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Remaining Useful Life Prediction With Fusing Failure Time Data and Field Degradation Data With Random Effects

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ABSTRACT Accurate remaining useful life (RUL) prediction has a great significance to improve the reliability and safety for key equipment. However, it often occur imperfect or even no prior degradation information in practical application for the existing RUL prediction methods, which could produce prediction error. To solve this issue, this paper proposes a two-step RUL prediction method based on Wiener processes with reasonably fusing the failure time data and field degradation data. First, we obtain some interesting natures of parameters estimation based on the basic linear Wiener process. These natures explain the relationship between the parameters estimation results and the feature of degradation data, i.e. item sample numbers, detection time and detect frequency, and give the basis regarding how to reasonably fuse the failure time data and field degradation data. Second, under the Bayesian framework, we further propose a two-step method by fusing the failure time data and field degradation data with considering the random effects based on the proposed natures of parameters estimation. In this method, we propose an EM algorithm to estimate the mean and variance drift parameter of Wiener processes by the failure time data. Next, we generalize this two-step RUL prediction method to the nonlinear Wiener process. Last, we use two case studies to demonstrate the usefulness and superiority of the proposed method.

INDEX TERMS Remaining useful life prediction, wiener processes, fusing, failure time data, field degradation data, random effects, Bayesian framework.

I. INTRODUCTION

Engineering practice shows that prognostics and health management (PHM) can reduce maintenance costs, improve the reliability and safety, and reduce the risk of failure events [1]. This is essential important for some areas with requirement of high safety and high reliability, such as electric vehicles, military and aerospace [2]–[5]. PHM mainly includes two parts: one is prognostics, i.e. remaining useful life (RUL) prediction, predicting the time when the health condition of equipment first crossing the failure threshold; the other is health management, i.e. making the optimal maintenance decision based on the prognostic information to achieve the lowest maintenance costs and minimum failure risk, which mainly includes determining the optimal maintenance time, formulating the order strategy of spare parts and providing the scheme for prolonging life [6]–[8].

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TABLE 1. Acronym.

ADT	accelerated degradation testing
CM	condition monitoring
EM	expectation maximization
MLE	maximum likelihood estimation
MSEs	mean square errors
PHM	prognostics and health management
REs	relative errors
RUL	remaining useful life
PDFs	probability distribution functions

TABLE 2. Notations.

$f(\cdot)$	Probability density distribution function of RUL
t	Cycle time
x	Condition monitoring data
λ	Drift coefficient of the linear Wiener process model
σ_B	Diffusion coefficient of the linear Wiener process model
σ_λ^2	Prior variance of λ
$\sigma_{\lambda,k}^2$	Posterior variance of λ
μ_λ	Prior mean of λ
$\mu_{\lambda,k}$	Posterior mean of λ
$\Lambda(t:\theta)$	Nonlinear coefficient of the nonlinear Wiener process
w	The failure threshold
θ	The fixed coefficient of nonlinear Wiener process
$\mathbf{\Theta}$	Prior parameters of the linear Wiener degradation model
$\mathbf{\Omega}$	Prior parameters of the nonlinear Wiener degradation model
$B(t)$	Brownian motion
$D(\cdot)$	The variance of estimation
$E(\cdot)$	The expectation of estimation
$M1$	The RUL prediction method proposed in this paper
$M2$	The traditional Bayes RUL prediction method
$N(\cdot)$	Normal distribution
$\boldsymbol{T}_{1:m}$	The failure time data
$X(t)$	Predicted condition data at time t

As the precondition of health management, RUL prediction is a key issue in PHM. A typical characteristic of RUL prediction is that it can make full use of condition monitoring (CM) data representing degradation to determine the remaining useful life time before failure at the current time [9]. CM data mainly includes historical degradation data of congeneric units and the field degradation information of the assessed unit. Usually, the historical degradation data of congeneric units is used to estimate the fixed parameters and prior information of random parameters of the degradation [10]. Then, the random parameters are updated based on the field degradation information of the assessed unit under the Bayesian framework. This Bayesian mechanism established a linkage between the past and current degradation data of the congeneric units.

A representative work regarding this strategy under Bayesian framework is first presented by Gebraeel *et al.* [11] for the regression-based model. Following Gebraeel *et al.* [11], some related issues and many variants and applications have been studied and reported [12]–[14]. Also, this strategy under Bayesian framework has been applied in Wiener processes based degradation model, which is a very popular model for modeling the degradation modeling and has been widely used to model the degradation process in systems [15], such as lithium-ion batteries [16], [17], LCDs (Liquid crystal display) [18], gyroscopes [19], bearings [20], etc. Si *et al.* [21] did a lot of interesting works in this research area for the degradation process with three sources of variance, e.g. linear Wiener process, nonlinear Wiener process [22], and the validity of these methods has also been verified by many simulation examples and case studies. This Bayesian mechanism has also been applied to the generalized Wiener process [23], Wiener process with skew-normal random effects [24], bivariate Wiener degradation process [25], nonlinear Wiener process [26], [27], additive Wiener process [28], Adaptive Wiener process [29], Wiener process with imperfect inspections [30], Wiener process with measurement error [10], etc. Under the Bayesian framework, the RUL estimation results include the degradation information of congeneric units and the field assessed unit.

However, it often occur imperfect prior information, such as inaccurate or absence of prior degradation information, in practical application of RUL prediction, which could result in RUL prediction error [31]–[33]. In the existing study, there are mainly two ways to solve this problem. The first way is combining the Bayesian updating and expectation maximization (EM) algorithm, which was first presented by Wang *et al.* [34] for fitting an adapted Brownian motion-based model with a drifting parameter. Then, this updating mechanism has been generalized to the degradation model based on the basic Wiener process [31], [32], nonlinear process [35], [36], degradation process with three sources of variance [37], [38]. Based on the EM algorithm, the RUL estimation results could overcome the influences of imperfect prior information. However, why the EM algorithm obtains better results than the traditional Bayesian method is still unclear. Recently, Tang *et al.* [39] proved an interesting rule that the estimation results based on the EM algorithm is equal to the classic MLE method for the single assessed unit based on the linear Wiener process with measurement error. The reason is that the EM algorithm itself is used to find the maximum likelihood estimation of unknown parameters. Based on this rule, Tang *et al.* [39] presented a heuristic parameter updating algorithm to reasonably integrate the prior information and field information. However, the iteration interval length and iteration times are subjectively determined, which needs further studies.

The second way is to deal with imperfect prior information by fusing the failure time data of congeneric units. To deal with the case in absence of prior degradation information, Gebraeel *et al.* [33] proposed a RUL prediction method by fitting the failure time data to a Bernstein distribution for obtaining prior distributions of regression-based degradation model. This mechanism has also been generalized to the Wiener process based model. Lehmann [40] derived the likelihood function for the joint observation of failure times and degradation data at discrete times. Wang *et al.* [41] proposed a Bayesian evaluation method to integrate the accelerated degradation testing (ADT) data with the failure time data based on a joint likelihood function. Zhang *et al.* [42] established a likelihood function by integrating bivariate degradation data with lifetime data. Similar works can be found in [43]–[46], and references therein.

About the above works regarding fusing failure time data based on Wiener process, there are mainly three limitations. The first limitation is that the random effects are not considered in the degradation modeling based on Wiener process. That is, the drift parameter is considered as a fixed value. However, the drift parameter is usually modeled as a random parameter to represent unit-to-unit variance. Ignoring the random effects could increase the RUL estimation error [21]. The second limitation is how to offline estimate the parameters when considering that the random effects are unclear. For the regression-based degradation model, the failure time data are used to estimate the prior information of drift parameter [33]. However, compared with the regression-based degradation model, there is one more parameter (i.e. diffusion parameter) for the Wiener process based model. This increases the difficulty of parameters estimation. The third limitation is lack of studies focus on nonlinear degradation process when fusing the failure time data. Nonlinear is a typical feature in degradation modeling and RUL estimation, which should be considered for the nonlinear degradation process [19]. To the best of our knowledge, current studies regarding fusing failure time data are all aimed at the linear degradation process, even for the regression-based degradation model [33].

From the above review over related works, it can be observed that the RUL prediction under imperfect prior information has not been researched thoroughly. Therefore, this paper attempts to reasonably fuse failure time data and field degradation data with random effects. First, we propose some interesting natures of parameters estimation based on the basic linear Wiener process. This contribution is important, and has not been reported before. Second, we present a RUL prediction method for the degradation model with random effects by these natures of parameters estimation. This method first applies field degradation data to estimate the fixed parameters that represent common characteristics of the model, and then use history failure time data to estimate the prior distributions of drift parameter that represent personality features based on the EM algorithm. This contribution is the first time that the random effects are considered in fusing failure time data and field degradation data. Next, we generalize this fusing method with consider random effects to the nonlinear degradation model, which is the third contribution of this paper. Last, we use two case studies to demonstrate the effectiveness and usefulness of the proposed RUL prediction method, which can not only overcome the imperfect prior degradation information, but also can effectively improve the accuracy of RUL prediction compared with the traditional Bayesian method.

The remainders of the paper are presented as follow. Section II analyzes the natures of the parameters estimation for the linear Wiener process based degradation model. Section III develops a two-step RUL prediction method with fusing failure time data and field degradation data to overcome the imperfect prior degradation information. In section IV, two real-world examples are carried out to illustrate the usefulness and superiority of the presented method.

II. NATURES OF PARAMETERS ESTIMATION FOR THE WIENER PROCESS BASED DEGRADATION MODEL

The Wiener process is a type of diffusion process driven by Brownian motion with a drift coefficient, which can provide a good description of a system's dynamic characteristics due to their non-monotonic property, infinite divisibility property, and physical interpretations. For the RUL prediction based on Wiener process, how to estimate the prior parameters estimation is a key issue. Therefore, the natures of parameters estimation based on the basic linear Wiener process are discussed in this section.

The degradation process based on the basic linear Wiener process can be expressed as follows:

$$X(t) = x_0 + \lambda t + \sigma_B B(t) \tag{1}$$

where x_0 is the initial state, λ is the drift coefficient, which characterizes the degradation rate of equipment, σ_B is the diffusion coefficient, and $B(t)$ is the standard Brownian motion representing the dynamic characteristics and uncertainty of the degradation process. Without loss of generality, we set $x_0 = 0$. In order to distinguish the individual difference, drift coefficient λ is regarded as a random variable and follows normal distribution, i.e. $\lambda \sim N(\mu_\lambda, \sigma_\lambda^2)$. Therefore, the prior model parameters are $\boldsymbol{\Theta}=\{\mu_\lambda, \sigma_\lambda^2, \sigma_B^2\}$.

Before researching the natures of parameters estimation based on the basic linear Wiener process, we first give the following lemma.

Lemma 1: Define $\boldsymbol{x}_{0:k} = \{x_0, x_1, x_2, \ldots, x_k\}$ are the field degradation data of the time $t_0, t_1, t_2, \ldots t_k$ for a specific item, the unbiased estimation of $\hat{\lambda}$ and $\hat{\sigma}_B^2$ can be obtained as follows.

$$\hat{\lambda} = x_k/t_k \tag{2}$$

$$\hat{\sigma}_B^2 = \frac{1}{k-1}\sum\nolimits_{i=1}^{k} \frac{1}{\Delta t_i}\left(\Delta x_i - \Delta t_i x_k/t_k\right)^2 \tag{3}$$

where

$$E(\hat{\lambda}) = \lambda, \quad E(\hat{\sigma}_B^2) = \sigma_B^2 \tag{4}$$

For more details about Lemma 1, refer to Theorem 1 and Theorem 2 in [47]. Lemma 1 gives the unbiased estimation of $\hat{\lambda}$ and $\hat{\sigma}_B^2$, however, the estimation accuracy is not discussed. To demonstrate the nature regarding estimation accuracy of $\hat{\lambda}$ and $\hat{\sigma}_B^2$, we give the following Theorem.

Theorem 1: For the unbiased estimation of $\hat{\lambda}$ and $\hat{\sigma}_B^2$ given in (2) and (3), the variance of estimation of $\hat{\lambda}$ and $\hat{\sigma}_B^2$ can be obtained as follows.

$$D(\hat{\lambda}) = \sigma_B^2/t_k \tag{5}$$

$$D(\hat{\sigma}_B^2) = \frac{2\sigma_B^4}{k-1} \tag{6}$$

Proof: Based on model (1) and the nature of Wiener process, it can be obtained that Δx_i obeys a normal distribution, i.e. $\Delta x_i \sim N(\lambda \Delta t_i, \sigma_B^2 \Delta t_i)$, and x_k also obeys a normal distribution, i.e. $x_k \sim N(\lambda t_k, \sigma_B^2 t_k)$. Therefore x_k/t_k obeys a normal distribution as follow:

$$x_k/t_k \sim N(\lambda, \sigma_B^2/t_k) \tag{7}$$

Then, the variance of $\hat{\lambda}$ can be obtained as follow:

$$D(\hat{\lambda}) = \sigma_B^2/t_k \tag{8}$$

Since $\Delta x_i \sim N(\lambda \Delta t_i, \sigma_B^2 \Delta t_i)$, we can obtain that

$$\sum\nolimits_{i=1}^{k} \frac{(\Delta x_i - \Delta t_i x_k/t_k)^2}{\sigma_B^2 \Delta t_i} \sim \chi^2(k-1) \tag{9}$$

where χ^2 denotes the Chi-square distribution. Thereby, the variance of (8) can be obtained as follows.

$$D\left(\sum\nolimits_{i=1}^{k} \frac{(\Delta x_i - \Delta t_i x_k/t_k)^2}{\sigma_B^2 \Delta t_i}\right) = 2(k-1) \tag{10}$$

Based on (10), we can obtain the variance of $\hat{\sigma}_B^2$ as follows.

$$\begin{aligned} D(\hat{\sigma}_B^2) &= D\left(\frac{1}{k-1}\sum\nolimits_{i=1}^{k} \frac{(\Delta x_i - \Delta t_i x_k/t_k)^2}{\Delta t_i}\right) \\ &= D\left(\frac{\sigma_B^2}{k-1}\sum\nolimits_{i=1}^{k} \frac{(\Delta x_i - \Delta t_i x_k/t_k)^2}{\sigma_B^2 \Delta t_i}\right) \\ &= \frac{2\sigma_B^4}{k-1} \end{aligned} \tag{11}$$

This completes the proof.

Remark 1: The (5) from Theorem 1 shows that the estimation accuracy of λ for a single unit is mainly effected by the length of detection time, i.e. the bigger the t_k, the more accurate estimation of λ. Additionally, it shows from (6) that the estimation accuracy of σ_B^2 is mainly effected by the detected number of the field degradation data $\boldsymbol{x}_{0:k}$, i.e. the bigger the k, the more accurate estimation of σ_B^2.

Theorem 1 is deduced for only one item. However, in a general way, offline parameters estimation is implemented with considering random effects, i.e. for a certain items. In order to analyze the nature of parameters estimation with considering the random effects, we first use a two-step maximum likelihood estimation (MLE) method presented in [39] to estimate the parameters for the basic linear Wiener process. Without loss of generality, the degradation of all items are detected at the same time $t_0, t_1, \ldots t_m$. Suppose that there are $n$items with the same type and the detected degradation data at time $t_0, t_1, \ldots t_m$ of the ith item is $\boldsymbol{x}_i = \{x_{0,i}, x_{1.i}, \ldots, x_{m,i}\}$, then all the historical degradation data can be expressed as $\boldsymbol{X} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N\}$. Based on this two step MLE method, we first give the log-likelihood function of $\lambda_1, \lambda_2, \ldots, \lambda_n, \sigma_B^2$ as follows [16]

$$\begin{aligned} &\ln L(\lambda_1, \lambda_2, \ldots, \lambda_n, \sigma_B^2|\mathbf{X}) \\ &= -\frac{mN}{2}\left(\ln 2\pi + \ln \sigma_B^2\right) \\ &\quad -\frac{N}{2}\sum\nolimits_{j=1}^{m} \ln \Delta t_j - \frac{1}{2\sigma_B^2}\sum\nolimits_{i=1}^{N}\sum\nolimits_{j=1}^{m} \\ &\quad \times \frac{1}{\Delta t_j}\left(\Delta x_{j,i} - \lambda_j \Delta t_j\right)^2 \end{aligned} \tag{12}$$

where $\Delta x_{j,i} = x_{j,i} - x_{j-1,i}$, $\Delta t_j = t_j - t_{j-1}$.

Then, $\Phi = \{\mu_\lambda, \sigma_\lambda^2, \sigma_B^2\}$ can be estimated by two steps as follows.

$$\hat{\mu}_\lambda = \frac{1}{N}\sum\nolimits_{i=1}^{N} \frac{x_{m,i}}{t_m} \tag{13}$$

$$\hat{\sigma}_\lambda^2 = \frac{1}{N-1}\sum\nolimits_{i=1}^{N}\left(\frac{x_{m,i}}{t_m} - \hat{\mu}_\lambda\right)^2 \tag{14}$$

$$\hat{\sigma}_B^2 = \frac{1}{N(m-1)}\sum\nolimits_{i=1}^{N}\sum\nolimits_{j=1}^{m}\left(\Delta x_{j,i} - \Delta t_{j,i} x_{m,i}/t_m\right) \tag{15}$$

Note that the estimation of σ_λ^2 and σ_B^2 are under the framework of unbiased estimation, which is different from the results in [48]. However, the estimation of σ_B^2 is consistent with the results in [47].

Then, we can obtain the following theorem.

Theorem 2: For the degradation model based on linear Wiener process described in (1), when using the two step MLE method to estimate the unknown parameters, the expectation and variance for $\hat{\mu}_\lambda$, $\hat{\sigma}_\lambda^2$ and $\hat{\sigma}_B^2$ can be obtained as follows:

$$E(\hat{\mu}_\lambda) = \mu_\lambda, \quad D(\hat{\mu}_\lambda) = \frac{1}{N}(\sigma_\lambda^2 + \frac{\sigma_B^2}{t_k}) \tag{16}$$

$$E(\hat{\sigma}_\lambda^2) = \sigma_\lambda^2 + \frac{\sigma_B^2}{t_k}, \quad D(\hat{\sigma}_\lambda^2) = \frac{2}{N-1}\left(\sigma_\lambda^2 + \frac{\sigma_B^2}{t_k}\right)^2 \tag{17}$$

$$E(\hat{\sigma}_B^2) = \sigma_B^2, \quad D(\hat{\sigma}_B^2) = \frac{2N}{m-1}\sigma_B^4 \tag{18}$$

Proof: Based on the nature of Wiener process in model (1), for the ith item, $x_{m,i}$ obeys a normal distribution, i.e. $x_{m,i} \sim N(\mu_\lambda t_{m,i}, \sigma_\lambda^2 t_m^2 + \sigma_B^2 t_m)$. Then, $x_{m,i}/t_m$ also obeys a normal distribution, i.e. $x_{m,i}/t_m \sim N(\mu_\lambda, \sigma_\lambda^2 + \sigma_B^2/t_m)$. Therefore, $\hat{\mu}_\lambda$ obeys a normal distribution as follow:

$$\hat{\mu}_\lambda = \frac{1}{N}\sum\nolimits_{i=1}^{N} \frac{x_{m,i}}{t_m} \sim N\left(\mu_\lambda, \frac{1}{N}\left(\sigma_\lambda^2 + \frac{\sigma_B^2}{t_m}\right)\right) \tag{19}$$

According to (19), the expectation and variance for $\hat{\mu}_\lambda$ can be obtained as follows:

$$E(\hat{\mu}_\lambda) = \mu_\lambda \tag{20}$$

$$D(\hat{\mu}_\lambda) = \frac{1}{N}\left(\sigma_\lambda^2 + \frac{\sigma_B^2}{t_m}\right) \tag{21}$$

Since $x_{m,i}/t_m \sim N(\mu_\lambda, \sigma_\lambda^2 + \sigma_B^2/t_m)$, we obtain that:

$$\sum\nolimits_{i=1}^{N} \frac{\left(x_{m,i}/t_m - \hat{\mu}_\lambda\right)^2}{\sigma_\lambda^2 + \sigma_B^2/t_m} \sim \chi^2(N-1) \tag{22}$$

where χ^2 denotes the Chi-square distribution. Thereby, the expectation and variance for (22) can be obtained as follows:

$$E\left[\sum\nolimits_{i=1}^{N} \frac{\left(x_{m,i}/t_m - \hat{\mu}_\lambda\right)^2}{\sigma_\lambda^2 + \sigma_B^2/t_m}\right] = N-1 \tag{23}$$

$$D\left(\sum\nolimits_{i=1}^{N} \frac{\left(x_{m,i}/t_m - \hat{\mu}_\lambda\right)^2}{\sigma_\lambda^2 + \sigma_B^2/t_m}\right) = 2(N-1) \tag{24}$$

Therefore, the expectation and variance for $\hat{\sigma}_\lambda^2$ can be obtained as follow:

$$E(\hat{\sigma}_\lambda^2) = \mathrm{E}\left(\frac{1}{N-1}\sum_{i=1}^{N}\left(x_{m,i}/t_m - \hat{\mu}_\lambda\right)^2\right)$$
$$= \mathrm{E}\left(\frac{\sigma_\lambda^2 + \sigma_B^2/t_m}{N-1}\sum_{i=1}^{N}\frac{(x_{m,i}/t_m - \hat{\mu}_\lambda))^2}{\sigma_\lambda^2 + \sigma_B^2/t_m}\right)$$
$$= \sigma_\lambda^2 + \frac{\sigma_B^2}{t_m} \tag{25}$$
$$D(\hat{\sigma}_\lambda^2) = \mathrm{D}\left(\frac{1}{N-1}\sum_{i=1}^{N}\left(x_{m,i}/t_m - \hat{\mu}_\lambda\right)^2\right)$$
$$= D\left(\frac{\sigma_\lambda^2 + \sigma_B^2/t_m}{N-1}\sum_{i=1}^{N}\frac{(x_{m,i}/t_m - \hat{\mu}_\lambda)^2}{\sigma_\lambda^2 + \sigma_B^2/t_m}\right)$$
$$= \frac{2}{N-1}\left(\sigma_\lambda^2 + \frac{\sigma_B^2}{t_m}\right)^2 \tag{26}$$

From Theorem 1, we can obtain (18) after some simplification.

This completes the proof.

Remark 2: From above two theorems, the following conclusions can be obtained:

(1) It shows from (16) and (17) that the accuracy of estimation for μ_λ and σ_λ^2 is mainly effected by the number of item samples and the length of the longest detection time t_m. That is, the more samples N or the longer t_m, the more accurate to the estimation of μ_λ and σ_λ^2. If the detection time t_m is not considered, the estimation of μ_λ and σ_λ^2 is mainly affected by the number of item samples. For the case that the prior degradation data is scarce, the failure time data could also increase the estimation accuracy of μ_λ and σ_λ^2. In other words, adding the failure time data into degradation modeling could increase the estimation accuracy of μ_λ and σ_λ^2.

(2) It can be observed from (18) that the accuracy of estimation for σ_B^2 is proportional to detection time $k-1$, but inversely proportional to the number of item samples. In other words, adding the failure time data into degradation modeling could decrease the estimation accuracy of σ_B^2.

(3) Based on the above conclusions, we can use the failure time data to estimate the prior information of drift parameter, and the field degradation data to estimate the diffusion parameter. This mechanism could effectively improve the RUL prediction accuracy when the prior information is scare or inaccurate, as discussed in Section III.

III. RUL PREDICTION METHOD WITH FUSING FAILURE TIME DATA AND FIELD DEGRADATION DATA

In practical use, it often occurs the situation with imperfect prior degradation information for the existing RUL prediction methods, such as inaccurate or even no prior degradation information at all. Therefore, traditional Bayesian methods have certain limitations for RUL prediction. To solve this problem, this section proposes a RUL prediction method with reasonably fusing failure time data and field degradation information based on the natures proposed in Section II.

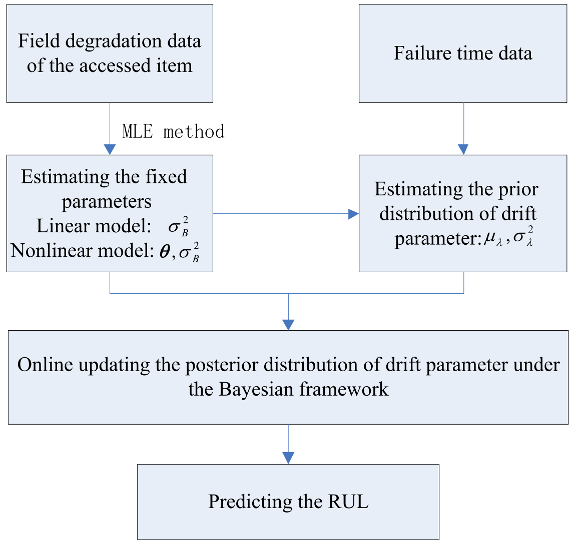

FIGURE 1. The RUL prediction algorithm flow with fusing failure time data and field degradation data.

The flow chart of this method is shown in Fig. 1 and the main steps are as follows:

Step 1: According to the field degradation data of the accessed item, the MLE for fixed parameters that represent common characteristics can be obtained. Then, the failure time data are used to estimate the prior distribution of random parameters by using the MLE or EM algorithm. That is, transforming the failure time data into prior distribution of λ.

Step 2: Online updating the posterior distribution of the random variable under the Bayesian framework, and then predicting the RUL.

During the implementation of this method, once a new field degradation data is detected, it needs to carry out the algorithm proposed in Fig. 1 again. Therefore, the estimation accuracy of the fixed parameters and prior information of random parameter can be improved with updating of field degradation data. In following, the RUL prediction methods with fusing failure time data and field degradation information are respectively developed for the degradation models based on linear Wiener processes and non-linear Wiener processes.

A. RUL PREDICTION BASED ON LINEAR WIENER PROCESS

1) PRIOR PARAMETERS ESTIMATION

For the degradation model based on linear Wiener process, as show in (1), a two-step prior parameters estimation method based on EM algorithm is proposed. The prior parameters based on linear Wiener process are $\boldsymbol{\Theta} = \{\mu_\lambda, \sigma_\lambda^2, \sigma_B^2\}$. Define that $\boldsymbol{x}_{0:k} = \{x_0, x_1, x_2, \ldots, x_k\}$ are the field degradation data of the time $t_1, t_2, \ldots t_k$ for one equipment. There are two steps for this method as described follows:

Step 1: Estimating the fixed parameter based on the field degradation data.

The log-likelihood function of the $\boldsymbol{x}_{0:k}$ with respect to σ_B^2 and λ can be written as follow:

$$\ln L(\lambda, \sigma_B^2|\boldsymbol{x}_{0:k}) = -\frac{k}{2}\left(\ln 2\pi + \ln\sigma_B^2\right) - \frac{1}{2}\sum_{j=1}^{k}\ln\Delta t_j - \frac{1}{2\sigma_B^2}\sum_{j=1}^{k}\frac{1}{\Delta t_j}\left(\Delta x_j - \lambda\Delta t_j\right)^2 \quad (27)$$

Maximizing (27), the fixed parameter σ_B can be obtained as follow:

$$\hat{\sigma}_B^2 = \frac{1}{k-1}\sum_{j=1}^{k}\frac{1}{\Delta t_j}(\Delta x_j - \Delta t_j x_k/t_k)^2 \quad (28)$$

Step 2: Transforming the failure time data into prior distribution of λ by using the EM algorithm.

Before estimating the prior distribution of λ, the following theorem is given firstly.

Theorem 3: Define that T_v is the failure time that the degradation first crosses through the failure threshold w of a specific item. Given the prior distribution of the λ, the posterior distribution of λ conditional on failure time T_v follows the normal distribution according to Bayesian theory. That is:

$$\lambda|T_v \sim N(\mu_{\lambda,v}, \sigma_{\lambda,v}^2) \quad (29)$$

$$\mu_{\lambda,v} = \frac{w\sigma_\lambda^2 + \mu_\lambda\hat{\sigma}_B^2}{T_v\sigma_\lambda^2 + \hat{\sigma}_B^2}, \quad \sigma_{\lambda,v}^2 = \frac{\sigma_\lambda^2\hat{\sigma}_B^2}{T_v\sigma_\lambda^2 + \hat{\sigma}_B^2} \quad (30)$$

Proof: Given the drift parameter λ_v of a specific item, according to the nature of Wiener process, the failure time T_v obeys inverse Gaussian distribution, and the likelihood function of T_v can be written as follow:

$$L(T_v|\lambda_v) = \frac{w}{\sqrt{2\pi T_v^3\hat{\sigma}_B^2}}\exp\left(-\frac{(w-\lambda_v T_v)^2}{2T_v\hat{\sigma}_B^2}\right) \quad (31)$$

Given the prior distribution of λ, i.e. $\lambda \sim N(\mu_\lambda, \sigma_\lambda^2)$, the posterior distribution of λ conditional on T_v can be calculated as follow:

$$\begin{aligned} p(\lambda|T_v) &\propto p(T_v|\lambda)p(\lambda) \\ &\propto \exp\left[-\frac{(w-\lambda T_v)^2}{2\hat{\sigma}_B^2 T_v} - \frac{1}{2\sigma_\lambda^2}(\lambda-\mu_\lambda)^2\right] \\ &\propto \exp\left[-\frac{1}{2}\left(\frac{T_v}{\hat{\sigma}_B^2} + \frac{1}{\sigma_\lambda^2}\right)\lambda^2 + \left(\frac{w}{\hat{\sigma}_B^2} + \frac{\mu_\lambda}{\sigma_\lambda^2}\right)\lambda\right] \\ &\propto \exp\left(-\frac{(\lambda-\mu_{\lambda,v})^2}{2\sigma_{\lambda,v}^2}\right) \end{aligned} \quad (32)$$

This completes the proof.

From theorem 3, the prior distribution of λ can be obtained by using the EM algorithm. Define that $\boldsymbol{T}_{1:m} = \{T_1, T_2, \ldots, T_m\}$ are the failure time data that the degradation first crosses through the failure threshold w of congeneric items. Then, the log-likelihood function of the failure time $\boldsymbol{T}_{1:m}$ and drift parameters $\boldsymbol{\lambda}$ with respect to μ_λ and σ_λ^2 can be written as follow:

$$\begin{aligned} &\ln L(\mu_\lambda, \sigma_\lambda^2|\boldsymbol{T}_{1:m}, \boldsymbol{\lambda}) \\ &= -\frac{m\ln 2\pi}{2} + m\ln w - \frac{m}{2}\ln\hat{\sigma}_B^2 \\ &\quad - \frac{3}{2}\sum_{v=1}^{m}\ln T_v - \sum_{v=1}^{m}\frac{(w-\lambda_v T_v)^2}{2\hat{\sigma}_B^2 T_v} - \frac{m}{2}\ln\sigma_\lambda^2 \\ &\quad - \frac{1}{2\sigma_\lambda^2}\sum_{v=1}^{m}(\lambda_v - \mu_\lambda)^2 \end{aligned} \quad (33)$$

Let $\hat{\Theta}^{(i)} = \{\hat{\mu}_\lambda^{(i)}, \hat{\sigma}_\lambda^{2(i)}\}$ are the parameters estimation results based on EM algorithm in the i-th step, then the $(i+1)$-th step of EM algorithm can be divided into E-step and M-step as follows:

E-step: Calculating the expectation for (33) as follows:

$$\begin{aligned} &L(\Theta|\boldsymbol{T}_{1:m}, \hat{\Theta}^{(i)}) \\ &= E_{\boldsymbol{\lambda}|\boldsymbol{T}_{1:m},\hat{\Theta}^{(i)}}\left[\ln L(\Theta|\boldsymbol{T}_{1:m})\right] \\ &= -\frac{m\ln 2\pi}{2} + m\ln w - \frac{m}{2}\ln\hat{\sigma}_B^2 - \frac{3}{2}\sum_{v=1}^{m}\ln T_v \\ &\quad - \sum_{v=1}^{m}\frac{(w-\lambda_v T_v)^2 + \sigma_\lambda^2 T_v^2}{2\hat{\sigma}_B^2 T_v} - \frac{m}{2}\ln\sigma_\lambda^2 \\ &\quad - \frac{1}{2\sigma_\lambda^2}\sum_{v=1}^{m}\left[(\lambda_{\lambda,v} - \mu_\lambda)^2 + \sigma_{\lambda,v}^2\right] \end{aligned} \quad (34)$$

where $\mu_{\lambda,v}$ and $\sigma_{\lambda,v}^2$ are the posterior distribution of λ conditional on $\boldsymbol{T}_{1:m}$ and $\hat{\Theta}^{(i)}$.

M-step: The $\hat{\Theta}^{(i+1)}$ can be obtained by maximizing (34). That is:

$$\hat{\Theta}^{(i+1)} = \arg\max_{\Theta} L(\Theta|\hat{\Theta}^{(i)}) \quad (35)$$

Taking the partial derivatives of μ_λ and σ_λ^2 for (34) and setting them to zero, gives the MLE for μ_λ and σ_λ^2 in the $(i+1)$-th step as follows:

$$\begin{aligned} \hat{\mu}_\lambda &= \frac{1}{m}\sum_{v=1}^{m}\mu_{\lambda,v}, \\ \hat{\sigma}_\lambda^2 &= \frac{1}{m-1}\sum_{v=1}^{m}\left[(\mu_{\lambda,v} - \hat{\mu}_\lambda)^2 + \sigma_{\lambda,v}^2\right] \end{aligned} \quad (36)$$

Note that the estimation of σ_λ^2 is under the framework of unbiased estimation.

2) RUL PREDICTION

Given the prior distribution of λ, the posterior distribution of λ conditional on $\boldsymbol{x}_{1:k}$ also follows normal distribution according to Bayesian theory [32]. That is:

$$\lambda|\boldsymbol{x}_{1:k} \sim N(\mu_{\lambda,k}, \sigma_{\lambda,k}^2) \quad (37)$$

where:

$$\mu_{\lambda,k} = \frac{x_k\hat{\sigma}_\lambda^2 + \hat{\mu}_\lambda\hat{\sigma}_B^2}{t_k\hat{\sigma}_\lambda^2 + \hat{\sigma}_B^2}, \quad \sigma_{\lambda,k}^2 = \frac{\hat{\sigma}_B^2\hat{\sigma}_\lambda^2}{t_k\hat{\sigma}_\lambda^2 + \hat{\sigma}_B^2} \quad (38)$$

After detecting the field degradation data $\boldsymbol{x}_{1:k}$, the degradation process when $t > t_k$ can be written as follow:

$$X(t|\boldsymbol{x}_{1:k}) = x_k + (\lambda|\boldsymbol{x}_{1:k})(t-t_k) + \hat{\sigma}_B B(t-t_k) = x_k + (\lambda|\boldsymbol{x}_{1:k}) l_k + \hat{\sigma}_B B(l_k) \tag{39}$$

Then the RUL at time t_k can be transformed into the time when the degradation process $\{X(l_k+t_k), l_k \geq 0\}$ first crosses the failure threshold $w_k = w - x_k$. And the corresponding RUL can be written as follow:

$$L_k = \inf\{l_k : X(l_k+t_k) \geq w|\boldsymbol{x}_{1:k}\} \tag{40}$$

Therefore, the probability density function (PDF) of the RUL can be written as follow [32]:

$$f_{L_k|\boldsymbol{x}_{1:k},w}(l_k|\boldsymbol{x}_{1:k}, w) = \frac{w_k}{\sqrt{2\pi l_k^2(\sigma_{\lambda,k}^2 l_k^2 + \hat{\sigma}_B^2 l_k)}} \exp\left(-\frac{(w_k - \mu_{\lambda,k} l_k)^2}{2(\sigma_{\lambda,k}^2 l_k^2 + \hat{\sigma}_B^2 l_k)}\right) \tag{41}$$

B. RUL PREDICTION BASED ON NONLINEAR WIENER PROCESS

1) DEGRADATION MODELING AND PRIOR PARAMETERS ESTIMATION

The degradation model based on nonlinear Wiener process can be expressed as follow [10], [19], [35]:

$$X(t) = x_0 + \lambda\Lambda(t;\theta) + \sigma_B B(t) \tag{42}$$

where x_0 is the initial state, $\lambda\Lambda(t;\boldsymbol{\theta})$ is the nonlinear drift part, and σ_B is the diffusion coefficient. The diffusion coefficient σ_B and nonlinear coefficient $\boldsymbol{\theta}$ are fixed coefficients, representing the common characteristics of the degradation process. The drift coefficient λ is a random variable to describe the individual difference among different equipment, which follows normal distribution, i.e. $\lambda \sim N(\mu_\lambda, \sigma_\lambda^2)$. Therefore, the prior parameters of the model based on nonlinear Wiener process are $\boldsymbol{\Omega} = \{\mu_\lambda, \sigma_\lambda^2, \boldsymbol{\theta}, \sigma_B\}$.

Compared with the linear model, the difference is that the close-form posterior distribution of the drift coefficient of the nonlinear model could not be obtained under the given failure time T_v. That is, Theorem 3 does not apply to the nonlinear model. Therefore, it is difficult to directly estimate the prior distribution of drift coefficients by using EM algorithm. To solve this problem, a two-step MLE method is developed for estimating the prior parameter for the nonlinear degradation model, and the specific steps of the two-step MLE method are given as follows.

Step 1: Estimating the fixed parameter

Given that $\boldsymbol{x}_{0:k} = \{x_0, x_1, x_2, \ldots, x_k\}$ are the field degradation data of an equipment, then the log-likelihood function of $\boldsymbol{x}_{0:k}$ with respect to λ, σ_B^2 and $\boldsymbol{\theta}$ can be written as follow:

$$\begin{aligned}\ln L(\lambda, \boldsymbol{\theta}, \sigma_B^2|\,\boldsymbol{x}_{0:k}) &= -\frac{k}{2}(\ln 2\pi + \ln\sigma_B^2) - \frac{1}{2}\sum_{i=1}^{k} \ln\Delta t_i \\ &\quad - \frac{1}{2\sigma_B^2}\sum_{i=1}^{k}\frac{1}{\Delta t_i}\left(\Delta x_i - \lambda\left(\Lambda(t_i;\boldsymbol{\theta}) - \Lambda(t_{i-1};\boldsymbol{\theta})\right)\right)^2\end{aligned} \tag{43}$$

Taking the partial derivatives of λ and σ_B^2 for (43) and setting them to zero, the restricted estimation for λ and σ_B^2 limited by $\boldsymbol{\theta}$ can be obtained as follows:

$$\hat{\lambda}(\boldsymbol{\theta}) = \frac{\sum_{i=1}^{k}\left(\Delta x_i\left(\Lambda(t_i;\boldsymbol{\theta}) - \Lambda(t_{i-1};\boldsymbol{\theta})\right)/\Delta t_i\right)}{\sum_{i=1}^{k}\left(\left(\Lambda(t_i;\boldsymbol{\theta}) - \Lambda(t_{i-1};\boldsymbol{\theta})\right)^2/\Delta t_i\right)} \tag{44}$$

$$\hat{\sigma}_B^2(\boldsymbol{\theta}) = \frac{1}{k}\sum_{i=1}^{k}\left(\frac{1}{\Delta t_i}\left(\Delta x_i - \hat{\lambda}(\boldsymbol{\theta})\left(\Lambda(t_i;\boldsymbol{\theta}) - \Lambda(t_{i-1};\boldsymbol{\theta})\right)\right)^2\right) \tag{45}$$

Bring (44) and (45) into (43), and simplifying, gives the log-likelihood function for b in terms of the estimated $\hat{\lambda}$ and $\hat{\sigma}_B^2$ as follow:

$$\ln L(\boldsymbol{\theta}|\,\boldsymbol{x}_{0:k}) = -\frac{k}{2}\left(\ln 2\pi + \ln\hat{\sigma}_B^2(\boldsymbol{\theta})\right) - \frac{1}{2}\sum_{i=1}^{k}\Delta t_i - \frac{k}{2} \tag{46}$$

The estimation of $\hat{\boldsymbol{\theta}}$ can be obtained by maximizing (46). Then the MLE for $\hat{\lambda}$ and $\hat{\sigma}_B^2$ can be obtained by bring $\hat{\boldsymbol{\theta}}$ into (44) and (45).

Step 2: Estimating the prior distribution of the drift parameter based on the failure time data.

Define the drift coefficient λ_v of one specific item, according to the nature of nonlinear Wiener process, the failure time T_v obeys the inverse Gaussian distribution, and the likelihood function of T_v can be written as follow:

$$L(T_v|\lambda_v) = \frac{w - \lambda_v\left(\Lambda(T_v;\hat{\boldsymbol{\theta}}) - \Lambda'(T_v;\hat{\boldsymbol{\theta}})T_v\right)}{\sqrt{2\pi T_v^3\hat{\sigma}_B^2}} \times \exp\left(-\frac{(w - \lambda_v\Lambda(T_v;\hat{\boldsymbol{\theta}}))^2}{2T_v\hat{\sigma}_B^2}\right) \tag{47}$$

According to (47), the log-likelihood function of the failure time T_v with respect to λ_v can be written as follow:

$$\begin{aligned}\ln L(T_v|\lambda_v) &= \ln\left(w - \lambda_v\left(\Lambda(T_v;\hat{\boldsymbol{\theta}}) - \Lambda'(T_v;\hat{\boldsymbol{\theta}})T_v\right)\right) \\ &\quad - \frac{1}{2}\ln 2\pi - \frac{3}{2}\ln T_v - \frac{1}{2}\ln\hat{\sigma}_B^2 - \frac{\left(w - \lambda_v\Lambda(T_v;\hat{\theta})\right)^2}{2\hat{\sigma}_B^2 T_v}\end{aligned} \tag{48}$$

The MLE for $\hat{\lambda}_v$ can be obtained by maximizing (48). Given that $\boldsymbol{T}_{1:m} = \{T_1, T_2, \ldots, T_m\}$ are the failure time data of the nonlinear model, then the MLE for $\hat{\boldsymbol{\lambda}} = \{\hat{\lambda}_1, \hat{\lambda}_2, \ldots, \hat{\lambda}_m\}$ can be obtained by (48). Therefore, the prior distribution of λ can be calculated as follows:

$$\hat{\mu}_\lambda = \frac{1}{m}\sum_{v=1}^{m}\hat{\lambda}_v, \quad \hat{\sigma}_\lambda^2 = \frac{1}{m-1}\sum_{v=1}^{m}(\hat{\lambda}_v - \hat{\mu}_\lambda)^2 \tag{49}$$

Note that the estimation of is $\hat{\sigma}_\lambda^2$ set to be unbiased.

2) RUL PREDICTION

Given the prior distribution of λ, i.e. $\lambda \sim N(\mu_\lambda, \sigma_\lambda^2)$, the posterior distribution of λ conditional on $\boldsymbol{x}_{1:k}$ also obeys a normal distribution according to Bayesian theory [10], that is:

$$\lambda \,|\boldsymbol{x}_{1:k} \sim N(\mu_{\lambda,k}, \sigma_{\lambda,k}^2) \tag{50}$$

where:

$$\mu_{\lambda,k} = \frac{B_k \hat{\sigma}_\lambda^2 + \hat{\mu}_\lambda \hat{\sigma}_B^2}{A_k \hat{\sigma}_\lambda^2 + \hat{\sigma}_B^2}, \quad \sigma_{\lambda,k}^2 = \frac{\hat{\sigma}_B^2 \hat{\sigma}_\lambda^2}{A_k \hat{\sigma}_\lambda^2 + \hat{\sigma}_B^2} \tag{51}$$

$$A_k = \sum_{i=1}^{k} \frac{\left(\Lambda(t_i; \hat{\boldsymbol{\theta}}) - \Lambda(t_{i-1}; \hat{\boldsymbol{\theta}})\right)^2}{t_i - t_{i-1}} \tag{52}$$

$$B_k = \sum_{i=1}^{k} \frac{\left(\Lambda(t_i; \hat{\boldsymbol{\theta}}) - \Lambda(t_{i-1}; \hat{\boldsymbol{\theta}})\right)(x_i - x_{i-1})}{t_i - t_{i-1}} \tag{53}$$

After detecting the field degradation data $\boldsymbol{x}_{1:k}$, the degradation process when $t > t_k$ can be written as follow:

$$\begin{aligned} X(t|\boldsymbol{x}_{1:k}) &= x_k + (\lambda|\boldsymbol{x}_{1:k})\left(\Lambda(t; \hat{\boldsymbol{\theta}}) - \Lambda(t_k; \hat{\boldsymbol{\theta}})\right) + \sigma_B B(t - t_k) \\ &= x_k + (\lambda|\boldsymbol{x}_{1:k})\left(\Lambda(l_k + t_k; \hat{\boldsymbol{\theta}}) - \Lambda(t_k; \hat{\boldsymbol{\theta}})\right) + \sigma_B B(l_k) \end{aligned} \tag{54}$$

Then, the RUL at time t_k can be defined as follow:

$$L_k = \inf\{l_k : X(l_k + t_k) \geq w|\boldsymbol{x}_{1:k}\} \tag{55}$$

Therefore, the PDF of the RUL can be written as follows [10]:

$$\begin{aligned} & f_{L_k|\boldsymbol{x}_{1:k},w}(l_k|\boldsymbol{x}_{1:k}, w) \\ & \approx \frac{1}{\sqrt{2\pi l_k^2 \left(\sigma_{\lambda,k}^2 \varphi(l_k)^2 + \hat{\sigma}_B^2 l_k\right)}} \\ & \quad \times \exp\left[-\frac{\left(w - \mu_{\lambda,k}\varphi(l_k)\right)^2}{2\left(\hat{\sigma}_\lambda^2 \varphi(l_k)^2 + \hat{\sigma}_B^2 l_k\right)}\right] \\ & \quad \times \left[w - x_k - \mu_{\lambda,k}\beta(l_k) - \frac{w - x_k - \mu_{\lambda,k}\varphi(l_k)}{\sigma_{\lambda,k}^2 \varphi(l_k)^2 + \hat{\sigma}_B^2 l_k} \right. \\ & \quad \left. \times\ \sigma_{\lambda,k}^2 \beta(l_k)\varphi(l_k)\right] \end{aligned} \tag{56}$$

where:

$$\beta(l_k) = \Lambda(l_k + t_k; \hat{\boldsymbol{\theta}}) - \Lambda(t_k; \hat{\boldsymbol{\theta}}) - \Lambda'(l_k + t_k; \hat{\boldsymbol{\theta}}) l_k \tag{57}$$

$$\varphi(l_k) = \Lambda(l_k + t_k; \hat{\boldsymbol{\theta}}) - \Lambda(t_k; \hat{\boldsymbol{\theta}}) \tag{58}$$

IV. EXPERIMENT

In this section, the experiments are carried out to illustrate the usefulness and superiority of the presented method in this paper. Firstly, we use Monte Carlo algorithm to simulate lasers' degradation data to demonstrate the validity of the Remark 1 and Remark 2 in section II. Then, two practical case studies are provided to illustrate the effectiveness and superiority of the RUL prediction method proposed in section III.

A. SIMULATION EXPERIMENTS

In this subsection, we use the prior parameters of the lasers, i.e. $\boldsymbol{\Theta}=\{\mu_\lambda = 2.04 \times 10^{-3}, \sigma_\lambda^2 = 4.33 \times 10^{-4}, \sigma_B^2 = 1.08 \times 10^{-2}\}$, to simulate lasers' degradation data. More details about the parameters estimation process can be found in [10].

1) EXPERIMENT FOR REMARK 1

In this experiment, $\lambda = 2.04 \times 10^{-3}, \sigma_B^2 = 1.08 \times 10^{-2}$ are used to simulate the degradation data of one laser as show in figure 2. Without losing generality, let $t_k = 2000$, $\Delta t_k = 1$.

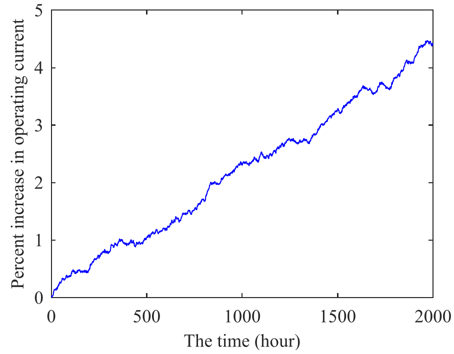

FIGURE 2. The simulation degradation data of one laser.

Then, the MLE for prior parameters can be obtained as show in Fig. 3, where the estimated parameters are updated in real-time with the change of t_k. Fig. 3(a) indicates that

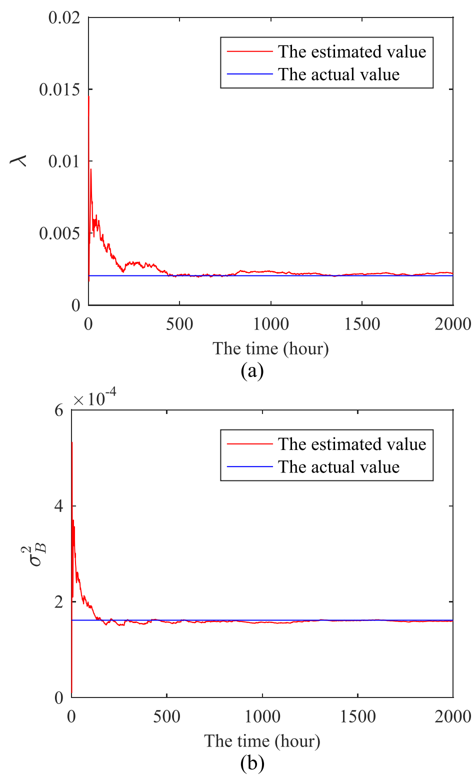

FIGURE 3. Parameters estimation of one laser: (a) λ and (b) σ_B^2.

the longer for the detection time t_k, the more accurate to the estimation of λ. And it shows from Fig. 3(b) that the more degraded data $\boldsymbol{x}_{0:k}$ are detected, the more accurate to the estimation of σ_B^2.

2) EXPERIMENTS FOR REMARK 2

In this experiment, we set $\mu_\lambda = 2.04 \times 10^{-3}$, $\sigma_\lambda^2 = 4.33 \times 10^{-4}$ and $\sigma_B^2 = 1.08 \times 10^{-2}$ to simulate the degradation data for N lasers with random effects, as show in Fig. 4. Without losing generality, here we set $N = 10$, $t_k = 2000$, $\Delta t_k = 100$.

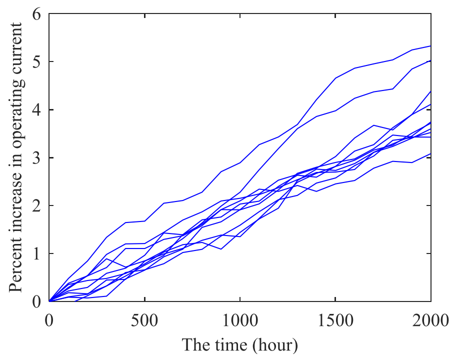

FIGURE 4. The simulation degradation data of 10 lasers.

Then, the MLE for μ_λ and σ_λ^2 can be obtained as show in Fig. 5, where the estimated results are real-time updated with the change of t_k. From Fig. 5, it can be observed that when the number of samples is constant, the longer t_k, the more accurate to the estimation of μ_λ and σ_λ^2.

In addition, let $N = 5, 10, 20, 40, 80$, $t_k = 300$, $\Delta t_k = 1$. The degradation laser data of different N samples are simulated respectively. Then, the MLE for μ_λ and σ_λ^2 of different $\boldsymbol{N}$ samples are obtained as show in Fig. 6. It can be observed that for the same detection time t_k, the more samples N, the more accurate to the estimation of μ_λ and σ_λ^2.

B. THE PRACTICAL CASE STUDY

In this subsection, two practical case studies are used to demonstrate the effectiveness and superiority of the proposed RUL prediction methods based on linear Wiener process and nonlinear Wiener process, respectively. For simplicity, the method proposed in this paper is referred to as M1, the method based on traditional Bayesian method is referred to as M2. The traditional Bayesian method for the nonlinear method can be referred to [10], [26], where the linear degradation is a special case. In order to better illustrate the estimated accuracy, the relative error (RE) and the mean square error (MSE) are defined as follows:

$$RE = RUL_{es} - RUL_{real} \tag{59}$$

$$MSE = \int_0^\infty (RUL_{es} - RUL_{real})^2 f_{RUL_{es}|\boldsymbol{x}_{1:k}} \times (RUL_{es}|\boldsymbol{x}_{1:k}) dRUL_{es} \tag{60}$$

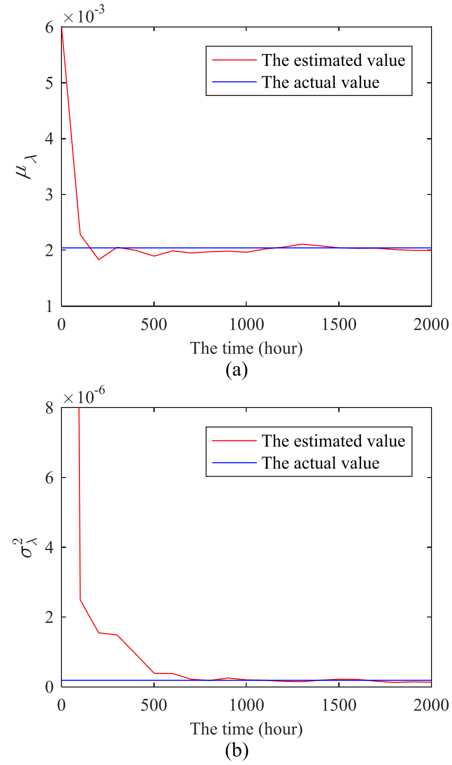

FIGURE 5. The prior distribution estimation of the random coefficient λ: (a) μ_λ and (b) σ_λ^2.

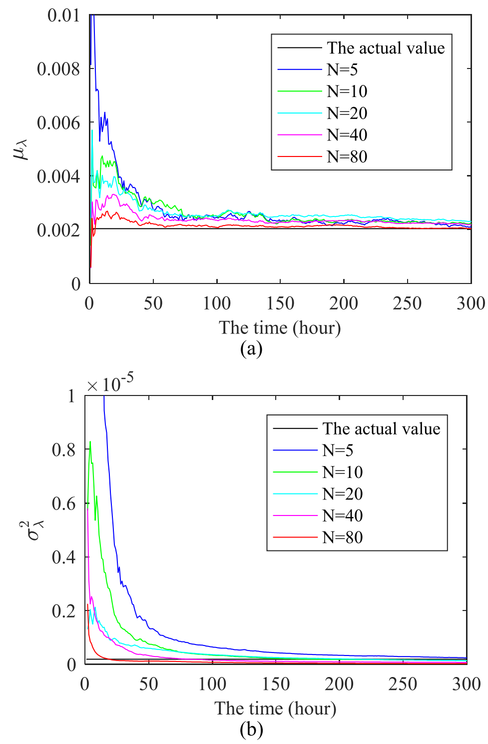

FIGURE 6. The prior distribution estimation of the random coefficient λ with different samples (N = 5, 10, 20, 40, 80): (a) μ_λ and (b) σ_λ^2.

1) RUL PREDICTION BASED ON LINEAR WIENER PROCESS UNDER ACCURATE PRIOR INFORMATION

For the RUL prediction based on linear Wiener process, the degradation data of lasers published by Meeker and

Escobar [49] are used to demonstrate the effectiveness and superiority of the proposed method, as show in Fig. 7.

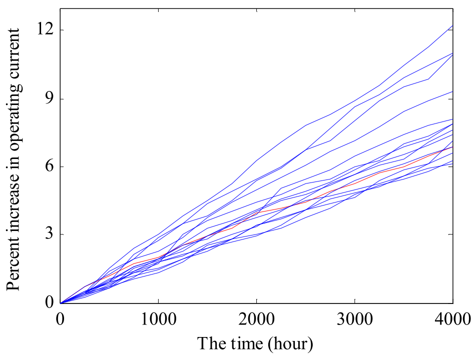

FIGURE 7. The degradation data of all lasers.

It can be observed from Fig. 7 that the linearity is obvious and the uncertainty is relatively small for all lasers. Therefore, it can be considered as the linear degradation process under accurate prior information. As most laser data do not cross the failure threshold which is usually set 10, here we set 6 as the failure threshold for an illustrative purpose. And, the laser data that plotted with red color is chosen as an accessed item with a final detected degradation data of 6.88 for a comparative study, and the rest lasers that plotted with blue color are used to estimate the prior parameters. The prior degradation parameters estimated by the MLE method [48] based on the traditional method are $\mu_\lambda = 2.04 \times 10^{-3}$, $\sigma_\lambda^2 = 1.88 \times 10^{-7}$ and $\sigma_B^2 = 1.17 \times 10^{-4}$. Unlike the traditional Bayesian method, the degradation data of congeneric items are used to estimate the failure time of each item, which are used as the prior failure time data information for the method presented in this paper. The specific failure time data are 2193.8, 2586.2, 3908.8, 3162.1, 2179.8, 3347.3, 3846.2, 3045.7 1965.6, 3234.5, 3045.7, 2966.6, 3488.4 and 3625.6, respectively.

In the following, we plot the RUL distributions of the two methods and the actual RUL at some different time points, as shown in Fig. 8. It can be seen that the RUL distribution calculated by the two methods can both cover the actual RUL.

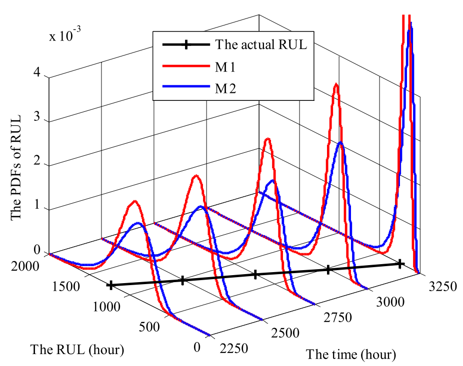

FIGURE 8. The PDF of RUL predicted by M1 and M2 under accurate prior information.

However, the RUL distribution of the proposed method is more concentrated on the actual RUL and more narrow than the traditional Bayes method, which indicates that our method has higher accuracy.

In addition, we further calculate the REs and MSEs at all different time points, as shown in Fig. 9 and Fig. 10, respectively. From Fig. 9, it can be observed that the REs of M1 at all different time are near but lower than M2. From Fig. 10, it shows that the MSEs of M1 at all different times are lower than M2.

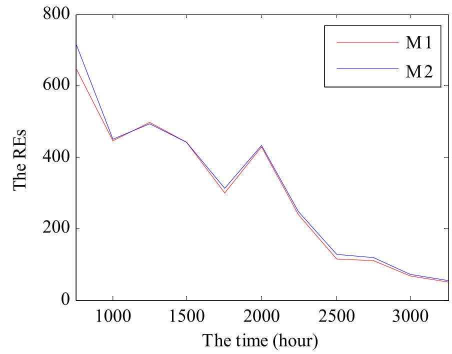

FIGURE 9. The RE by M1 and M2 under accurate prior information.

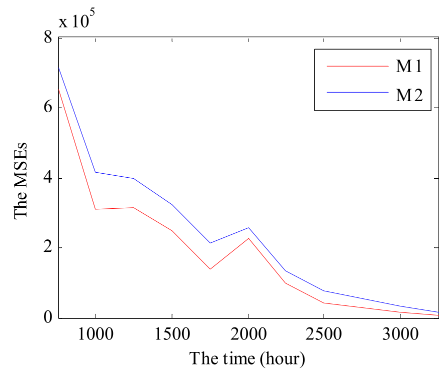

FIGURE 10. The MSE by M1 and M2 under accurate prior information.

To further illustrate the differences between these two methods, we plot the estimation of the modeling parameters that change over time, as shown in Fig. 11. From Fig. 11, we observe that the estimation of μ_λ and σ_λ^2 are consistent with that based on the Bayesian method, which demonstrate the validity of our method regarding fusing the failure time data into the prior information of the drift parameter. Additionally, as the estimation of σ_B^2 is relatively small compared with the value based on the Bayesian method, the estimation PDF of RUL of our method is sharper and more concentrated on the mean of the estimated RUL. The reason is that the estimation of σ_B^2 by our method is determined by the field degradation data. However, the estimation of σ_B^2 by the Bayesian method is determined by congeneric items, which has no concern with the field data. This is a flaw of the traditional Bayesian method, as the estimation accuracy of σ_B^2

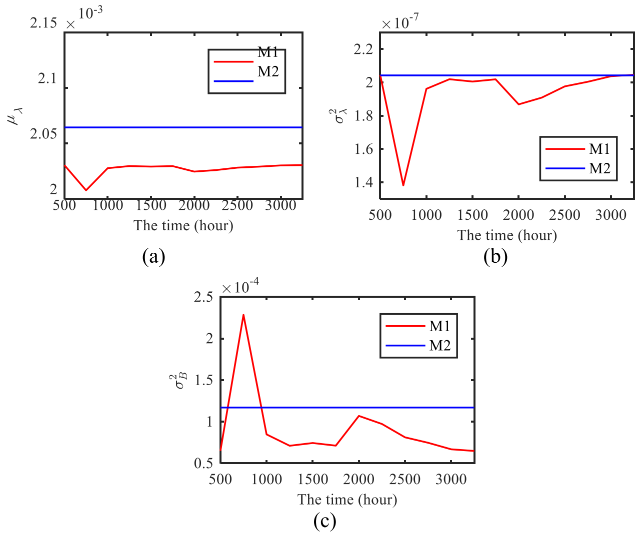

FIGURE 11. The prior parameters estimation by M1 and M2 at different time points: (a) μ_λ, (b) σ_λ^2 and (c) σ_B^2.

is mainly determined by the detection time. This flaw doesn't cause too much error for the degradation data with accurate prior information. However, this estimation error increases when the prior information is imperfect, as illustrated in the following.

2) RUL PREDICTION BASED ON LINEAR WIENER PROCESS UNDER IMPERFECT PRIOR INFORMATION

Without losing of generality, in this experiment we use the Monte Carlo method to simulated degradation data under imperfect prior information by increasing the value of σ_B^2, as show in Fig. 12. Here, we set $\sigma_B^2 = 3.6 \times 10^{-3}$.

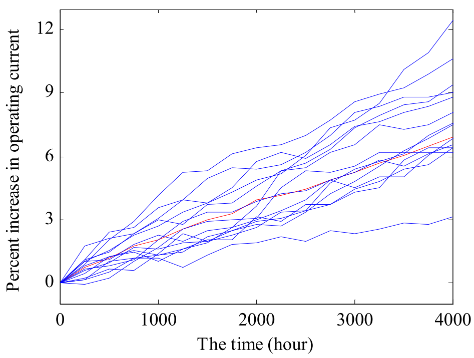

FIGURE 12. The simulated laser data under imperfect prior information.

Here, the laser data that plotted with red color is chosen to compare these two methods, and the other lasers that plotted with blue color are used to estimate the prior parameters. The failure time data for the proposed method are 3515.3, 2729.4, 1930.0, 2654.1, 2966.0, 3171.0, 2565.0, 3204.2, 3669.2, 2253.7, 3741.9, 3737.2, 7676.8 and 3861.2, respectively. Then, we calculate the estimated RUL, and plot the PDF of RUL at some different time points in Fig. 13. It can

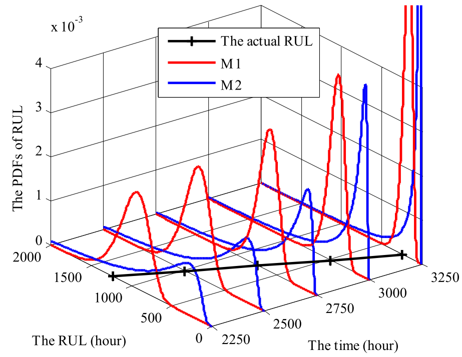

FIGURE 13. The PDF of RUL predicted by M1 and M2 under imperfect prior information.

be observed that the mode and expectation of M1 is closer to the actual RUL than M2, and the distribution interval by M1 is smaller than M2. This results indicates the superiority of the proposed method.

Then, we calculate the REs and MSEs at all different time points, as shown in Fig. 14 and Fig. 15, respectively. It can be obviously seen that the proposed method has a higher accuracy than traditional Bayesian method for RUL prediction under the imperfect prior information.

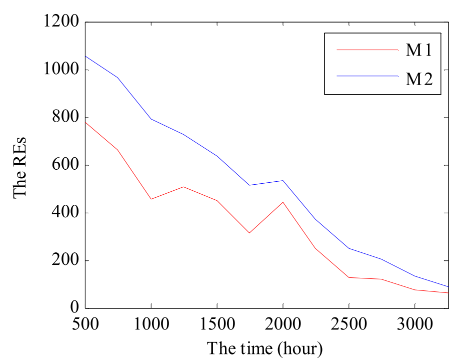

FIGURE 14. The RE by M1 and M2 under imperfect prior information.

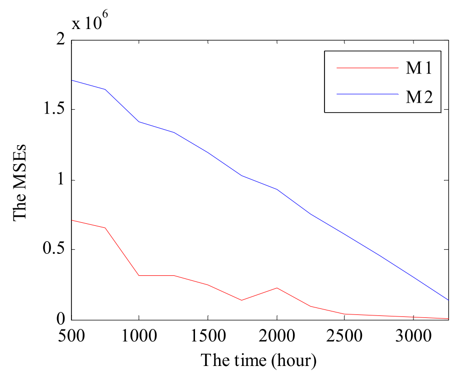

FIGURE 15. The MSE by M1 and M2 under imperfect prior information.

We further plot the estimation of the modeling parameters that change over time, as shown in Fig. 16. We observe that

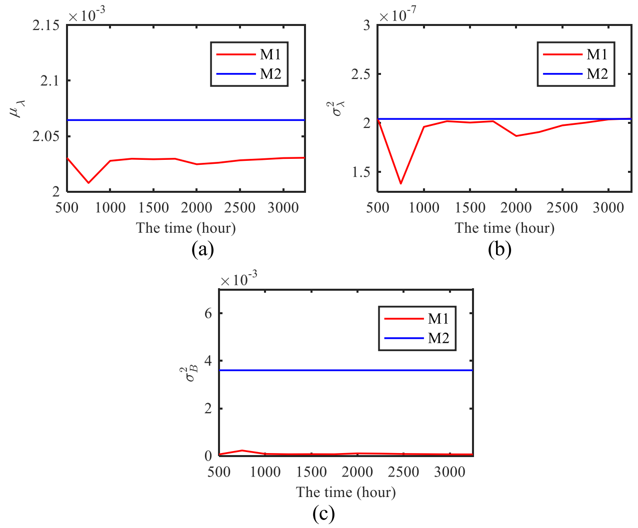

FIGURE 16. The prior parameters estimation by M1 and M2 at different time points: (a) μ_λ, (b) σ_λ^2 and (c) σ_B^2.

the estimation of μ_λ and σ_λ^2 are consistent with that based on the Bayesian method, which is consistent with the results under the accurate prior information. However, the estimation of σ_B^2 by the Bayesian method is over estimated, which could result in big error for RUL prediction. This over estimation could cause the estimation of RUL distribution to be closer to zero and wider. For example, at the time of 2250 hours, the confidence interval of the RUL distribution is about 2000 hours based on the Bayesian method, which is greater than 1500 hours of our proposed method. Additionally, under the concept of first hitting time, the RUL distribution of the Bayesian method is closer to zero, which is to be right in Fig. 13. This estimation error could lead to premature maintenance, thus reducing the utilization rate of equipment and increasing the total cost of product operation.

3) RUL PREDICTION BASED ON NONLINEAR WIENER PROCESS

In this subsection, we apply the degradation data published by NASA to illustrate the RUL prediction based on nonlinear Wiener process, as show in Fig. 17. The degradation data shown in Fig. 17 is affected by the relaxation effect, which could be occurred during some long rest time, which could lead to capacity regenerated phenomenon and recovery process [50]. Therefore, in this experiment we apply the degradation data after eliminating the relaxation effect for an illustrate purpose. For more details about how to eliminate the relaxation effect, see [39], [50] and references therein. The degradation data with eliminating the relaxation effect are shown in Fig. 18.

We set 70% capacity as the failure threshold of lithium-ion batteries, and let $\Lambda(t;\boldsymbol{\theta}) = t^b$. B0005 battery is chosen to compare the two methods and the other batteries are used to estimate the prior parameters. Firstly, we calculate the prior parameters based on the degradation data of B0006, B0007,

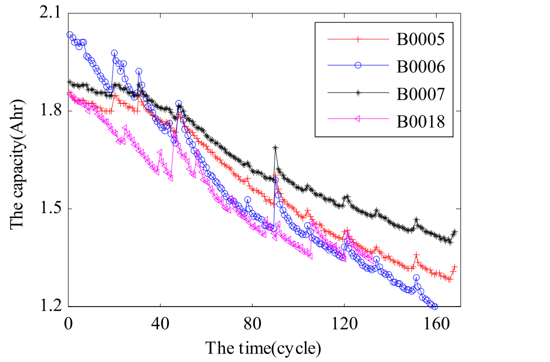

FIGURE 17. The degradation data of lithium-ion batteries based on cycle time.

and B00018 are as follows: $\mu_\lambda = 8.31 \times 10^{-3}$, $\sigma_\lambda^2 = 8.82 \times 10^{-6}$, $\sigma_B^2 = 2.04 \times 10^{-4}$ and $b = 0.977$. The failure times of B0006, B0007, and B00018 batteries are 69.5, 110.3 and 51 respectively, which can be used as the prior failure time data of the proposed method.

In the following, we calculate the RUL distributions of M1 and M2, as shown in Fig. 18. It can be seen that the RUL distribution calculated by these two methods can cover the actual RUL. Nevertheless, the RUL distribution of the proposed method is more concentrated on the actual RUL and more narrow than traditional Bayesian method, which shows that our proposed prediction method has higher accuracy.

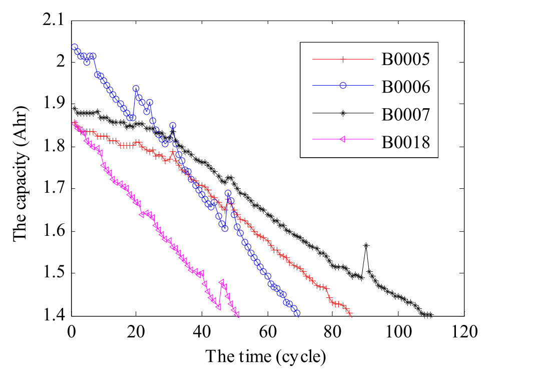

FIGURE 18. The degradation data with eliminating the relaxation effect.

We further calculate the REs and MSEs at some different time points, as shown in Fig. 19 and Fig. 20, respectively. The results show that the REs and MSEs of M1 at all the time points are much lower than M2. This indicates the superiority of our proposed method.

To further illustrate the differences between these two methods, we plot the estimation of the modeling parameters that change over time, as shown in Fig. 21. From Fig. 21, we can observe that the estimation of σ_B^2 by M2 is much bigger than M1. As discussed above, this over estimation could cause the estimation of RUL distribution to be closer to zero and wider, which could lead to premature maintenance. The reason why M1 and M2 could obtain different results is

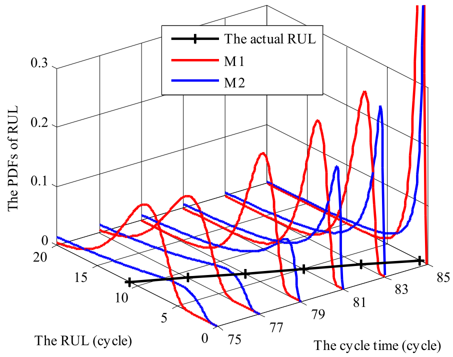

FIGURE 19. The PDF of RUL prediction by M1 and M2 based on the degradation data of lithium-ion batteries

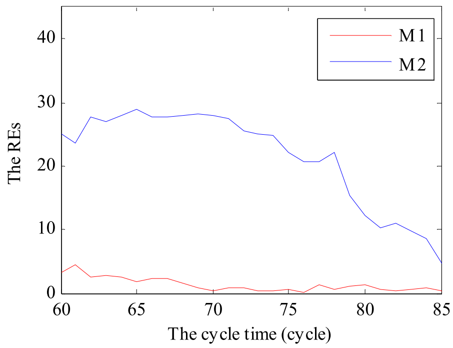

FIGURE 20. The RE by M1 and M2 with accurate prior information.

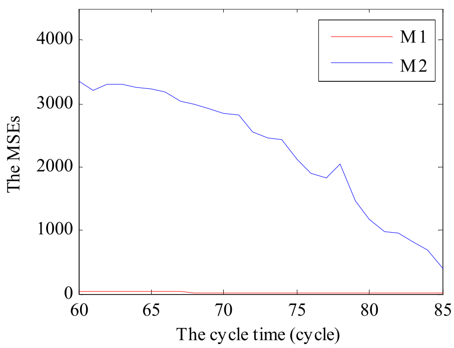

FIGURE 21. The MSE by M1 and M2 with accurate prior information.

due to imperfect prior information, which can be explained as follows.

As can be seen from Fig. 22, the degradation data of N0006 and N0018 batteries have obvious linear degradation characteristics, which is the reason why the estimated value of the nonlinear coefficient b is 0.977. This estimation is much smaller than the estimated $\hat{b}$ of N0005, which is 1.436. In addition, the diffusion coefficient σ_B^2 of N0005 itself is smaller. These two reasons lead to the overestimate of σ_B^2 based on the Bayesian method.

From the above works of the RUL prediction based on linear Wiener process and nonlinear Wiener process, we can observe that when the prior information is accurate, M1 and M2 obtain similar results. However, under the imperfect prior

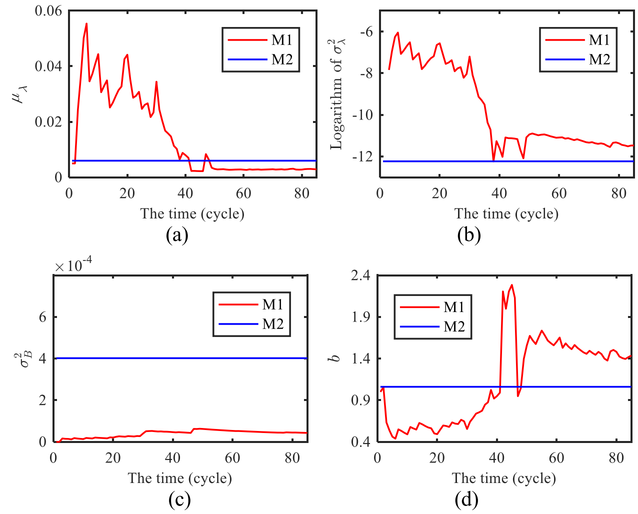

FIGURE 22. The prior estimation by M1 and M2 at different time points: (a) μ_λ, (b) log (σ_λ^2) (c) σ_B^2, (d) b.

information, M1 obtains better results than M2. The reason for this phenomenon can be summarized as follows. Under the imperfect prior information, the estimation of the diffusion coefficient σ_B^2 could be overestimated. However, the estimation accuracy of σ_B^2 is determined by the detection times. And thus, for the assessed item with many detection times, using the imperfect prior information could result in prediction error. In this case, using the field degradation information could ensure the prediction accuracy. Additionally, fusing the failure time data could obtain the similar estimation of μ_λ and σ_λ^2 with the traditional Bayesian method. This implies the effectiveness regarding the fusing method on the failure time data. For the reason why M2 obtains a lower prediction accuracy can be explained as follows. Under the Bayesian framework, the estimation of σ_B^2 is not updated with the field degradation data. This leads to prediction error, especially for the case under the imperfect prior information.

V. CONCLUSION

RUL prediction is great important in PHM. In this paper, a novel online RUL prediction method based on Wiener process is proposed by fusing the failure time data and field degradation information. Experiments are carried out to illustrate the usefulness and superiority of the presented method, and the results show better accuracy comparing with the traditional Bayesian method. From above works, the main contributions can be summarized as follows:

(1) Based on the basic linear Wiener process, this paper studies the relationship between the parameters estimation and the feature of degradation data, i.e. item sample numbers, detection time and detect frequency. These natures of parameters estimation are interesting, and give the basis regarding how to reasonably fuse the failure time data and field degradation data.

(2) Based on the proposed natures of parameters estimation, we propose a novel two-step method by fusing the failure time data and field degradation data with considering the random effects for the linear Wiener process. The EM algorithm is used to estimate the mean and variance drift parameter. We also generalize this mechanism to the nonlinear Wiener process. The proposed method could overcome the effect of imperfect prior information.

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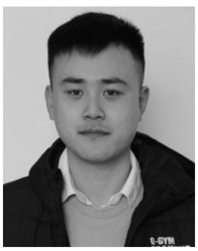

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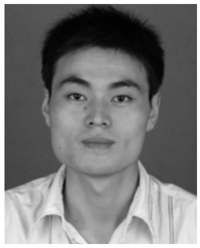

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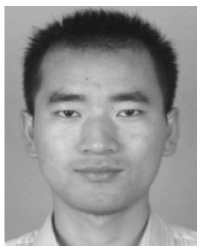

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