Relic abundance of asymmetric dark matter

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October 14, 2013

References

- "Relic abundance of Asymmetric Dark Matter" arXiv:1104.5548, JCAP 1107 (2011) 003
- "Relic Abundance of Asymmetric Dark Matter in Quintessence," arXiv:1308.0353 [hep-ph].

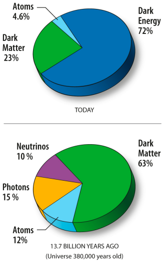
Part I

- Relic abundance of asymmetric dark matter in the standard cosmological scenario
- Constraints on parameter space

Part II

- Relic abundance of asymmetric dark matter in quintessence model with a kination phase
- Conclusions

Motivation



http://map.nasa.gov

- CMB, large-scale structure of the universe, etc. $\Rightarrow$ Relic abundance of non-baryonic cold dark matter (CDM): $\Omega_{\rm CDM} h^2 = 0.1109 \pm 0.0056$.
- Weakly interacting massive particles (WIMPs) are good candidates for CDM, e.g. neutralino which is Majorana particle.
- Dark matter particles can be asymmetric particles.
- $\Omega_b \approx 0.046$, $\Omega_{DM} \approx 5\Omega_b \rightarrow$ common mechanism $\rightarrow$ baryon asymmetry and a postulated asymmetry in the Dark Matter sector.

- In symmetric dark matter case, if the WIMP annihilation cross section is of the weak size, WIMPs scenario predicts relic abundance which naturally falls around the observed CDM abundance.
- In our work we generalize this to the case of asymmetric dark matter in the standard cosmological scenario in first part. We assume if there is dark matter asymmetry, it is created well before dark matter annihilation reactions freeze-out. In the second part, we extend this discussion to the kination model.

Part I: Relic abundance of asymmetric dark matter

- Considering a Dark Matter particle denoted by χ, $\bar{\chi} \neq \chi$.
  The relic densities of χ and $\bar{\chi}$ are determined by the Boltzmann equations:

$$\begin{aligned} \frac{\mathrm{d}n_\chi}{\mathrm{d}t} + 3Hn_\chi &= -\langle \sigma v \rangle (n_\chi n_{\bar{\chi}} - n_{\chi,\mathrm{eq}} n_{\bar{\chi},\mathrm{eq}}) \; ; \\ \frac{\mathrm{d}n_{\bar{\chi}}}{\mathrm{d}t} + 3Hn_{\bar{\chi}} &= -\langle \sigma v \rangle (n_\chi n_{\bar{\chi}} - n_{\chi,\mathrm{eq}} n_{\bar{\chi},\mathrm{eq}}) \; . \end{aligned} \qquad (1)$$

  The expansion rate is

$$H = \frac{\pi T^2}{M_{\mathrm{Pl}}} \sqrt{\frac{g_*}{90}} \, , \qquad (2)$$

  where $M_{\mathrm{Pl}} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass, and g_* is the effective number of the relativistic degrees of freedom.

The equilibrium number densities $n_{\chi,\mathrm{eq}}$ and $n_{\bar{\chi},\mathrm{eq}}$ are then given by

$$\begin{aligned} n_{\chi,\mathrm{eq}} &= g_\chi \left(\frac{m_\chi T}{2\pi}\right)^{3/2} \mathrm{e}^{(-m_\chi+\mu_\chi)/T}, \\ n_{\bar{\chi},\mathrm{eq}} &= g_\chi \left(\frac{m_\chi T}{2\pi}\right)^{3/2} \mathrm{e}^{(-m_\chi-\mu_\chi)/T}, \end{aligned} \quad (3)$$

$\mu_{\bar{\chi}} = -\mu_\chi$ in equilibrium.

- At high temperatures, χ and $\bar{\chi}$: in thermal equilibrium in the early universe,
- When $T < m_\chi$, $n_{\chi,\mathrm{eq}}$ and $n_{\bar{\chi},\mathrm{eq}} \searrow$ exponentially, as long as $m_\chi > |\mu_\chi|$. Eventually $\Gamma = n_{\bar{\chi}}\langle\sigma v\rangle$, $\bar{\Gamma} = n_\chi\langle\sigma v\rangle < H$.
- χ and $\bar{\chi}$ distributions: no longer in chemical equilibrium, and their co–moving number densities $\rightarrow$ constants. T_F, $\bar{T}_F$: freeze-out temperatures.

- Introducing the dimensionless quantities $Y_\chi = n_\chi/s$, $Y_{\bar{\chi}} = n_{\bar{\chi}}/s$, and $x = m_\chi/T$, where $s = (2\pi^2/45)g_* T^3$ is the entropy density and assuming that the universe expands adiabatically during the radiation dominated epoch, the Boltzmann equations become

$$\frac{dY_\chi}{dx} = -\frac{\lambda\langle\sigma v\rangle}{x^2}\left(Y_\chi\ Y_{\bar{\chi}} - Y_{\chi,\mathrm{eq}}\ Y_{\bar{\chi},\mathrm{eq}}\right); \tag{4}$$

$$\frac{dY_{\bar{\chi}}}{dx} = -\frac{\lambda\langle\sigma v\rangle}{x^2}\left(Y_\chi\ Y_{\bar{\chi}} - Y_{\chi,\mathrm{eq}}\ Y_{\bar{\chi},\mathrm{eq}}\right), \tag{5}$$

where

$$\lambda = \frac{4\pi}{\sqrt{90}}\ m_\chi M_{\mathrm{Pl}}\ \sqrt{g_*}\,. \tag{6}$$

- Subtracting above equations each other

$$\frac{dY_\chi}{dx} - \frac{dY_{\bar\chi}}{dx} = 0\,. \tag{7}$$

This implies

$$Y_\chi - Y_{\bar\chi} = C\,, \tag{8}$$

C: a constant, i.e. the difference of the co–moving densities of the particles and anti–particles is conserved.

- Then the Boltzmann equations become

$$\frac{dY_\chi}{dx} = -\frac{\lambda\langle\sigma v\rangle}{x^2}\left(Y_\chi^2 - CY_\chi - P\right); \tag{9}$$

$$\frac{dY_{\bar\chi}}{dx} = -\frac{\lambda\langle\sigma v\rangle}{x^2}\left(Y_{\bar\chi}^2 + CY_{\bar\chi} - P\right), \tag{10}$$

where

$$P = Y_{\chi,\mathrm{eq}} Y_{\bar\chi,\mathrm{eq}} = (0.145 g_\chi/g_*)^2\, x^3\, \mathrm{e}^{-2x}\,. \tag{11}$$

- $$\langle\sigma v\rangle = a + 6\, bx^{-1} + \mathcal{O}(x^{-2})\,.$$

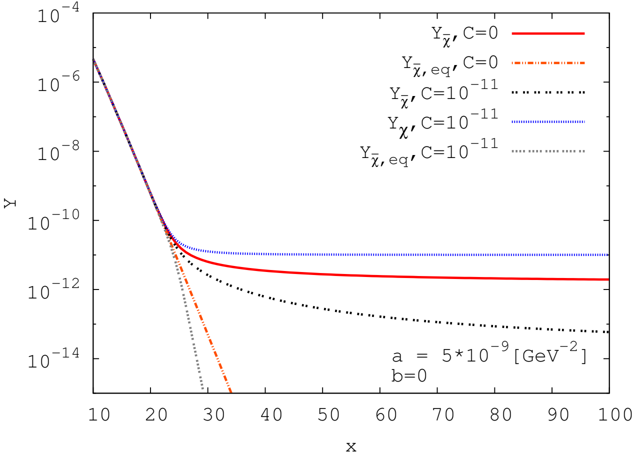


Figure: $a = 5.0 \times 10^{-9}$ GeV^{-2} $= 5.9 \times 10^{-26}$ cm^3 s^{-1}, $b = 0$, $m = 100$ GeV and $C = 10^{-11}$ or zero.

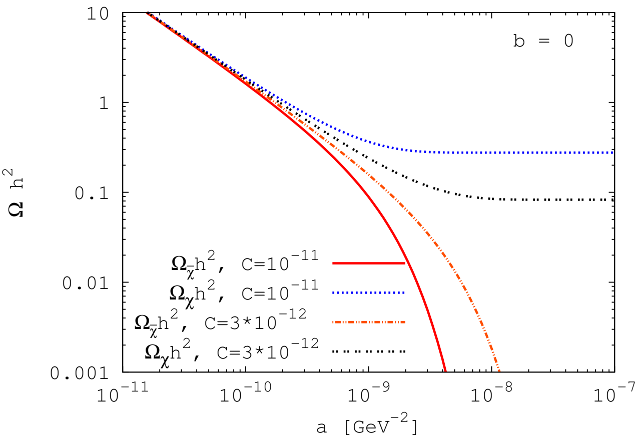


Figure: $m_\chi = 100$ GeV, $g_\chi = 2$ and $g_* = 90$.

Analytic solution

- In terms of the quantity $\Delta_{\bar\chi} = Y_{\bar\chi} - Y_{\bar\chi,\mathrm{eq}}$, the Boltzmann equation for $\bar\chi$ becomes:

$$\frac{d\Delta_{\bar\chi}}{dx} = -\frac{dY_{\bar\chi,\mathrm{eq}}}{dx} - \frac{\lambda\langle\sigma v\rangle}{x^2}\left[\Delta_{\bar\chi}(\Delta_{\bar\chi} + 2Y_{\bar\chi,\mathrm{eq}}) + C\Delta_{\bar\chi}\right]. \quad (12)$$

- At sufficiently high temperature $Y_{\bar\chi} \rightarrow Y_{\bar\chi,\mathrm{eq}}$. $\Delta_{\bar\chi}$ is small, and $d\Delta_{\bar\chi}/dx$, $\Delta^2_{\bar\chi} \rightarrow 0$, then

$$\frac{dY_{\bar\chi,\mathrm{eq}}}{dx} = -\frac{\lambda\langle\sigma v\rangle}{x^2}\left(2\Delta_{\bar\chi} Y_{\bar\chi,\mathrm{eq}} + C\Delta_{\bar\chi}\right). \quad (13)$$

- When particle were in thermal equilibrium, $Y_\chi = Y_{\chi,\mathrm{eq}}$ and $Y_{\bar{\chi}} = Y_{\bar{\chi},\mathrm{eq}}$, then

$$Y^2_{\bar{\chi},\mathrm{eq}} + CY_{\bar{\chi},\mathrm{eq}} - P = 0\,. \tag{14}$$

There are two solutions, but only one of them yields a positive $\bar{\chi}$ equilibrium density:

$$Y_{\bar{\chi},\mathrm{eq}} = -\frac{C}{2} + \sqrt{\frac{C^2}{4} + P}\,. \tag{15}$$

$$\Delta_{\bar{\chi}} \simeq \frac{2x^2 P}{\lambda\langle\sigma v\rangle\left(C^2 + 4P\right)}\,. \tag{16}$$

This solution will be used to determine the freeze–out temperature $\bar{x}_F$ for $\bar{\chi}$.

- At sufficiently low temperature, i.e. for $x > \bar{x}_F$, $\propto Y_{\bar{\chi},\mathrm{eq}} \to 0$ in the Boltzmann equation (12),

$$\frac{d\Delta_{\bar{\chi}}}{dx} = -\frac{\lambda\langle\sigma v\rangle}{x^2}\left(\Delta_{\bar{\chi}}^2 + C\Delta_{\bar{\chi}}\right) \ . \tag{17}$$

- Integrating Eq.(17) from $\bar{x}_F$ to ∞ then yields

$$Y_{\bar{\chi}}(x \to \infty) = \frac{C}{\exp\left(C\lambda \int_{\bar{x}_F}^{\infty} \langle\sigma v\rangle \, x^{-2} dx\right) - 1}. \tag{18}$$

Using

$$\langle \sigma v \rangle = a + 6\, b x^{-1} + \mathcal{O}(x^{-2})\,.$$

$$Y_{\bar{\chi}}(x \to \infty) = \frac{C}{\exp\left[C(4\pi/\sqrt{90})\, m_\chi M_{\rm Pl}\, \sqrt{g_*}\,(a\ \bar{x}_F^{-1} + 3b\ \bar{x}_F^{-2})\right] - 1}\,,$$

$$Y_{\chi}(x \to \infty) = \frac{C}{1 - \exp\left[-C(4\pi/\sqrt{90})\, m_\chi M_{\rm Pl}\, \sqrt{g_*}\,(a x_F^{-1} + 3b x_F^{-2})\right]}\,,$$

For convenience, we express the final abundance in terms of

$$\Omega_\chi h^2 = \frac{m_\chi s_0 Y_\chi(x \to \infty) h^2}{\rho_{\rm crit}}\,, \tag{19}$$

$s_0 = 2.9 \times 10^3\ \mathrm{cm}^{-3}$: present entropy density, $\rho_{\rm crit} = 3M_{\rm Pl}^2 H_0^2$: present critical density.

$$\Omega_{\rm DM} h^2 = 2.76 \times 10^8\ m_\chi \left[Y_\chi\ (x \to \infty) + Y_{\bar{\chi}}\ (x \to \infty)\right] \mathrm{GeV}^{-1}\,. \tag{20}$$

Fixing freeze-out temperature $\bar{x}_F$

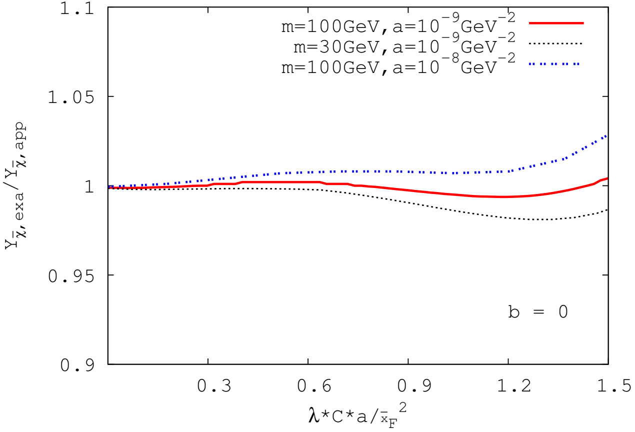
- In standard definition we assume that freeze–out occurs when the deviation $\Delta_{\bar{\chi}}$ is of the same order as the equilibrium value of $Y_{\bar{\chi}}$:

$$\xi Y_{\bar{\chi},\mathrm{eq}}(\bar{x}_{F_0}) = \Delta_{\bar{\chi}}(\bar{x}_{F_0}) \,, \tag{21}$$

$\xi = \sqrt{2} - 1$: a numerical constant of order unity.

- Making some corrections to this result

$$\bar{x}_F = \bar{x}_{F_0} \left(1 + \frac{0.285\lambda\, aC}{\bar{x}_{F_0}^3} + \frac{1.350\lambda\, bC}{\bar{x}_{F_0}^4} \right) . \tag{22}$$



1.1
1.05
1
0.95
0.9
m=100GeV,a=10^-9GeV^-2
m=30GeV,a=10^-9GeV^-2
m=100GeV,a=10^-8GeV^-2
b = 0
0.3
0.6
0.9
1.2
1.5
λ*C*a/x̄F^2
Yχ̄,exa/Yχ̄,app

Constraints on parameter space and indirect detection signals

$$0.10 < \Omega_{\rm DM} h^2 < 0.12 \tag{23}$$

For asymmetric dark matter, the χ and $\bar{\chi}$ contributions have to be added:

$$\Omega_{\rm DM} = \Omega_\chi + \Omega_{\bar{\chi}}\,. \tag{24}$$

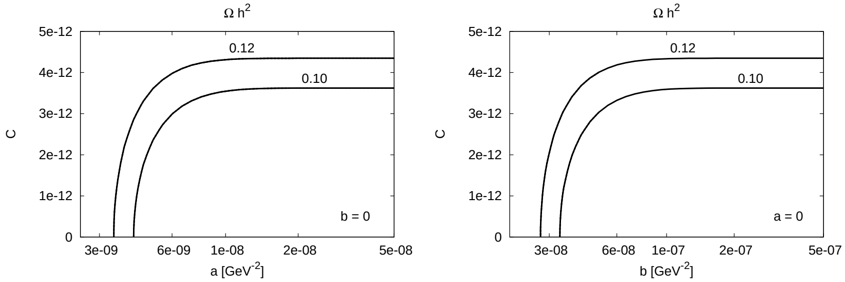


Figure: $m_\chi = 100$ GeV, $g_\chi = 2$ and $g_* = 90$.

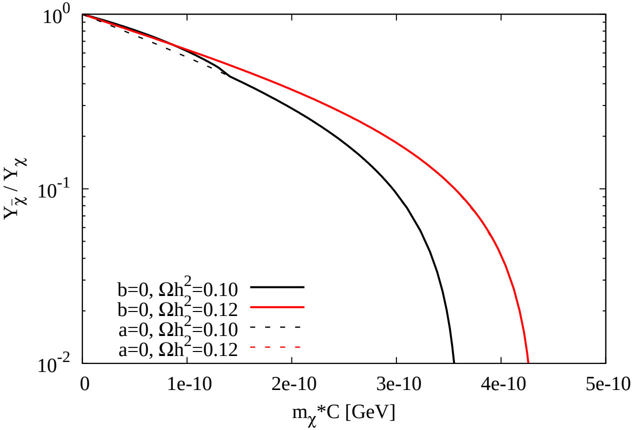


Figure: $g_* = 90$ and $m_\chi = 100$ GeV.

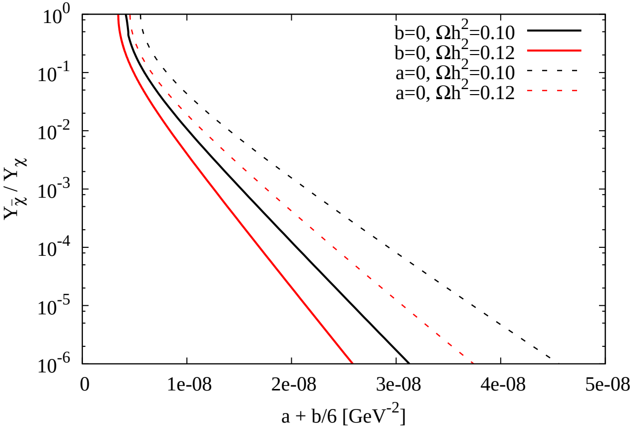


Figure: $g_* = 90$ and $m_\chi = 100$ GeV.

Part II: Relic abundance of asymmetric dark matter in quintessence model with a kination phase

- The Dark Matter relic density is changed by the modified expansion rate of the universe.
- One reason for that might be the additional contributions to the total energy density from quintessence model, a modification of general relativity and etc.
- In quintessence model with kination phase, the kinetic energy of a scalar field $\rho_\phi \simeq \dot{\phi}^2/2$ dominates over the potential energy density $V(\phi)$ and the radiation energy density ρ_{rad}.
- The overall energy density decreases as $\rho_{tot} \simeq \dot{\phi}^2/2 \sim R^{-6}$. $T \sim R^{-1}$, thus $H^2 \sim \rho_{tot} \sim T^6$.

The ratio of the expansion rate H during kination period and the expansion rate of the standard case $H_{\rm std}$:

$$\frac{H^2}{H_{\rm std}^2} = 1 + \frac{\rho_\phi}{\rho_r}\,, \tag{25}$$

where ρ_ϕ/ρ_r:

$$\frac{\rho_\phi}{\rho_r} =\simeq \eta \left(\frac{T}{T_0}\right)^2\,, \tag{26}$$

$\eta = \rho_\phi(T_0)/\rho_r(T_0)$. T_0: some reference temperature.

We can rewrite Eq.(25) as

$$H = A(T) H_{\rm std}, \tag{27}$$

$$A(T) = \sqrt{1 + \eta \left(\frac{T}{T_0} \right)^2}. \tag{28}$$

In order not to spoil the successful prediction of BBN, $A(T)$ must return to 1 at the low temperature around 1 MeV.

With the modified expansion rate, the Boltzmann Eqs.(9), (10) become

$$\frac{dY_\chi}{dx} = -\frac{\lambda\langle\sigma v\rangle}{x^2 A(x)} \left(Y_\chi^2 - CY_\chi - P\right); \tag{29}$$

$$\frac{dY_{\bar\chi}}{dx} = -\frac{\lambda\langle\sigma v\rangle}{x^2 A(x)} \left(Y_{\bar\chi}^2 + CY_{\bar\chi} - P\right).$$

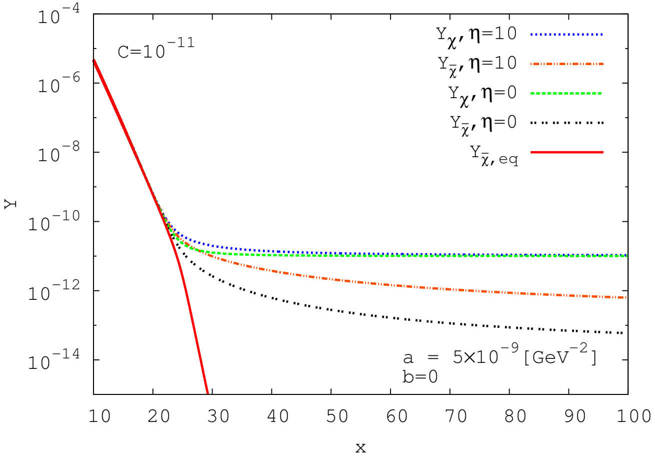


Figure: $a = 5.0 \times 10^{-9}$ GeV^{-2} $= 5.9 \times 10^{-26}$ cm^3 s^{-1}, $b = 0$, $m = 100$ GeV, $x_0 = 20$.

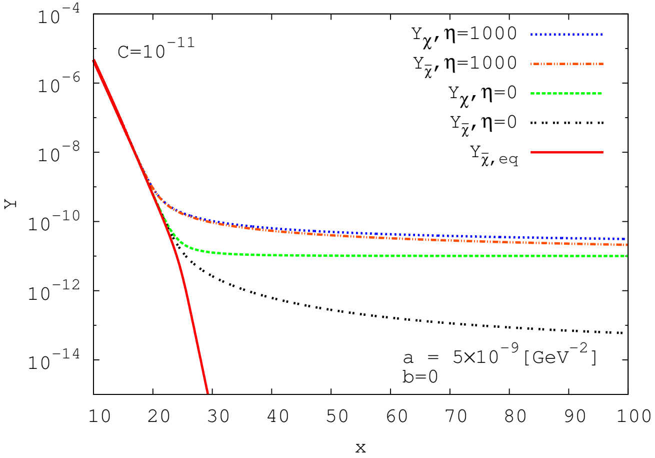


Figure: $a = 5.0 \times 10^{-9}$ GeV^{-2} $= 5.9 \times 10^{-26}$ cm^3 s^{-1}, $b = 0$, $m = 100$ GeV, $x_0 = 20$.

 Relic abundance of asymmetric dark matter in quintessence

$$Y_{\bar{\chi}}(x \to \infty) = \frac{C}{\exp\left[1.32\, C\, m_\chi M_{\rm Pl} \sqrt{g_*}\, I(\bar{x}_F)\right] - 1}\,, \qquad (30)$$

here

$$\begin{aligned} I(\bar{x}_F) &= \int_{\bar{x}_F}^{\infty} \frac{\langle \sigma v \rangle}{x^2\ A(x)}\ dx \\ &= \frac{a}{\sqrt{\eta}\ x_0} \ln\left(\sqrt{\eta}\, \frac{x_0}{\bar{x}_F} + \sqrt{1 + \eta\, \frac{x_0^2}{\bar{x}_F^2}}\right) + \frac{6b}{\eta x_0^2}\left(\sqrt{1 + \eta \frac{x_0^2}{\bar{x}_F^2}} - 1\right) \end{aligned}$$

$$Y_\chi(x \to \infty) = \frac{C}{1 - \exp\left[-1.32\, C\, m_\chi M_{\rm Pl} \sqrt{g_*}\, I(x_F)\right]}\,, \qquad (31)$$

where $I(x_F)$ is given by

$$\begin{aligned} I(x_F) &= \int_{x_F}^{\infty} \frac{\langle \sigma v \rangle}{x^2\ A(x)}\ dx \\ &= \frac{a}{\sqrt{\eta}\ x_0} \ln\left(\sqrt{\eta}\,\frac{x_0}{x_F} + \sqrt{1 + \eta\,\frac{x_0^2}{x_F^2}}\right) + \frac{6\,b}{\eta x_0^2}\left(\sqrt{1 + \eta\frac{x_0^2}{x_F^2}} - 1\right) \end{aligned}$$

Relic abundance of asymmetric dark matter in quintessence

$$\Omega_{\rm DM} h^2 = \frac{2.76 \times 10^8 \ m_\chi \ C}{\exp\left[1.32\, C\, m_\chi M_{\rm Pl}\, \sqrt{g_*}\, I(\bar{x}_F)\right] - 1} + \frac{2.76 \times 10^8 \ m_\chi \ C}{1 - \exp\left[-1.32\, C\, m_\chi M_{\rm Pl}\, \sqrt{g_*}\, I(x_F)\right]} .$$

 Relic abundance of asymmetric dark matter in quintessence

$$0.10 < \Omega_{\rm DM} h^2 < 0.12$$

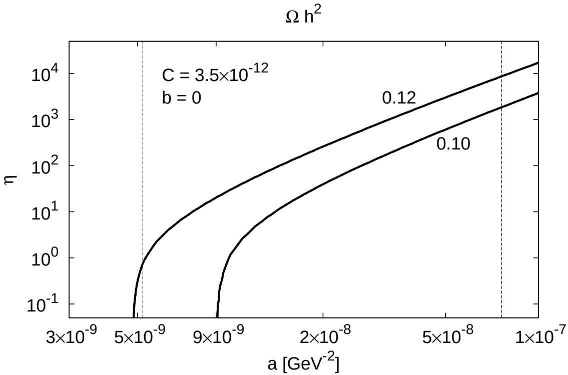


Figure: The two vertical lines 6.0×10^{-26} cm^3 s^{-1}$= 5.2 \times 10^{-9}$ GeV^{-2} and 8.8×10^{-25} cm^3 s^{-1}$= 7.6 \times 10^{-8}$ GeV^{-2} are the upper limits for the cross sections for mass 100 GeV from the *Fermi*-LAT collaboration[arXiv:1108.3546 [astro-ph.HE]].

6. Summary and Conclusions

- We discussed relic abundance of Asymmetric Dark Matter in the standard cosmological scenario.
- The final abundance is determined not only by the cross section as in the symmetric Dark Matter case, but also by the asymmetry.
- We extend our discussion to the quintessence model with a kination phase. We found in this case, the enhancement factor affects the relic density of asymmetric dark matter, it increases the particle and anti–particle relic density, the increase amount depends on the enhancement factor.

Thank you