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Reliability prediction of engineering systems with competing failure modes due to component degradation †

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Abstract

Reliability of an engineering system depends on two reliability metrics: the mechanical reliability, considering component failures, that a functional system topology is maintained and the performance reliability of adequate system performance in each functional configuration. Component degradation explains not only the component aging processes leading to failure in function, but also system performance change over time. Multiple competing failure modes for systems with degrading components in terms of system functionality and system performance are considered in this paper with the assumption that system functionality is not independent of system performance. To reduce errors in system reliability prediction, this paper tries to extend system performance reliability prediction methods in open literature through combining system mechanical reliability from component reliabilities and system performance reliability. The extended reliability prediction method provides a useful way to compare designs as well as to determine effective maintenance policy for efficient reliability growth. Application of the method to an electro-mechanical system, as an illustrative example, is explained in detail, and the prediction results are discussed. Both mechanical reliability and performance reliability are compared to total system reliability in terms of reliability prediction errors.

Keywords: Component reliability; Degradation; Mechanical reliability; Performance reliability; Sample-based method; Total system reliability

1. Introduction

In reliability engineering, there are two major failure cases of practical importance. These are a) hard failures that imply complete breakdown in functionality, and b) soft failures whereby the system is functional but the performance measures are out of conformance due to mainly component degradations [1]. For hard failures, most traditional reliability studies have focused on so-called mechanical reliability and use binary state reliability models based on parallel/series/ complex configurations [2]. For soft failures, performance reliability is defined as the probability that system performance measures are within specification limits for the lifetime [3]. There have been research activities to relate reliability concepts to performance reliability. Styblinski [4] defined "drift reliability" as a probability that the system will perform satisfactorily (will not fail) for a specific period of time under the stated environmental conditions, provided that the only cause of failure is drift (or degradation) of component values in time. Drift reliability considers soft failure in the circuit and does not include hard failure. Condra [5] stated "Reliability is quality over time." The definition is a much simpler customer-oriented definition, and it is mainly based on quality over time, not functionality. Yang et al. [2] decomposed system failure mode into degradation failure mode and catastrophic failure mode, and they defined total system reliability function $R^T(t)$ at a time t as

$$R^T(t) = R_D(t) R_C(t) \tag{1}$$

where $R_D(t)$ is related to system response degradation, and $R_C(t)$ to catastrophic failure in component. In addition, component or device reliability has been evaluated through integrating catastrophic failures and degradation failures in the aspect of multiple competing failure modes [6, 7].

Savage and Carr revisited definition of quality and reliability and related them through a framework that emphasized conformance and functionality [8]. They proposed that a structure of reliability consisted of functionality and quality, wherein functionality is related to component reliability and quality to system responses. They defined total system reliability as a probability that system performance standard is satisfied, conditional on the system being in a functional topology, and it can be rewritten as

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$$R^{T}(t)=\sum_{i=1}^{ft}\left\{\Pr(S(t)\,|\,T_i(t))\cdot\Pr(T_i(t))\right\} \tag{2}$$

where S(t) is the event of successful system performance and $T_i(t)$ is the event that the system exists in functional topology (ft). Pr(S(t)|$T_i(t)$) is the probability of conformance for each functional topology, and Pr($T_i(t)$) is the topological probability for each functional topology T_i. Therefore, total system reliability becomes system performance reliability when the term Pr($T_i(t)$) in Eq. (2) is assumed to be unity, and it becomes system mechanical reliability when the term Pr(S(t)|$T_i(t)$) is assumed to be unity. It is necessary to consider the term Pr(S(t)|$T_i(t)$) for accurate reliability evaluation if mechanical reliability is not independent of performance reliability.

Consideration of component degradation links to reliability problems. Modeled degradation data have been used to infer:

- Case (a): component lifetime distribution or reliability function,
- Case (b): system mechanical reliability from inferred component reliabilities,
- Case (c): system mechanical reliability from component degradation data using analytical system response models,
- Case (d): system performance reliability using both component degradation data and mechanistic system models for multi-response systems, and
- Case (e): system dependability through integrating system mechanical reliability and system performance reliability.

For Case (a), using both predicted degradation models $X(t)$ and the pre-defined particular critical levels of degradation, either life-time distribution or reliability is inferred. Degradation data over time are fitted to degradation path curves or degradation distributions, and then the fitted degradation models are used to infer failure times.

For Case (b), component reliability functions are inferred as mentioned in Case (a). Then, system reliability is predicted using series/parallel system reliability models assuming that Pr(S(t)) = 1 in Eq. (2). In the prediction, all components are assumed to be independently working and degrading over time. For examples, Liao and Elsayed [9] predicted system mechanical reliability of a brake system using component reliabilities and parallel/series system reliability models. Coit et al. [10] inferred component reliabilities using a distribution-based approach, and then predicted system mechanical reliability function through applying a series system reliability model. For example, predicted reliability $R^M(t)$ of a system comprising two independent components (monotonically decreasing $X_1(t)$ and $X_2(t)$ with each lower critical level ζ_1 and ζ_2) has the form

$$\begin{aligned}R^{M}(t)&=\Pr\left\{\left(X_1(t)>\zeta_1\right)\cap\left(X_2(t)>\zeta_2\right)\right\}\\&=\Pr\left\{X_1(t)>\zeta_1\right\}\Pr\left\{X_2(t)>\zeta_2\right\}.\end{aligned} \tag{3}$$

For Case (c), mechanical reliability of a system has been analyzed using degrading characteristics of components and analytical system responses (i.e., strength of a structure). System reliability was predicted through applying traditional stress-strength interference (SSI) reliability models presented in Ref. [11] assuming that Pr(S(t)) = 1 in Eq. (2). In the models, one-dimensional probability distribution was used to describe each stochastic process such as stochastic loading and strength aging, and both processes were assumed to be statistically independent. As well, Andrieu-Renaud et al. proposed a time-variant FORM (first order reliability method) to evaluate an out-crossing rate [12]. They predicted reliability of a structural system with strength aging due to corrosion. The reliability function was built numerically at discrete time intervals.

For Case (d), there are two different approaches: tracking-based approach and non-tracking based approach, assuming that there is no failure in system functionality due to degradations i.e. Pr($T_i(t)$) = 1 in Eq. (2). The tracking-based approach that uses Monte-Carlo simulation (MCS) takes samples of the component distributions at t = 0 and traces their paths using their degradation models to provide time-variant system responses [4, 13, 14]. Through tracking and comparing the time-variant system responses with response specifications, system performance reliability is predicted. For the non-tracking based approach, system performance reliability has been predicted through relating time-variant system responses to their response specifications, by time-variant limit-state functions. Then, based on a set-theoretic concept, the incremental failure region, emerging from success region in the limit-state functions during a time interval, is approximated using only two contiguous discrete times. Finally, system cumulative distribution function at a time is evaluated through summing up probabilities of the incremental failure regions over time. Either non-sample based method using FORM [1, 12, 13] or sampling method based on MCS [16] has been used to evaluate the incremental failure probability.

For Case (e), the fundamental idea of evaluating system dependability comes from Case (d); system mechanical reliability is evaluated considering a system-level response such as a mechanic stress that component interactions provide, not component reliabilities related to degradation [17]. Cases (c) and (d) are integrated in system level to evaluate system dependability that considers both mechanical and performance reliability simultaneously. In general, system maintenance policy is based on component reliability information. Thus, it would be difficult to determine maintenance periods to either fix or replace critical components using the system mechanical reliability based on a system-level response. It follows that system reliability evaluation based on Cases (b) and (d) would be required for efficient maintenance policy in order to perform effective reliability growth. Therefore, it is necessary to evaluate system reliability through integrating both system mechanical reliability from component reliabilities and system performance reliability.

This paper, related to Cases (b) and (d), considers multiple competing failure modes for systems with degrading components connected in series in terms of system functionality and system performance, assuming that system functionality is not independent of system performance. System performance reliability prediction methods in open literature are extended to predict total system reliability of systems. The extended method combines system mechanical reliability from component reliabilities and system performance reliability, and it would provide an efficient way to evaluate Eq. (2).

Total system reliability modeling using component degradations is described in Sec. 2. Specifically, mechanical reliability modeling using component reliability and performance reliability modeling using system responses are shown in Sec. 2.1 and Sec. 2.2, respectively, and total system reliability modeling is given in Sec. 2.3. The total system reliability assessment based on a sampling approach is explained in Sec. 3. Application of total system reliability prediction method to an electromechanical system is discussed with some error analyses in Sec. 4.

2. Total system reliability modeling using component degradations

2.1 System mechanical reliability modeling

Most components exhibit unavoidable degradations over time. Degradation arises from environmental conditions or stresses under which a component operates. There are two general types of degradation modeling being widely used [18]: the degradation path curve approach, and the degradation distribution-based approach known as a graphical approach.

The degradation path curve approach starts with a deterministic description of degradation path (i.e., a known physical model of degradation over time). In the approach, it is widely assumed that each sample degrades in the same way under fixed environmental conditions, and each degradation path has an identical functional form [18]. Random coefficients are introduced to describe variations due to manufacturing processes in the path curve. Then, statistical distribution parameters of random coefficients are numerically estimated using observed degradation data [10]. A single degradation path curve $X(t)$ of V (= $X(t=0)$) under a deterministic environmental condition $\mathbf{c}$ is a function of both a vector $\boldsymbol{\theta}$ and time: $X(t) = f(\boldsymbol{\theta}(V, \mathbf{c}), t)$, where the vector $\boldsymbol{\theta}$ is composed of constant and random coefficients. For example, a single linear degradation path model with fixed coefficient θ_0 and random coefficient θ_1, $\boldsymbol{\theta} = [\theta_0, \theta_1]$, has the form $X(t) = \theta_0 + \theta_1 t$.

The degradation distribution-based approach is based on statistical models, i.e., distribution parameters. In the approach, degradation is characterized by change of distribution parameters versus time. A probability distribution function is chosen to adequately describe the degradation data at each observation time. A two-step statistical analysis: (a) estimating the distribution parameters at each observation time supposing that the degradation data at each time follows the chosen distribution with time-variant distribution parameters, and then (b) fitting time-dependent distribution parameter functions is carried out to model degradation data [18]. Degradation data $X(t)$ with distribution parameters versus time ($\mathbf{p}'(t)$) are expressed as a function of initial distribution parameters $\mathbf{p}$ and time t, and have the form $\mathbf{p}'(t) = f(\mathbf{p}, t)$. For example, Yang et al. [19] used the s-normal random process to model degradation data under the assumption that at an individual time t_i, the observed degradation data follows a s-normal distribution with mean $\mu'(t_i)$ and standard deviation $\sigma'(t_i)$ at time t_i.

For the two types of degradation modeling, Son et al. proposed a unified general model based on Rosenblatt transformation denoted as $\Gamma(\mathbf{u}, \mathbf{v}, \mathbf{p}) = 0$ with standard normal variables $\mathbf{u}$ and distribution parameters $\mathbf{p}$ of $\mathbf{v}$ [20]. From the unified degradation model, the degradation sample $x(t)$ of $X(t)$ in $\mathbf{u}$-space is expressed as

$$x(t) = f(\mathbf{p}, \mathbf{u}, t)\,. \tag{4}$$

From the point of view of hard failure related to degradation, component reliability has been evaluated using a pre-defined particular critical level of degradation that would lead to hard failure. Component degradation model $X(t)$ with a critical level ζ provides a time-variant limit-state function, and the functions for a upper critical level ζ_U and a lower critical level ζ_L have the forms, respectively,

$$\begin{aligned} g(X(t)) &= \zeta_U - X(t) \\ g(X(t)) &= X(t) - \zeta_L. \end{aligned} \tag{5}$$

Thus, reliability of a component with a degradation model $X(t)$ can be defined as

$$R(t) = \Pr\big(g(X(t)) > 0\big)\,. \tag{6}$$

Let us consider a system comprising m degrading components ($X_1(t), \cdots X_m(t)$) connected in series, and the components fail independently. The system mechanical reliability function $R^M(t)$ can be rewritten in terms of m limit-state functions as, for $\forall \tau \in [0, t]$

$$R^M(t) = \Pr\big((g_1(X_1(\tau)) > 0) \cap \cdots \cap (g_m(X_m(\tau)) > 0)\big) \tag{7}$$

where g_i represents the limit-state function for $X_i(t)$. For example, system mechanical reliability function expressed as Eq. (3), can be rewritten as, for $\forall \tau \in [0, t]$

$$R^M(t) = \Pr\big((g_1(X_1(t)) > 0) \cap (g_2(X_2(t)) > 0)\big) \tag{8}$$

where $g_i = X_i(t) - \zeta_i$ for $i = 1, 2$.

Now, system mechanical reliability $R^M(t)$ in terms of the unified degradation model denoted as Eq. (4) can be expressed as

$$R^M(t) = \Pr\left(\bigcap_{i=1}^{m} (g_i((\mathbf{p})_i, (\mathbf{u})_i, \tau) > 0),\ \text{for } \forall \tau \in [0, t]\right) \tag{9}$$

where $X_i(t)$ is mapped to the standard normal vector $(\mathbf{u})_i$ using the distribution parameter vector $(\mathbf{p})_i$. The vector $(\mathbf{u})_i$ are elements of $\mathbf{u}$, and $(\mathbf{p})_i$ are also elements of $\mathbf{p}$. Hence, $R^M(t)$ has a general form

$$R^M(t) = \Pr\left(\bigcap_{i=1}^{m} (g_i(\mathbf{p},\mathbf{u},\tau) > 0), \text{for } \forall \tau \in [0,\ t]\right). \tag{10}$$

2.2 System performance reliability modeling

A system model relating outputs to inputs could be formed by either a mechanistic approach using the interactions of components, or an empirical approach using response surface methodology. In both approaches, the q uncertain performance measures such as responses, $\mathbf{Z} = [Z_1, Z_2, \dots Z_q]$ can be written as functions of the m system variables $\mathbf{V}$ in the explicit form,

$$Z_i = z_i(\mathbf{V}) \text{ for } i = 1, 2, \cdots, q. \tag{11}$$

The system variables represent component characteristics such as (a) resistance for a resistor in electrical circuits, and (b) either torque constant or winding resistance for a servo motor in electro-mechanical system. System responses depend on time when component degradations $\mathbf{X}(t)$ for $\mathbf{V}$ are considered in Eq. (11). Now, the i^{th} time-variant system response $z_i(\mathbf{x}(t))$ has a form in terms of degradation models following Eq. (4) as [20]:

$$z_i(\mathbf{x}(t)) = z_i(\mathbf{p},\ \mathbf{u},\ t). \tag{12}$$

Relating the response to a specification limit by a limit-state function leads to a time-variant limit-state function of the form

$$g_i(\mathbf{x}(t)) = \pm\{z_i(\mathbf{p},\mathbf{u},\ t) - \zeta\} \tag{13}$$

where z_i is a response and ζ is either a lower or an upper limit-specification.

System performance reliability has been interpreted using a series system concept [1, 3-4]. That is, performance reliability at time t is the probability that all time-variant responses satisfy their critical limits before time t. Thus, system performance reliability for q responses with n limit-state functions can be expressed as [1]

$$R^P(t) = \Pr\left(\bigcap_{i=1}^{n} (g_i(\mathbf{p},\mathbf{u},\tau) > 0), \text{for } \forall \tau \in [0,\ t]\right). \tag{14}$$

2.3 Total system reliability modeling

If components comprising a system degrade and fail without leading to system performance change, system mechanical reliability is independent of system performance reliability, and thus total system reliability has the form

$$R^T(t) = R^M(t) \cdot R^P(t). \tag{15}$$

In general, component degradation explains not only component aging processes leading to failure in function, but system performance change over time. It follows that system mechanical reliability would not be independent of system performance reliability. For more accurate reliability estimation, a total system reliability prediction method to combine system mechanical reliability from component reliabilities and system performance reliability is required.

Let us consider a system wherein system mechanical reliability is not independent of system performance reliability. Combination of system mechanical reliability $R^M(t)$ in Eq. (10) with system performance reliability $R^P(t)$ in Eq. (14) provides total system reliability. For notation convenience, setting g_i (the limit-state function related to system performance) as g_{i+m} for $i = 1, 2, \dots n$, we have the total system reliability in terms of $(m + n)$ limit-state functions as

$$R^T(t) = \Pr\left\{\bigcap_{i=1}^{n+m} \left(g_i(\mathbf{p},\mathbf{u},\tau) > 0\right),\ \text{for } \forall \tau \in [0,\ t]\right\}. \tag{16}$$

The reliability model denoted as Eq. (16) provides an efficient way to evaluate total system reliability in Eq. (2) since reliability of both system functionality and system performance can be simultaneously investigated in the same space, i.e., **u**-space.

3. Total system reliability assessment

Total system reliability expressed as Eq. (16), can be more easily evaluated by considering non-conformance, i.e., $g(\mathbf{x}(t)) < 0$, and thus we have the cumulative distribution function, complement of total system reliability, as

$$\begin{aligned} F^T(t) &= 1 - R^T(t) \\ &= \Pr\left\{\bigcup_{i=1}^{n+m} \left(g_i(\mathbf{p},\ \mathbf{u},\ \tau) \le 0\right),\ \text{for } \exists \tau \in [0,\ t]\right\}. \end{aligned} \tag{17}$$

Probability in Eq. (17) herein is numerically approximated using discrete time events based on a finite time step [1]. Consider a selected time t_L using a fixed time step h. For a time index $l = 0, 1, \dots L$, t_l $(= l \cdot h)$ denotes the time at the l^{th} step. Now Eq. (17) becomes

$$F^T(t_L) = \Pr\left\{\bigcup_{l=0}^{L} \left(\bigcup_{i=1}^{n+m} (g_i(\mathbf{p},\mathbf{u},t_l) \le 0\right)\right\}. \tag{18}$$

For an instantaneous failure region of the i^{th} limit-state function at a discrete time t_l, $E_{l,i} = \{\mathbf{u} \in \mathbf{U} : g_i(\mathbf{p},\ \mathbf{u},\ t_l) \le 0\}$, we have a system instantaneous failure region $\mathbf{E}_l$ at time t_l as

$$\mathbf{E}_l = E_{l,1} \cup E_{l,2} \cup \cdots \cup E_{l,n+m} = \bigcup_{i=1}^{n+m} E_{l,i}. \tag{19}$$

System incremental failure probability from time t_l during the time step h is written from Refs. [1, 16] as

$$\Delta F(\mathbf{p},t_l) = \Pr(\mathbf{E}_{l+1} \cup \mathbf{E}_l) - \Pr(\mathbf{E}_l) . \tag{20}$$

A random sampling method such as Monte Carlo simulation is herein applied to evaluate probabilities in Eq. (20) even though FORM could be applicable [1, 15]. Probabilities in Eq. (20) may be evaluated directly by sampling the sign of the appropriate limit-state functions [16]. To obtain the signs, two test functions for $\exists i \in [1, n+m]$ are defined as

$$\phi_1(\mathbf{u},\, t_l) = \begin{cases} 1 & \text{if } g_i(\mathbf{p},\, \mathbf{u},\, t_l) \le 0 \\ 0 & \text{otherwise} \end{cases} , \tag{21}$$

$$\phi_2(\mathbf{u},\, t_l) = \begin{cases} 1 & \text{if } g_i(\mathbf{p},\, \mathbf{u},\, t_l) \text{ or } g_i(\mathbf{p},\, \mathbf{u},\, t_{l+1}) \le 0 \\ 0 & \text{otherwise} \end{cases} . \tag{22}$$

Based on a random sampling method with N samples for **u**, the number of systems $N_f(t_l)$ that do not conform to the specifications at time t_l, and the number of systems $N_f(t_l, t_{l+1})$ that do not conform to the specifications at times t_l and t_{l+1} are evaluated respectively as

$$N_f(t_l) = \sum_{k=1}^{N} \phi_1(\mathbf{u}^k,\, t_l) , \tag{23}$$

$$N_f(t_l,\, t_{l+1}) = \sum_{k=1}^{N} \phi_2(\mathbf{u}^k,\, t_l) . \tag{24}$$

Now, we have the approximate incremental failure over time h, $\Delta \hat{F}(t_l)$, as

$$\Delta \hat{F}^T(t_l) = \frac{N_f(t_l,\, t_{l+1}) - N_f(t_l)}{N} . \tag{25}$$

Then, total system reliability is evaluated using both Eq. (23) with $l = 0$ and Eq. (25) as

$$\begin{aligned} \hat{R}^T(t) &= 1 - \hat{F}^T(t) \\ &= 1 - \left[\frac{N_f(t_0)}{N} + \sum_{l=0}^{L} \{\Delta \hat{F}^T(t_l)\} \right]. \end{aligned} \tag{26}$$

Eq. (26) is also used to evaluate both mechanical reliability including component reliability and performance reliability in this paper since $N_f(t_l)$ and $N_f(t_l, t_{l+1})$ are determined by the corresponding limit-state functions.

4. Case studies

The servo system of interest is shown in Fig. 1 with components and interconnection models taken from [15]. A voltage supply v_1 acts as the input, and at the output an applied torque τ_{15} models the load arising from some arbitrary connected subsystem. The electro-mechanical characteristics of the system arise from three subsystems; the difference amplifier, the motor and tacho-generator ($M_{7,9}$ and $G_{8,10}$), and the gear train shown as $G_{12,13}$. The difference amplifier consists of the three resistors R_2, R_3 and R_4 along with the operational amplifier ($O_{5,6}$) of a very large closed-loop gain. System variables of interest comprise the torque constants K, the rotational inertias J, the winding resistances R_m, the resistance for R_2, R_3 and R_4, and the gear ratio r (= r_{12}/r_{13}) for the gear train.

Table 1. Mechanistic models and performance specifications of a servo system.

Response	Mechanistic model	Specification
$Z_1(t_c)$	$t_c = \dfrac{4JR_m(R_m + R_4)}{K^2(2R_m + R_4 + R_3)}$	Smaller-is-best $USL_1 = 0.034$ [*sec*]
$Z_2(\omega_{ss})$	$\omega_{ss} = \dfrac{rR_3(R_m + R_4)}{KR_2(2R_m + R_4 + R_3)} v_1 - \dfrac{r^2 R_m (R_m + R_4)}{K^2(2R_m + R_4 + R_3)} \tau_{15}$	Target-is-best $LSL_2 = 540$, $T_2 = 570$, $USL_2 = 590$ [*rad/sec*]
$Z_3(\tau_o)$	$\tau_o = \dfrac{KR_3}{rR_m R_2} v_1$	Larger-is-best $LSL_3 = 0.24$ [*N-m*]

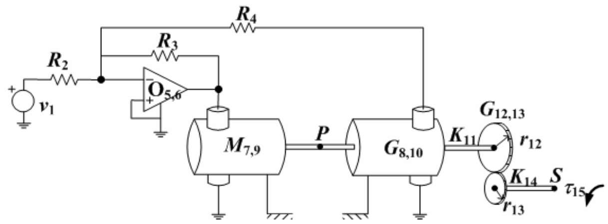

Fig. 1. Schematic of mechatronic servo system.

The three system performance measures related system performance reliability in this work are 1) the time constant t_c related to the time for the shaft speed at S to reach steady-state speed, 2) the steady-state shaft speed at S denoted as ω_{ss}, and 3) the required initial, or starting torque τ_o to supply the load at point S. The mechanistic models in terms of the electro-mechanical characteristics with the response specification are given in Table 1. The value of voltage v_1 supplied from a known power supply is uncertain owing to manufacturing variations. The value for load torque τ_{15} of a known range is uncertain owing to the particular end-use. Thus, both v_1 and τ_{15} are designated as random variables. Also, the resistances of resistors R_2, R_3, and R_4, are uncertain due to variation in their manufacturing process, and thus they are considered as random variables. Moreover, values for the torque constant K, the motor resistance R_m, and the gear ratio r, respectively, that are critical system variables, are considered as random variables. However, the value for the rotor inertia J of both the motor and tacho-generator is fixed at the nominal value 1/1000000 $kg\text{-}m^2$.

According to Bonnett and Soukup [21], electric motor problems occur for a variety of reasons, ranging from basic design faults and poor manufacturing quality to problems caused by

Table 2. Component information for a servo system.

System variable (component)	Distribution	Distribution parameters p_i	Degradation model, t [year]
V_1 (K)	Normal	$\mu_1 = 0.008534$ $tol_1 = 2\%$ ($\sigma_1 = tol_1\mu_1/300$)	None
V_2 (R_m)	Normal	$\mu_2 = 2.9\ \Omega$ $tol_2 = 2\%$ ($\sigma_2 = tol_2\mu_2/300$)	$\mu_2'(t) = \mu_2 \exp(9\times10^{-3}t)$ $\sigma_2'(t) = \sigma_2 \exp(1\times10^{-3}t)$
V_3 (r)	Normal	$\mu_3 = 0.52575$ $tol_3 = 2\%$ ($\sigma_3 = tol_3\mu_3/300$)	None
V_4 (v_1)	Normal	$\mu_4 = 12$ volts $tol_4 = 1\%$ ($\sigma_4 = tol_4\mu_4/300$)	None
V_5 (τ_{15})	Uniform	$\mu_5 = 1/100$ N-m $tol_5 = 2\%$	None
V_6 (R_2)	Normal	$\mu_6 = 10{,}000\ \Omega$ $tol_6 = 2\%$ ($\sigma_6 = tol_6\mu_6/300$)	$\mu_6'(t) = \mu_6 \exp(1.2\times10^{-2}t)$ $\sigma_6'(t) = \sigma_6 \exp(4.5\times10^{-3}t)$
V_7 (R_3)	Normal	$\mu_7 = 40{,}000\ \Omega$ $tol_7 = 2\%$ ($\sigma_7 = tol_7\mu_7/300$)	$\mu_7'(t) = \mu_7 \exp(1.3\times10^{-2}t)$ $\sigma_7'(t) = \sigma_7 \exp(5.0\times10^{-3}t)$
V_8 (R_4)	Normal	$\mu_8 = 10{,}000\ \Omega$ $tol_8 = 2\%$ ($\sigma_8 = tol_8\mu_8/300$)	$\mu_8'(t) = \mu_8 \exp(1.2\times10^{-2}t)$ $\sigma_6'(t) = \sigma_6 \exp(4.5\times10^{-3}t)$

application and site conditions. Specifically, they are most likely to arise from bearing failures - probably the most common cause - with stator winding insulation degradation a close second. The critical degradation limit for motor winding resistance R_m is related to the insulation system in the motor. The degradation of motor winding resistance above a certain critical limit is supposed to provide hard failure of the insulation system in the motor [22]. This hard failure would cause the motor to fail, and thus the servo system fails in system functionality. For resistors, failure causes of resistors may be degradation in resistance as well as dielectric breakdown and either short or open circuit. Abrupt increase in resistance of a resistor might cause an open circuit problem. Thus, resistance degradation for each resistor R_2, R_3, and R_4 above each certain critical limit is supposed to provide hard failure of the resistor. The servo system is assumed to fail in system functionality if the motor and resistors fail due to degradation in resistance.

Distribution and degradation information for each variable is given in Table 2. Four resistances R_m, R_2, R_3, and R_4 are assumed to degrade. The assumed degradation characteristics in terms of both means and tolerances (i.e. $\mathbf{p} = [\mu_1, tol_1, \mu_2, tol_2, \cdots \mu_8, tol_8]$) are given in Table 2. The eight u-v mappings for the system variables are given explicitly as

$$\begin{aligned} v_i &= \mu_i + \sigma_i u_i \quad \text{for } i = 1, 2, 3, 4, 6, 7 \text{ and } 8 \\ v_5 &= \mu_5 - (tol_5\mu_5) + 2(tol_5\mu_5)\Phi(u_5) \end{aligned} \tag{27}$$

where Φ represents a cumulative standard normal distribution function.

Thus, the degradation models for the winding resistance R_m, and resistors R_2, R_3, and R_4 versus usage time are written as

$$x_k(t) = \mu_k'(t) + \sigma_k'(t)u_k \quad \text{for } k = 2, 6, 7 \text{ and } 8. \tag{28}$$

For other system variables that have no degradation, we use $x(t) = v$. For component failure, critical upper specification limits for resistance degradations $X_6(t)$, $X_7(t)$, and $X_8(t)$ are considered as 11000, 44000, and 11000 [Ω], respectively, and the limit for initial winding resistance $X_2(t)$ is as 3.103 [Ω]. Thus, each limit-state function has is expressed as

$$\begin{aligned} g_1(X_2(t)) &= 3.103 - (\mu_2'(t) + \sigma_2'(t)u_2), \\ g_2(X_6(t)) &= 11000 - (\mu_6'(t) + \sigma_6'(t)u_6), \\ g_3(X_7(t)) &= 44000 - (\mu_7'(t) + \sigma_7'(t)u_7), \\ g_4(X_8(t)) &= 11000 - (\mu_8'(t) + \sigma_8'(t)u_8). \end{aligned} \tag{29}$$

Thus, component reliability from Eq. (6) with each specification limit ζ_i is defined as

$$R_i(t) = \Phi\left(\frac{\zeta_i - \mu_i'(t)}{\sigma_i'(t)}\right) \quad \text{for } i = 2, 6, 7, 8. \tag{30}$$

The system mechanical reliability $R^M(t)$ from Eq. (10) is defined as, for $\forall\tau\in[0, t]$

$$R^M(t) = \Pr\begin{pmatrix} (g_1(X_2(\tau)) > 0) \cap (g_2(X_6(\tau)) > 0) \cap \\ (g_3(X_7(\tau)) > 0) \cap (g_4(X_8(\tau)) > 0) \end{pmatrix}. \tag{31}$$

The four time-variant limit-state functions for three responses have the forms

$$\begin{aligned} g_5(\mathbf{p},\mathbf{u},\ t) &= 0.045 - z_1(\mathbf{p},\mathbf{u},\ t), \\ g_6(\mathbf{p},\mathbf{u},\ t) &= 589 - z_2(\mathbf{p},\mathbf{u},\ t), \\ g_7(\mathbf{p},\mathbf{u},\ t) &= z_2(\mathbf{p},\mathbf{u},\ t) - 551, \\ g_8(\mathbf{p},\mathbf{u},\ t) &= z_3(\mathbf{p},\mathbf{u},\ t) - 0.22. \end{aligned} \tag{32}$$

The terms $z_i(\mathbf{p}, \mathbf{u}, t)$ for i = 1, 2, and 3 in Eq. (32) represent a time-variant response affecting system performance reliability. For example, we have the time-variant response for starting torque τ_o in g_8 as

$$z_3(\mathbf{p},\mathbf{u},\ t) = \frac{(\mu_1 + \sigma_1 u_1)(\mu_7'(t) + \sigma_7'(t)u_7)}{(\mu_3 + \sigma_3 u_3)(\mu_2'(t) + \sigma_2'(t)u_2)(\mu_6'(t) + \sigma_6'(t)u_6)}(\mu_4 + \sigma_4 u_4). \tag{33}$$

System performance reliability is expressed using the limit-state functions in Eq. (32) as

$$R^P(t) = \Pr\left(\bigcap_{i=5}^{8}(g_i(\mathbf{p},\mathbf{u},\tau) > 0), \text{for } \forall\tau\in[0,\ t]\right). \tag{34}$$

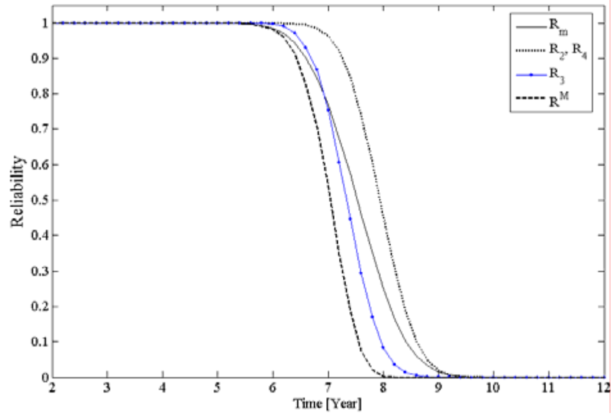

Fig. 2. System mechanical reliability with each component's reliability.

Now, total system reliability for the servo system can be written as

$$R^T(t) = \Pr\left\{\bigcap_{i=1}^{8}\left(g_i(\mathbf{p},\mathbf{u},\tau) > 0\right),\ \text{for}\ \forall\tau\in[0,\ t]\right\}. \qquad (35)$$

We predict total system reliability for **p** = [0.008534, 2, 2.9, 2, 0.52575, 2, 12, 1, 1/100, 2, 10000, 2, 40000, 2, 10000, 2] using $N = 10{,}000$ samples up to the time $t = 14$ [year] with the time step $h = 0.2$ [year]. First, predicted component reliability values for the motor (R_m), and resistors (R_2, R_3, R_4) using Eq. (30) are shown in Fig. 2. Reliability of R_2 is similar to one for R_4 within the maximum probability difference of 0.008. The system mechanical reliability is evaluated using Eq. (31), and it is added to Fig. 2. System mechanical reliability (hereinafter referred to as $R^M(t)$) is mainly determined by both R_m and R_3 up to 7 years, but resistance degradation of R_3 has a greater effect on $R^M(t)$ after 7 years than other components. Thus, it can be stated that reliability of R_3 would be critical to $R^M(t)$.

Prediction results of total system reliability (hereinafter referred to as $R^T(t)$) using Eq. (35) and performance reliability (hereinafter referred to as $R^P(t)$) using Eq. (34) are shown in Fig. 3, where $R^P(t)$ is critical to $R^T(t)$ up to 6 years, but $R^M(t)$ is critical to $R^T(t)$ mainly after 6 years. Thus, soft failures related to system performance, causing system failures, are expected to occur in most cases up to 6 years. But, system hard failures related to system functionality are predicted to occur frequently after 6 years. In this work, repair or replacement of the resistor R_3 would provide more efficient reliability growth than any other components in the aspect of system maintenance, since reliability of R_3 is critical to $R^M(t)$.

There would be error in evaluating $R^T(t)$ with some assumptions. Prediction of $R^T(t)$ using $R^M(t)$ with the assumption of Pr(S(t)) = 1 in Eq. (2) causes errors in system reliability estimation. And prediction of $R^T(t)$ as $R^P(t)$ with the assumption of Pr($T_i(t)$) = 1 in Eq. (2) also leads to erroneous reliability estimation results. These errors are evaluated using difference in probability, $[R^M(t) - R^T(t)]$ and $[R^P(t) - R^T(t)]$, and they are shown in Fig. 4. The error denoted as $[R^M(t) - R^T(t)]$ is within

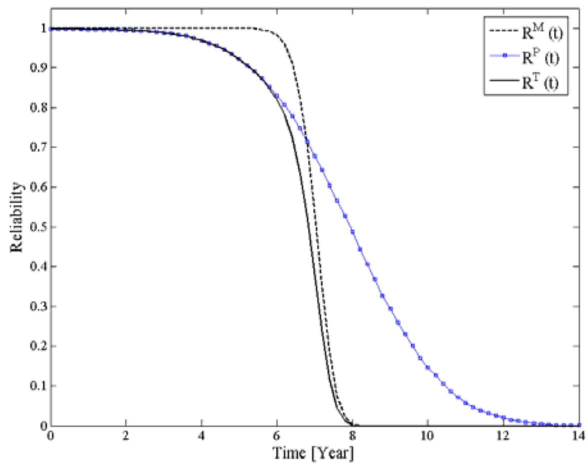

Fig. 3. Total system reliability with mechanical reliability and performance reliability.

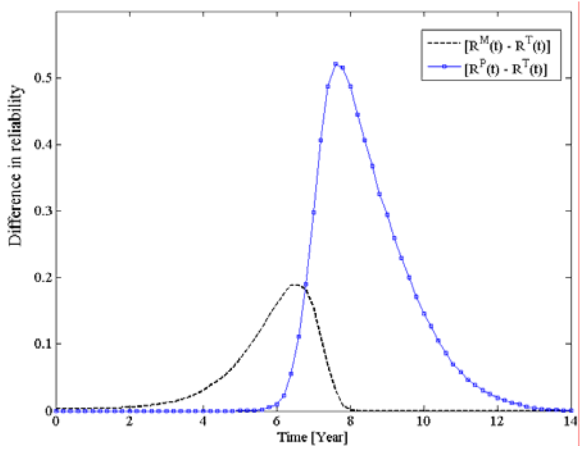

Fig. 4. Error due to consideration of either $R^M(t)$ or $R^P(t)$ as $R^T(t)$.

0.2 in probability. The error increases before about 6 years and then decreases after that time, and then becomes zero at 8 years. The error denoted as $[R^P(t) - R^T(t)]$ is within 0.6 in probability, and it increases before around 8 years and then decreases after that time. These error characteristics would be explained by the points that (a) $R^P(t)$ has more influence on $R^T(t)$ mainly before 6 years, and (b) $R^M(t)$ does on $R^T(t)$ after 6 years.

Prediction of $R^T(t)$ using a simple product of $R^M(t)$ and $R^P(t)$, i.e., $R^M(t)\cdot R^P(t)$ assuming that $R^M(t)$ is independent of $R^P(t)$, instead of Eq. (35) leads to a prediction error.

Fig. 5 represents the prediction error that has a maximum value of about 0.019 in probability. The error is negligible when $R^M(t) \approx 0$ or $R^M(t) \approx 1$ where the intersection probability of failure regions in the limit-state functions related to soft and hard failures from Eq. (35) is very small. However, the error is not negligible in other cases. Thus, we may conclude that $R^T(t)$ is not a simple product of $R^M(t)$ and $R^P(t)$, and failure for the servo system in functionality is not independent of failure in performance.

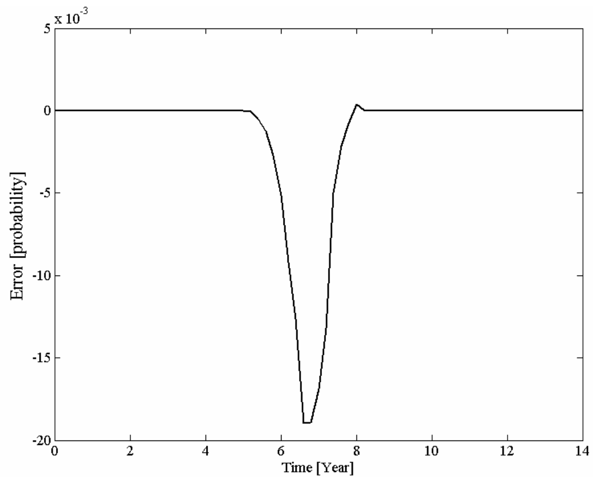

Fig. 5. Error due to the assumption of independence in failure.

5. Conclusions

This paper considered competing failure modes for systems with degraded components in terms of system functionality and system performance. To reduce errors in reliability prediction, system performance reliability prediction methods in the open literature were extended to predict total system reliability. The extended total system reliability prediction method combines system mechanical reliability from component reliabilities and system performance reliability in case system functionality is not independent of system performance. Application of the prediction method to the servo system showed the importance of this work in reducing reliability prediction errors.

The extended reliability prediction method could provide an efficient way to evaluate total system reliability using not Eq. (2) but Eq. (26). The prediction method might be extended to reliability prediction of general systems with components connected in series and/or parallel, and provide a useful way to compare designs as well as to determine both maintenance policy and warranty time for efficient reliability growth. Total system reliability prediction approach for general engineering systems with multi-competing failure modes involving non-degrading, stress-related failure mechanisms as well as component degradation is an on-going research topic.

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