# Reliability-Oriented Electricity Distribution System Switch and Tie Line Optimization 

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#### Abstract

In the past decade, enhancing the reliability of distribution networks by means of optimal switch placement has attracted much attention. In the case of failures in a distribution feeder, such disconnect switches will isolate the faulted section, and the customers downstream of the faulted point can be supplied by neighboring feeders through tie lines. Nevertheless, such reserve branches not only might experience failures themselves but also may not even exist prior to the switch placement. Accordingly, this paper presents a mathematical-programming-based model for the concurrent placement of disconnect switches and tie lines in the distribution networks to enhance the service reliability, considering both practical benefits and drawbacks of such reserve branches. In the proposed model, installation of remote-controlled and manual switches at various locations of distribution feeders together with potential tie lines are considered. Also, practical operational constraints regarding the utilization of tie lines, and the impact of failures in such reserve branches on the reliability indices are meticulously modeled in the proposed formulation. Unreliability cost is estimated based on a reward-penalty scheme and the revenue lost due to the not supplied demand during the network contingencies. As an instance of mixed-integer linear programming, the proposed optimization model can be efficiently solved to the global optimality using commercially available software. Aiming at investigating the applicability of the proposed model, it is implemented on a test network, and the results are thoroughly analyzed through various case studies.


INDEX TERMS Electricity distribution system, mixed-integer linear programming, reliability, rewardpenalty scheme, switch optimization, tie line.

## NOMENCLATURE

INDICES
$i$ Index for zones of the reward-penalty scheme.
$j$ Index for sending or receiving end of tie lines.
$l$ Index for feeder sections and tie lines.
$r$ Index for tie lines.
$n$ Index for load nodes.

## SETS

$L \quad$ Set of all feeder sections.
$\Psi \quad$ Set of tie lines.
$\Psi_{n} \quad$ Subset of $\Psi$ containing the tie line corresponding to node $n$.

[^0]$\Gamma_{l, n} \quad$ Subset of $L$, which is comprised of feeder sections between feeder section $l$ and node $n$.
$\Omega \quad$ Set of all load nodes.
$\Omega_{l}^{D n} \quad$ Subset of $\Omega$, which includes nodes downstream of feeder section $l$.
$\Omega_{r}^{R} \quad$ Subset of $\Omega$, which consists of nodes in the feeder connected to the receiving side of tie line $r$.
$\Omega_{r}^{S} \quad$ Subset of $\Omega$, which consists of nodes in the feeder connected to the sending side of tie line $r$.
$\Omega_{l}^{U p}$ Subset of $\Omega$ containing nodes upstream of feeder section $l$.

## PARAMETERS

$g \quad$ Annual load growth rate.
$I C^{M}$, Investment costs for an MS and an RCS,
$I C^{R C}$ respectively.

| $I C_{r}^{R}$ | Investment cost for tie line $r$. |
| :---: | :---: |
| IPR | Incentive penalty rate. |
| IRR | Incentive reward rate. |
| M | A sufficiently large number. |
| $N_{n}$ | Total number of customers connected to lode node $n$. |
| $\begin{aligned} & O M^{M} \\ & O M^{R C} \end{aligned}$ | Operation and maintenance costs for an MS and an RCS, respectively. |
| OM $M_{r}^{R}$ | Operation and maintenance cost for tie line $r$. |
| $P_{n}$ | Average demand at node $n$. |
| PCap | Penalty cap. |
| RCap | Reward cap. |
| $R T_{l}$ | Repair time for feeder section $l$. |
| $S T^{R C}$, | Switching times for RCSs and MSs, |
| $S T^{M}$ | respectively. |
| $T$ | Load growth period. |
| $U_{R}, U_{S}$ | Useful lifetime of tie lines and switches, respectively. |
| $\alpha$ | Annual interest rate. |
| $\delta^{(.)}$ | Annuity factor. |
| $\lambda_{l}$ | Failure rate of feeder section $l$. |
| $\rho$ | Expected revenue from delivering one unit of electrical energy to the customers. |

## VARIABLES

EENS Expected energy not supplied.
Inv ${ }^{R}$, Inv ${ }^{S}$ Investment cost of tie lines and distribution switches, respectively.
$O F \quad$ Objective function.
$O p \quad$ Operational cost of distribution switches.
PRS Cost imposed by the reward-penalty scheme.
$R R C \quad$ Reliability-related costs.
SAIDI System average interruption duration index.
$x^{r} \quad$ Binary investment variable, which becomes 1 if tie line $r$ should be constructed, being 0 otherwise.
$x_{l}^{R, M} \quad$ Binary investment variable, which is equal to 1 if an MS is installed at the receiving end of feeder section or tie line $l$, being 0 otherwise.
$x_{r}^{R, N O} \quad$ Binary variable, which is equal to 1 if receiving side switch of tie line $r$ is normally-open, being 0 otherwise.
Binary investment variable, which is equal to 1 if an RCS is installed at the receiving end of feeder section or tie line $l$, being 0 otherwise.
$x_{l}^{S, M} \quad$ Binary investment variable, which is equal to 1 if an MS is installed at the sending end of feeder section or tie line $l$, being 0 otherwise. $x_{r}^{S, N O} \quad$ Binary variable, which is equal to 1 if sending side switch of tie line $r$ is normally-open, being 0 otherwise.
$x_{l}^{S, R C} \quad$ Binary investment variable, which is equal to 1 if an RCS is installed at the sending end of feeder section or tie line $l$, being 0 otherwise.

| $\beta^{D Z}, \beta^{P C}$ | Binary variables indicating whether their cor- <br> responding auxiliary variables can have a <br> non-zero value. |
| :--- | :--- |
| $\sigma_{i}$ | Non-negative auxiliary variable related to each <br> zone of the reward-penalty scheme. |
| $\tau_{l, n}$ | The expected annual interruption duration for <br> the customers at load node $n$ due to the fault |
| on feeder section $l$. |  |

## I. INTRODUCTION

Distribution system reliability has attracted notable attention in recent years, owing to the significant portion of power outages attributed to failures in medium voltage networks [1]. Aiming to unleash the great potentials for reliability enhancement at the distribution level, a plethora of researchers have studied various approaches, among which installation of disconnect switches has always been considered an effective strategy for outage mitigation [2]. Efficient placement of such switches harmonized with redundant feeder sections or tie lines can significantly reduce the duration of customer interruptions. In this respect, several models have been developed to optimize the investment in distribution network switches.

The methodology presented in [2] exists in the context of a network planning algorithm, and starts with full manual switching before placing tie lines, removing non-costeffective manual switches (MSs), upgrading MSs to remote or network circuit breakers where cost-effective. This is clearly unlikely to reach the theoretical optimum as the approach consists of a series of benefit to cost decisions. With the primary goal of minimizing unsupplied loads in the case of faults with a minimum number of switches, Calderaro et al. developed a mixed-integer non-linear programming (MINLP) model for the optimal switch placement problem [3]. Authors in [4] proposed a particle swarm optimization (PSO)-based model, which minimizes both the number of installed disconnect switches and the number of not-served customers. Ray et al. in [5] presented a multi-objective formulation for optimal placement of remote-controlled switches (RCSs), intending to minimize the cost of expected energy not served (EENS) as well as the cost of installed switches. In [6], a non-linear method was presented for determining the set of MSs which should be upgraded to RCSs so as to restore the most amount of load possible by operating RCSs during network contingencies. A mathematical formulation in the form of mixed-integer linear programming (MILP) was proposed for the optimal placement of distribution switches in [7], where the costs of switch deployment and customer outages are minimized. Authors in [8] also provided an MILP model to determine the arrangement of RCSs, while minimizing the unreliability cost as well as the installation and operational costs of the switches.

Unlike the non-linear formulations devised in [3]-[6], which required heuristic methods to solve them, the MILP models proposed in [7], [8] could be efficiently solved with guaranteed convergence by the branch-and-cut algorithm. Thus, in contrast to heuristic methods which not only may
require operator experience for parameter adjustment, but also their solutions might be far from the optimum, the MILP models can be readily leveraged to obtain the global optimal solution for the switch optimization problem. Nevertheless, as the MILP model introduced in [7], [8] was pioneering, there has been much room for further development. As a result, many research studies have been carried out in recent years to expand the MILP model for the optimal switch placement problem.

In this regard, Heidari et al. extended the model by considering the presence of distributed generation in the network [9]. Authors in [10] and [11] developed methods to consider uncertainties in the MILP switch optimization problem. Also, in [12], the MILP model was extended so that it would determine the RCS placement both in the main feeders and in the laterals. While the authors in [7]-[12] only considered the investments in one type of disconnect switches, Farajollahi et al. in [13] developed an MILP model for the optimal placement of both the MSs and RCSs, which also took into account switch malfunction probability. They also extended the MILP formulation in [14] to determine the optimal placement of fault indicators, MSs, and RCSs concurrently. Failure of the switches was also taken into account in the formulation devised in [15]. Reference [16] extended the previous models so as to conduct simultaneous placement of fuses, reclosers, MSs, and RCSs. Also, in [17], Li et al. proposed an MILP model for simultaneous deployment of fault indicators, MSs, and RCSs in a distribution network with branch lines. Finally, authors in [18] developed an MILP formulation for the optimal switch placement problem, which determined the type of tie switches at the reserve connection points.

Even though all of the studies mentioned above made significant contributions in expanding the reliability-oriented switch placement models, those models share quite similar, but unpragmatic, assumptions regarding the tie lines in the distribution network. Accordingly, in [3], [5], [9], no tie line exists at the end of network feeders. Also, the existence of tie lines and type of tie switches in the network are determined prior to the placement of other switches [4], [6]-[8], [10]-[17]. Lastly, although the MILP model in [18] determines the type of tie switches as a result of the optimization, it assumes that the existence of tie lines is specified prior to the optimization. In addition to these drawbacks, in the studies which considered the existence of tie lines in the distribution network [4], [6]-[8], [10]-[18], the potential failures in the tie line itself, where the tie switch is placed, is not taken into consideration. However, in practice, in order to monitor the availability of the tie switch in case of faults, the tie line should be energized by keeping one end of it connected to a feeder and the other end open for the sake of network radiality. As a result, faults in the tie line will impact the feeder which is connected to the normally-closed side of the tie line. Thus, such simplifications may not only lead to over-optimistic solutions but might also diverge the model from finding the most economical solution for the problem.

The former is due to the fact that the faults in the tie line itself and their effect on the customers' interruption are not considered, and the latter is because, in some cases, constructing a tie line might be a more efficient solution for the sake of enhancing the reliability. Hence, not only should the installation of tie switches be decided by the distribution company (DISCO), but also the option of constructing a new tie line with consideration of both its benefits and drawbacks.

To address such deficiencies, this paper considers concurrent deployment of disconnect switches, including RCSs and MSs, and tie lines in the distribution network, aiming at enhancing the system reliability. Moreover, the optimization will specify not only from what type but also on which side of the tie line the tie switches should be installed. In other words, in case of constructing a tie line, the normally-open and normally-closed sides of tie line, and the switch types installed on each side, if any, will be determined by the optimization. In addition to the benefits of tie lines, the impact of their potential failures is also considered in the proposed model.

Furthermore, unlike [7], [8], [10]-[17], which used the total customer interruption cost as the unreliability cost of the DISCO, we leverage other reliability indices, based on which the unreliability cost a DISCO may incur in practical applications is calculated [18]. To be more specific, although, according to its classical definition in [19], the total customers' interruption cost can appropriately reflect the network performance in terms of service reliability, this index cannot be accurately calculated in practice owing to its dependence on the customer damage functions of load nodes, the values of which are difficult to assess. Also, it goes without saying that the DISCOs are not obliged to consider such unreliability costs in their optimization models in practice, whereas they are typically under reliability incentive schemes [18], [20], [21]. These schemes are typically applied to system-oriented reliability indices such as system average interruption duration index (SAIDI) [21]. Thus, as done in [18], in this paper, the unreliability cost imposed on the DISCOs is determined through two more pragmatic reliability indices, namely SAIDI and EENS. While the former is typically used in the reliability incentive schemes based upon which the regulators penalize (or reward) the DISCOs, the latter is used to calculate the lost revenue because of the energy not delivered to the customers during supply outages. Since disconnect switches and tie lines affect the duration of customer interruptions, SAIDI and EENS satisfactorily reflect the benefits of installing such assets in the network. However, if the switching times could be less than the minimum duration threshold considered for the interruption frequency indices, e.g., system average interruption frequency index (SAIFI), they should also be included into the model to more accurately evaluate the reliability-related costs. In that case, the proposed reliability evaluation technique can be readily modified to model interruption frequency indices.

Considering the mentioned points, the main contributions of this paper are provided in what follows:

1) Proposing an MILP model for optimizing concurrent disconnect switch and tie line placement in distribution networks, aiming at minimizing the investment, installation, operational, and maintenance costs of MSs, RCSs, and tie lines as well as the reliability-related cost.
2) Developing a novel reliability assessment technique to model SAIDI and EENS considering the effects of tie lines.
3) Modeling practical planning and operational considerations of tie lines, including the decision on normally-open and normally-closed ends of tie lines as well as type of the switch installed at each side, tie line failures, and normal operating condition of tie lines.
The remainder of this paper is organized as follows. Section II describes the modeling concepts based upon which the mathematical formulations are developed in Section III. Section IV is devoted to the implementation of the model on a test network. Finally, the concluding remarks are presented in Section V.


FIGURE 1. A typical distribution feeder.

## II. PROBLEM MODELING

Fig. 1 represents the typical distribution feeder model considered in this paper. As per this figure, the first feeder section $l 1$ is equipped with a circuit breaker at its sending end. Marked with small circles, connection points of all feeder section to the load nodes are considered candidate locations for installing disconnect switches. At each candidate location, either an RCS or an MS may be installed. Thus, two binary investment variables are assumed for each candidate location, i.e., $x_{l}^{S, R C}$ and $x_{l}^{S, M}$ if the candidate location is at the sending end of feeder section $l$, or $x_{l}^{R, R C}$ and $x_{l}^{R, M}$ in case the candidate location corresponds to the receiving end of feeder section $l$. Thus, superscripts $S$ and $R$ stand for sending and receiving ends of feeder sections, whereas $R C$ and $M$ indicate type of the switch, i.e., remote-controlled or manual, respectively.

As another investment plan for enhancing the network reliability, installing tie lines, e.g., $r 1$ in Fig. 1, is considered. Each tie line connects the last node of a feeder to that of its adjacent feeder. In order to keep the radial topology of the network, such tie lines must be kept open during normal operation. However, for post-fault network reconfiguration, these tie lines are leveraged to transfer a portion or all of the interrupted demand to its adjacent feeder. On the other hand, if both ends of the tie lines are opened, the network operators cannot monitor the availability of tie lines. In other words, if a tie line is not live, its faults cannot be detected by the protection system. Thus, in practice, each tie line is energized
from one side, whereas a normally-open switch is installed at the other end. Thus, in addition to the four binary variables indicating the investments made in RCSs and MSs at both ends of each tie line, we need a binary variable, $x_{r}$, as the investment variable for constructing tie line $r$, and two binary variables, $x_{r}^{S, N O}$ and $x_{r}^{R, N O}$, to determine the normally-open side. For instance, if $x_{r}^{S, N O}$ becomes 1 , it implies that the switch at the sending side of tie line $r$ is normally-open. It should be emphasized that the sending and receiving sides of tie lines are selected arbitrarily and do not indicate their operating conditions in any way. In order to ensure the radial network operation while a tie line is live, only one of binary variables $x_{r}^{S, N O}$ and $x_{r}^{R, N O}$ can be equal to 1 . Such constraints are meticulously captured in the mathematical model presented in the next section.

## III. PROBLEM FORMULATION

While employing disconnect switches, either RCSs or MSs, and leveraging tie lines in all potential locations undoubtedly leads to the best service reliability, this will leave the distribution network with extensive costs, including investment as well as operation and maintenance costs of those devices. As a result, making a trade-off between service reliability and distribution network costs is inevitable.

In this section, a mathematical model is developed to determine the optimal plan for installation of RCSs and MSs as well as the construction of tie lines in the distribution network, for the sake of enhancing the system reliability. In this regard, the proposed MILP model determines the optimal location and type of disconnect switches and specifies if constructing tie lines are an efficient solution for enhancing the service reliability of customers.

## A. OBJECTIVE FUNCTION

The objective of the model is to minimize the total costs of the distribution network, which is formulated in (1). The objective function is comprised of four terms, namely the investment cost of switches, $\operatorname{In} v^{S}$, the investment cost of tie lines, $I n v^{R}$, the system operational cost, $O p$, and the reliability-related cost, $R R C$. Since the objective function minimizes the annualized system cost, each term of the objective function is calculated for one year. In this regard, the annuity factors for the investment cost of the switches and tie lines, i.e., $\delta^{U_{S}}$ and $\delta^{U_{R}}$, respectively, are defined in (2), which are calculated based on the annual interest rate and useful lifetime of the assets.

The investment cost of disconnect switches is determined in (3). Similarly, (4) calculates the investment cost required for tie lines. Equation (5) projects the system operational cost, which is comprised of operation and maintenance cost of all installed RCSs, MSs, and tie lines. It is worth noting that, in (3)-(5), without loss of generality, the investment or operation and maintenance cost of assets of the same type in all candidate locations is assumed to be identical; however, for the assets of the same type installed in different locations, non-identical costs can readily be considered in the proposed
model. Expression (6) defines binary decision variables for the deployment of disconnect switches and tie lines.

In (7), the reliability-related cost, $R R C$, is formulated, which includes the annualized value of revenue lost due to undelivered energy and the cost burdened by the reward-penalty scheme, $P R S$. The former is a linear function of EENS, and the latter is determined based on SAIDI. It is assumed that the network demand grows at a rate of $g$ annually for $T$ consecutive years, on the basis of which, (8) calculates the annuity factor of the revenue lost due to undelivered energy, i.e., $\delta^{T}$. However, the load growth does not have an impact on the PRS since it is determined based on the SAIDI, which is a reliability index indicating the average duration of interruption per customer, and, therefore, the annual load growth does not affect it.

$$
\begin{align*}
& \text { Minimize } O F= \delta^{U_{S}} I n v^{S}+\delta^{U_{R}} I n v^{R}+O p+R R C  \tag{1}\\
& \delta^{U}= \frac{\alpha}{1-(1+\alpha)^{-U}} ; \forall U \in\left\{U_{S}, U_{R}\right\}  \tag{2}\\
& I n v^{S}= \sum_{l \in L \cup \Psi}\left(x_{l}^{S, R C}+x_{l}^{R, R C}\right) I C^{R C} \\
&+\sum_{l \in L \cup \Psi}\left(x_{l}^{S, M}+x_{l}^{R, M}\right) I C^{M}  \tag{3}\\
& I n v^{R}= \sum_{r \in \Psi}\left(x_{r} I C_{r}^{R}\right)  \tag{4}\\
& O p= \sum_{l \in L \cup \Psi}\left(x_{l}^{S, R C}+x_{l}^{R, R C}\right) O M^{R C} \\
&+\sum_{l \in L \cup \Psi}\left(x_{l}^{S, M}+x_{l}^{R, M}\right) O M^{M} \\
&+\sum_{r \in \Psi}\left(x_{r} O M_{r}^{R}\right)  \tag{5}\\
& x_{l}^{S, R C}, x_{l}^{R, R C}, x_{l}^{S, M}, x_{l}^{R, M}, x_{r} \in\{0,1\} ; \\
& \forall l \in L \cup \Psi, \forall r \in \Psi  \tag{6}\\
& R R C= \delta^{T} \rho E E N S+P R S  \tag{7}\\
& \delta^{T}= \alpha\left(\frac{\left(\frac{1+g}{1+\alpha}\right)^{T}-1}{g-\alpha}+\frac{(1+g)^{T-1}}{\alpha(1+\alpha)^{T}}\right) \tag{8}
\end{align*}
$$

## B. RELIABILITY ASSESSMENT MODEL

In the proposed model, unlike the state-of-the-art on optimal switch placement, the existence of tie lines at the end of feeders are not taken for granted, and the optimization model determines if the installation of such lines are efficient for enhancing the service reliability. In this respect, a novel reliability assessment model is developed in this section, which not only considers the installation of disconnect switches in the feeder sections and tie lines but also takes into account the deployment of new tie lines. As noted earlier, to calculate the reliability-related cost, $R R C$, two reliability indices, EENS and SAIDI, should be determined. Accordingly, equations (9) and (10) respectively calculate EENS and SAIDI.

$$
\begin{equation*}
E E N S=\sum_{l \in L \cup \Psi} \sum_{n \in \Omega} \tau_{l, n} P_{n} \tag{9}
\end{equation*}
$$

$$
\begin{equation*}
S A I D I=\frac{\sum_{l \in L \cup \Psi} \sum_{n \in \Omega} \tau_{l, n} N_{n}}{\sum_{n \in \Omega} N_{n}} \tag{10}
\end{equation*}
$$

The annual interruption duration of the customers at each load node due to the feeder section faults is determined by jointly considering (11)-(29). It is worth noting that the optimization sets annual interruption duration variables, $\tau_{l, n}$, to their lower bounds, since the proposed optimization problem minimizes an objective function which is monotonically increasing with respect to the reliability indices, and, therefore, those corresponding variables.

Equation (11) ensures that the lower bound for annual interruption duration of load node $n$ due to faults in feeder section $l$ of the node's supplying feeder equals the failure rate of that section multiplied by the minimum restoration time, i.e., the RCS switching time. The case of equality happens only when node $n$ is placed at the upstream of feeder section $l$, and at least one RCS exists between the node and faulted section; or when node $n$ is downstream of feeder section $l$, and not only at least one RCS exists in between, but also a tie line is constructed at the end of the faulted distribution feeder, equipped with an RCS at its normally-open side. If not, other constraints will determine the lower bound for $\tau_{l, n}$.

Accordingly, when no RCS exists between the faulted feeder section and its upstream node, expressions (12) and (13) determine the annual interruption duration for the nodes located upstream of faulty feeder section $l$. If there is at least one MS, in the absence of RCSs, between them, (12) determines the lower bound for $\tau_{l, n}$. This is because, in this case, the right-hand side of (13) becomes zero, while the lower bound enforced by (12) is $\lambda_{l} S T^{M}$. On the contrary, if no disconnect switch exists between a faulted feeder section and its upstream node, (13) governs the lower bound for $\tau_{l, n}$. This is due to the fact that the repair time, $R T$, is higher than the switching times for RCSs and MSs, $S T^{(.)}$, and since the lower bounds imposed by (11)-(13) are respectively $\lambda_{l} S T^{R C}$, $\lambda_{l} S T^{M}$, and $\lambda_{l} R T$, constraint (13) dictates the lower bound.

$$
\begin{align*}
& \tau_{l, n} \geq \lambda_{l} S T^{R C} ; \quad \forall l \in L, \quad \forall n \in \Omega_{l}^{U p} \cup \Omega_{l}^{D n}  \tag{11}\\
& \tau_{l, n} \geq \lambda_{l} S T^{M}\left[1-x_{l}^{S, R C}-\sum_{\bar{l} \in \Gamma_{l, n}}\left(x_{\bar{l}}^{S, R C}+x_{\bar{l}}^{R, R C}\right)\right] \\
& \forall l \in L, \forall n \in \Omega_{l}^{U p}  \tag{12}\\
& \tau_{l, n} \geq \lambda_{l} R T_{l}\left[1-x_{l}^{S, R C}-\sum_{\bar{l} \in \Gamma_{l, n}}\left(x_{\bar{l}}^{S, R C}+x_{\bar{l}}^{R, R C}\right)\right. \\
&\left.-x_{l}^{S, M}-\sum_{\bar{l} \in \Gamma_{l, n}}\left(x_{\bar{l}}^{S, M}+x_{\bar{l}}^{R, M}\right)\right] ; \forall l \in L, \forall n \in \Omega_{l}^{U p} \tag{13}
\end{align*}
$$

To calculate the annual interruption duration, $\tau_{l, n}$, for the nodes, $n$, located downstream of faulted feeder section, $l$, in addition to the disconnect switches, the operationality of tie lines and the switches installed in them should be taken into consideration. This is done by jointly considering (14)-(18) in the mathematical formulation. As mentioned before, if a tie line, equipped with an RCS at its normally-open side,
is installed at the end of a faulty distribution feeder and at least one RCS exists between the faulted feeder section and the downstream node, $\tau_{l, n}$ is equal to the switching time of an RCS. However, in other cases, the lower bound of $\tau_{l, n}$ for the node downstream of a faulted feeder section is determined by (14)-(17).

Structurally identical to (12) and (13), expressions (14) and (15) determine the restoration time of node $n$ downstream of faulted branch $l$, when there is a tie line at the end of the faulty feeder, but no RCS exists between the node and faulted feeder section. In this respect, (14) sets the lower bound for $\tau_{l, n}$ in the case where no RCS, but at least one MS, exists between the faulted branch and the downstream node, and (15) determines $\tau_{l, n}$ when there is no disconnect switch in between. This is based on the fact that when there is at least one RCS between the faulted branch and its downstream node, the right-hand-side of both (14) and (15) becomes zero, and none of them determines the lower bound for $\tau_{l, n}$. However, if no RCS, but at least an MS, exists in between, only the right-hand-side of (15) is zero, and (14) determines the lower bound of $\tau_{l, n}$. Finally, if no disconnect switch is installed in between, the lower bound of $\tau_{l, n}$ is dictated by (15).

$$
\begin{align*}
& \tau_{l, n} \geq \lambda_{l} S T^{M}\left[1-x_{l}^{R, R C}-\sum_{\bar{l} \in \Gamma_{l, n}}\left(x_{\bar{l}}^{S, R C}+x_{\bar{l}}^{R, R C}\right)\right] \\
& \forall l \in L, \forall n \in \Omega_{l}^{D n}  \tag{14}\\
& \tau_{l, n} \geq \\
& \quad \lambda_{l} R T_{l}\left[1-x_{l}^{R, R C}-\sum_{\bar{l} \in \Gamma_{l, n}}\left(x_{\bar{l}}^{S, R C}+x_{\bar{l}}^{R, R C}\right)\right.  \tag{15}\\
& \left.\quad-x_{l}^{R, M}-\sum_{\bar{l} \in \Gamma_{l, n}}\left(x_{\bar{l}}^{S, M}+x_{\bar{l}}^{R, M}\right)\right] ; \quad \forall l \in L, \forall n \in \Omega_{l}^{D n}
\end{align*}
$$

Nevertheless, the service restoration time for the nodes downstream of a faulted feeder section also depends on whether there is a tie line at the end of faulty feeder and on the type of switch installed at the normally-open side of the tie line. Constraints (16)-(18) jointly set the lower bound of $\tau_{l, n}$ when an RCS is not installed at the normally-open side of the tie line. In case the tie line is constructed (i.e., $x_{r}$ equals 1 ) and an MS is installed at its normally-open side, (16) determines the lower bound of $\tau_{l, n}$. This is because, in this case, the right-hand side of (17) is 0 or less, while, in (16), it is not, so the latter governs the lower bound. However, if neither an MS nor an RCS is installed at the normally-open side of tie line (i.e., the tie line has not been constructed), (17) dictates the lower bound of $\tau_{l, n}$. This is due to the fact that if no tie line is installed (i.e., $x_{r}$ equals 0 ), (18) forces all investment variables of disconnect switches at tie line $r$ to be 0 , and, therefore, the right-hand side of (17) would be $\lambda_{l} R T_{l}$. This means that, in case of faults, the restoration time of customers located downstream of the faulted feeder section equals the repair time of the faulty branch, which is quite sensible since until the faulted feeder section is not repaired, the service of
the customers located downstream cannot be restored as there is no tie line.

$$
\begin{align*}
\tau_{l, n} \geq & \lambda_{l} S T^{M}\left[1-\sum_{r \in \Psi_{n}}\left(x_{r}^{S, R C} x_{r}^{S, N O}+x_{r}^{R, R C} x_{r}^{R, N O}\right)\right] ; \\
& \forall l \in L, \forall n \in \Omega_{l}^{D n}  \tag{16}\\
\tau_{l, n} \geq & \lambda_{l} R T_{l}\left[1-\sum_{r \in \Psi_{n}}\left(\left(x_{r}^{S, R C}+x_{r}^{S, M}\right) x_{r}^{S, N O}\right.\right. \\
& \left.\left.+\left(x_{r}^{R, R C}+x_{r}^{R, M}\right) x_{r}^{R, N O}\right)\right] ; \\
& \forall l \in L, \forall n \in \Omega_{l}^{D n}  \tag{17}\\
x_{r}^{j, R C}+x_{r}^{j, M} \leq & x_{r} ; \quad \forall r \in \Psi, \quad \forall j \in\{S, R\} \tag{18}
\end{align*}
$$

It goes without saying that multiplication of two binary variables, e.g., $x_{r}^{S, R C} x_{r}^{S, N O}$ in (16), makes the mathematical model non-linear. Thus, in order to linearize (16) and (17) in the model, multiplication of two given binary variables, $a$ and $b$, can be equivalently modeled as below:

$$
\begin{align*}
& c \leq a  \tag{19}\\
& c \leq b  \tag{20}\\
& c \geq a+b-1  \tag{21}\\
& 0 \leq c \leq 1 \tag{22}
\end{align*}
$$

where $c$ is a binary-valued continuous variable that equals the multiplication of $a$ and $b$. Thus, by replacing the non-linear terms with continuous variables in the non-linear equations and considering mentioned constraints in the mathematical model, both (16) and (17) can be modeled as linear constraints.

Even though installing a tie line at the end of a distribution feeder can significantly increase the speed of service restoration for the customers downstream of the faulted feeder section, a fault may happen in the tie line itself. Consequently, faults happening in a tie line interrupt the service of customers which are located in the feeder connected to the normally-closed side of that tie line. However, the service of customers connected to the normally-open side of the faulted tie line would not be interrupted.

Equations (23)-(26) jointly determine the annual interruption duration of network customers because of the faults in tie lines. Expression (23), which represents the non-negativity of $\tau_{r, n}$, determines its lower bound only when load node $n$ is not located in the feeder connected to the normally-closed side of tie line $r$. Constraint (24) sets the minimum of $\tau_{r, n}$ if node $n$ is connected to the normally-closed side of tie line $r$. However, if an RCS is neither installed at the normally-closed side of faulted tie line nor exists between load node $n$ and the tie line, (25) and (26) determine the lower bound of $\tau_{r, n}$. In this circumstance, if an MS exists either at the normally-closed side or between the faulted tie line and the load node, (25) dictates the lower bound, since the right-hand side of (26) would be 0 or less. This means that the service to the customers located at node $n$ can be restored after the MS isolates the faulted section from the rest of the feeder, so the interruption duration
is equal to the MS switching time. On the other hand, when there is no disconnect switch either at the normally-closed side of faulted tie line $r$ or between node $n$ and the tie line, (26) determines that $\tau_{r, n}$ is not lower than $\lambda_{r} R T_{r}$. This is because, in case of faults in the tie line, the customers located at node $n$ cannot be restored until the tie line is fully repaired, as there are no switches in between.

$$
\begin{align*}
\tau_{r, n} \geq & 0 ; \forall r \in \Psi, \forall n \in \Omega_{r}^{S} \cup \Omega_{r}^{R}  \tag{23}\\
\tau_{r, n} \geq & \lambda_{r} S T^{R C}\left(1-x_{r}^{j, N O}\right) ; \\
& \forall r \in \Psi, \forall n \in \Omega_{r}^{j}, \forall j \in\{S, R\}  \tag{24}\\
\tau_{r, n} \geq & \lambda_{r} S T^{M}\left[1-x_{r}^{j, N O}-x_{r}^{j, R C}\right. \\
& \left.-\sum_{\bar{l} \in \Gamma_{r, n}}\left(x_{\bar{l}}^{S, R C}+x_{\bar{l}}^{R, R C}\right)\right] \\
& \forall r \in \Psi, \forall n \in \Omega_{r}^{j}, \forall j \in\{S, R\}  \tag{25}\\
\tau_{r, n} \geq & \lambda_{r} R T_{r}\left[1-x_{r}^{j, N O}-x_{r}^{j, R C}-x_{r}^{j, M}\right. \\
& \left.-\sum_{\bar{l} \in \Gamma_{r, n}}\left(x_{\bar{l}}^{S, R C}+x_{\bar{l}}^{R, R C}+x_{\bar{l}}^{S, M}+x_{\bar{l}}^{R, M}\right)\right] \\
& \forall r \in \Psi, \quad \forall n \in \Omega_{r}^{j}, \forall j \in\{S, R\} \tag{26}
\end{align*}
$$

Expressions (27)-(29) are logical constraints related to the investments in tie lines and deployment of corresponding switches. In this regard, (27) sets both $x_{r}^{R, N O}$ and $x_{r}^{S, N O}$ to 1 (i.e., both ends of tie line $r$ are considered open) when tie line $r$ is not constructed and ensures that exactly one of the two binary variables, either $x_{r}^{R, N O}$ or $x_{r}^{S, N O}$, equals one (i.e., only one side of a constructed tie line must be normally-open) when the tie line is installed. Equations (28) and (29) together with (27) serve to guarantee that a disconnect switch, either an RCS or an MS, is installed at the normally-open side of tie line $r$ in case it is constructed.

$$
\begin{align*}
x_{r}^{R, N O}+x_{r}^{S, N O} & =2-x_{r} ; \quad \forall r \in \Psi  \tag{27}\\
x_{r}^{S, R C}+x_{r}^{S, M} & \geq x_{r}^{S, N O}-x_{r}^{R, N O} ; \quad \forall r \in \Psi  \tag{28}\\
x_{r}^{R, R C}+x_{r}^{R, M} & \geq x_{r}^{R, N O}-x_{r}^{S, N O} ; \quad \forall r \in \Psi \tag{29}
\end{align*}
$$

## C. REWARD-PENALTY SCHEME MODELING

As noted earlier, the proposed MILP model considers a reward-penalty scheme, on the basis of which the distribution regulator rewards the distribution utility if it maintains a sufficient level of reliability and penalizes it if it fails. It is worth mentioning that reward-penalty schemes are regulatory tools that have been implemented in many countries, to ensure that distribution utilities provide a reliable service for their customers [23]-[25].

Accordingly, the general structure of the reward-penalty scheme is represented in Fig. 2. The figure depicts the relation between the amount of penalty or reward and a reliability index (i.e., SAIDI in this paper). According to


FIGURE 2. Reward-penalty graph.
this figure, a lower value of SAIDI brings about more reward, or less penalty for the DISCO. However, the amounts of both reward and penalty are limited to specific levels so as to restrict the financial risks associated with the reward-penalty scheme [24].

To be more specific, the reward-penalty scheme is comprised of five zones in terms of how the DISCO is rewarded or penalized with respect to its SAIDI. In the zone where SAIDI is less than the reward cap point (RCP), the DISCO receives a specified reward, the reward cap (RCap). Similarly, in the zone where SAIDI is more than the penalty cap point (PCP), the DISCO will suffer a definite amount of penalty, the penalty cap (PCap). According to Fig. 2, the reward point $(\mathrm{RP})$ and penalty point ( PP ) separate three other zones located between the PCP and RCP. For the SAIDI values in the zone between RP and PP, known as the dead zone [20], neither penalty nor reward is implemented. Also, the slopes of the straight lines between RCP and RP, and PCP and PP are equal to the incentive reward rate and incentive penalty rate, respectively.

Based on the above-mentioned framework, (30)-(41) jointly calculate the cost applied through the reward-penalty scheme (i.e., $P R S$ ). In this regard, five non-negative continuous variables, $\sigma_{i}$, are utilized, each of which is related to one of the zones in the reward-penalty scheme depicted in Fig. 2. By aggregating all five variables, (30) calculates the value of SAIDI, and (31) determines the cost imposed by the reward-penalty scheme, $P R S$. It goes without saying that as the objective function is strictly increasing with respect to $P R S$, the optimization solver sets $P R S$ to its minimum value. Constraints (32)-(36) specify the maximum values for variables $\sigma_{i}$. Expression (37) serves to guarantee that $\sigma_{3}$ (the variable associated with the dead zone) can be set to a non-zero value only if $\sigma_{2}$ reaches its maximum value imposed by (33). In the same way, (38) ensures that $\sigma_{5}$ can take a non-zero value only after $\sigma_{4}$ reaches its maximum specified in (35). As an extra logical constraint, (39) is added to the model to enhance the efficiency of the optimization solver. Expression (40) represents the non-negativity of variables $\sigma_{i}$, while (41) implies the binary nature of variables $\beta^{D Z}$
and $\beta^{P C}$.

$$
\begin{align*}
S A I D I & =\sum_{i=1}^{5} \sigma_{i}  \tag{30}\\
P R S & \geq-R C a p+\sigma_{2} I R R+\sigma_{4} I P R  \tag{31}\\
\sigma_{1} & \leq R C P  \tag{32}\\
\sigma_{2} & \leq \mathrm{RP}-\mathrm{RCP}  \tag{33}\\
\sigma_{3} & \leq \beta^{D Z}(\mathrm{PP}-\mathrm{RP})  \tag{34}\\
\sigma_{4} & \leq \mathrm{PCP}-\mathrm{PP}  \tag{35}\\
\sigma_{5} & \leq \beta^{P C} M  \tag{36}\\
\beta^{D Z} & \leq 1+\frac{\sigma_{2}-(\mathrm{RP}-\mathrm{RCP})}{R P-R C P}  \tag{37}\\
\beta^{P C} & \leq 1+\frac{\sigma_{4}-(\mathrm{PCP}-\mathrm{PP})}{P C P-P P}  \tag{38}\\
\beta^{D Z} & \geq \beta^{P C}  \tag{39}\\
\sigma_{i} & \geq 0 ; \quad \forall i \in\{1, \ldots, 5\}  \tag{40}\\
\beta^{D Z}, \beta^{P C} & \in\{0,1\} \tag{41}
\end{align*}
$$

## IV. NUMERICAL STUDY

To assess the applicability of the proposed switch and tie line placement model, it is implemented on a modified version of the test distribution network connected to bus 2 of the Roy Billinton test system (RBTS). Thus, in the following, the test network data and other details of the simulations are provided thoroughly for the sake of reproducibility; afterwards, two case studies have been conducted, and the results are comprehensively discussed so as to scrutinize the impact of the key parameters in the model on the obtained solutions. Since the proposed model is in an MILP form, it can be readily solved by commercially available solvers. We implemented the model in GAMS 24.9 and solved it by CPLEX 12.6, where the optimality gap was set to 0 .

## A. TEST NETWORK AND SIMULATION ASSUMPTIONS

The test network consists of 4 feeders, 14 feeder sections, and 14 load nodes, as represented in Fig. 3. Also, 2 candidate tie lines and a total of 28 candidate locations for the installation of disconnect switches (RCSs or MSs) are taken into account.


FIGURE 3. Single-line diagram of the distribution network connected to RBTS Bus 2.

In the implemented simulations, the expected revenue from delivering one unit of electrical energy to the customers, $\rho$, is assumed $0.12 \mathrm{k} \$ / \mathrm{MWh}$. Also, the useful lifetime of disconnect switches and tie lines, $U_{S}$ and $U_{R}$, are considered to be 15 and 35 years, respectively, while an annual interest rate of $8 \%$ is taken into account. Additionally, it is assumed that the network demand will rise for ten years with a constant annual growth rate of $3 \%$.

TABLE 1. Economic data for disconnect switches and tie lines.

|  | Switches |  | Tie Lines |  |
| :--- | ---: | ---: | ---: | ---: |
|  | MS | RCS | $r 1$ | $r 2$ |
| Investment and installation cost $(\mathrm{k} \$)$ | 0.5 | 4.7 | 43.0 | 51.0 |
| Operation and maintenance cost $(\mathrm{k} \$)$ | 0.010 | 0.094 | 0.430 | 0.510 |

TABLE 2. Specifications of distribution line types.

|  | Overhead Lines | Underground Cables |
| :--- | :---: | :---: |
| Repair time (h) | 3 | 12 |
| Failure rate (failures/km/year) | 0.065 | 0.040 |

TABLE 3. Length and type of test network lines.

| Feeder Section | Type | Length $(\mathrm{km})$ |
| :--- | :---: | :---: |
| $l 1, l 3, l 7, l 12$ | Overhead line | 0.75 |
| $l 2, l 5, l 10, l 13$ | Underground cable | 0.75 |
| $l 4, l 9, l 14$ | Underground cable | 0.60 |
| $l 6$ | Overhead line | 0.60 |
| $l 8, l 11$ | Underground cable | 0.80 |
| $r 1$ | Overhead line | 0.70 |
| $r 2$ | Overhead line | 0.85 |

Table 1 represents the economic data for the disconnect switches and potential tie lines. Also, the distribution lines of the test network, including both feeder sections and tie lines, are assumed to be of two different types, the specifications of which are provided in Table 2. The data for distribution lines and load nodes of the test network are presented in Tables 3 and 4, respectively. The switching times for RCSs and MSs are also assumed 0.1 hour and 1 hour, respectively.

As for the reward-penalty scheme, the values of PCP, PP, RP, and RCP are assumed to be equal to $0.90,0.40,0.37$ and 0.05 of an hour per customer per year, respectively. Also, the incentive reward rate, $I R R$, and incentive penalty rate, $I P R$, are set at $25 \mathrm{k} \$$ and $35 \mathrm{k} \$$ per unit of SAIDI, respectively.

## B. INVESTIGATING THE IMPACT OF THE REWARD-PENALTY SCHEME

In order to numerically substantiate the beneficial impact of the reward-penalty scheme on the service reliability level, the proposed switch and tie line placement model has been solved for two cases. In Case I, the reward-penalty scheme is considered, while it is eliminated in Case II. The results

TABLE 4. Data for network load nodes.

| Load Node | Number of Customers | Average Demand (MW) |
| :--- | :---: | :---: |
| $n 1$ | 420 | 1.070 |
| $n 2$ | 211 | 1.101 |
| $n 3$ | 11 | 1.020 |
| $n 4$ | 10 | 0.454 |
| $n 5$ | 1 | 1.000 |
| $n 6$ | 1 | 1.150 |
| $n 7$ | 210 | 0.535 |
| $n 8$ | 410 | 0.985 |
| $n 9$ | 2 | 1.132 |
| $n 10$ | 10 | 0.454 |
| $n 11$ | 210 | 0.904 |
| $n 12$ | 400 | 0.900 |
| $n 13$ | 1 | 0.566 |
| $n 14$ | 1 | 1.020 |

extracted from the simulations of the two mentioned cases are represented in Table 5. As per this table, the EENS and especially the SAIDI are significantly lower (i.e., the service reliability is higher) in Case I compared to Case II, which is due to the higher investments made on switches and tie lines in the former case.

TABLE 5. Numerical results for the distribution network connected to RBTS Bus 2.

|  | Case I | Case II |
| :--- | ---: | ---: |
| Objective function $O F(\mathrm{k} \$)$ | 4.384 | 1.329 |
| Annualized investment cost of switches $\delta^{U_{S}} \operatorname{Inv} v^{S}(\mathrm{k} \$)$ | 2.115 | 0.058 |
| Operational cost of switches $O p^{S}(\mathrm{k} \$)$ | 0.352 | 0.010 |
| Annualized investment cost of tie lines $\delta^{U_{R}} \operatorname{Inv} v^{R}(\mathrm{k} \$)$ | 4.376 | 0 |
| Operational cost of tie lines $O p^{R}(\mathrm{k} \$)$ | 0.510 | 0 |
| Penalty-reward costs $P R S(\mathrm{k} \$)$ | -3.701 | - |
| Revenue lost from undelivered energy $\delta^{T} \rho E E N S(\mathrm{k} \$)$ | 0.732 | 1.261 |
| SAIDI (hours/customer/year) | 0.222 | 0.714 |
| EENS (MWh) | 5.047 | 8.694 |

The optimal tie line and switch arrangements obtained in Case I are depicted in Fig. 4. According to this figure, in Case I, eleven disconnect switches (seven MSs and three RCSs) and one tie line are installed, whereas in Case II, only one MS is placed at the sending end of feeder section $l 3$, which means that neither a tie line nor an RCS is installed in this case. As a result of more investments made in the switches and the tie line, not only are the investment costs higher in Case I, but also the total cost (i.e., the objective function) is increased in Case I compared to that of Case II. This means that, when the reward-penalty scheme is imposed on the DISCO, it has no choice but to increase the service reliability of its customers through investing in more switches and tie lines, and, in this case, the investment and operation costs of installed assets outweigh the incentive earned through the reward-penalty scheme. Thus, through implementing such schemes, the trade-off between the costs and the reliability for the DISCO has changed in a way that the optimal switch and tie line arrangement brings more service reliability for the customers.


FIGURE 4. The optimal switch and tie line plan for Case I.

As per Fig. 4, it is evident that when the reward-penalty scheme is considered (i.e., Case I), eleven disconnect switches are invested in. Most of these switches are installed close to the load nodes with a high number of customers, namely $n 1, n 2, n 7, n 8, n 11$, and $n 12$, to protect them from lengthy service interruptions in case of faults. However, when the reward-penalty scheme is eliminated (in Case II), only one switch is installed in the middle of distribution feeder 1 (more precisely, at the sending end of feeder section $l 3$ ). This is because, in this case, the only reliability index that is of importance is EENS, which is related to the amount of load not supplied as a result of faults. Thus, protecting the feeder with the most demand (i.e., feeder 1) is of the highest priority, and the DISCO does so by installing a switch in that feeder. However, due to the high switch investment cost and limited incentives for reliability enhancement, the DISCO lacks the motivation for investing in switches or tie lines in other potential locations.

Nonetheless, implementing the reward-penalty scheme in Case I motivates the DISCO to install eleven disconnect switches in the network. As can be seen in Fig. 4, three of them are remote-controlled, interestingly, all of which are installed on underground cables. The main reason is that despite their lower failure rate, the repair time of faulted underground cables is four times that of the overhead lines. Thus, in order to protect load nodes with a high number of customers from the long duration of service interruption in the case of faults in underground cables, RCSs are installed in the locations where they can minimize the restoration time of customers the most. As an illustration for this, due to the optimal placement of RCSs in Case I, the network operator can rapidly restore the electrical power of many customers located at both load nodes $n 7$ and $n 8$, if a fault happens in feeder sections $l 9$ or $l 10$, both of which are underground cables.

Based on the obtained results, the only investment in tie lines has been made in Case I for tie line $r 2$, which is depicted in Fig. 4. In this case, tie line $r 2$ (which connects feeders 3 and 4) has been chosen for construction over $r 1$, since both the aggregated demand and number of customers in feeders 3 and 4 are higher than those for feeders 1 and 2. While installation of tie line $r 2$ has contributed to enhancing
the network reliability, the tie line itself accounts for 0.003 of the SAIDI value and 0.272 MWh of the EENS due to potential faults in it. Nevertheless, as its beneficial impact far outweighs the disadvantage, the optimization solver determines that tie line $r 2$ should be constructed. Finally, although construction of tie lines considerably increases the rate of service restoration for the customers downstream of a faulted feeder section, their high investment cost causes them not to be an efficient alternative in every potential location. Hence, investing in tie lines should also be considered as an available option, similar to RCSs and MSs, in the reliability-oriented optimization models so as to guarantee the optimal plan for placement of both switches and tie lines for the sake of enhancing the reliability.

## C. SENSITIVITY ANALYSIS ON THE INCENTIVE REWARD RATE

In this section, the impact of the incentive reward rate, $I R R$, as a key parameter in the reward-penalty scheme is scrutinized. To do so, we have carried out the simulations for various values of $I R R$, from $17.5 \mathrm{k} \$$ to $32.5 \mathrm{k} \$$. Table 6 presents the number of switches and tie lines that has been installed in each of the cases. Also, Fig. 5 and Fig. 6 depict EENS and SAIDI of the system in the investigated cases, respectively.

TABLE 6. Number of switches and tie lines installed in the test network for various incentive reward rate values.

| IRR (k\$) | 17.5 | 20.0 | 22.5 | 25.0 | 27.5 | 30.0 | 32.5 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Number of RCSs | 3 | 1 | 2 | 3 | 3 | 3 | 3 |
| Number of MSs | 3 | 10 | 9 | 8 | 8 | 8 | 14 |
| Number of tie lines | 0 | 1 | 1 | 1 | 1 | 1 | 2 |

According to Table 6, in the cases with a higher incentive reward rate value, more investments are made in disconnect switches and tie lines, as expected. Still, the number of installed switches and tie lines have no specified relationship with the value of $I R R$. This is due to the fact that for each $I R R$, a specific arrangement of switches and tie lines is the optimal plan, and even a relatively small increase or decrease in $I R R$ may change the arrangement thoroughly. For example, when the $I R R$ increases from $17.5 \mathrm{k} \$$ to $20 \mathrm{k} \$$, the number of RCSs decreases by two while seven more MSs and one more tie line are installed compared to the previous case.

With the increase of $I R R$, more switches and tie lines are installed, and more installation of such assets in the network results in lower EENS and SAIDI (i.e., higher reliability) in the cases with a higher incentive reward rate, which can be seen in Fig. 5 and Fig. 6. However, the trend of changes in the reliability indices is not linear with respect to the incentive reward rate value, $I R R$. As an example, the increase of $I R R$ from $17.5 \mathrm{k} \$$ to $20 \mathrm{k} \$$ and from $20 \mathrm{k} \$$ to $22.5 \mathrm{k} \$$ can be compared. In the former, both SAIDI and EENS are decreased significantly, whereas, in the latter, both do not change considerably. For the values of $I R R$ in the interval between $20 \mathrm{k} \$$ and $30 \mathrm{k} \$$, the changes in both of them,


FIGURE 5. EENS for various incentive reward rate values.


FIGURE 6. SAIDI for various incentive reward rate values.
especially EENS, are negligible, which can be observed in Fig. 5 and Fig. 6. Nonetheless, similar to the transition of $I R R$ from $17.5 \mathrm{k} \$$ to $20 \mathrm{k} \$$, with the increase from $30 \mathrm{k} \$$ to $32.5 \mathrm{k} \$$, SAIDI and EENS of the system decline considerably.

Based on these observations, it can be perceived that in some intervals, variations in $I R R$ do not lead into significant changes in the DISCO's optimal plan for switch and tie line placement and, therefore, the reliability indices. In some other situations, even a relatively small variation in the $I R R$ may result in a considerable change in SAIDI or EENS. This means that although the values of reliability indices (e.g., EENS or SAIDI) do not have a direct relationship with the amount of incentive, in some cases, those indices will be changed meaningfully even with a small increase or decrease in the incentive. Thus, it is evident that increasing the amount of incentive will result in a relatively higher reliability level, yet the extent to which the incentives should be increased for each situation is not apparent. As a result, determining the appropriate amount of incentive, such as $I R R$ in a reward-penalty scheme, is of great importance if the regulators aim to efficiently enhance the reliability of distribution networks to a specific level but not unduly incentivize over-investment.

## V. CONCLUSION

An MILP model has been proposed in this paper to optimize the placement of disconnect switches, i.e., MSs and RCSs, and tie lines within the distribution network. In this model,
the connection point of every feeder section or potential tie line to a load node was considered a candidate location for installation of such switches. Minimizing investment and operational costs of the switches and tie lines as well as the reliability-related cost was regarded as the objective of the optimization model. The reliability-related cost was estimated based upon a reward-penalty scheme based on SAIDI and also the revenue lost due to the undelivered energy, which was a function of EENS. A reliability evaluation model was, then, developed to calculate SAIDI and EENS, considering the impact of potential disconnect switches and tie lines on the customer interruptions. The model was finally applied to a test distribution network, and the obtained results were analyzed in detail. More specifically, it was shown that the placement of tie lines at the end of feeders might not always be an optimal strategy for enhancing the reliability. Thus, unlike the state-of-the-art on MILP distribution switch placement models, decision on the construction of tie lines should also be incorporated into the model. The outcomes also revealed the significant, yet nonlinear impact of incentive reward rate on the DISCO's motivations for enhancing the network reliability. Future research will address the inclusion of lateral distributors into the presented model.

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