Reliable Fuzzy Control for Active Suspension Systems with Actuator Delay and Fault

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***Abstract*—This paper is focused on reliable fuzzy H_∞ controller design for active suspension systems with actuator delay and fault. Takagi-Sugeno (T-S) fuzzy model approach is adapted in the study with consideration of the sprung and unsprung masses variation, the actuator delay and fault, and other suspension performances. By utilizing parallel-distributed compensation scheme, a reliable fuzzy H_∞ performance analysis criterion is derived for the proposed T-S fuzzy model. Then, a reliable fuzzy H_∞ controller is designed such that the resulting T-S fuzzy system is reliable in the sense that it is asymptotically stable and has a prescribed H_∞ performance under given constraints. The existence condition of the reliable fuzzy H_∞ controller is obtained in terms of linear matrix inequalities Finally, a quarter-vehicle suspension model is used to demonstrate the effectiveness and potential of the proposed design techniques.**

***Index Terms*—H_∞ control, actuator delay, actuator fault, active suspension systems with uncertainty, Fuzzy control.**

I. INTRODUCTION

VEHICLE engineering has approved the crucial role of a vehicle suspension playing in evaluating the vehicle dynamics performance. A suspension component has vital functions: for instance, to support the vehicle weight, to provide effective isolation of the chassis from road excitations, to keep tyre contact with the ground, and to maintain the wheels in appropriate position on the road surface. The roles of a vehicle suspension system are to adequately guarantee the stability of the vehicle, while to provide as much comfort as possible for the passengers by serving the basic function of isolating passengers from road-induced vibration and shocks [1]–[4]. Considerable attentions and efforts have been paid to the challenging issue of how to optimize the required suspension performances, namely, ride comfort, road handling, and suspension deflection. It is evident that many vehicle models and controller design methods have been reported in [5]–[7]. On the other hand, many active suspension control approaches have been presented to handle the tradeoff by utilizing various control techniques such as fuzzy logic and neural network control [8], gain scheduling control [9], linear optimal control [10], adaptive control [11] and H_∞ control [2], [12], [13] and their combined methods.

It should be noticed, however, that all the aforementioned suspension control results are under a full reliability assumption that all control components of the systems are in ideal working conditions. Due to the growing complexity of automated control systems, various faults are likely to be encountered, especially faults from actuators and sensors [14]–[16]. During the past few decades, many researches have attempted to resolve the reliable and fault tolerant control problems for dynamic systems with uncertainty such as actuator and sensor faults, a great number of theoretic results have also been presented [17]–[20]. For instance, the reliable H_∞ controller design problem was been investigated at a context of linear systems [21], and a controller was designed ensuring the resulting control system reliable, i.e., guaranteed asymptotic stability and H_∞ performance, under the assumption that all control components of sensors and actuators are operational. As a matter of fact, an active suspension system is different from its counterpart of a passive suspension system in that its actuator has capability of adjusting force to meet the criteria of the vehicle dynamics, such as guaranteeing the stability of the vehicle, securing passenger comfort and satisfying the suspension performance.

However, when either the actuator or sensor faults occur in an active suspension system, the conventional controllers can not achieve better performance in comparison with the reliable and fault-tolerant controllers as discussed in [22]. Therefore, it is challenging to design a reliable controller such that the system stability and performance of the active suspension closed-loop system can be tolerated with sensor or actuator faults. Due to the electrical and electromagnetic characteristics of the actuators and transmission of the measurement information, electrohydraulic actuators are preferably employed to track the desired forces in order to avoid input time delays, it is a commonly key factor to degrade the control performances and even cause instability in the control systems. Controller design schemes recently have been presented for linear systems with different types of delays [23]–[25]. There exist two mainstreams of controller design methods involving

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actuator delays. One is to design a controller by using the integrated system model including actuator dynamics [26]; the other is to consider the actuator time delay in the controller design process in order to design a controller that can stabilize the system and guarantee the closed-loop performance in spite of the existence of time delay [27].

An active suspension system has the ability to enhance vehicle dynamics by relaxing external impact such as road surface on vehicle travel comfort. In terms of its control design, uncertainty of vehicle sprung and unsprung masses such as its loading conditions should be taken into account to meet vehicle travel performance criteria. For instance, the polytopic parameter uncertainties was been employed to model the varying vehicle sprung or unsprung masses [2], [28], [29]. The parameter-dependent controllers was proposed for the quarter-car suspension systems with sprung mass variation [29]. The parameter-independent sampled-data H_∞ controller design strategy was presented to handle both sprung and unsprung mass variations in a case study of a quarter-car suspension system [2]. The state of the art in suspension control design in these scenarios, however, could not provide feasible performance for uncertain active suspension systems with actuator delay and fault. Clearly, there is a need for a new controller design method which has capability of satisfying the control condition. On the other hand, since fuzzy sets were proposed by Zadeh [30], fuzzy logic control has developed into a conspicuous and successful branch of automation and control theory. The T-S fuzzy model has been proved as an effective theoretical method and practical tool for representing complex nonlinear systems and applications [31]–[34].

T-S fuzzy model based systems are described as a weighted sum of some simple linear subsystems, and thus are easily analyzable, the success on control analysis and synthesis problems have been also demonstrated by various techniques [35]–[37], [37]–[40]. Recently, research has been conducted to challenge the reliability for the continuous-time T-S fuzzy systems [41]–[44]. However, in the context of vehicle suspension control design, there are few results on reliable fuzzy H_∞ controller design for T-S fuzzy systems with both actuator delay and fault. On the other hand, fuzzy controller design had been investigated for suspension systems in the past years, for example, [45]–[47]. In particular, a T-S model-based fuzzy control design approach was presented for electrohydraulic active suspension systems with input constraints [47]. It is evident, however, there are few results on fuzzy H_∞ controller design for uncertain active suspension systems with actuator delay and fault.

This paper is concerned with the problem of reliable fuzzy H_∞ control for uncertain active suspension systems with actuator delay and fault based on the T-S fuzzy model approach. The vehicle dynamic system is established by the fact that vehicle sprung and unsprung mass variations, the actuator delay and fault have been taken account into the suspension performances. The parallel-distributed compensation (PDC) scheme is, then, used to develop reliable fuzzy H_∞ performance analysis condition for the proposed T-S fuzzy system, the reliable fuzzy H_∞ controller is designed to guarantee the systems asymptotic stability and H_∞ performance, simultaneously satisfying the constraint performances. Further, the linear matrix inequality (LMI)-based condition of reliable fuzzy H_∞ controller design is derived. Finally, the proposed method is evaluated on a quarter-car suspension model. Simulation results demonstrate the designed reliable fuzzy H_∞ controller has robust capability of guaranteeing better suspension performance with uncertainty of the sprung and unsprung mass variations, the actuator delay and fault.

The reminder of this paper is organized as follows. The problem to be addressed is formulated in Section 2. Section 3 presents the reliable fuzzy H_∞ controller design results and Section 4 provides fuzzy H_∞ controller design scheme. Simulation results are provided to evaluate the proposed method in Section 5, finally the paper is concluded in Section 6.

Notation: The notation used throughout the paper is presented. The superscript T stands for matrix transposition. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\|\cdot\|_\infty$ denotes the H_∞ norm for matrices. The notation $P > 0$ (≥ 0) is used to denote a symmetric and positive definite (semi-definite) matrix. In symmetric block matrices or complex matrix expressions, an asterisk $*$ is employed to represent a term that is readily induced by symmetry and diag$\{\dots\}$ stands for a block-diagonal matrix. sym(A) is used to denote $A + A^T$ for simplicity. Matrices, if the dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. The space of square-integrable vector functions over $[0,\infty)$ is denoted by $L_2[0,\infty)$, and for $w = \{w(t)\} \in L_2[0,\infty)$, its norm is denoted by $\|w\|_2 = \sqrt{\int_{t=0}^{\infty} |w(t)|^2 dt}$.

II. PROBLEM FORMULATION

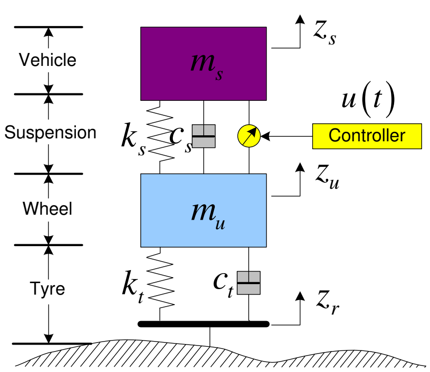


Fig. 1. A quarter-car model

There is a substantially growing interest in investigating the modeling and active control design for active suspension systems in the past three decades. It is due to the fact that these systems play an important role in ensuring the suspension performance, such as ride comfort, road holding, and suspension deflection. A quarter-vehicle model has been used widely in the literature for designing active suspension controller, as shown in Fig. 1, where m_s is used to denote the sprung mass, which represents the car chassis; m_u is the unsprung mass, which represents mass of the wheel assembly; $u(t)$ stands for the active input of the suspension system; z_s and z_u denote the displacements of the sprung and unsprung masses, respectively; z_r is used to denote the road displacement input; c_s and k_s are damping and stiffness of the suspension system, respectively; k_t and c_t stand for compressibility and damping of the pneumatic tyre, respectively. Then, the dynamic equation of the suspension model is established as follows:

$$\begin{aligned} & m_u\ddot{z}_u(t)+c_s[\dot{z}_u(t)-\dot{z}_s(t)]+k_s[z_u(t)-z_s(t)] \\ & +k_t[z_u(t)-z_r(t)]+c_t[\dot{z}_u(t)-\dot{z}_r(t)] \\ = & -u(t), \\ & m_s\ddot{z}_s(t)+c_s[\dot{z}_s(t)-\dot{z}_u(t)]+k_s[z_s(t)-z_u(t)] \\ = & u(t). \end{aligned} \tag{1}$$

Denote $x_1(t) = z_s(t) - z_u(t)$ as the suspension deflection, $x_2(t) = z_u(t) - z_r(t)$ as the tire deflection, $x_3(t) = \dot{z}_s(t)$ as the sprung mass speed, $x_4(t) = \dot{z}_u(t)$ as the unsprung mass speed, and $w(t) = \dot{z}_r(t)$ as the disturbance input, respectively. The equations in (1) can be rewritten as:

$$\dot{x}(t) = A(t)x(t) + B_1(t)w(t) + B(t)u(t), \tag{2}$$

where

$$\begin{aligned} A(t) &= \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \\ \frac{k_s}{m_u} & -\frac{k_t}{m_u} & \frac{c_s}{m_u} & -\frac{c_s+c_t}{m_u} \end{bmatrix}, \\ B(t) &= \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_s} \\ -\frac{1}{m_u} \end{bmatrix}, \quad B_1(t) = \begin{bmatrix} 0 \\ -1 \\ 0 \\ \frac{c_t}{m_u} \end{bmatrix}, \\ x(t) &= \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & x_4(t) \end{bmatrix}^T. \end{aligned}$$

For the control design problems of suspension systems, their performances, such as ride comfort and suspension deflection, road holding are the fundamentals being taken into account. It is widely accepted that ride comfort can be generally quantified by the body acceleration in the vertical direction in the context of a quarter-vehicle model, hence it is practical to choose body acceleration, $\ddot{z}_s(t)$, as the first control output. It indicates that one of the objectives is to minimize the vertical acceleration, $\ddot{z}_s(t)$, to secure vehicle travel comfort.

Recall H_∞ control method, the value of H_∞ norm is defined as an upper bound of the root mean square gain, the main objective is to minimize the H_∞ norm of the transfer function from the disturbance $w(t)$ to the control output $z_1(t) = \ddot{z}_s(t)$ with an emphasis on ride comfort improving. Meanwhile, the following required performances have to be taken into account as well:

I) The suspension deflection cannot exceed a maximum value constrained by mechanical structure, that is,

$$|z_s(t) - z_u(t)| \le z_{\max}, \tag{3}$$

where $z_{\max}$ is the maximum suspension deflection.

II) The dynamic tyre load has to be less than the static tyre load in order to ensure a firm uninterrupted contact of the wheels on the road,

$$k_t(z_u(t) - z_r(t)) < (m_s + m_u)g. \tag{4}$$

Based on the above criteria, the body acceleration $\ddot{z}_s(t)$ is chosen as the performance control output, the suspension stroke $z_s(t) - z_u(t)$ and relative dynamic tire load $k_t(z_u(t) - z_r(t))/(m_s + m_u)g$ are chosen as the second control output $z_2(t)$. Therefore, the following system is derived to present the active vehicle suspension system:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B_1(t)w(t) + B(t)u(t), \\ z_1(t) &= C_1(t)x(t) + D_1(t)u(t), \\ z_2(t) &= C_2(t)x(t), \end{aligned} \tag{5}$$

where $A(t)$, $B_1(t)$ and $B(t)$ are defined in (2), and

$$\begin{aligned} C_1(t) &= \begin{bmatrix} -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \end{bmatrix}, \quad D_1(t) = \frac{1}{m_s}, \\ C_2(t) &= \begin{bmatrix} \frac{1}{z_{\max}} & 0 & 0 & 0 \\ 0 & \frac{k_t}{(m_s+m_u)g} & 0 & 0 \end{bmatrix}. \end{aligned} \tag{6}$$

Note that the suspension suspension system in (5) is a model with uncertainty in that the sprung mass m_s and the unsprung mass m_u vary in the given ranges, in which m_s and m_u denote $m_s(t)$ and $m_u(t)$ respectively. In the meantime, the actuator delay and fault should be taken into account since the suspension performance could be affected by these factors. It leads to the system as:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B_1(t)w(t) + B(t)u_f(t-d(t)), \\ z_1(t) &= C_1(t)x(t) + D_1(t)u_f(t-d(t)), \\ z_2(t) &= C_2(t)x(t), \\ x(t) &= \phi(t), \quad t \in [-\bar{d}, 0], \end{aligned} \tag{7}$$

where $\phi(t)$ is a vector-valued initial continuous function defined on $t \in [-\bar{d}, 0]$. $d(t)$ denotes the time-varying delay satisfying

$$0 \le d(t) \le \bar{d}, \quad d(t) \le \mu. \tag{8}$$

Considering the fault channel from controller to actuator,

$$u^f(t) = m_a u(t), \tag{9}$$

m_a is used to represent the possible fault of the corresponding actuator $u^f(t)$. $\check{m}_a \le m_a \le \hat{m}_a$, where $\check{m}_a$ and $\hat{m}_a$ are constant scalars and used to constrain lower and upper bounds of the actuator faults. Three following cases are considered corresponding to three different actuator conditions:

1) $\check{m}_a = \hat{m}_a = 0$, then $m_a = 0$, which implies that the corresponding actuator $u^f(t)$ is completely failed.

2) $\check{m}_a = \hat{m}_a = 1$, thus we obtain $m_a = 1$, which represents the case of no fault in the actuator $u^f(t)$.

3) $0 < \check{m}_a < \hat{m}_a < 1$, which means that there exists partial fault in the corresponding actuator $u^f(t)$.

The sprung mass $m_s(t)$ and the unsprung mass $m_u(t)$ are uncertainties, which vary in a given range, i.e. $m_s(t) \in [m_{s\min}, m_{s\max}]$ and $m_u(t) \in [m_{u\min}, m_{u\max}]$. It is to say that the uncertainty scenarios of the mass $m_s(t)$ is bounded by its minimum value $m_{s\min}$ and its maximum value $m_{s\max}$. In addition, the mass $m_u(t)$ is bounded by its minimum value $m_{u\min}$ and its maximum value $m_{u\max}$. Next, we obtain the values of $\frac{1}{m_s(t)}$ and $\frac{1}{m_u(t)}$ from $m_s(t) \in [m_{s\min}, m_{s\max}]$ and $m_u(t) \in [m_{u\min}, m_{u\max}]$. Then we have

$$\begin{aligned} \max\frac{1}{m_s(t)} &= \frac{1}{m_{s\min}} =: \hat{m}_s, \quad \min\frac{1}{m_s(t)} = \frac{1}{m_{s\max}} =: \check{m}_s, \\ \max\frac{1}{m_u(t)} &= \frac{1}{m_{u\min}} =: \hat{m}_u, \quad \min\frac{1}{m_u(t)} = \frac{1}{m_{u\max}} =: \check{m}_u. \end{aligned}$$

The sector nonlinear method [32] is employed to represent $\frac{1}{m_s(t)}$ and $\frac{1}{m_u(t)}$ by,

$$\begin{aligned} \frac{1}{m_s(t)} &= M_1(\xi_1(t))\hat{m}_s + M_2(\xi_1(t))\check{m}_s, \\ \frac{1}{m_u(t)} &= N_1(\xi_2(t))\hat{m}_u + N_2(\xi_2(t))\check{m}_u, \end{aligned}$$

where $\xi_1(t) = \frac{1}{m_s(t)}$ and $\xi_2(t) = \frac{1}{m_u(t)}$ are premise variables,

$$\begin{aligned} M_1(\xi_1(t)) + M_2(\xi_1(t)) &= 1, \\ N_1(\xi_2(t)) + N_2(\xi_2(t)) &= 1. \end{aligned}$$

The membership functions $M_1(\xi_1(t))$, $M_2(\xi_1(t))$, $N_1(\xi_2(t))$ and $N_2(\xi_2(t))$ can be calculated as

$$\begin{aligned} M_1(\xi_1(t)) &= \frac{\frac{1}{m_s(t)} - \check{m}_s}{\hat{m}_s - \check{m}_s}, \quad M_2(\xi_1(t)) = \frac{\hat{m}_s - \frac{1}{m_s(t)}}{\hat{m}_s - \check{m}_s}, \\ N_1(\xi_2(t)) &= \frac{\frac{1}{m_u(t)} - \check{m}_u}{\hat{m}_u - \check{m}_u}, \quad N_2(\xi_2(t)) = \frac{\hat{m}_u - \frac{1}{m_u(t)}}{\hat{m}_u - \check{m}_u}. \end{aligned}$$

The member functions are labelled as **Heavy**, **Light**, **Heavy** and **Light** as shown in Fig.2. Then, the system with uncertainty in (7) is represented by the following fuzzy model:

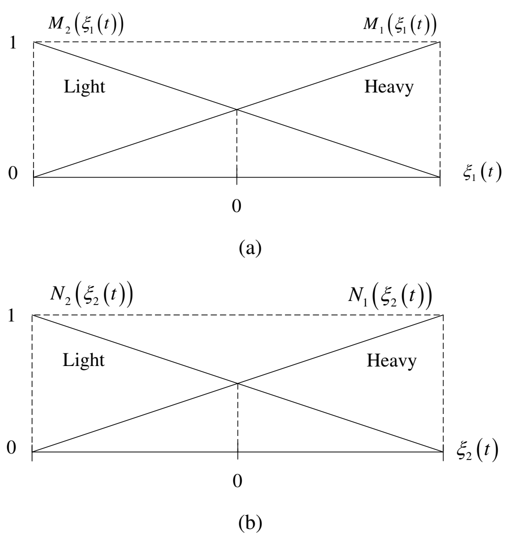


Fig. 2. (a) Membership functions $M_1(\xi_1(t))$ and $M_2(\xi_1(t))$ (b) Membership functions $N_1(\xi_2(t))$ and $N_2(\xi_2(t))$

Model Rule 1: **IF** $\xi_1(t)$ is **Heavy** and $\xi_2(t)$ is **Heavy**, **THEN**

$$\begin{aligned} \dot{x}(t) &= A_1x(t) + B_1u_f(t-d(t)) + B_{11}w(t), \\ z_1(t) &= C_{11}x(t) + D_{11}u_f(t-d(t)), \\ z_2(t) &= C_{21}x(t), \end{aligned}$$

matrices A_1, B_1, B_{11}, C_{11}, D_{11} and C_{21} are obtained by replacing $\frac{1}{m_s(t)}$ and $\frac{1}{m_u(t)}$ with matrices $A(t)$, $B(t)$, $B_1(t)$, $C_1(t)$, $D_1(t)$ and $C_2(t)$ with $\hat{m}_s$ and $\hat{m}_u$ respectively.

Model Rule 2: **IF** $\xi_1(t)$ is **Heavy** and $\xi_2(t)$ is **Light**, **THEN**

$$\begin{aligned} \dot{x}(t) &= A_2x(t) + B_2u_f(t-d(t)) + B_{12}w(t), \\ z_1(t) &= C_{12}x(t) + D_{12}u_f(t-d(t)), \\ z_2(t) &= C_{22}x(t), \end{aligned}$$

matrices A_2, B_2, B_{12}, C_{12}, D_{12} and C_{22} are obtained by replacing $\frac{1}{m_s(t)}$ and $\frac{1}{m_u(t)}$ with matrices $A(t)$, $B(t)$, $B_1(t)$, $C_1(t)$, $D_1(t)$ and $C_2(t)$ with $\hat{m}_s$ and $\check{m}_u$ respectively.

Model Rule 3: **IF** $\xi_1(t)$ is **Light** and $\xi_2(t)$ is **Heavy**, **THEN**

$$\begin{aligned} \dot{x}(t) &= A_3x(t) + B_3u_f(t-d(t)) + B_{13}w(t), \\ z_1(t) &= C_{13}x(t) + D_{13}u_f(t-d(t)), \\ z_2(t) &= C_{23}x(t), \end{aligned}$$

matrices A_3, B_3, B_{13}, C_{13}, D_{13} and C_{23} are obtained by replacing $\frac{1}{m_s(t)}$ and $\frac{1}{m_u(t)}$ with matrices $A(t)$, $B(t)$, $B_1(t)$, $C_1(t)$, $D_1(t)$ and $C_2(t)$ with $\check{m}_s$ and $\hat{m}_u$ respectively.

Model Rule 4: **IF** $\xi_1(t)$ is **Light** and $\xi_2(t)$ is **Light**, **THEN**

$$\begin{aligned} \dot{x}(t) &= A_4x(t) + B_4u_f(t-d(t)) + B_{14}w(t), \\ z_1(t) &= C_{14}x(t) + D_{14}u_f(t-d(t)), \\ z_2(t) &= C_{24}x(t), \end{aligned}$$

matrices A_4, B_4, B_{14}, C_{14}, D_{14} and C_{24} are obtained by replacing $\frac{1}{m_s(t)}$ and $\frac{1}{m_u(t)}$ with matrices $A(t)$, $B(t)$, $B_1(t)$, $C_1(t)$, $D_1(t)$ and $C_2(t)$ wit $\check{m}_s$ and $\check{m}_u$ respectively.

Fuzzy blending allows to infer the overall fuzzy model as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{4} h_i(\xi(t))\left[A_ix(t) + B_iu_f(t-d(t)) + B_{1i}w(t)\right], \\ z_1(t) &= \sum_{i=1}^{4} h_i(\xi(t))\left[C_{1i}x(t) + D_{1i}u_f(t-d(t))\right], \\ z_2(t) &= \sum_{i=1}^{4} h_i(\xi(t))C_{2i}x(t), \end{aligned} \tag{10}$$

where

$$\begin{aligned} h_1(\xi(t)) &= M_1(\xi_1(t)) \times N_1(\xi_2(t)), \\ h_2(\xi(t)) &= M_1(\xi_1(t)) \times N_2(\xi_2(t)), \\ h_3(\xi(t)) &= M_2(\xi_1(t)) \times N_1(\xi_2(t)), \\ h_4(\xi(t)) &= M_2(\xi_1(t)) \times N_2(\xi_2(t)). \end{aligned}$$

It is apparent that the fuzzy weighting functions $h_i(\xi(t))$ satisfy $h_i(\xi(t)) \geq 0$, $\sum_{i=1}^{4} h_i(\xi(t)) = 1$. In order to design a fuzzy reliable controllers, PDC is adapted and the following fuzzy controller is obtained:

Control Rule 1: **IF** $\xi_1(t)$ is **Heavy** and $\xi_2(t)$ is **Heavy**, **THEN** $u(t) = K_{a1}x(t)$.

Control Rule 2: **IF** $\xi_1(t)$ is **Heavy** and $\xi_2(t)$ is **Light**, **THEN** $u(t) = K_{a2}x(t)$.

Control Rule 3: **IF** $\xi_1(t)$ is **Light** and $\xi_2(t)$ is **Heavy**, **THEN** $u(t) = K_{a3}x(t)$.

Control Rule 4: **IF** $\xi_1(t)$ is **Light** and $\xi_2(t)$ is **Light**, **THEN** $u(t) = K_{a4}x(t)$.

Hence, the overall fuzzy control law is represented by

$$u(t) = \sum_{j=1}^{4} h_j(\xi(t))K_{aj}x(t) \tag{11}$$

where K_{aj} $(j = 1,2,3,4)$ are the local control gains and $u(t-d(t)) = \sum_{j=1}^{4} h_j(\xi(t-d(t)))K_{aj}x(t-d(t))$. Therefore, in this paper, we assume that $h_j(\xi(t-d(t)))$ is well defined for $t \in [-\bar{d}, 0]$, and $h_j(\xi(t-d(t))) \geq 0$, $(j = 1,2,3,4)$ $\sum_{j=1}^{4} h_j(\xi(t-d(t))) = 1$. For simplicity, the following notations will be used:

$$h_i =: h_i(\xi(t)), \quad h_j^d =: h_j(\xi(t-d(t))).$$

Applying the fuzzy controller (11) to system (10) yields the closed-loop system:

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{4}\sum_{j=1}^{4} h_i h_j^d [A_i x(t) + B_i m_a(t) K_{aj} x(t-d(t)) \\
&\quad + B_{1i} w(t)], \\
z_1(t) &= \sum_{i=1}^{4}\sum_{j=1}^{4} h_i h_j^d [C_{1i} x(t) + D_{1i} m_a K_{aj} x(t-d(t))], \\
z_2(t) &= \sum_{i=1}^{4} h_i C_{2i} x(t).
\end{aligned} \tag{12}$$

The T-S fuzzy system in (12) is established based on the practically measurable sprung $m_s(t)$ and unsprung $m_u(t)$. The sector nonlinearity method [32] is employed to analyze the variation of the sprung $m_s(t)$ and unsprung $m_u(t)$ and present the T-S fuzzy system in (12).

Without loss of generality, it is assumed, $w \in L_2[0,\infty)$, and $\|w\|_2^2 \leq w_{\max} < \infty$. The objective in this subsection is to design the feedback gain matrices K_{aj} $(j = 1,2,3,4)$ such that the following requirements are satisfied:

(1) the closed-loop system is asymptotically stable;

(2) under zero initial condition, the closed-loop system guarantees that $\|z_1\|_2 < \gamma\|w\|_2$ for all nonzero $w \in L_2[0,\infty)$, where $\gamma > 0$ is a prescribed scalar;

(3) the following control output constraints are guaranteed:

$$\left|\{z_2(t)\}_q\right| \leq 1, \quad q = 1,2. \tag{13}$$

III. Reliable Fuzzy Controller Design

In this section, reliable fuzzy H_∞ state-feedback controller is derived for the active suspension system with actuator delay and fault. It ensures that the closed-loop system in (12) is asymptotically stable, and it also guarantees a prescribed gain from disturbance $w(t)$ to performance output $z_1(t)$, under the condition that the suspension stroke and tire deflection constraints are satisfied. First, the following lemma is presented,

Lemma 1: ([21]) For a time-varying diagonal matrix $\Phi(t) = \mathrm{diag}\{\sigma_1(t), \sigma_2(t), \cdots, \sigma_p(t)\}$ and two matrices R and S with appropriate dimensions, if $|\Phi(t)| \leq V$, where $V > 0$ is a known diagonal matrix, then for any scalar $\varepsilon > 0$, it is true that

$$R\Phi S + S^T\Phi^T R^T \leq \varepsilon RVR^T + \varepsilon^{-1}S^T VS.$$

Next, the following scalars is introduced which will be used in the later development in this paper. $M_{a0} = (\check{m}_a + \hat{m}_a)/2$, $L_a = [m_a - M_{a0}]/m_{a0}$ and $J_a = (\hat{m}_a - \check{m}_a)/(\hat{m}_a + \check{m}_a)$. Thus, one has $m_a = M_{a0}(I + L_a)$ and $L_a^T L_a \leq J_a^T J_a \leq I$. Then, it leads to the following theorem.

Theorem 1: Consider the closed-loop system in (12). For given scalars $\bar{d} > 0$, μ and matrices K_{aj}, if there exist matrices $P > 0$, $Q > 0$, $S > 0$, $R > 0$, N_j, and M_j with appropriate dimensions and positive scalars $\varepsilon_{1ij} > 0$ and $\varepsilon_{2ij} > 0$ $(i,j = 1,2,3,4)$ such that the following LMIs hold for $q = 1,2$:

$$\begin{bmatrix}
\Phi_{11}^{ij} & \sqrt{\bar{d}}M & \Phi_{13}^{ij} & \Phi_{14}^{ij} & \Phi_{15}^{ij} & \Phi_{16}^{1ij} \\
0 & -R & 0 & 0 & 0 & 0 \\
0 & 0 & -I & 0 & D_{1i} & 0 \\
0 & 0 & 0 & -R & \sqrt{\bar{d}}RB_i & 0 \\
0 & 0 & 0 & 0 & -\varepsilon_{1ij}J_a^{-1} & 0 \\
0 & 0 & 0 & 0 & 0 & -\varepsilon_{1ij}J_a^{-1}
\end{bmatrix} < 0, \tag{14}$$

$$\begin{bmatrix}
\Phi_{11}^{ij} & \sqrt{\bar{d}}N & \Phi_{13}^{ij} & \Phi_{14}^{ij} & \Phi_{15}^{ij} & \Phi_{16}^{2ij} \\
0 & -R & 0 & 0 & 0 & 0 \\
0 & 0 & -I & 0 & D_{1i} & 0 \\
0 & 0 & 0 & -R & \sqrt{\bar{d}}RB_i & 0 \\
0 & 0 & 0 & 0 & -\varepsilon_{2ij}J_a^{-1} & 0 \\
0 & 0 & 0 & 0 & 0 & -\varepsilon_{2ij}J_a^{-1}
\end{bmatrix} < 0, \tag{15}$$

$$\begin{bmatrix}
-P & \sqrt{\rho}\{C_{2i}\}_q^T \\
* & -I
\end{bmatrix} < 0, \tag{16}$$

where

$$\begin{aligned}
\Phi_{11}^{ij} &= \Xi_{11}^{ij} + \mathrm{sym}(\Xi_2), \quad \Xi_{11}^{ij} = \begin{bmatrix} \Theta_{11}^{ij} & \Theta_{12}^{ij} \\ * & -\gamma^2 I \end{bmatrix}, \\
\Theta_{11}^{ij} &= \begin{bmatrix} \mathrm{sym}(PA_i) + Q + S & PB_i M_{a0} K_{aj} & 0 \\ * & -(1-\mu)S & 0 \\ * & * & -Q \end{bmatrix}, \\
\Theta_{12}^{ij} &= \begin{bmatrix} PB_{1i} \\ 0 \\ 0 \end{bmatrix}, \Xi_2 = \begin{bmatrix} M & N-M & -N & 0 \end{bmatrix}, \\
\Phi_{13}^{ij} &= \begin{bmatrix} C_{1i} & D_{1i}M_{a0}K_{aj} & 0 & 0 \end{bmatrix}^T, \\
\Phi_{14}^{ij} &= \begin{bmatrix} \sqrt{\bar{d}}RA_i & \sqrt{\bar{d}}RB_i M_{a0}K_{aj} & 0 & \sqrt{\bar{d}}RB_{1i} \end{bmatrix}^T, \\
\Phi_{15}^{ij} &= \begin{bmatrix} B_i^T P & 0 & 0 & 0 \end{bmatrix}^T, \\
\Phi_{16}^{1ij} &= \begin{bmatrix} 0 & \varepsilon_{1ij}M_{a0}K_{aj} & 0 & 0 \end{bmatrix}^T, \\
\Phi_{16}^{2ij} &= \begin{bmatrix} 0 & \varepsilon_{2ij}M_{a0}K_{aj} & 0 & 0 \end{bmatrix}^T, \\
M &= \begin{bmatrix} M_1^T & M_2^T & M_3^T & M_4^T \end{bmatrix}^T, \\
N &= \begin{bmatrix} N_1^T & N_2^T & N_3^T & N_4^T \end{bmatrix}^T.
\end{aligned}$$

Furthermore,

(1) the closed-loop system is robustly asymptotically stable;

(2) the performance $\|T_{z_1 w}\|_\infty < \gamma$ is minimized subject to output constraints (13) with the disturbance energy under the bound $w_{\max} = (\rho - V(0))/\gamma^2$, where $T_{z_1 w}$ denotes the closed-loop transfer function from the road disturbance $w(t)$ to the control output $z_1(t)$.

Proof: Considering the Lyapunov-Krasovskii functional as follows:

$$\begin{aligned} V(t) &= x^T(t)Px(t) + \int_{t-\bar{d}}^{t} x^T(s)Qx(s)\,\mathrm{d}s \\ &\quad + \int_{t-d(t)}^{t} x^T(s)Sx(s)\,\mathrm{d}s \\ &\quad + \int_{-\bar{d}}^{0}\int_{t+\alpha}^{t} \dot{x}^T(s)R\dot{x}(s)\,\mathrm{d}s\mathrm{d}\alpha. \end{aligned} \tag{17}$$

The derivative of $V(t)$ along the solution of system (12) is expressed as

$$\begin{aligned} \dot{V}(t) &\leq 2x^T(t)P\dot{x}(t) + x^T(t)(Q+S)x(t) \\ &\quad - x^T(t-\bar{d})Qx(t-\bar{d}) + \bar{d}\dot{x}^T(t)S\dot{x}(t) \\ &\quad - (1-\mu)x^T(t-d(t))Qx(t-d(t)) \\ &\quad - \int_{t-d(t)}^{t} \dot{x}^T(s)R\dot{x}(s)\,\mathrm{d}s \\ &\quad - \int_{t-\bar{d}}^{t-d(t)} \dot{x}^T(s)R\dot{x}(s)\,\mathrm{d}s. \end{aligned} \tag{18}$$

To develop H_∞ performance analysis criterion, the system (12) is stable with $w(t) = 0$; then the H_∞ performance index is satisfied. For any appropriately dimensioned matrices $\hat{M}$ and $\hat{N}$, the following equalities hold directly according to Newton-Leibniz formula:

$$\begin{aligned} \eta_1(t) &= 2\xi^T(t)\hat{M} \\ &\quad \times \left(x(t) - x(t-d(t)) - \int_{t-d(t)}^{t} \dot{x}(s)\,\mathrm{d}s \right) = 0, \\ \eta_2(t) &= 2\xi^T(t)\hat{N} \\ &\quad \times \left(x(t-d(t)) - x(t-\bar{d}) - \int_{t-\bar{d}}^{t-d(t)} \dot{x}(s)\,\mathrm{d}s \right) = 0, \end{aligned}$$

where

$$\begin{aligned} \xi^T(t) &= \begin{bmatrix} x^T(t) & x^T(t-d(t)) & x^T(t-\bar{d}) \end{bmatrix}, \\ \hat{M} &= \begin{bmatrix} M_1^T & M_2^T & M_3^T \end{bmatrix}^T, \hat{N} = \begin{bmatrix} N_1^T & N_2^T & N_3^T \end{bmatrix}^T. \end{aligned}$$

Adding $\eta_1(t)$ and $\eta_2(t)$ into the right hand side of (18), the following inequalities is obtained:

$$\begin{aligned} \dot{V}(t) &\leq \sum_{i=1}^{4}\sum_{j=1}^{4} h_i h_j^d \xi^T(t)\left[\hat{\Xi}_{ij} + d(t)\hat{M}R^{-1}\hat{M}^T \right. \\ &\quad \left. + \left(\bar{d} - d(t)\right)\hat{N}R^{-1}\hat{N}^T\right]\xi(t) \\ &\quad - \int_{t-d(t)}^{t} \left[\xi^T(t)\hat{M} + \dot{x}^T(s)R\right]R^{-1} \\ &\quad \times \left[\hat{M}^T\xi(t) + R\dot{x}(s)\right]\mathrm{d}s \\ &\quad - \int_{t-\bar{d}}^{t-d(t)} \left[\xi^T(t)\hat{N} + \dot{x}^T(s)R\right]R^{-1} \\ &\quad \times \left[\hat{N}^T\xi(t) + R\dot{x}(s)\right]\mathrm{d}s \\ &\leq \sum_{i=1}^{4}\sum_{j=1}^{4} h_i h_j^d \xi^T(t)\left[\hat{\Xi}_{ij} + d(t)\hat{M}R^{-1}\hat{M}^T \right. \\ &\quad \left. + \left(\bar{d} - d(t)\right)\hat{N}R^{-1}\hat{N}^T\right]\xi(t) \\ &= \sum_{i=1}^{4}\sum_{j=1}^{4} h_i h_j^d \xi^T(t)\left[\frac{d(t)}{\bar{d}}\left(\hat{\Xi}_{ij} + \bar{d}\hat{M}R^{-1}\hat{M}^T\right) \right. \\ &\quad \left. + \frac{\bar{d} - d(t)}{\bar{d}}\left(\hat{\Xi}_{ij} + \bar{d}\hat{N}R^{-1}\hat{N}^T\right)\right]\xi(t), \end{aligned}$$

where

$$\hat{\Xi}_{ij} = \hat{\Theta}_{11}^{ij} + \mathrm{sym}\left(\hat{\Pi}_2\right) + \Upsilon\bar{d}R\Upsilon^T,$$

and

$$\hat{\Pi}_2 = \begin{bmatrix} \hat{M} & \hat{N} - \hat{M} & -\hat{N} \end{bmatrix}, \quad \Upsilon = \begin{bmatrix} A_i & B_i m_a K_{aj} & 0 \end{bmatrix}^T,$$

where the matrix $\hat{\Theta}_{11}^{ij}$ is the matrix Θ_{11}^{ij}, where the term $PB_iM_{a0}K_{aj}$ is replaced by $PB_im_aK_{aj}$. It is found that

$$\begin{aligned} \tilde{\Xi}_{ij}^1 &= \begin{bmatrix} \hat{\Theta}_{11}^{ij} + \mathrm{sym}\left(\hat{\Pi}_2\right) & \sqrt{\bar{d}}\hat{M} & \sqrt{\bar{d}}\Upsilon R \\ * & -R & 0 \\ * & * & -R \end{bmatrix} \\ &\leq \begin{bmatrix} \Theta_{11}^{ij} & \sqrt{\bar{d}}\hat{M} & \tilde{\Phi}_{14}^{ij} \\ * & -R & 0 \\ * & * & -R \end{bmatrix} + \varepsilon_{1ij}^{-1}\Lambda^T J_a \Lambda + \varepsilon_{1ij}\Delta J_a \Delta^T, \\ \tilde{\Xi}_{ij}^2 &= \begin{bmatrix} \hat{\Theta}_{11}^{ij} + \mathrm{sym}\left(\hat{\Pi}_2\right) & \sqrt{\bar{d}}\hat{N} & \sqrt{\bar{d}}\Upsilon R \\ * & -R & 0 \\ * & * & -R \end{bmatrix} \\ &\leq \begin{bmatrix} \Theta_{11}^{ij} & \sqrt{\bar{d}}\hat{N} & \tilde{\Phi}_{14}^{ij} \\ * & -R & 0 \\ * & * & -R \end{bmatrix} + \varepsilon_{2ij}^{-1}\Lambda^T J_a \Lambda + \varepsilon_{2ij}\Delta J_a \Delta^T, \end{aligned}$$

and

$$\begin{aligned} \tilde{\Phi}_{14}^{ij} &= \begin{bmatrix} \sqrt{\bar{d}}RA_i & \sqrt{\bar{d}}RB_iM_{a0}K_{aj} & 0 \end{bmatrix}^T, \\ \Lambda &= \begin{bmatrix} B_i^T P & 0 & 0 & \sqrt{\bar{d}}B_i^T R & 0 \end{bmatrix}, \\ \Delta^T &= \begin{bmatrix} 0 & M_{a0}K_{aj} & 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

From (14)–(15) and according to Schur complement, $\tilde{\Xi}_{ij}^1 < 0$ and $\tilde{\Xi}_{ij}^2 < 0$ are obtained, it is to say that

$$\hat{\Xi}_{ij} + \bar{d}\hat{M}R^{-1}\hat{M}^T < 0, \quad \hat{\Xi}_{ij} + \bar{d}\hat{N}R^{-1}\hat{N}^T < 0.$$

It leads to $\dot{V}(t) < 0$, then the system in (12) is asymptotically stable for the delay $d(t)$ satisfying (8). Next, the H_∞ performance of the system in (12) is established under zero initial conditions. Firstly, the Lyapunov functional is defined as shown in (17). It is not difficult to achieve:

$$\begin{aligned} &\dot{V}(t) + z_1^T(t)z_1(t) - \gamma^2 w^T(t)w(t) \\ &\leq \sum_{i=1}^{4}\sum_{j=1}^{4} h_i h_j^d \bar{\xi}^T(t)\left[\check{\Xi}_{ij} + d(t)MR^{-1}M^T \right. \\ &\quad \left. + \left(\bar{d} - d(t)\right)NR^{-1}N^T\right]\bar{\xi}(t) \\ &= \sum_{i=1}^{4}\sum_{j=1}^{4} h_i h_j^d \bar{\xi}^T(t)\left[\frac{d(t)}{\bar{d}}\left(\check{\Xi}_{ij} + \bar{d}MR^{-1}M^T\right) \right. \\ &\quad \left. + \frac{\bar{d} - d(t)}{\bar{d}}\left(\check{\Xi}_{ij} + \bar{d}NR^{-1}N^T\right)\right]\bar{\xi}(t), \end{aligned}$$

where

$$\check{\Xi}_{ij} = \check{\Phi}_{11}^{ij} + \check{\Phi}_{13}^{ij}\check{\Phi}_{13}^{ijT} + \check{\Phi}_{14}^{ij}\check{\Phi}_{14}^{ijT}, \quad \bar{\xi}^T(t) = \begin{bmatrix} \xi^T(t) & w^T(t) \end{bmatrix},$$

and $\check{\Phi}_{11}^{ij}$, $\check{\Phi}_{13}^{ij}$ and $\check{\Phi}_{14}^{ij}$ are the matrices Φ_{11}^{ij}, Φ_{13}^{ij} and Φ_{14}^{ij} in which the terms $PB_iM_{a0}K_{aj}$, $K_{aj}^TM_{a0}D_{1i}^T$ and $\sqrt{\bar{d}}K_{aj}^TM_{a0}B_i^TR$ are replaced by the terms $PB_im_aK_{aj}$, $K_{aj}^Tm_aD_{1i}^T$ and $K_{aj}^Tm_aB_i^T$ respectively. According to Schur complement and the above method, we develop

$$\dot{V}(t)+z_1^T(t)z_1(t)-\gamma^2w^T(t)w(t)<0, \tag{19}$$

for all nonzero $w\in L_2[0,\infty)$. Under zero initial conditions, we have $V(0)=0$ and $V(\infty)\geq 0$. Integrating both sides of (19) yields $\|z_1\|_2<\gamma\|w\|_2$ for all nonzero $w\in L_2[0,\infty)$, and the H_∞ performance is established.

In what follows, we will show that the hard constraints in (13) are guaranteed. Inequality (19) guarantees $\dot{V}(t)-\gamma^2w^T(t)w(t)<0$. Integrating both sides of the above inequality from zero to any $t>0$, we obtain

$$V(t)-V(0)<\gamma^2\int_0^t w^T(s)w(s)\mathrm{d}s<\gamma^2\|w\|_2^2. \tag{20}$$

From the definition of the Lyapunov functional in (17), we obtain that $x^T(t)Px(t)<\rho$ with $\rho=\gamma^2w_{\max}+V(0)$. Similar to [2], the following inequality hold

$$\begin{aligned}
&\max_{t>0}\left|\{z_2(t)\}_q\right|^2\\
\leq\;&\max_{t>0}\left\|\sum_{i=1}^4 h_ix^T(t)\{C_{2i}\}_q^T\{C_{2i}\}_qx(t)\right\|_2\\
=\;&\max_{t>0}\left\|\sum_{i=1}^4 h_ix^T(t)P^{\frac{1}{2}}P^{-\frac{1}{2}}\{C_{2i}\}_q^T\{C_{2i}\}_qP^{-\frac{1}{2}}P^{\frac{1}{2}}x(t)\right\|_2\\
<\;&\rho\cdot\theta_{\max}\left(\sum_{i=1}^4 h_iP^{-\frac{1}{2}}\{C_{2i}\}_q^T\{C_{2i}\}_qP^{-\frac{1}{2}}\right),\quad q=1,2,
\end{aligned}$$

where $\theta_{\max}(\cdot)$ represents maximal eigenvalue. From the above inequality, it leads to that the constraints in (13) are guaranteed, if

$$\rho\cdot\sum_{i=1}^4 h_iP^{-\frac{1}{2}}\{C_{2i}\}_q^T\{C_{2i}\}_qP^{-\frac{1}{2}}<I, \tag{21}$$

which means

$$\sum_{i=1}^4 h_i\left(\rho\cdot P^{-\frac{1}{2}}\{C_{2i}\}_q^T\{C_{2i}\}_qP^{-\frac{1}{2}}-I\right)<0,$$

which is guaranteed by the feasibility of (16). The proof is completed. ■

Remark 1: In this paper, the free-weight matrices method [48] has been utilized to propose the delay-dependent H_∞ performance analysis condition for the time-varying actuator delay $d(t)$. How to develop the less conservative condition is still a challenging research topic. The interval time-varying delay [49] and present less conservative results have been targeted in our future work.

In what follows, the reliable fuzzy H_∞ controller existence condition is presented for the active suspension system in (12), it is based on reliable fuzzy H_∞ performance analysis criterion in Theorem 1.

Theorem 2: Consider the closed-loop system in (12). For given scalars $\bar{d}>0$ and μ, if there exist matrices $\bar{P}>0$, $\bar{Q}>0$, $\bar{S}>0$, $\bar{R}>0$, Y_{aj}, $\bar{N}_j$, and $\bar{M}_j$ with appropriate dimensions and positive scalars $\bar{\varepsilon}_{1ij}>0$ and $\bar{\varepsilon}_{2ij}>0$ $(i,j=1,2,3,4)$ such that the following LMIs hold for $q=1,2$:

$$\begin{bmatrix}
\bar{\Phi}_{11}^{ij} & \sqrt{\bar{d}}\bar{M} & \bar{\Phi}_{13}^{ij} & \bar{\Phi}_{14}^{ij} & \bar{\Phi}_{15}^{1ij} & \bar{\Phi}_{16}^{ij}\\
0 & \bar{R}-2\bar{P} & 0 & 0 & 0 & 0\\
0 & 0 & -I & 0 & D_{1i} & 0\\
0 & 0 & 0 & -R & \bar{\varepsilon}_{1ij}\sqrt{\bar{d}}B_i & 0\\
0 & 0 & 0 & 0 & -\bar{\varepsilon}_{1ij}J_a^{-1} & 0\\
0 & 0 & 0 & 0 & 0 & -\bar{\varepsilon}_{1ij}J_a^{-1}
\end{bmatrix}<0, \tag{22}$$

$$\begin{bmatrix}
\bar{\Phi}_{11}^{ij} & \sqrt{\bar{d}}\bar{N} & \bar{\Phi}_{13}^{ij} & \bar{\Phi}_{14}^{ij} & \bar{\Phi}_{15}^{2ij} & \bar{\Phi}_{16}^{ij}\\
0 & \bar{R}-2\bar{P} & 0 & 0 & 0 & 0\\
0 & 0 & -I & 0 & D_{1i} & 0\\
0 & 0 & 0 & -R & \bar{\varepsilon}_{2ij}\sqrt{\bar{d}}B_i & 0\\
0 & 0 & 0 & 0 & -\bar{\varepsilon}_{2ij}J_a^{-1} & 0\\
0 & 0 & 0 & 0 & 0 & -\bar{\varepsilon}_{2ij}J_a^{-1}
\end{bmatrix}<0, \tag{23}$$

$$\begin{bmatrix}-\bar{P} & \sqrt{\rho}\bar{P}\{C_{2i}\}_q^T\\ * & -I\end{bmatrix}<0, \tag{24}$$

where

$$\begin{aligned}
\bar{\Phi}_{11}^{ij}&=\bar{\Xi}_{11}^{ij}+\mathrm{sym}\left(\bar{\Xi}_2\right),\quad \bar{\Xi}_{11}^{ij}=\begin{bmatrix}\bar{\Theta}_{11}^{ij} & \bar{\Theta}_{12}^{ij}\\ * & -\gamma^2I\end{bmatrix},\\
\bar{\Theta}_{11}^{ij}&=\begin{bmatrix}\mathrm{sym}(A_i\bar{P})+\bar{Q}+\bar{S} & B_iY_{aj} & 0\\ * & -(1-\mu)\bar{S} & 0\\ * & * & -\bar{Q}\end{bmatrix},\\
\bar{\Theta}_{12}^{ij}&=\begin{bmatrix}B_{1i}\\0\\0\end{bmatrix},\bar{\Xi}_2=\begin{bmatrix}\bar{M} & \bar{N}-\bar{M} & -\bar{N} & 0\end{bmatrix},\\
\bar{\Phi}_{13}^{ij}&=\begin{bmatrix}C_{1i}\bar{P} & D_{1i}Y_{aj} & 0 & 0\end{bmatrix}^T,\\
\bar{\Phi}_{14}^{ij}&=\begin{bmatrix}\sqrt{\bar{d}}A_i & \sqrt{\bar{d}}B_iY_{aj} & 0 & \sqrt{\bar{d}}B_{1i}\end{bmatrix}^T,\\
\bar{\Phi}_{15}^{1ij}&=\begin{bmatrix}\bar{\varepsilon}_{1ij}B_i^T & 0 & 0 & 0\end{bmatrix}^T,\\
\bar{\Phi}_{15}^{2ij}&=\begin{bmatrix}\bar{\varepsilon}_{2ij}B_i^T & 0 & 0 & 0\end{bmatrix}^T,\\
\bar{\Phi}_{16}^{ij}&=\begin{bmatrix}0 & Y_{aj} & 0 & 0\end{bmatrix}^T,\\
\bar{M}&=\begin{bmatrix}\bar{M}_1^T & \bar{M}_2^T & \bar{M}_3^T & \bar{M}_4^T\end{bmatrix}^T,\\
\bar{N}&=\begin{bmatrix}\bar{N}_1^T & \bar{N}_2^T & \bar{N}_3^T & \bar{N}_4^T\end{bmatrix}^T.
\end{aligned}$$

Then a reliable controller in the form of (11) exists, such that

(1) the closed-loop system is asymptotically stable;

(2) the performance $\|T_{z_1w}\|_\infty<\gamma$ is minimized subject to output constraints (13) with the disturbance energy under the bound $w_{\max}=(\rho-V(0))/\gamma^2$.

Moreover, if inequalities (22)–(24) have a feasible solution, then the control gain K_{aj} in (11) is given by $K_{aj}=M_{a0}^{-1}Y_{aj}\bar{P}^{-1}$.

Proof: From $(\bar{R}-\bar{P})\bar{R}^{-1}(\bar{R}-\bar{P})\geq 0$, we have $-\bar{P}\bar{R}^{-1}\bar{P}\leq\bar{R}-2\bar{P}$. After replacing $\bar{R}-2\bar{P}$ in (22)–(23) with $-\bar{P}\bar{R}^{-1}\bar{P}$ and performing corresponding congruence transformation by

$$\mathrm{diag}\left\{\bar{P}^{-1},\bar{P}^{-1},\bar{P}^{-1},I,\bar{P}^{-1},I,\bar{R}^{-1},\bar{\varepsilon}_{1ij}^{-1}I,\bar{\varepsilon}_{1ij}^{-1}I\right\},$$

and by

$$\mathrm{diag}\left\{\bar{P}^{-1},\bar{P}^{-1},\bar{P}^{-1},I,\bar{P}^{-1},I,\bar{R}^{-1},\bar{\varepsilon}_{2ij}^{-1}I,\bar{\varepsilon}_{2ij}^{-1}I\right\},$$

together with the change of matrix variables defined by

$$\begin{aligned}
P &= \bar{P}^{-1}, \quad R = \bar{R}^{-1}, \quad Q = \bar{P}^{-1}\bar{Q}\bar{P}^{-1}, \\
K_j &= M_{a0}^{-1} Y_j \bar{P}^{-1}, \qquad S = \bar{P}^{-1}\bar{S}\bar{P}^{-1}, \quad \varepsilon_{1ij} = \bar{\varepsilon}_{1ij}^{-1}, \\
\varepsilon_2 &= \bar{\varepsilon}_{2ij}^{-1}, \quad M = \mathrm{diag}\left\{\bar{P}^{-1}, \bar{P}^{-1}, \bar{P}^{-1}, I\right\}\bar{M}\bar{P}^{-1}, \\
N &= \mathrm{diag}\left\{\bar{P}^{-1}, \bar{P}^{-1}, \bar{P}^{-1}, I\right\}\bar{N}\bar{P}^{-1}.
\end{aligned}$$

It is concluded that the conditions in (14) and (15) hold. On the other hand, (24) is equivalent to (16) by performing a simple congruence transformation with diag$\left\{\bar{P}^{-1}, I\right\}$. Therefore, all the conditions in Theorem 1 are satisfied. The proof is completed. ■

Remark 2: In the study, the conservative will be reduced if the matrices Q, S, R, M and N are replaced by $\sum_{i=1}^4 h_i Q_i$, $\sum_{i=1}^4 h_i S_i$, $\sum_{i=1}^4 h_i R_i$, $\sum_{i=1}^4 h_i M_i = \sum_{i=1}^4 h_i \begin{bmatrix} M_{1i}^T & M_{2i}^T & M_{3i}^T & M_{4i}^T \end{bmatrix}^T$ and $\sum_{i=1}^4 h_i N_i = \sum_{i=1}^4 h_i \begin{bmatrix} N_{1i}^T & N_{2i}^T & N_{3i}^T & N_{4i}^T \end{bmatrix}^T$. However, computation complexion of the existence condition in Theorem 2 of reliable fuzzy H_∞ controller design will be increased intensively. Thus, the above proof is employed to handle the tradeoff in this study.

IV. Fuzzy H_∞ Controller Design

In the section, fuzzy H_∞ controller design is presented for active suspension systems with actuator delay based on T-S fuzzy model method. If there is no actuator fault in the active suspension system, then we obtain,

$$\begin{aligned}
\dot{x}(t) &= A(t)x(t) + B_1(t)w(t) + B(t)u(t-d(t)), \\
z_1(t) &= C_1(t)x(t) + D_1(t)u(t-d(t)), \\
z_2(t) &= C_2(t)x(t),
\end{aligned} \tag{25}$$

Based on the above presented fuzzy modeling, the overall fuzzy model is inferred as follows:

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{4} h_i(\xi(t))\left[A_i x(t) + B_i u(t-d(t)) + B_{1i} w(t)\right], \\
z_1(t) &= \sum_{i=1}^{4} h_i(\xi(t))\left[C_{1i} x(t) + D_i u(t-d(t))\right], \\
z_2(t) &= \sum_{i=1}^{4} h_i(\xi(t)) C_{2i} x(t).
\end{aligned} \tag{26}$$

In addition, the overall fuzzy control law is represented by

$$u(t) = \sum_{j=1}^{4} h_j(\xi(t)) K_{sj} x(t) \tag{27}$$

For the case of the standard controller (27), the closed-loop system is given by

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{4}\sum_{j=1}^{4} h_i h_j^d \left[A_i x(t) + B_i K_{sj} x(t-d(t)) + B_{1i} w(t)\right], \\
z_1(t) &= \sum_{i=1}^{4}\sum_{j=1}^{4} h_i h_j^d \left[C_{1i} x(t) + D_{1i} K_{sj} x(t-d(t))\right], \\
z_2(t) &= \sum_{i=1}^{4} h_i C_{2i} x(t).
\end{aligned} \tag{28}$$

Employing the similar method proposed in the previous section, the following corollary is obtained for the fuzzy H_∞ performance analysis at the context of the system in (28) with actuator delay.

Corollary 1: Consider the closed-loop system in (28). Given scalars $\bar{d} > 0$, μ and matrices K_{sj}, if there exist matrices $P > 0$, $Q > 0$, $S > 0$, $R > 0$, N_j, and M_j $(j = 1,2,3,4)$ with appropriate dimension such that the following LMIs hold for $q = 1,2$:

$$\begin{bmatrix} \acute{\Phi}_{11}^{ij} & \sqrt{\bar{d}}M & \acute{\Phi}_{13}^{ij} & \acute{\Phi}_{14}^{ij} \\ 0 & -R & 0 & 0 \\ 0 & 0 & -I & 0 \\ 0 & 0 & 0 & -R \end{bmatrix} < 0, \tag{29}$$

$$\begin{bmatrix} \acute{\Phi}_{11}^{ij} & \sqrt{\bar{d}}N & \acute{\Phi}_{13}^{ij} & \acute{\Phi}_{14}^{ij} \\ 0 & -R & 0 & 0 \\ 0 & 0 & -I & 0 \\ 0 & 0 & 0 & -R \end{bmatrix} < 0, \tag{30}$$

$$\begin{bmatrix} -P & \sqrt{\rho}\{C_{2i}\}_q^T \\ * & -I \end{bmatrix} < 0, \tag{31}$$

where

$$\begin{aligned}
\acute{\Phi}_{11}^{ij} &= \acute{\Xi}_{11}^{ij} + \mathrm{sym}(\Xi_2), \acute{\Xi}_{11}^{ij} = \begin{bmatrix} \acute{\Theta}_{11}^{ij} & \Theta_{12}^{ij} \\ * & -\gamma^2 I \end{bmatrix}, \\
\acute{\Phi}_{13}^{ij} &= \begin{bmatrix} C_{1i} & D_{1i}K_{sj} & 0 & 0 \end{bmatrix}^T, \\
\acute{\Theta}_{11}^{ij} &= \begin{bmatrix} \mathrm{sym}(PA_i) + Q + S & PB_iK_{sj} & 0 \\ * & -(1-\mu)S & 0 \\ * & * & -Q \end{bmatrix}, \\
\acute{\Phi}_{14}^{ij} &= \begin{bmatrix} \sqrt{\bar{d}}RA_i & \sqrt{\bar{d}}RB_iK_{sj} & 0 & \sqrt{\bar{d}}RB_{1i} \end{bmatrix}^T,
\end{aligned}$$

Take into account the matrices Ξ_2, Θ_{12}^{ij}, M and N in Theorem 1, we obtain,

(1) the closed-loop system is asymptotically stable;

(2) the performance $\|T_{z_1 w}\|_\infty < \gamma$ is minimized subject to output constraints (13)..

Similarly, the fuzzy H_∞ controller design condition as below is derived from Theorem 2.

Corollary 2: Consider the closed-loop system in (28). Given scalars $\bar{d} > 0$ and μ, the closed-loop system (12) is asymptotically stable with an H_∞ disturbance attenuation level γ, if there exist matrices $\bar{P} > 0$, $\bar{Q} > 0$, $\bar{S} > 0$, $\bar{R} > 0$, Y_{sj}, $\bar{N}_j$, and $\bar{M}_j$ $(j = 1,2,3,4)$ with appropriate dimensions such that the following LMIs hold for $q = 1,2$:

$$\begin{bmatrix} \grave{\Phi}_{11}^{ij} & \bar{\Phi}_{12}^{1ij} & \grave{\Phi}_{13}^{ij} & \grave{\Phi}_{14}^{ij} \\ 0 & \bar{R} - 2\bar{P} & 0 & 0 \\ 0 & 0 & -I & 0 \\ 0 & 0 & 0 & -R \end{bmatrix} < 0, \tag{32}$$

$$\begin{bmatrix} \grave{\Phi}_{11}^{ij} & \bar{\Phi}_{12}^{2ij} & \grave{\Phi}_{13}^{ij} & \grave{\Phi}_{14}^{ij} \\ 0 & \bar{R} - 2\bar{P} & 0 & 0 \\ 0 & 0 & -I & 0 \\ 0 & 0 & 0 & -R \end{bmatrix} < 0, \tag{33}$$

$$\begin{bmatrix} -\bar{P} & \sqrt{\rho}\bar{P}\{C_{2i}\}_q^T \\ * & -I \end{bmatrix} < 0, \tag{34}$$

where

$$\begin{aligned}
\grave{\Phi}_{11}^{ij} &= \grave{\Xi}_{11}^{ij} + \mathrm{sym}\left(\bar{\Xi}_2\right), \quad \grave{\Xi}_{11}^{ij} = \begin{bmatrix} \grave{\Xi}_{11}^{ij} & \bar{\Theta}_{12}^{ij} \\ * & -\gamma^2 I \end{bmatrix}, \\
\grave{\Phi}_{13}^{ij} &= \begin{bmatrix} C_{1i}\bar{P} & D_{1i}Y_{sj} & 0 & 0 \end{bmatrix}^T, \\
\Theta_{11}^{ij} &= \begin{bmatrix} \mathrm{sym}\left(A_i\bar{P}\right)+\bar{Q}+\bar{S} & B_iY_{sj} & 0 \\ * & -\left(1-\mu\right)\bar{S} & 0 \\ * & * & -\bar{Q} \end{bmatrix}, \\
\grave{\Phi}_{14}^{ij} &= \begin{bmatrix} \sqrt{\bar{d}}A_i & \sqrt{\bar{d}}B_iY_{sj} & 0 & \sqrt{\bar{d}}B_{1i} \end{bmatrix}^T,
\end{aligned}$$

$\bar{\Xi}_2$, $\bar{\Theta}_{12}^{ij}$, $\bar{M}$ and $\bar{N}$ are defined in Theorem 2. Then a standard controller in the form of (27) exists, such that

(1) the closed-loop system is asymptotically stable;

(2) the performance $\|T_{z_1w}\|_\infty < \gamma$ is minimized subject to output constraint (13).

Moreover, if inequalities (32)–(34) have a feasible solution, then the control gain K_{sj} in (27) is given by $K_{sj} = Y_{sj}\bar{P}^{-1}$.

Remark 3: When the derivative of $d(t)$ is unknown, and the delay $d(t)$ satisfies $0 < d(t) \leq \bar{d}$, by setting $S = 0$ in (18) and the LMIs-based conditions in Theorems 1–2 and Corollary 1–2, the reliable fuzzy H_∞ controller and fuzzy H_∞ controller can be obtained for the systems in (12) and (28) under the condition that the actuator delay $d(t)$ satisfies $0 < d(t) \leq \bar{d}$ respectively.

It is can be seen from the LMI-based conditions in Theorem 2 and Corollary 2 both dependent on the matrix variables and the objective scalar γ, which implies that γ can be included as an optimization variable to obtain a lower bound of the guaranteed H_∞ performance. Based on the different conditions, reliable fuzzy H_∞ controller and fuzzy H_∞ controller can be designed with the minimal γ by solving the following convex optimization problems:

$$\begin{aligned}
& \min \gamma \quad \text{s.t.} \quad (22)-(24). \\
\bar{P} &> 0, \bar{Q} > 0, \bar{S} > 0, \bar{R} > 0, \bar{\varepsilon}_{1ij} > 0, \bar{\varepsilon}_{2ij} > 0, Y_{aj}, \bar{M}, \bar{N} \quad (35)
\end{aligned}$$

and

$$\begin{aligned}
& \min \gamma \quad \text{s.t.} \quad (32)-(34). \\
\bar{P} &> 0, \bar{Q} > 0, \bar{S} > 0, \bar{R} > 0, Y_{sj}, \bar{M}, \bar{N}. \quad (36)
\end{aligned}$$

V. Simulation Results

A quarter-vehicle active suspension system is exploited to demonstrate the effectiveness of the proposed approach in this section. The quarter-vehicle suspension model parameters in Table 1 are used for this study. The sprung mass $m_s(t)$ is assumed to set as the range $[873\text{kg}, 1073\text{kg}]$ and the unsprung mass $m_u(t)$ to $[104\text{kg}, 124\text{kg}]$. In this study, the maximum allowable suspension stroke is set as $z_{\max} = 0.1$ m with $\rho = 1$. For the actuator delay $d(t) = 5 + 5\sin(\frac{1}{50})$ ms satisfying $\bar{d} = 10$ ms and $\mu = 0.1$, we consider fuzzy H_∞ controller design for the uncertain active suspension systems in (28). By using the convex optimization in (36), it is found that the minimum guaranteed closed-loop H_∞ performance index γ_{min} is 5.3011 and the fuzzy controller gain matrices

TABLE I
Quarter-car model parameters

k_s	k_t	c_s	c_t
42720N/m	101115N/m	1095Ns/m	14.6Ns/m

$$K_{si} = 10^4 \times \begin{bmatrix} -3.3260 & 5.6998 & -2.5167 & 0.2824 \end{bmatrix}, \quad (37)$$

where $i = 1, 2, 3, 4$.

It is expected that the desired fuzzy H_∞ controller in (27) with the parameters in (37) can be designed such that: 1) the sprung mass acceleration $z_1(t)$ is as small as possible; 2) the suspension deflection is below the maximum allowable suspension stroke $z_{\max} = 0.1$ m, which means that $x_1(t)/z_{\max}$ below 1; 3) the relation dynamic tire load $k_tx_2(t)/(m_s(t) + m_u(t))g < 1$. We first consider the following test road disturbance as

$$z_r(t) = 0.0254\sin 2\pi t + 0.005\sin 10.5\pi t + 0.001\sin 21.5\pi t(\text{m}). \quad (38)$$

According to [47], the road disturbance has a similar frequency as the car body resonance frequency (1Hz) under the condition that high-frequency disturbance added to simulate the rough road surface. In order to carry out the simulation for the fuzzy H_∞ controller as in (28), the variational sprung mass $m_s(t)$ and the variational unsprung mass $m_u(t)$ are set as: $m_s(t) = 973 + 100\sin(t)$ kg and $m_u(t) = 114 + 10\cos(t)$ kg, for deriving the fuzzy membership functional $h_i(\xi(t))$ $(i = 1, 2, 3, 4)$. By using the fuzzy H_∞ controller in (27) with the parameters in (37), we derive the corresponding closed-loop fuzzy system. Fig. 3 depicts the responses of body vertical accelerations and the actuator force for the open- (e.g., passive) and closed-loop (e.g., active) systems. Fig. 4 demonstrates the responses of suspension stroke and tire deflection constraint for both the passive and active systems. It is observed from Fig. 3 that the proposed fuzzy H_∞ control strategy reduces the sprung mass acceleration significantly in comparison with the passive suspension under the same road disturbance. The designed fuzzy H_∞ controller can achieve the less value of the maximum body acceleration for the active suspension system than the passive system, and passenger acceleration in the active suspension system is reduced significantly, which guarantees better ride comfort. In addition, it can be seen that, from Fig. 4, the suspension deflection constraint $x_1(t)/z_{\max} < 1$ and the relation dynamic tire load constraint $k_tx_2(t)/(m_s(t) + m_u(t))g < 1$ are guaranteed, which implies the road holding capability is ensured by the desired fuzzy controller. These two figures confirm that the designed standard state-feedback fuzzy H_∞ controller can achieve better ride comfort and road handling, guarantee constraint suspension deflection for the active suspension system.

To further evaluate the effectiveness of the proposed fuzzy H_∞ controller design strategy with actuator delays, the road disturbance as below is taken into account. In the context of active suspension performance, the road disturbance can be generally assumed as discrete events of relatively short duration and high intensity, caused by, for example, a pronounced bump or pothole on an otherwise smooth road surface. The

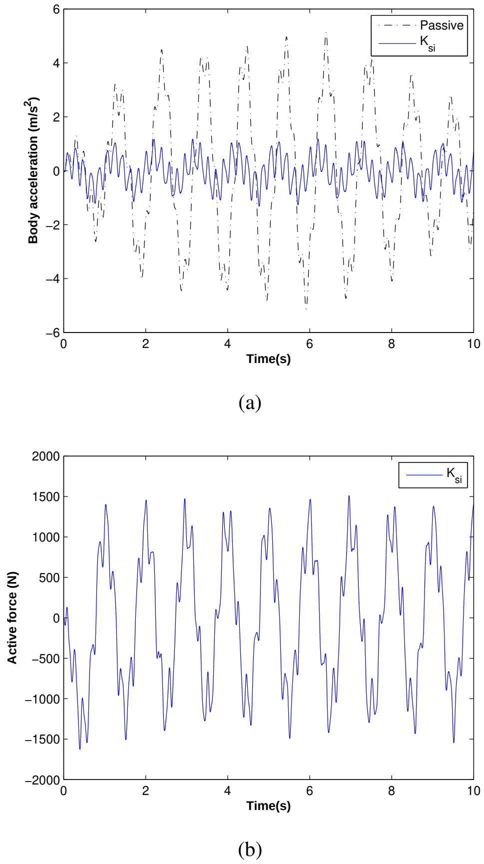


Fig. 3. (a) Responses of body vertical accelerations, (b) Response of active force.

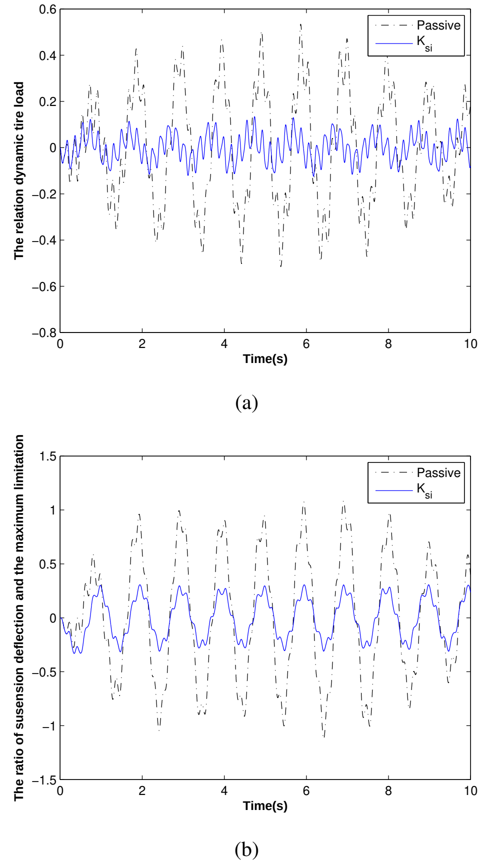


Fig. 4. (a) Responses of suspension deflection constraint, (b) Responses of tire stroke constraint.

road surface is represented by,

$$z_r(t) = \begin{cases} \frac{A}{2}(1-\cos(\frac{2\pi V}{L}t)), & \text{if } 0 \le t \le \frac{L}{V}, \\ 0, & \text{if } t > \frac{L}{V}, \end{cases} \tag{39}$$

where A and L are the height and the length of the bump. Assume $A = 50$ mm, $L = 6$ m and the vehicle forward velocity as $V = 35$ (km/h). Fig. 5 illustrates the the responses to body vertical accelerations and the actuator force; Fig. 6 presents the responses to suspension stroke and tire deflection constraint for the passive and active systems under the introduced road disturbance, respectively. The simulation results convincingly demonstrate that the fuzzy H_∞ controller offers better suspension performance than the open-loop suspension system.

The effectiveness and advantages of the proposed reliable fuzzy H_∞ controller design for active suspension systems with actuator delay and fault will be demonstrated in what follows. The parameters notation in the fuzzy H_∞ controller design in the above section is applied here as well. It is assumed that there exists the following actuator fault, namely, $\check{m}_a = 0.1$, $\hat{m}_a = 0.5$, which implies $M_{a0} = 0.3$ and $J_a = 0.2$. Based on the convex optimization presented in (35), we can obtain the minimum guaranteed closed-loop H_∞ performance index γ_{min} is 28.6991 and the reliable fuzzy controller gain matrices

$$\begin{aligned} K_{a1} &= 10^4 \times \begin{bmatrix} 4.1910 & -0.9700 & -2.5381 & 0.5713 \end{bmatrix}, \\ K_{a2} &= 10^4 \times \begin{bmatrix} 4.1916 & -0.9829 & -2.5381 & 0.5711 \end{bmatrix}, \\ K_{a3} &= 10^4 \times \begin{bmatrix} 4.1964 & -0.9751 & -2.5382 & 0.5706 \end{bmatrix}, \\ K_{a4} &= 10^4 \times \begin{bmatrix} 4.2149 & -0.9439 & -2.5388 & 0.5701 \end{bmatrix}. \end{aligned} \tag{40}$$

For the two kinds of road disturbances, namely, the first case road disturbance as shown in (38) and the second case road disturbance as given in (39). In Figs. 7–10, the responses to the open and closed-loop systems with the actuator delay and fault via the standard fuzzy H_∞ controller K_{si} and reliable controller K_{ai} $(i = 1,2,3,4)$ are based on the two different types of road disturbances. These figures show that the less value of the maximum body acceleration is achieved for the active suspension system, the suspension deflection constraint $x_1(t)/z_{\max} < 1$ is guaranteed and the relation dynamic tire load $k_t x_2(t)/(m_s(t)+m_u(t))g$ is below 1 in comparison with the passive suspension system, by utilizing the standard fuzzy H_∞ controller K_{si} and reliable controller K_{ai} $(i = 1,2,3,4)$ for different three types road disturbances respectively. However, it can be observed from Figs. 7 and 9 that the reliable fuzzy H_∞ controller achieves less value of the maximum body acceleration than the standard H_∞ controller for the active suspension system with actuator delay and fault. From Fig.

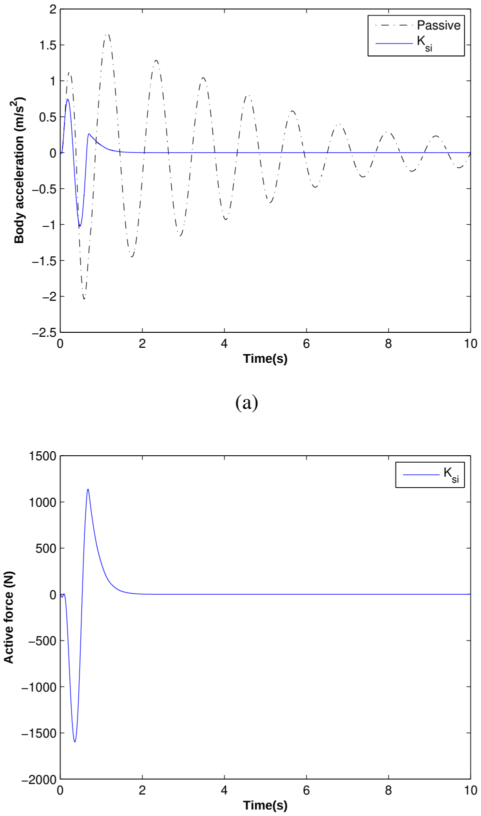


Fig. 5. (a) Responses of body vertical accelerations, (b) Response of active force.

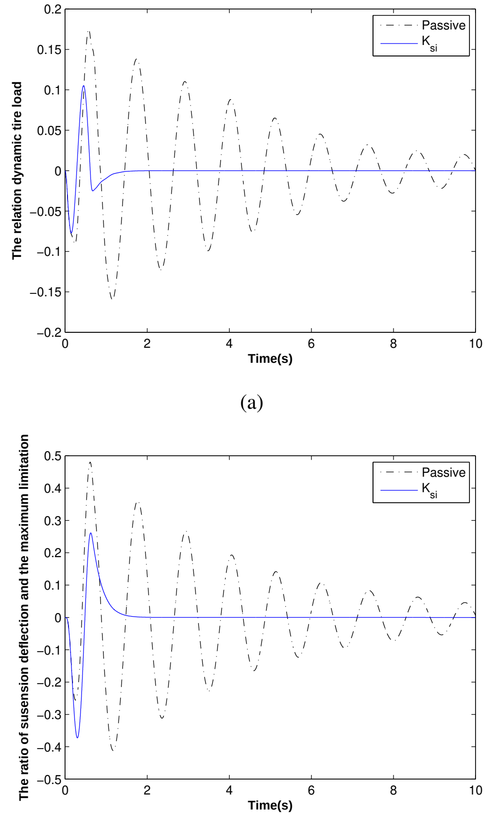


Fig. 6. (a) Responses of suspension deflection constraint, (b) Responses of tire stroke constraint.

8 and 10, it can be seen that K_{ai} $(i = 1,2,3,4)$ is capable to provide a much more steady control force in fault condition than conventional controller K_{si} $(i = 1,2,3,4)$.

To further evaluate the suspension system performance under different fuzzy controllers K_{si} and K_{ai} $(i = 1,2,3,4)$, the root mean square (RMS) values of the body acceleration are exploited to demonstrate its advantages. The road disturbances can also be generally assumed as random vibrations, which are consistent and typically specified as random process with a given ground displacement power spectral density (PSD) of

$$G_q(n) = G_q(n_0)\left(\frac{n}{n_0}\right)^{-c}, \tag{41}$$

where n_0 denotes the spatial frequency and n_0 is the reference spatial frequency of $n_0 = 0.1$ (1/m); $G_q(n_0)$ is used to stand for the road roughness coefficient; $c = 2$ is the road roughness constant. Related to the time frequency f, we have $f = nV$ with V for the vehicle forward velocity. Based on the equation (41), we can obtain the PSD ground displacement:

$$G_q(f) = G_q(n_0)\, n_0^{-2} \frac{V}{f^2}. \tag{42}$$

Accordingly, PSD ground velocity is given by

$$G_{\dot{q}}(f) = (2\pi f)^2 G_q(f) = 4\pi G_q(n_0)\, n_0^2 V, \tag{43}$$

which is only related with the vehicle forward velocity. When the vehicle forward velocity is fixed, the ground velocity can be viewed as a white-noise signal. We choose the four difference road roughness $G_q(n_0) = 16\times10^{-6}$ m^3, 64×10^{-6} m^3, 256×10^{-6} m^3 and 1024×10^{-6} m^3, which are corresponded to B Grade (Good), C Grade (Average), D Grade (Poor) and E Grade (Very Poor) for the vehicle forward velocity $V = 35$ (km/h), respectively.

RMS are strictly related to the ride comfort, which are often used to quantify the amount of acceleration transmitted to the vehicle body. The RMS value of variable $x(t)$ is calculated as $\mathrm{RMS}_x = \sqrt{(1/T)\int_0^T x^T(t)x(t)\mathrm{d}t}$. In our study, we choose $T = 100$ s to calculate the RMS values of the body acceleration, suspension stroke and relative dynamics tire load for different road roughness coefficient $G_q(n_0)$, which are listed in Tables II–IV by using the fuzzy controller K_{si} and reliable fuzzy controller K_{ai}, respectively. It can be observed that these tables indicate that the improvement in ride comfort and the satisfaction of hard constraints can be achieved for the different load conditions by using reliable fuzzy controller K_{ai} compared with the fuzzy controller K_{si} for the uncertain suspension systems with actuator delay and fault.

VI. Conclusions

This paper has investigated the problem of reliable fuzzy H_∞ control for active suspension systems with actuator delay and fault. The sprung and unsprung mass variations, the actuator delay and fault, and the suspension performance

TABLE II
RMS Body Acceleration

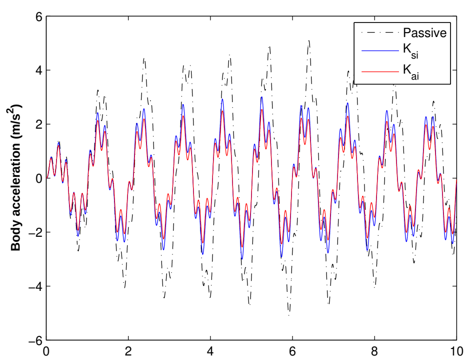
Parameter	Passive systems	Fuzzy Controller	Reliable Fuzzy Controller
$G_q(n_0) = 16 \times 10^{-6}$ m^3	0.0081	0.0046	0.0041
$G_q(n_0) = 64 \times 10^{-6}$ m^3	0.0152	0.0092	0.0083
$G_q(n_0) = 256 \times 10^{-6}$ m^3	0.0284	0.0183	0.0166
$G_q(n_0) = 1024 \times 10^{-6}$ m^3	0.0644	0.0387	0.0351

TABLE III
RMS Suspension Stroke

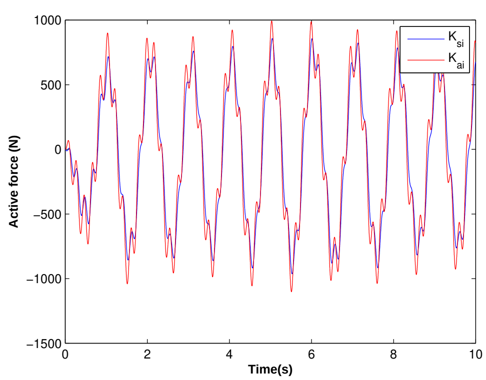
Parameter	Passive systems	Fuzzy Controller	Reliable Fuzzy Controller
$G_q(n_0) = 16 \times 10^{-6}$ m^3	1.7635×10^{-4}	9.7651×10^{-5}	9.5584×10^{-5}
$G_q(n_0) = 64 \times 10^{-6}$ m^3	3.3536×10^{-4}	1.9626×10^{-4}	1.9057×10^{-4}
$G_q(n_0) = 256 \times 10^{-6}$ m^3	6.2909×10^{-4}	3.9088×10^{-4}	3.8283×10^{-4}
$G_q(n_0) = 1024 \times 10^{-6}$ m^3	0.0014	8.2616×10^{-4}	8.0992×10^{-4}

TABLE IV
RMS Relative Dynamics Tire Load

Parameter	Passive systems	Fuzzy Controller	Reliable Fuzzy Controller
$G_q(n_0) = 16 \times 10^{-6}$ m^3	8.3596×10^{-4}	5.2554×10^{-4}	4.9612×10^{-4}
$G_q(n_0) = 64 \times 10^{-6}$ m^3	0.0016	0.0010	9.9561×10^{-4}
$G_q(n_0) = 256 \times 10^{-6}$ m^3	0.0030	0.0021	0.0020
$G_q(n_0) = 1024 \times 10^{-6}$ m^3	0.0067	0.0044	0.0042

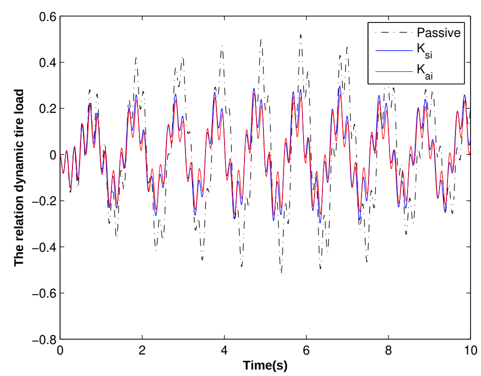


(a)

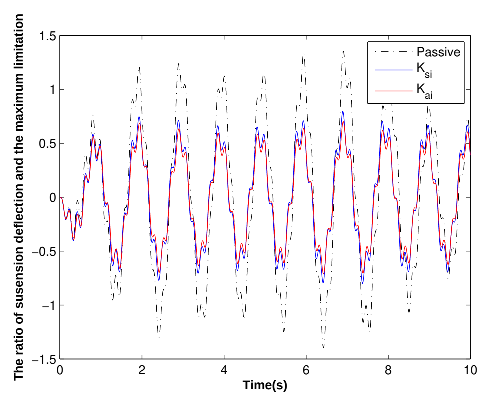


(b)

Fig. 7. (a) Responses of body vertical accelerations, (b) Response of active force.

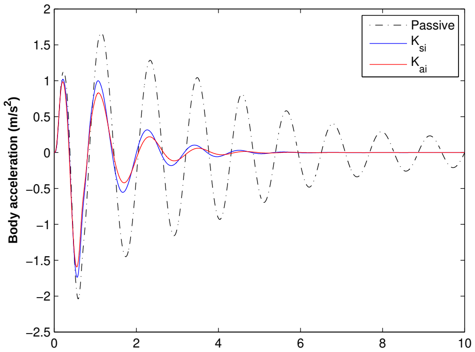


(a)

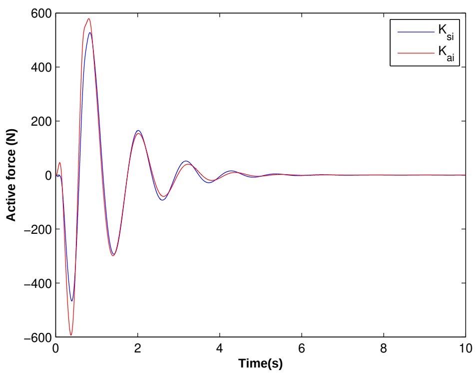


(b)

Fig. 8. (a) Responses of suspension deflection constraint, (b) Responses of tire stroke constraint.

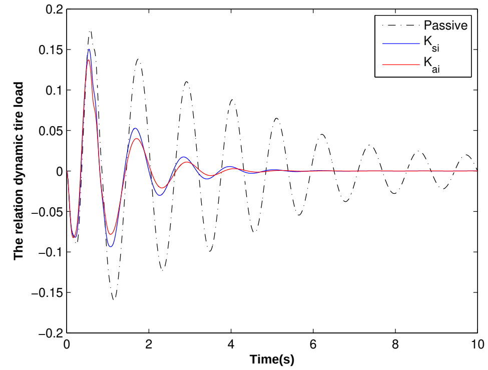


(a)

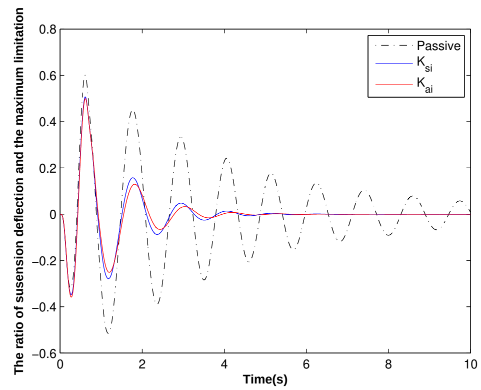


(b)

Fig. 9. (a) Responses of body vertical accelerations, (b) Response of active force.



(a)



(b)

Fig. 10. (a) Responses of suspension deflection constraint, (b) Responses of tire stroke constraint.

have all been taken into account to construct the T-S fuzzy system for the control design objective. Based on the PDC scheme and stability theory, the reliable fuzzy H_∞ performance analysis condition has been derived for the proposed T-S fuzzy system presenting the active suspension system with uncertainty. Then, the reliable fuzzy H_∞ controller has been designed such that the resulting closed-loop T-S fuzzy system is asymptotically stable with H_∞ performance, and simultaneously satisfies the constraint suspension performance. A quarter-vehicle suspension model has been used to validate the effectiveness of the proposed design method. Simulation results have clearly demonstrated that the designed reliable fuzzy controller has the capability of guaranteeing a better suspension performance under sprung and unsprung mass variations, actuator delay and fault.

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