deviation). Therefore, the required sample size (TBF data) for the unknown population is given as follows:

$$n = \frac{z^2 \times \sigma^2}{e^2} = \frac{1.96^2 \times 1900^2}{190^2} = 384.16 \cong 385 \tag{1}$$

The required sample size is 385. The present study uses 959 TBF and TTR data of 50 CNCMTs operated in similar environmental conditions that are appropriate, and the sampling distribution of the CNCMT represents the distribution of the population of CNCMT$_2$. In this view, the reliability, maintainability, and availability analysis is carried out based on the following assumptions:

- Working temperature varies from 0° to 50° C.
- Relative humidity is less than 75%.
- Vibration level during the transportation is 3.5G or less.
- Vibration level during operation is 0.5G or less.
- Foundation precision level graduated to 0.02/0.05 mm/m.
- Capacity of the foundation capacity is more than 4000 kg.
- Spindle working temperature varies between -60°C to + 130°C.
- Maximum spindle speed is 5,500 rpm.
- Recommended lubricant, coolant, and hydraulic oil are used.
- Lubrication of various parts is done at suggested intervals with the suggested quantity.
- Hydraulic oil and coolant is replaced at regular intervals.
- Failed component/sub-system is replaced with the same and new component.
- Maintenance activities are carried out by using prescribed procedures.

5 Steady-state Availability Analysis of CNCMT

Availability analysis can be used to identify critical, sub-critical components/ equipment/sub-system of the CNCMT from the reliability and maintainability point of view. The CNCMT's availability is significantly influenced by the sub-system's failure and repair rates. The developed data analysis framework is used to estimate Steady-State Availability (SSA), and the effects of sub-system's failure and repair rates on the SSA of the CNCMT are investigated. For this purpose, the availability analysis of the CNCMT under consideration is presented using the Markov chain.

5.1 System Description for Markov Modeling

The fourteen sub-systems of the CNCMT, and notations for the operational and failed states, failure, and repair rates are defined and given in Table 6. These codes and notations are used for modeling the CNCMT using Markov chains. The availability modeling and analysis are carried out under the following assumptions:

- Sub-system's failure rates and repair rates are constant
- The failures and repairs are statistically independent and identically distributed (iid).

- Only one failure occurs at a time.
- After repair action, the state of the sub-system is assumed to be as good as new (renewal approach).

Table 6
Description for modeling of the CNCMT using Markov chains

Sr. No.	**Sub-system**	**Code**		**Failure rate (λ_i)**	**Repair rate (μ_i)**
		Operational state	**Failed state**		
1	Main Transmission (MT)	A	a	λ_1	μ_1
2	Spindle Sub-system (SS)	B	b	λ_2	μ_2
3	Chuck Sub-system (ChS)	C	c	λ_3	μ_3
4	X and Z Axis Sub-system (XZAS)	D	d	λ_4	μ_4
5	Turret Sub-system (TS)	E	e	λ_5	μ_5
6	Cooling Sub-system (CS)	F	f	λ_6	μ_6
7	Lubrication Sub-system (LS)	G	g	λ_7	μ_7
8	Hydraulic Sub-system (HS)	H	h	λ_8	μ_8
9	CNC Sub-system (CNCS)	I	i	λ_9	μ_9
10	Electrical and Electronic Sub-system (EES)	J	j	λ_{10}	μ_{10}
11	Swarf Conveyor (SC)	K	k	λ_{11}	μ_{11}
12	Pneumatic Sub-system (PS)	L	l	λ_{12}	μ_{12}
13	Tail-stock Sub-system (TSS)	M	m	λ_{13}	μ_{13}
14	Other Sub-system (OS)	N	n	λ_{14}	μ_{14}

5.2 Development of Transition Diagram and Mathematical Modeling

Figure 3 gives the notations and symbols used for representing the states of the subsystems. Figure 4 shows the transition diagram or state-space model and the logical representation of CNCMT's failures. The transition diagram defines the transitions of sub-system's one state to another (operational to failed and failed to operational). Here, $P_i(t)$ is the probability that at any time t the system is in the ith state and $(\cdot)'$ is the derivative with respect to time t.

Figure 3
Used symbols

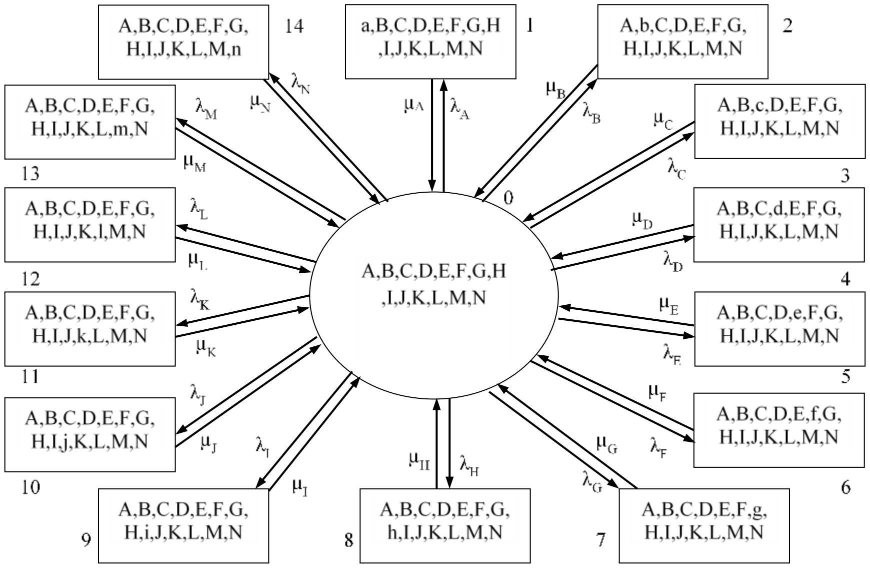

Figure 4
Transition diagram of CNCMT using Markov chains

The laws of probability and transition diagram are used, and equations (2)-(16) are developed. The steady state availability equation for the CNCMT is then developed as follows:

$$
\begin{aligned}
&P_0'(t) \\
&+ (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} \\
&+ \lambda_{14})P_0(t) \\
&= \mu_1 P_1(t) + \mu_2 P_2(t) + \mu_3 P_3(t) + \mu_4 P_4(t) + \mu_5 P_5(t) + \mu_6 P_6(t) + \mu_7 P_7(t) \\
&+ \mu_8 P_8(t) + \mu_9 P_9(t) + \mu_{10} P_{10}(t) + \mu_{11} P_{11}(t) + \mu_{12} P_{12}(t) + \mu_{13} P_{13}(t) \\
&+ \mu_{14} P_{14}(t)
\end{aligned}
\tag{2}
$$

$$P_1'(t) + \mu_1 P_1(t) = \lambda_1 P_0(t) \tag{3}$$

$$P_2'(t) + \mu_2 P_2(t) = \lambda_2 P_0(t) \tag{4}$$

$$P_3'(t) + \mu_3 P_3(t) = \lambda_3 P_0(t) \tag{5}$$

$$P_4'(t) + \mu_4 P_4(t) = \lambda_4 P_0(t) \tag{6}$$

$$P_5'(t) + \mu_5 P_5(t) = \lambda_5 P_0(t) \tag{7}$$

$$P_6'(t) + \mu_6 P_6(t) = \lambda_6 P_0(t) \tag{8}$$

$$P_7'(t) + \mu_7 P_7(t) = \lambda_7 P_0(t) \tag{9}$$

$$P_8'(t) + \mu_8 P_8(t) = \lambda_8 P_0(t) \tag{10}$$

$$P_9'(t) + \mu_9 P_9(t) = \lambda_9 P_0(t) \tag{11}$$

$$P_{10}'(t) + \mu_{10} P_{10}(t) = \lambda_{10} P_0(t) \tag{12}$$

$$P_{11}'(t) + \mu_{11} P_{11}(t) = \lambda_{11} P_0(t) \tag{13}$$

$$P_{12}'(t) + \mu_{12} P_{12}(t) = \lambda_{12} P_0(t) \tag{14}$$

$$P_{13}'(t) + \mu_{13} P_{13}(t) = \lambda_{13} P_0(t) \tag{15}$$

$$P_{14}'(t) + \mu_{14} P_{14}(t) = \lambda_{14} P_0(t) \tag{16}$$

The initial conditions are: $t = 0,\ P_i(t) = 1\ for\ i = 0,$ otherwise $P_i(t) = 0$. The life of the CNCMT is approximately taken as 12 years. Therefore, for such a long duration of time, the SSA of the CNCMT can be calculated by setting $\frac{d}{dt} \to 0$ and $t \to \infty$, into all the differential (Equations (2)-(16)). Thus, Equations ((17)-(31)) give the limiting state probabilities:

$$\begin{aligned}(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} + \lambda_{14})P_0 \\ = \mu_1 P_1 + \mu_2 P_2 + \mu_3 P_3 + \mu_4 P_4 + \mu_5 P_5 + \mu_6 P_6 + \mu_7 P_7 + \mu_8 P_8 \\ + \mu_9 P_9 + \mu_{10} P_{10} + \mu_{11} P_{11} + \mu_{12} P_{12} + \mu_{13} P_{13} \\ + \mu_{14} P_{14}\end{aligned} \tag{17}$$

$$\mu_1 P_1 = \lambda_1 P_0 \tag{18}$$

$$\mu_2 P_2 = \lambda_2 P_0 \tag{19}$$

$$\mu_3 P_3 = \lambda_3 P_0 \tag{20}$$

$$\mu_4 P_4 = \lambda_4 P_0 \tag{21}$$

$$\mu_5 P_5 = \lambda_5 P_0 \tag{22}$$

$$\mu_6 P_6 = \lambda_6 P_0 \tag{23}$$

$$\mu_7 P_7 = \lambda_7 P_0 \tag{24}$$

$$\mu_8 P_8 = \lambda_8 P_0 \tag{25}$$

$$\mu_9 P_9 = \lambda_9 P_0 \tag{26}$$

$$\mu_{10} P_{10} = \lambda_{10} P_0 \tag{27}$$

$$\mu_{11} P_{11} = \lambda_{11} P_0 \tag{28}$$

$$\mu_{12}P_{12} = \lambda_{12}P_0 \tag{29}$$

$$\mu_{13}P_{13} = \lambda_{13}P_0 \tag{30}$$

$$\mu_{14}P_{14} = \lambda_{14}P_0 \tag{31}$$

For analysis purpose and simplification, the values $(P_1, P_2, P_3 \ldots, P_{14})$ are respectively introduced in Equations (17)-(31):

$$P_1 = K_1P_0;\ P_2 = K_2P_0;\ P_3 = K_3P_0;\ P_4 = K_4P_0;\ P_5 = K_5P_0;\ P_6 = K_6P_0;\ P_7 = K_7P_0;\ P_8 = K_8P_0;\ P_9 = K_9P_0;\ P_{10} = K_{10}P_0;\ P_{11} = K_{11}P_0;\ P_{12} = K_{12}P_0;\ P_{13} = K_{13}P_0;\ P_{14} = K_{14}P_0;\ P_7 = K_7P_0;\ P_8 = K_8P_0;$$

For normalized conditions, the sum of all the probabilities is equal to one:

$$\sum_{i=0}^{15} P_i = 1$$

The sum of all the operating state probabilities gives the model for the SSA of the CNCMT as given as follows:

$$SSA = P_0 = [1 + K_1 + K_2 + K_3 + K_4 + K_5 + K_6 + K_7 + K_8 + K_9 + K_{10} + K_{11} + K_{12} + K_{13} + K_{14}]^{-1}$$

$$\therefore P_0 = [1 + A]^{-1} \tag{32}$$

where,

$$A = K_1 + K_2 + K_3 + K_4 + K_5 + K_6 + K_7 + K_8 + K_9 + K_{10} + K_{11} + K_{12} + K_{13} + K_{14} \tag{33}$$

5.3 Results and Analysis

This Section illustrates the results with an analysis of the developed SSA model of the CNCMT given by Equation (33). The influence of sub-system's failure rate and repair rates on the SSA of the CNCMT is also analyzed. Table 7 reports the sub-system's failure rate and repair rate per hour that is generally affected by various factors such as operating conditions, maintenance procedures, errors during maintenance, and entry of data in the maintenance register. Sub-system's failure and repair rates are varied by ± 5% and ± 10% to study its effect on the SSA of the CNCMT.

Figure 5 shows the effect of variation of the failure rates of the sub-systems on the availability of the CNCMT. In this case, the repair rates of the sub-systems are kept as it is to identify the severe sub-system of the CNCMT from a reliability point of view. It is seen that the SSA of the CNCMT gives a range of failure rates of the sub-systems excluding TS that varies from 0.9053 to 0.9073 (change in SSA = 0.221%).

Table 7

Failure rate (λ) and repair rate (μ) of CNCMT's sub-systems

Sr. No.	Equipment Name	Failure Rate (λ_i) per hour	Repair Rate (μ_i) per hour
1	Main Transmission (MT)	0.000434028	0.263157895
2	Spindle Sub-system (SS)	0.000262467	0.02173913
3	Chuck Sub-system (ChS)	0.000442478	0.142857143
4	*X* and *Z* Axis Sub-system (XZAS)	0.000338409	0.029411765
5	Turret Sub-system (TS)	0.000576037	0.0625
6	Cooling Sub-system (CS)	0.000516529	0.27027027
7	Lubrication Sub-system (LS)	0.00035727	0.011764706
8	Hydraulic Sub-system (HS)	0.000428449	0.263157895
9	CNC Sub-system (CNCS)	0.000485201	0.080645161
10	Electrical and Electronic Sub-system (EES)	0.000483559	0.095238095
11	Swarf Conveyor (SC)	0.000217817	0.333333333
12	Pneumatic Sub-system (PS)	0.000341997	0.5
13	Tail-stock Sub-system (TSS)	0.000119904	0.25
14	Other Sub-system (OS)	0.000295858	0.434782609

However, as the failure rate of the TS increases from 0.010588235 to 0.012941177, the SSA of the CNCMT decreases from 0.9111 to 0.9016 (change in SSA = 1.042%). Therefore, it can be seen that the SSA of the CNCMT is mostly influenced by the failure rate TS. Furthermore, the failure rate of the XZAS also affects the SSA of the CNCMT to a certain extent.

Figure 6 shows the effect of the variation of the repair rates of sub-systems on the SSA of the CNCMT. Here, the failure rates of all the sub-systems are kept as it is to identify the severe sub-system of the CNCMT from a maintainability point of view. It is observed that the SSA of the CNCMT for the given range of repair rates of the sub-systems excluding LS varies from 0.9052 to 0.9072 (change in SSA = 0.22%). However, as the repair rate of LS improves from 0.010588235 to 0.012941177, the SSA of the CNCMT increases from 0.9019 to 0.91 (change in SSA = 0.90%). Therefore, it can be concluded that the repair rate of LS has the highest effect on the SSA of the CNCMT. Furthermore, XZAS and TS also affect the SSA of the CNCMT to a certain extent.

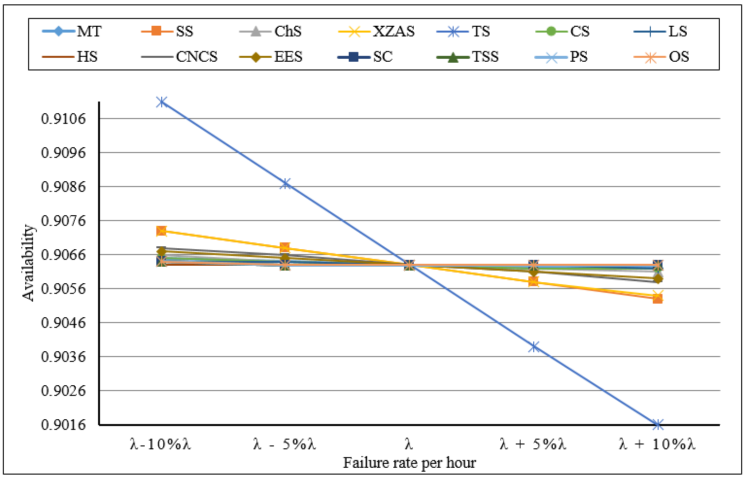

Figure 5

Effect of sub-system's failure rate on the SSA of the CNCMT

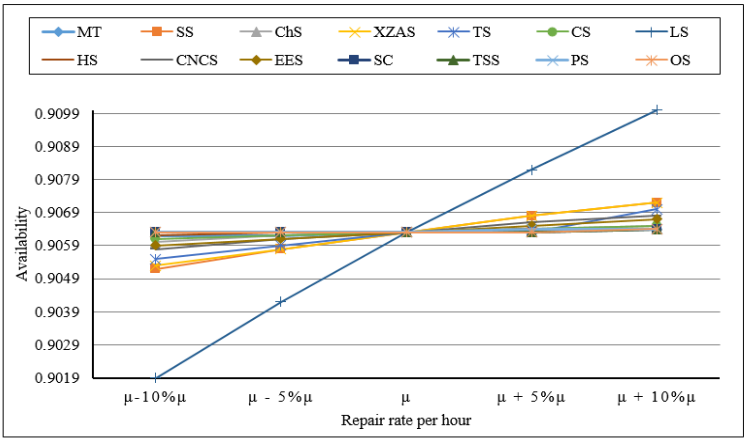

Figure 6

Effect of sub-system's repair rate on the SSA of the CNCMT

The effect of variation of sub-systems failure and repair rates on the SSA of the CNCMT is also presented in Table 8 and Figure 7. It is observed that the SSA of the CNCMT is 0.9063 (90.63%). It varies from 0.922 for minimum failure rate and maximum repair rate to 0.8882 for maximum failure rate and minimum repair rate (change in SSA = 3.8055%). The SSA matrix of the CNCMT is given in Table 8. This availability variation is greatly influenced due to the considerable variation in the failure rate of TS and repair rate of LS. The SSA of CNCMT can be improved to a large extent by improving the failure rate of TS and repair rate of LS. The failure and repair rates of other sub-systems can also be improved to maximize the SSA of the CNCMT.

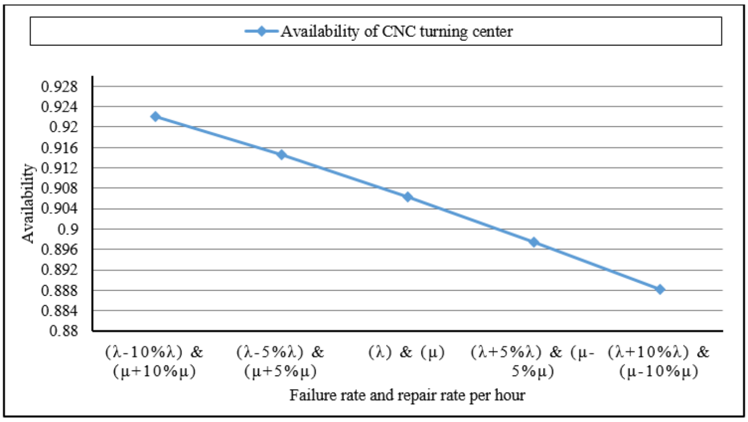

Figure 7
Effect of sub-system's failure and repair rates on the SSA of the CNCMT

Table 8
Availability matrix of CNCMT

λ_i μ_i	**λ_i - 10% λ_i**	**λ_i - 5% λ_i**	**λ_i**	**λ_i + 5% λ_i**	**λ_i + 10% λ_i**
μ_i - 10% μ_i	0.9063	--	--	--	0.8882
μ_i - 5% μ_i	--	0.9063	--	0.8975	--
μ_i	--	--	0.9063	--	--
μ_i + 5% μ_i	--	0.9145	--	0.9063	--
μ_i + 10% μ_i	0.9220	--	--	--	0.9063

Conclusions

Reliability, maintainability, and availability modeling and analysis are the integral parts of the design of any Computerized Numerical Control Machine Tool (CNCMT). This paper aimed to investigate the steady-state availability of a typical CNCMT using a developed TBF and TTR data analysis framework. The results obtained from reliability analysis show that the failure rate of the CNCMT is almost constant as the value of shape parameter (β) is very close to 1. The system MTBF is almost 2000 hours, which means that nearly four failures of the CNCMT will occur per year. CNC Sub-system (CNCS), Chuck Sub-system (ChS), Electrical and Electronic Sub-system (EES), Hydraulic Sub-system (HS), Main Transmission (MT), Turret Sub-system (TS) and *X* and *Z*-axis Sub-system (XZAS) are the critical sub-systems of the CNCMT from a reliability perspective. Lubrication Sub-system (LS), Spindle Sub-system (SS), and XZAS are the sub-systems that require considerable time from a maintenance perspective. The SSA of the CNCMT is estimated to be 0.9063 (90.63%). It varies from 0.922 to 0.8882 (change in SSA = 3.8055%) for 90% confidence level. This variation in the availability value is due to the large variation in the failure rate of TS and repair rate of LS. Therefore, it is concluded that the SSA of the CNCMT is significantly affected by the repair rate of the LS and the failure rate of the TS. This is useful in deciding the optimum values of failure and repair rates of these sub-systems for maximum availability. The results of reliability, maintainability, and availability analysis can be used further to develop the life cycle costing model of the CNCMT. Future works will be devoted to the development of a dynamic reliability model for the CNCMTs.

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