Comments on the Bell-Clauser-Horne-Shimony-Holt inequality

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Abstract

We discuss the relationship between the Bogoliubov transformations, squeezed states, entanglement and maximum violation of the Bell-CHSH inequality. In particular, we point out that the construction of the four bounded operators entering the Bell-CHSH inequality can be worked out in a simple and general way, covering a large variety of models, ranging from Quantum Mechanism to relativistic Quantum Field Theories. Various examples are employed to illustrate the above mentioned framework. We start by considering a pair of entangled spin 1 particles and a squeezed oscillator in Quantum Mechanics, moving then to the relativistic complex quantum scalar field and to the analysis of the vacuum state in Minkowski space-time in terms of the left and right Rindler modes. In the latter case, the Bell-CHSH inequality turns out to be parametrized by the Unruh temperature.

1 Introduction

The aim of this work is that of pointing out that the construction of the four Hermitian operators A_i , B_i , i = 1, 2

$$A_i^2 = B_i^2 = 1 , \qquad [A_i, B_k] = 0 . \tag{1}$$

characterizing the quantum violation of the Bell-Clauser-Horne-Shimony-Holt inequality [1, 2, 3], i.e.

$$|\langle \psi | \mathcal{C}_{CHSH} | \psi \rangle| = |\langle \psi | (A_1 + A_2) B_1 + (A_1 - A_2) B_2 | \psi \rangle| > 2 , \qquad (2)$$

can be achieved in a rather simple and elegant way, which turns out to have general applicability, covering models ranging from Quantum Mechanics to more sophisticated examples such as: relativistic Quantum Field Theories.

In order to illustrate how the setup works, we proceed first by discussing two examples from Quantum Mechanics.

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2 Entangled pair of spin 1 particles

Let us start by considering a pair of entangled spin 1 particles. As entangled state $|\psi_s\rangle$, we take the singlet state:

$$|\psi_s\rangle = \left(\frac{|1\rangle_A| - 1\rangle_B - |0\rangle_A|0\rangle_B + |-1\rangle_A|1\rangle_B}{\sqrt{3}}\right).$$
(3)

It is easily checked that expression (3) can be thought as the vacuum state of the spin Hamiltonian

$$H = \vec{S}_A \cdot \vec{S}_B = \frac{1}{2} \left(\vec{S}_A + \vec{S}_B \right)^2 - 2 .$$
(4)

with $\left(\vec{S}_A, \vec{S}_B\right)$ denoting the spin 1 matrices. For the operators (A_i, B_i) we write

$$\begin{aligned}
A_i|-1\rangle_A &= e^{i\alpha_i}|0\rangle_A, & A_i|0\rangle_A = e^{-i\alpha_i}|-1\rangle_A, & A_i|1\rangle_A = |1\rangle_A, \\
B_i|1\rangle_B &= e^{i\beta_i}|0\rangle_B, & B_i|0\rangle_B = e^{-i\beta_i}|1\rangle_B, & B_i|-1\rangle_B = |-1\rangle_B,
\end{aligned}$$
(5)

where (α_i, β_i) are arbitrary real coefficients. The Hermitian operators $A_i, B_i, i = 1, 2$ fulfill the requirement (1).

For the Bell-CHSH correlator we obtain

$$\langle \psi_s | \mathcal{C}_{CHSH} | \psi_s \rangle = \frac{2}{3} \left(1 - \cos(\alpha_1 + \beta_1) - \cos(\alpha_2 + \beta_1) - \cos(\alpha_1 + \beta_2) + \cos(\alpha_2 + \beta_2) \right) . \tag{6}$$

Choosing, for example, $\alpha_2 = \beta_2 = 0$, $\alpha_1 = \frac{\pi}{2}$, $\beta_1 = \frac{3\pi}{4}$, one gets the violation

$$|\langle \Omega | \mathcal{C}_{CHSH} | \Omega \rangle| = \frac{2(2+\sqrt{2})}{3} \approx 2.27 , \qquad (7)$$

which compares well with the value reported by [4].

The same reasoning applies to the Bell spin 1/2 singlet state

$$|\psi_s\rangle = \left(\frac{|+\rangle_A|-\rangle_B - |-\rangle_A|+\rangle_B}{\sqrt{2}}\right) \,. \tag{8}$$

For the operators (A_i, B_i) we have now

$$A_{i}|+\rangle_{A} = e^{i\alpha_{i}}|-\rangle_{A}, \qquad A_{i}|-\rangle_{A} = e^{-i\alpha_{i}}|+\rangle_{A}, ,$$

$$B_{i}|+\rangle_{B} = e^{i\beta_{i}}|-\rangle_{B}, \qquad B_{i}|-\rangle_{B} = e^{-i\beta_{i}}|+\rangle_{B}.$$
(9)

Setting $\alpha_1 = 0$, $\alpha_2 = \frac{\pi}{2}$, $\beta_1 = \frac{\pi}{4}$, $\beta_2 = -\frac{\pi}{4}$, one recovers Tsirelson's bound [3], namely

$$|\langle \psi_s | \mathcal{C}_{CHSH} | \psi_s \rangle| = 2\sqrt{2} . \tag{10}$$

3 Two degrees of freedom squeezed oscillator

In this second example we consider a two degrees of freedom squeezed oscillator. Let us begin by introducing two annihilation operators (a, b) satisfying

$$\begin{bmatrix} a, a^{\dagger} \end{bmatrix} = 1, \qquad \begin{bmatrix} a^{\dagger}, a^{\dagger} \end{bmatrix} = 0, \qquad \begin{bmatrix} a, a \end{bmatrix} = 0, \begin{bmatrix} b, b^{\dagger} \end{bmatrix} = 1, \qquad \begin{bmatrix} b^{\dagger}, b^{\dagger} \end{bmatrix} = 0, \qquad \begin{bmatrix} b, b \end{bmatrix} = 0, \begin{bmatrix} a, b \end{bmatrix} = 0, \qquad \begin{bmatrix} a, b^{\dagger} \end{bmatrix} = 0.$$
 (11)

The operators (a, b) annihilate the state $|0\rangle$, which is not the vacuum state of the system, *i.e.*

$$a|0\rangle = b|0\rangle = 0.$$
⁽¹²⁾

Let us introduce the new operators (α, β) , obtained through the Bogoliubov transformations

$$\alpha = \frac{(a - \eta b^{\dagger})}{\sqrt{1 - \eta^2}}, \qquad \beta = \frac{(b - \eta a^{\dagger})}{\sqrt{1 - \eta^2}}, \qquad (13)$$

where η is a real parameter, $0 < \eta < 1$.

It is easily checked that the operators (α, β) fulfill the same commutation relations of eq.(11), namely

$$\begin{bmatrix} \alpha, \alpha^{\dagger} \end{bmatrix} = 1, \qquad [\alpha^{\dagger}, \alpha^{\dagger}] = 0, \qquad [\alpha, \alpha] = 0, \begin{bmatrix} \beta, \beta^{\dagger} \end{bmatrix} = 1, \qquad [\beta^{\dagger}, \beta^{\dagger}] = 0, \qquad [\beta, \beta] = 0, \begin{bmatrix} \alpha, \beta \end{bmatrix} = 0, \qquad [\alpha, \beta^{\dagger}] = 0.$$
 (14)

We consider now the normalized squeezed state

$$|\eta\rangle = \sqrt{(1-\eta^2)} e^{\eta a^{\dagger} b^{\dagger}} |0\rangle , \qquad \langle \eta |\eta\rangle = 1 .$$
(15)

Let us show that the state $|\eta\rangle$ is annihilated by the Bogoliubov operators (α, β) :

$$\alpha |\eta\rangle = \beta |\eta\rangle = 0.$$
 (16)

In fact, we have

$$(a - \eta b^{\dagger}) e^{\eta a^{\dagger} b^{\dagger}} |0\rangle = \left[a, e^{\eta a^{\dagger} b^{\dagger}} \right] |0\rangle - \eta b^{\dagger} e^{\eta a^{\dagger} b^{\dagger}} |0\rangle$$

$$= \sum_{n=1}^{\infty} \frac{\eta^{n}}{n!} (b^{\dagger})^{n} \left[a, (a^{\dagger})^{n} \right] |0\rangle - \eta b^{\dagger} e^{\eta a^{\dagger} b^{\dagger}} |0\rangle$$

$$= \eta b^{\dagger} \sum_{n=1}^{\infty} \frac{\eta^{n-1}}{(n-1)!} (b^{\dagger})^{(n-1)} (a^{\dagger})^{(n-1)} |0\rangle - \eta b^{\dagger} e^{\eta a^{\dagger} b^{\dagger}} |0\rangle = 0.$$

$$(17)$$

Similarly for the operator β . As a consequence, the squeezed state $|\eta\rangle$ turns out to be the vacuum state of the Hamiltonian

$$H = \alpha^{\dagger} \alpha + \beta^{\dagger} \beta = \frac{(1+\eta^2)}{(1-\eta^2)} \left(a^{\dagger} a + b^{\dagger} b \right) - \frac{2\eta}{(1-\eta^2)} \left(a^{\dagger} b^{\dagger} + a b \right) + 2\eta^2.$$

$$H|\eta\rangle = 0.$$
(18)

This expression is nothing but the Hamiltonian worked out in [5] in order to establish the violation of the Bell-CHSH inequality in relativistic free Quantum Field Theory. It becomes apparent now that the role of the parameter η , $0 < \eta < 1$, is that of a coupling constant.

3.1 Maximum violation of the Bell-CHSH inequality

We are ready now to discuss the violation of the Bell-CHSH exhibited by this model. To introduce the four Hermitian operators A_i , B_i , i = 1, 2

$$A_i^2 = B_i^2 = 1 , \qquad [A_i, B_k] = 0 , \qquad (19)$$

we proceed in exactly the same way as done in the case of the spin 1 example. We write the squeezed state $|\eta\rangle$ as

$$\begin{aligned} |\eta\rangle &= \sqrt{(1-\eta^2)} \sum_{n=0}^{\infty} \eta^n \frac{(a^{\dagger}b^{\dagger})^n}{n!} |0\rangle \\ &= \sqrt{(1-\eta^2)} \sum_{n=0}^{\infty} \left(\eta^{(2n)} |(2n_a)(2n_b)\rangle + \eta^{(2n+1)} |(2n_a+1)(2n_b+1)\rangle \right) , \end{aligned}$$
(20)

where $|m_a m_b\rangle$ stands for the normalized state

$$|m_a m_b\rangle = \frac{(a^{\dagger})^m (b^{\dagger})^m}{m!} |0\rangle .$$
⁽²¹⁾

For the operators A_i , B_i , i = 1, 2, we have

$$\begin{aligned}
A_i | (2n_a) \ m_b \rangle &= e^{i\alpha_i} | (2n_a + 1) \ m_b \rangle, & A_i | (2n_a + 1) \ m_b \rangle = e^{-i\alpha_i} | (2n_a) \ m_b \rangle, \\
B_i | m_a \ (2n_b) \rangle &= e^{i\beta_i} | m_a \ (2n_b + 1) \rangle, & B_i | m_a \ (2n_b + 1) \rangle = e^{-i\beta_i} | m_a \ (2n_b) \rangle.
\end{aligned}$$
(22)

The operator A_i acts only on the first entry, while B_i only on the second one.

A quick calculation gives

$$\langle \eta | A_k B_i | \eta \rangle = \frac{2\eta}{1+\eta^2} \cos(\alpha_k + \beta_i) .$$
⁽²³⁾

Therefore, for the Bell-CHSH correlator, one finds

$$\langle \eta | \mathcal{C}_{CHSH} | \eta \rangle = \langle \eta | (A_1 + A_2) B_1 + (A_1 - A_2) B_2 | \eta \rangle = \frac{2\eta}{1 + \eta^2} \left(\cos(\alpha_1 + \beta_1) + \cos(\alpha_2 + \beta_1) + \cos(\alpha_1 + \beta_2) - \cos(\alpha_2 + \beta_2) \right) .$$
 (24)

Setting [5]

$$\alpha_1 = 0, \qquad \beta_1 = -\frac{\pi}{4}, \qquad \alpha_2 = \frac{\pi}{2}, \qquad \beta_2 = \frac{\pi}{4},$$
(25)

expression (24) becomes

$$\langle \eta | \mathcal{C}_{CHSH} | \eta \rangle = 2 \, \frac{2\sqrt{2}\eta}{1+\eta^2} \,, \tag{26}$$

implying in a violation of the Bell-CHSH inequality whenever

$$\sqrt{2} - 1 < \eta < 1 . \tag{27}$$

In particular, the maximum violation is attained for values of $\eta \approx 1$, namely

$$\langle \eta | \mathcal{C}_{CHSH} | \eta \rangle \approx 2\sqrt{2} .$$
 (28)

4 The relativistic complex scalar quantum Klein-Gordon field

Let us move now to Quantum Field Theory, by considering the example of a free complex massive scalar Klein-Gordon field, *i.e.*

$$\mathcal{L} = \left(\partial^{\mu}\varphi^{\dagger}\partial_{\mu}\varphi - m^{2}\varphi^{\dagger}\varphi\right) \,. \tag{29}$$

Expanding φ in terms of annihiliation and creation operators, one gets

$$\varphi(t,\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{2\omega_k} \left(e^{-ikx} a_k + e^{ikx} b_k^{\dagger} \right) , \qquad k^0 = \omega_k = \sqrt{\vec{k}^2 + m^2} , \qquad (30)$$

where

$$[a_k, a_q^{\dagger}] = [b_k, b_q^{\dagger}] = (2\pi)^3 2\omega_k \delta^3(\vec{k} - \vec{q}) , \qquad (31)$$

are the non vanishing canonical commutation relations. Expression (30) is a too singular object, being in fact an operator valued distribution [6]. In order to give a well defined meaning to eq.(30), one introduces the smeared operators

$$a_f = \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{1}{2\omega_k} \hat{f}(\omega_k, \vec{k}) a_k , \qquad b_g = \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{1}{2\omega_k} \hat{g}(\omega_k, \vec{k}) b_k , \qquad (32)$$

with

$$\hat{f}(p) = \int d^4x \ e^{-ipx} f(x) , \qquad \hat{g}(p) = \int d^4x \ e^{-ipx} g(x) .$$
(33)

where (f(x), g(x)) are test functions belonging to the space of compactly supported smooth functions $C_0^{\infty}(\mathbb{R}^4)$. The support of f(x), $supp_h$, is the region in which the test function f(x) is non-vanishing.

When rewritten in terms of the operators (a_f, b_g) , the canonical commutation relations (31) read

$$\left[a_{h}, a_{h'}^{\dagger}\right] = \left[b_{h}, b_{h'}^{\dagger}\right] = \langle h|h'\rangle , \qquad (34)$$

where $\langle h|h'\rangle$ denotes the Lorentz invariant scalar product between the test functions h and h'. *i.e.*

$$\langle h|h'\rangle = \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{2\omega_k} \hat{h}(\omega_k, \vec{k}) \hat{h}^{'*}(\omega_k, \vec{k}) = \int \frac{d^4k}{(2\pi)^4} 2\pi \ \theta(k^0) \delta(k^2 - m^2) \hat{h}(k) \hat{h}^{'*}(k) \ . \tag{35}$$

In particular, from eq.(34), it follows that

$$\left[a_f, a_f^{\dagger}\right] = \left[b_f, b_f^{\dagger}\right] = ||f||^2 , \qquad (36)$$

where $||f||^2$ is the norm of the test function f, *i.e.*

$$||f||^{2} = \int \frac{d^{4}k}{(2\pi)^{4}} 2\pi \ \theta(k^{0})\delta(k^{2} - m^{2})\hat{f}(k)\hat{f}^{*}(k) \ . \tag{37}$$

Let us remark that, by a simple rescaling, the test functions can be taken to be normalized to 1, namely

$$f(x) \to \frac{1}{||f||} f(x) , \qquad ||f|| = 1 .$$
 (38)

Therefore, expression (36) becomes

$$\left[a_f, a_f^{\dagger}\right] = \left[b_f, b_f^{\dagger}\right] = ||f||^2 = 1.$$
(39)

The vacuum state $|0\rangle$ of the theory is defined as

$$a_f|0\rangle = b_g|0\rangle = 0 \qquad \forall (f,g) \in \mathcal{C}_0^{\infty}(\mathbb{R}^4) .$$
 (40)

The scalar complex KG field φ , eq.(29), enables us to introduce a squeezed state very similar to that introduced in the previous section,

$$|\sigma\rangle = \sqrt{(1-\sigma^2)} e^{\sigma a_f^{\dagger} b_g^{\dagger}} |0\rangle , \qquad \langle \sigma |\sigma\rangle = 1 , \qquad 0 < \sigma < 1 .$$
(41)

We write thus

$$\begin{aligned} |\sigma\rangle &= \sqrt{(1-\sigma^2)} \sum_{n=0}^{\infty} \sigma^n \frac{(a_f^{\dagger} b_g^{\dagger})^n}{n!} |0\rangle \\ &= \sqrt{(1-\sigma^2)} \sum_{n=0}^{\infty} \left(\sigma^{(2n)} |(2n_f)(2n_g)\rangle + \sigma^{(2n+1)} |(2n_f+1)(2n_g+1)\rangle \right) , \end{aligned}$$
(42)

where $|m_f m_g \rangle$ stands for the normalized state

$$|m_f m_g\rangle = \frac{(a_f^{\dagger})^m (b_g^{\dagger})^m}{m!} |0\rangle .$$
(43)

Proceeding as in the previous example, the operators A_i , B_i , i = 1, 2, are given by

$$\begin{aligned} A_{i}|(2n_{f}) \ m_{g}\rangle &= e^{i\alpha_{i}}|(2n_{f}+1) \ m_{g}\rangle, \qquad A_{i}|(2n_{f}+1) \ m_{g}\rangle = e^{-i\alpha_{i}}|(2n_{f}) \ m_{g}\rangle, \\ B_{i}|m_{f} \ (2n_{g})\rangle &= e^{i\beta_{i}}|m_{f} \ (2n_{g}+1)\rangle, \qquad B_{i}|m_{f} \ (2n_{g}+1)\rangle = e^{-i\beta_{i}}|m_{f} \ (2n_{g})\rangle. \end{aligned}$$
(44)

Again, the operator A_i acts only on the first entry, while B_i only on the second one.

It turns out that

$$\langle \sigma | A_k B_i | \sigma \rangle = \frac{2\sigma}{1+\sigma^2} \cos(\alpha_k + \beta_i) , \qquad (45)$$

so that, for the Bell-CHSH correlator, one has precisely the expression obtained in the case of the squeezed oscillator, namely

$$\langle \sigma | \mathcal{C}_{CHSH} | \sigma \rangle = \langle \sigma | (A_1 + A_2) B_1 + (A_1 - A_2) B_2 | \sigma \rangle$$

= $\frac{2\sigma}{1 + \sigma^2} \left(\cos(\alpha_1 + \beta_1) + \cos(\alpha_2 + \beta_1) + \cos(\alpha_1 + \beta_2) - \cos(\alpha_2 + \beta_2) \right) .$ (46)

Therefore, using

$$\alpha_1 = 0, \qquad \beta_1 = -\frac{\pi}{4}, \qquad \alpha_2 = \frac{\pi}{2}, \qquad \beta_2 = \frac{\pi}{4},$$
(47)

expression (46) becomes

$$\langle \sigma | \mathcal{C}_{CHSH} | \sigma \rangle = 2 \, \frac{2\sqrt{2}\sigma}{1+\sigma^2} \,, \tag{48}$$

implying in a violation of the Bell-CHSH inequality for

$$\sqrt{2} - 1 < \sigma < 1. \tag{49}$$

The maximum violation is attained for values of $\sigma \approx 1$:

$$\langle \eta | \mathcal{C}_{CHSH} | \eta \rangle \approx 2\sqrt{2} .$$
 (50)

It is worth reminding here that the violation of the Bell-CHSH in free Quantum Field Theory has been established several decades ago by using Algebraic Quantum Field Theory techniques, see [5] and refs.therein. See also [7] for a more recent discussion.

5 Some remarks on the Minkowski vacuum and the Rindler wedges

Let us conculde with a few remarks on the origin of the violation of the Bell-CHSH inequality in relativistic Quantum Field Theory. We consider here the case of a real scalar KG field. It can be argued that the violation of the Bell-CHSH inequality can be understood in a simple way as a consequence of the entanglement properties of the vacuum state $|0\rangle$, which can be expressed as a squeezed state in terms of left and right Rindler modes [8, 9], *i.e.*

$$|0\rangle = \prod_{i} \left((1 - e^{-\frac{2\pi\omega_i}{a}})^{\frac{1}{2}} \sum_{n_i=0}^{\infty} e^{-\frac{\pi n_i \omega_i}{a}} |n_i\rangle_L |n_i\rangle_R \right) , \qquad (51)$$

where $(|n_i\rangle_L, |n_i\rangle_R)$ are the left and right Rindler modes and $T = \frac{a}{2\pi}$ is the Unruh temperature. The relation (51) follows from the use of a Bogoliubov transformation applied to the the quantization of the real scalar KG field in the Rindler wedges [8, 9]. Proceeding as in the previous cases, for the four operators $(A_k, B_k), k = 1, 2$, we have

$$A_{k}|2n_{i}\rangle_{L} = e^{i\alpha_{k}}|2n_{i}+1\rangle_{L}, \qquad A_{k}|2n_{i}+1\rangle_{L} = e^{-i\alpha_{k}}|2n_{i}\rangle_{L},$$

$$B_{k}|2n_{i}\rangle_{R} = e^{i\beta_{k}}|2n_{i}+1\rangle_{R}, \qquad B_{k}|2n_{i}+1\rangle_{R} = e^{-i\beta_{k}}|2n_{i}\rangle_{R}$$
(52)

$$\sum_{k=1}^{K} \sum_{i=1}^{K} \sum_{j=1}^{K} \sum_{k=1}^{K} \sum_{i=1}^{K} \sum_{j=1}^{K} \sum_{i=1}^{K} \sum_{i=1}^{K} \sum_{i=1}^{K} \sum_{j=1}^{K} \sum_{i$$

As a consequence, setting again

$$\alpha_1 = 0, \qquad \alpha_2 = \frac{\pi}{2}, \qquad \beta_1 = -\frac{\pi}{4}, \qquad \beta_2 = \frac{\pi}{4},$$
(54)

the Bell-CHSH inequality can be parametrized as

$$|\langle \Omega | \mathcal{C}_{CHSH} | \Omega \rangle| = 2\sqrt{2} \tau(T) , \qquad (55)$$

where the form factor τ reads

$$\tau(T) = 2\sum_{i} \frac{\left(e^{\frac{\pi\omega_i}{a}} - e^{-\frac{\pi\omega_i}{a}}\right)}{\left(e^{\frac{2\pi\omega_i}{a}} - e^{-\frac{2\pi\omega_i}{a}}\right)} = \sum_{i} \frac{1}{\cosh(\frac{\omega_i}{2T})},$$
(56)

from which it follows that the violation of the Bell-CHSH inequality in relativistic Quantum Field Theory can be analyzed in terms of the Unruh temperature [10].

6 Conclusion

In this work we have pointed out that the four operators (A_i, B_i) , i = 1, 2 entering the Bell-CHSH inequality can be constructed in a simple and elegant way. The setup turns out to be of general applicability, ranging from Quantum Mechanics examples to relativistic Quantum Field Theory.

We have argued that the violation of the Bell-CHSH inequality in relativistic Quantum Field Theory can be understood as a direct consequence of the decomposition of the vacuum sate $|0\rangle$ as a deeply entangled squeezed state in terms of left and right Rindler modes.

These considerations might result in applications in several Quantum Field Theory models, including Abelian and non-Abelian gauge theories [10].

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