$T(\rho) \geq \frac{1}{2}I$. With these properties and based on the direct method of the Lyapunov stability theory, a new sliding vector and two significant Lyapunov functions are introduced in the controller design and system stability analysis. Besides, the convergent rate of the error signal can be determined by suitably choosing the sliding vector. As for the chattering problem, the saturation functions have been suggested to replace the sign functions in the control laws. Finally, the example of spacecraft driven by pairs of opposing thrusters verifies the success and robustness of the sliding-mode controller.

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Removal of Alignment Errors in an Integrated System of Two 3-D Sensors

An algorithm is presented to relatively align two 3-D sensors using targets that are tracked by both sensors. The algorithm estimates and removes sensor biases and sensor frame orientation errors. For illustrative purposes, the alignment algorithm is applied to simulated track data from two sensors.

I. INTRODUCTION

Interest in integrating stand-alone sensors into multisensor systems for command, control, and communications (C^3) has been increasing in recent years. Rather than develop new sensors to achieve more accurate tracking and improved surveillance in C^3 systems, it is less costly to integrate existing stand-alone sensors into a single system to obtain performance improvements and enhanced capabilities for tracking and surveillance. However, before the benefits of multisensor integration can be realized, the sensor registration (or alignment) problem must be addressed. Registration refers to the process of expressing the multisensor data in a common reference frame, where the data is free from errors due to improper alignment of the sensors, orientation errors in the reference frames of the sensors, and sensor location errors [1]. That is, the data from each sensor must be transformed to a common reference frame that is free from errors in the transformation process. Unfortunately, attempts to integrate multiple sensors into a single system for C^3 have had limited success, due largely to a failure to solve the registration problem [1, 2].

One source of registration errors is sensor calibration errors (i.e., offsets). Although the sensors are usually calibrated in an initial calibration procedure, the calibration may deteriorate over time. Another source of registration errors is attitude (or orientation) errors in the reference frames of the sensors. Attitude errors can be caused by bias errors

Manuscript received January 26, 1993.

IEEE Log No. T-AES/29/4/10994.

This work was supported by the Independent Research Program at the Naval Surface Warfare Center Dahlgren Division (NSWCDD), Dahlgren, VA.

in the gyros in the inertial measurement unit (IMU) of the sensor. Other sources of registration errors include sensor location errors caused by bias errors in the navigation systems associated with the sensors, and timing errors caused by bias errors in the clocks of the sensors. Sensor misalignment in a system of land-based sensors has been documented with real data collected during experiments [1, 3–5].

Some work has been done on the removal of registration errors [1–11]. Offset errors, attitude errors, location errors, and timing errors for a netted system of widely separated radars were considered in [1]. However, all of the individual registration errors could not be determined because of the coupling between the errors. Most of the other work includes only sensor offset errors [2–5] or only attitude errors [7, 8]. The inclusion of both offset and attitude errors was considered in [11], but their method first removes the offset errors assuming there are no attitude errors, and then removes the attitude errors. This approach ignores the coupling between the angular offset errors and the attitude errors; that is, the offset errors are treated independently of the attitude errors. Sensor location errors were considered in [9, 10] and the registration of dissimilar sensors in [6].

The purpose of this work is to include both offset and attitude errors simultaneously in the formulation of the registration problem for 3-D sensors, where a 3-D sensor measures range, azimuth, and elevation. For example, this problem occurs in the integration of 3-D radars. An algorithm is developed to relatively align two 3-D sensors using common targets that are tracked by both sensors. This algorithm uses the difference in the position of a target, as reported by the two sensors, to compute a set of bias parameters. These bias parameters are then applied to one of the sensor's track positional data to align it in the reference frame of the other sensor. This algorithm is applicable to those situations where there are no sensor location errors, the distance between the sensors is small (e.g., sensors located on the same platform), the magnitude of the attitude and sensor offset errors are small, and these errors do not change with time or vary slowly with time. It is also assumed that the sensors are synchronized in time and have the same update rates.

This paper is organized as follows. In Section II, a mathematical model is developed for this problem. The alignment algorithm is developed in Section III, and it is tested in Section IV with simulated track data. Finally, Section V summarizes the results of this study.

II. MATHEMATICAL DEVELOPMENT

The problem addressed in this work can be stated as follows. Given the position measurements from two 3-D sensors over time for a specific target, estimate the parameters that will align one sensor relative to the other one. In this problem, it is assumed that the locations of the sensors are known (i.e., no location errors) and the relative distance between the sensors is small. Also, it is assumed that the alignment errors are small. This allows the use of a first-order Taylor series approximation, and it results in a linearized version of the alignment problem. Below, the transformation between the reference frames of the sensors will be considered first. This is followed by the inclusion of attitude and sensor offset errors.

A. Transformation Between Reference Frames of Sensors

Consider a particular sensor, say the kth sensor, where $k = 1,2$. A reference frame is necessary in describing the measurements of the kth sensor. The reference frame in which the measurements of this sensor are made is called the measurement frame of the sensor. There is also a stabilized frame associated with this sensor. The stabilized frame is aligned to the true north-south horizontal line, the true east-west horizontal line, and the axis that is orthogonal to the horizontal plane formed by the north-south and east-west lines. Both frames have the same origin, but one frame is tilted with respect to the other one.

The stabilized frame at the kth sensor can be represented by the three mutually orthogonal unit vectors $\mathbf{e}_{x'k}$, $\mathbf{e}_{y'k}$, and $\mathbf{e}_{z'k}$. The subscript k denotes the kth sensor, and the subscripts x', y', and z' refer to the directions of east, north, and up, respectively. Similarly, the measurement frame can be represented by the three mutually orthogonal unit vectors $\mathbf{e}_{xk}$, $\mathbf{e}_{yk}$, and $\mathbf{e}_{zk}$. The transformation between these frames can be described by a set of Eulerian angles. The xyz-convention [12] is employed in this paper. In the xyz-convention, the transformation from the stabilized frame to the measurement frame is accomplished by first rotating about the z-axis of the stabilized frame by the yaw angle ϕ_k, then rotating about the intermediate y-axis by the pitch angle η_k, and rotating about the final x-axis by the roll angle ψ_k (see Fig. 1). It is assumed that the yaw, pitch, and roll angles at each sensor are known, e.g., from the IMUs of the sensors.

Let the column vectors $\mathbf{r}_k = [x_k y_k z_k]^{\mathrm{T}}$ and $\mathbf{r}'_k = [x'_k y'_k z'_k]^{\mathrm{T}}$ (the superscript T denotes matrix transposition) represent the rectangular coordinates of a point in the measurement and stabilized frames, respectively, of the kth sensor. The transformation from the measurement frame to the stabilized frame is given by

$$\mathbf{r}'_k = \mathbf{R}_k \mathbf{r}_k \tag{1}$$

where $\mathbf{R}_k$ is the 3×3 orthogonal matrix given by

$$\mathbf{R}_k = \begin{bmatrix} \cos\eta_k\cos\phi_k & \sin\psi_k\sin\eta_k\cos\phi_k - \cos\psi_k\sin\phi_k & \cos\psi_k\sin\eta_k\cos\phi_k + \sin\psi_k\sin\phi_k \\ \cos\eta_k\sin\phi_k & \sin\psi_k\sin\eta_k\sin\phi_k + \cos\psi_k\cos\phi_k & \cos\psi_k\sin\eta_k\sin\phi_k - \sin\psi_k\cos\phi_k \\ -\sin\eta_k & \cos\eta_k\sin\psi_k & \cos\eta_k\cos\psi_k \end{bmatrix}. \tag{2}$$

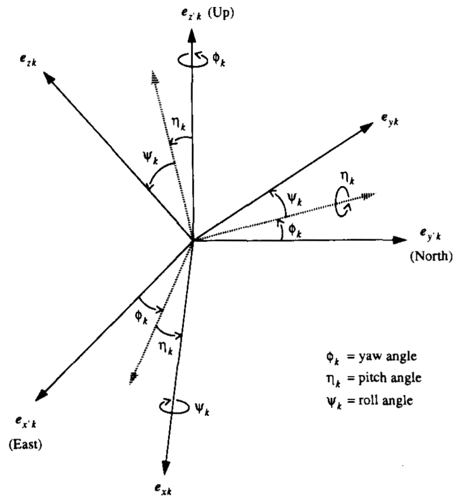

Fig. 1. Stabilized and measurement frames for kth sensor.

Let the second sensor ($k = 2$) be located at the point $\mathbf{t} = [t_x t_y t_z]^{\mathrm{T}}$ in the stabilized frame of the first sensor ($k = 1$). Since there are no location errors, the vector $\mathbf{t}$ is assumed to be known. The transformation from the stabilized frame of the second sensor to the stabilized frame of the first sensor is given by

$$\mathbf{r}'_{12} = \mathbf{r}'_2 + \mathbf{t} \tag{3}$$

where $\mathbf{r}'_2$ is a position vector in the stabilized frame of the second sensor, and $\mathbf{r}'_{12}$ is the corresponding position vector in the stabilized frame of the first sensor. Using (1) in (3) gives

$$\mathbf{R}_1\mathbf{r}_{12} = \mathbf{R}_2\mathbf{r}_2 + \mathbf{t} \tag{4}$$

where $\mathbf{r}_2$ and $\mathbf{r}_{12}$ are the corresponding position vectors in the measurement frames, respectively, of the second and first sensor. Since $\mathbf{R}_1$ is an orthogonal matrix, (4) can be expressed as

$$\mathbf{r}_{12} = \mathbf{R}\mathbf{r}_2 + \mathbf{R}_1^{\mathrm{T}}\mathbf{t} \tag{5}$$

where

$$\mathbf{R} = \mathbf{R}_1^{\mathrm{T}}\mathbf{R}_2. \tag{6}$$

Equation (5) represents the transformation from the measurement frame of the second sensor to the measurement frame of the first sensor.

It is convenient to assume that the tilt of the measurement frame with respect to the stabilized frame is small (i.e., the yaw, pitch, and roll angles are small). This simplifies the development of the alignment equations below. Assuming that the yaw, pitch, and roll angles describing the orientation of the measurement frame in the stabilized frame are small, the transformation matrix $\mathbf{R}_k$ in (2) can be approximated by the first-order Taylor series

$$\mathbf{R}_k \approx \mathbf{I} + \mathbf{dR}_k \tag{7}$$

where $\mathbf{I}$ is the 3×3 identity matrix, and $\mathbf{dR}_k$ is the matrix differential of $\mathbf{R}_k$, which is given by

$$\mathbf{dR}_k = \frac{\partial \mathbf{R}_k}{\partial \phi_k}\phi_k + \frac{\partial \mathbf{R}_k}{\partial \eta_k}\eta_k + \frac{\partial \mathbf{R}_k}{\partial \psi_k}\psi_k = \begin{bmatrix} 0 & -\phi_k & \eta_k \\ \phi_k & 0 & -\psi_k \\ -\eta_k & \psi_k & 0 \end{bmatrix}. \tag{8}$$

Here, all of the partial derivatives are evaluated at $(\phi_k, \eta_k, \psi_k) = (0,0,0)$. Substituting (7) into (6) gives the first-order approximation of $\mathbf{R}$ by

$$\mathbf{R} \approx \mathbf{I} + \mathbf{dR} \tag{9}$$

where $\mathbf{dR}$ is the matrix differential of $\mathbf{R}$ and it is given by

$$\mathbf{dR} = \mathbf{dR}_1^{\mathrm{T}} + \mathbf{dR}_2 = \begin{bmatrix} 0 & (\phi_1 - \phi_2) & -(\eta_1 - \eta_2) \\ -(\phi_1 - \phi_2) & 0 & (\psi_1 - \psi_2) \\ (\eta_1 - \eta_2) & -(\psi_1 - \psi_2) & 0 \end{bmatrix}. \tag{10}$$

Using these results in (5), the first-order approximation of the transformation from the measurement frame of the second sensor to the measurement frame of the first sensor is given by

$$\mathbf{r}_{12} = \mathbf{r}_2 + \mathbf{t} + \mathbf{dR}\mathbf{r}_2 + \mathbf{dR}_1^{\mathrm{T}}\mathbf{t}. \tag{11}$$

Although this transformation does not hold if the yaw, pitch, or roll angles are large, it serves as the model for developing the alignment equation below.

B. Attitude and Sensor Offset Errors

The attitude bias errors are modeled as additive constant biases to the reported values of the yaw, pitch, and roll angles. That is,

$$\begin{aligned} \phi_{k,\text{true}} &= \phi_k + \Delta\phi_k; \\ \eta_{k,\text{true}} &= \eta_k + \Delta\eta_k; \\ \psi_{k,\text{true}} &= \psi_k + \Delta\psi_k \end{aligned} \tag{12}$$

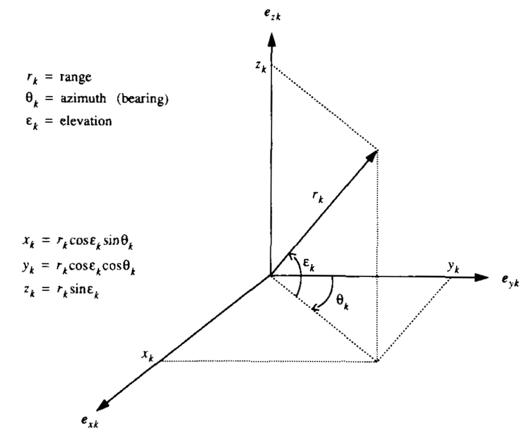

Fig. 2. Measurements in measurement frame of kth sensor.

where $\phi_{k,\text{true}}$, $\eta_{k,\text{true}}$, and $\psi_{k,\text{true}}$ are the true values of the yaw, pitch, and roll angles of the kth sensor, and $\Delta\phi_k$, $\Delta\eta_k$, and $\Delta\psi_k$ are the bias errors in these angles. The kth sensor measures the range r_k, azimuth θ_k, and elevation ε_k of a target. These measurements are obtained in the measurement frame of the sensor (see Fig. 2). The sensor offset errors are also modeled as additive constant biases to the measurements,

$$\begin{aligned} r_{k,\text{true}} &= r_k + \Delta r_k; \\ \theta_{k,\text{true}} &= \theta_k + \Delta\theta_k; \\ \varepsilon_{k,\text{true}} &= \varepsilon_k + \Delta\varepsilon_k. \end{aligned} \tag{13}$$

Here, the effects of random errors in the measurements are being ignored; they are included later when estimating the biases.

The transformation from the measurement frame of the first sensor to the measurement frame of the second sensor using the reported values of the yaw, pitch, and roll angles is given in (11). The transformation using the true values is given by

$$\mathbf{r}_{12} = \mathbf{r}_2 + \mathbf{t} + \mathbf{dR}_{\text{true}}\mathbf{r}_2 + \mathbf{dR}_{1,\text{true}}^{\mathrm{T}}\mathbf{t} \tag{14}$$

where $\mathbf{dR}_{1,\text{true}}$ and $\mathbf{dR}_{\text{true}}$ are the matrices defined in (8) and (10), respectively, but evaluated using the true values of the yaw, pitch, and roll angles. Equation (14) represents the transformation from the true measurement frame of the first sensor to the true measurement frame of the second sensor.

Let $\mathbf{r}_{1,\text{true}}$ denote the true position vector of the target in the first sensor's true measurement frame of the first sensor, and $\mathbf{r}_{2,\text{true}}$ is the true position vector of the target in the true measurement frame of the second sensor. Equation (14) can be used to transform $\mathbf{r}_{2,\text{true}}$ to the true measurement frame of the first sensor,

$$\mathbf{r}_{12,\text{true}} = \mathbf{r}_{2,\text{true}} + \mathbf{t} + \mathbf{dR}_{\text{true}}\mathbf{r}_{2,\text{true}} + \mathbf{dR}_{1,\text{true}}^{\mathrm{T}}\mathbf{t}. \tag{15}$$

By definition,

$$\mathbf{r}_{1,\text{true}} = \mathbf{r}_{12,\text{true}}. \tag{16}$$

This gives

$$\mathbf{r}_{1,\text{true}} = \mathbf{r}_{2,\text{true}} + \mathbf{t} + \mathbf{dR}_{\text{true}}\mathbf{r}_{2,\text{true}} + \mathbf{dR}_{1,\text{true}}^{\mathrm{T}}\mathbf{t} \tag{17}$$

which is used below to obtain a transformation to align the sensors.

None of the quantities in (17), except for $\mathbf{t}$ is known. The true values of these quantities must be related to the measured values and the biases. This is accomplished by assuming that the biases are small quantities and using first-order Taylor series approximations. Using (8), (10), and (12), the matrices $\mathbf{dR}_{1,\text{true}}$ and $\mathbf{dR}_{\text{true}}$ are approximated by

$$\mathbf{dR}_{1,\text{true}} \approx \mathbf{dR}_1 + \mathbf{A}_1; \qquad \mathbf{dR}_{\text{true}} \approx \mathbf{dR} + \mathbf{A} \tag{18}$$

where the matrix differentials $\mathbf{A}_1$ and $\mathbf{A}$ are given by

$$\begin{aligned} \mathbf{A}_1 &= \frac{\partial(\mathbf{dR}_1)}{\partial\phi_1}\Delta\phi_1 + \frac{\partial(\mathbf{dR}_1)}{\partial\eta_1}\Delta\eta_1 + \frac{\partial(\mathbf{dR}_1)}{\partial\psi_1}\Delta\psi_1 \\ &= \begin{bmatrix} 0 & -\Delta\phi_1 & \Delta\eta_1 \\ \Delta\phi_1 & 0 & -\Delta\psi_1 \\ -\Delta\eta_1 & \Delta\psi_1 & 0 \end{bmatrix} \end{aligned} \tag{19}$$

$$\begin{aligned} \mathbf{A} &= \sum_{k=1}^{2}\left[\frac{\partial(\mathbf{dR})}{\partial\phi_k}\Delta\phi_k + \frac{\partial(\mathbf{dR})}{\partial\eta_k}\Delta\eta_k + \frac{\partial(\mathbf{dR})}{\partial\psi_k}\Delta\psi_k\right] \\ &= \begin{bmatrix} 0 & \Delta\phi & -\Delta\eta \\ -\Delta\phi & 0 & \Delta\psi \\ \Delta\eta & -\Delta\psi & 0 \end{bmatrix} \end{aligned} \tag{20}$$

and

$$\begin{aligned} \Delta\phi &= \Delta\phi_1 - \Delta\phi_2; \\ \Delta\eta &= \Delta\eta_1 - \Delta\eta_2; \\ \Delta\psi &= \Delta\psi_1 - \Delta\psi_2. \end{aligned} \tag{21}$$

The rectangular coordinates x_k, y_k, and z_k of a position vector $\mathbf{r}_k = [x_k y_k z_k]^{\mathrm{T}}$ are related to the range r_k, azimuth θ_k, and elevation ε_k by (see Fig. 2)

$$\begin{aligned} x_k &= r_k\cos\varepsilon_k\sin\theta_k; \\ y_k &= r_k\cos\varepsilon_k\cos\theta_k; \\ z_k &= r_k\sin\varepsilon_k. \end{aligned} \tag{22}$$

Using (13), the position vector $\mathbf{r}_{k,\text{true}}$ is approximated by

$$\mathbf{r}_{k,\text{true}} \approx \mathbf{r}_k + \mathbf{dr}_k \tag{23}$$

where the vector differential $\mathbf{dr}_k$ is given by

$$\mathbf{dr}_k = \frac{\partial\mathbf{r}_k}{\partial r_k}\Delta r_k + \frac{\partial\mathbf{r}_k}{\partial\theta_k}\Delta\theta_k + \frac{\partial\mathbf{r}_k}{\partial\varepsilon_k}\Delta\varepsilon_k. \tag{24}$$

Here, all of the partial derivatives are evaluated at the target measurements of the kth sensor.

Using the results from the previous paragraph in (17) gives the following first-order approximation

$$\begin{aligned} \mathbf{r}_1 = {} & (\mathbf{r}_2 + \mathbf{t} + \mathbf{dR}\mathbf{r}_2 + \mathbf{dR}_1^{\mathrm{T}}\mathbf{t}) \\ & + (\mathbf{dr}_2 - \mathbf{dr}_1 + \mathbf{A}\mathbf{r}_2 + \mathbf{A}_1^{\mathrm{T}}\mathbf{t}). \end{aligned} \tag{25}$$

From (11), the terms in the first set of parentheses are $\mathbf{r}_{12}$, which represents the measured position of the target, $\mathbf{r}_2$, of the second sensor, transformed to the frame of the first sensor using the reported values of the yaw, pitch, and roll. The terms in the other set of parentheses represent the effects of the bias errors on the transformation. The term $\mathbf{A}_1^{\mathrm{T}}\mathbf{t}$ in (25) is negligible under the assumptions made for this problem. Assuming that the maximum values of the yaw, pitch, and roll bias errors are 1°, and the maximum separation between the sensors is 100 m, it can be shown that $\|\mathbf{A}_1^{\mathrm{T}}\mathbf{t}\| < 6$ m. Thus, this term is ignored and (25) becomes

$$\mathbf{r}_1 = \mathbf{r}_{12} + \mathbf{a} \tag{26}$$

where the alignment vector $\mathbf{a}$ is defined by

$$\mathbf{a} = \mathbf{dr}_2 - \mathbf{dr}_1 + \mathbf{Ar}_2. \tag{27}$$

If the bias errors are known, then $\mathbf{a}$ is known and (26) represents the transformation that aligns the second sensor to the first sensor. Conversely, if the bias errors are not known, but the position vectors of the target are measured, then (26) could be used to determine the biases. Thus, (26) is the basic equation for both aligning the sensors and also estimating the biases. For reasons discussed below, (26) is not used for this. Rather, it is used to derive other equations that serve this purpose.

Using (20) and (24) in (27), and some manipulations, the alignment vector $\mathbf{a}$ can be expressed in terms of the biases by

$$\begin{aligned}\mathbf{a} = {}& \mathbf{c}_1\Delta\phi + \mathbf{c}_2\Delta\eta + \mathbf{c}_3\Delta\psi \\ &+ \frac{\partial\mathbf{r}_2}{\partial r_2}\Delta r_2 - \frac{\partial\mathbf{r}_1}{\partial r_1}\Delta r_1 + \frac{\partial\mathbf{r}_2}{\partial\theta_2}\Delta\theta_2 \\ &- \frac{\partial\mathbf{r}_1}{\partial\theta_1}\Delta\theta_1 + \frac{\partial\mathbf{r}_2}{\partial\varepsilon_2}\Delta\varepsilon_2 - \frac{\partial\mathbf{r}_1}{\partial\varepsilon_1}\Delta\varepsilon_1\end{aligned} \tag{28}$$

where

$$\begin{aligned}\mathbf{c}_1 &= [y_2 \quad -x_2 \quad 0]^{\mathrm{T}};\\ \mathbf{c}_2 &= [-z_2 \quad 0 \quad x_2]^{\mathrm{T}};\\ \mathbf{c}_3 &= [0 \quad z_2 \quad -y_2]^{\mathrm{T}}.\end{aligned} \tag{29}$$

Since the sensors are close to each other (much closer than the distances to targets of interest) and the biases are assumed to be small, it is difficult to separate the effects of the individual biases in the data. That is, observability problems are encountered if an attempt is made to estimate all of the biases. This occurs because some of the vectors multiplying the biases in (28) are nearly the same. In particular, $\partial\mathbf{r}_1/\partial r_1 \approx \partial\mathbf{r}_2/\partial r_2$, $\partial\mathbf{r}_1/\partial\theta_1 \approx \partial\mathbf{r}_2/\partial\theta_2$, and $\partial\mathbf{r}_1/\partial\varepsilon_1 \approx \partial\mathbf{r}_2/\partial\varepsilon_2$. Also, it can be shown that $\mathbf{c}_1 = \partial\mathbf{r}_2/\partial\theta_2$. Thus, $\mathbf{a}$ can be approximated by

$$\mathbf{a} \approx \mathbf{c}_1\Delta\theta + \mathbf{c}_2\Delta\eta + \mathbf{c}_3\Delta\psi + \mathbf{c}_4\Delta\varepsilon + \mathbf{c}_5\Delta r \tag{30}$$

where

$$\begin{aligned}\Delta\theta &= \Delta\phi + \Delta\theta_2 - \Delta\theta_1;\\ \Delta\varepsilon &= \Delta\varepsilon_2 - \Delta\varepsilon_1;\\ \Delta r &= \Delta r_2 - \Delta r_1\end{aligned} \tag{31}$$

and

$$\begin{aligned}\mathbf{c}_4 &= \frac{\partial\mathbf{r}_2}{\partial\varepsilon_2}\\ &= [-r_2\sin\varepsilon_2\sin\theta_2 \quad -r_2\sin\varepsilon_2\cos\theta_2 \quad r_2\cos\varepsilon_2]^{\mathrm{T}}\\ \mathbf{c}_5 &= \frac{\partial\mathbf{r}_2}{\partial r_2} = \frac{\mathbf{r}_2}{r_2}.\end{aligned} \tag{32}$$

The alignment vector $\mathbf{a}$ depends on five parameters: the azimuth bias $\Delta\theta$, pitch bias $\Delta\eta$, roll bias $\Delta\psi$, elevation bias $\Delta\varepsilon$, and range bias Δr. The definitions of these five biases imply that the individual bias errors cannot be determined, e.g., the individual azimuth and yaw biases cannot be separated and they are lumped together in the azimuth bias $\Delta\theta$. Similarly, the individual elevation biases cannot be separated, nor the individual range biases, nor the individual pitch biases, nor the individual roll biases. The alignment vector $\mathbf{a}$ in (30) can be expressed in matrix notation as

$$\mathbf{a} = \mathbf{Cb} \tag{33}$$

where $\mathbf{b} = [\Delta\theta \quad \Delta\eta \quad \Delta\psi \quad \Delta\varepsilon \quad \Delta r]^{\mathrm{T}}$ is the bias vector and $\mathbf{C}$ is the matrix given by

$$\begin{aligned}\mathbf{C} &= [\mathbf{c}_1 \quad \mathbf{c}_2 \quad \mathbf{c}_3 \quad \mathbf{c}_4 \quad \mathbf{c}_5]\\ &= \begin{bmatrix} y_2 & -z_2 & 0 & -z_2\sin\theta_2 & x_2/r_2\\ -x_2 & 0 & z_2 & -z_2\cos\theta_2 & y_2/r_2\\ 0 & x_2 & -y_2 & r_2\cos\varepsilon_2 & z_2/r_2\end{bmatrix}.\end{aligned} \tag{34}$$

The basic alignment equation in (26) can be expressed as

$$\mathbf{r}_1 = \mathbf{r}_{12} + \mathbf{Cb} \tag{35}$$

The matrix $\mathbf{C}$ is evaluated using the measurements of the second sensor and they are expressed in the frame of the second sensor. Only a small amount of error is introduced in (35) if $\mathbf{C}$ is evaluated using the measurements of the second sensor, but expressed in the frame of the first sensor. That is, $\mathbf{C}$ will be evaluated as

$$\begin{aligned}\mathbf{C} &= [\mathbf{c}_1 \quad \mathbf{c}_2 \quad \mathbf{c}_3 \quad \mathbf{c}_4 \quad \mathbf{c}_5]\\ &= \begin{bmatrix} y_{12} & -z_{12} & 0 & -z_{12}\sin\theta_{12} & x_{12}/r_{12}\\ -x_{12} & 0 & z_{12} & -z_{12}\cos\theta_{12} & y_{12}/r_{12}\\ 0 & x_{12} & -y_{12} & r_{12}\cos\varepsilon_{12} & z_{12}/r_{12}\end{bmatrix}.\end{aligned} \tag{36}$$

This has the advantage of expressing all of the positional data in (35) in the frame of the first sensor.

Assuming that the bias vector $\mathbf{b}$ is known, the alignment of the second sensor to the first sensor is accomplished by the following procedure. First, (11) is used to transform the measured target position, $\mathbf{r}_2$, of the second sensor, to the measurement frame of the first sensor, using the reported values of the yaw, pitch, and roll angles; this produces $\mathbf{r}_{12}$. Then, the matrix $\mathbf{C}$ in (36), which depends only on $\mathbf{r}_{12}$, is calculated. Finally, (35) is used to compute $\mathbf{r}_1$, which represents $\mathbf{r}_2$, but aligned to the measurement frame of the first sensor. This alignment can be performed even if the first sensor is not tracking this particular target. Of course, a common target must be used to estimate the bias vector $\mathbf{b}$.

III. ALGORITHM DEVELOPMENT

An algorithm to estimate $\mathbf{b}$ is presented in this section. Assuming that both sensors are tracking a common target (so that $\mathbf{r}_1$, $\mathbf{r}_{12}$, and $\mathbf{C}$ are known), (35) can be thought of as a measurement of the bias vector $\mathbf{b}$, which can be expressed as

$$\mathbf{y} = \mathbf{C}\mathbf{b} + \mathbf{e} \tag{37}$$

where $\mathbf{y} = \mathbf{r}_1 - \mathbf{r}_{12}$ is the measurement of $\mathbf{b}$ and $\mathbf{e}$ is the random measurement error. Following a procedure similar to the one outlined in the previous section for the bias errors, the random vector $\mathbf{e}$ can be approximated by

$$\mathbf{e} \approx \mathbf{c}_1 e_\theta + \mathbf{c}_4 e_\varepsilon + \mathbf{c}_5 e_r \tag{38}$$

where

$$\begin{aligned} e_\theta &= e_{\theta 2} - e_{\theta 1}; \\ e_\varepsilon &= e_{\varepsilon 2} - e_{\varepsilon 1}; \\ e_r &= e_{r2} - e_{r1}. \end{aligned} \tag{39}$$

Here, it is assumed that all of the measurement noise adds to the measurements of the sensor, and e_{rk}, $e_{\theta k}$, and $e_{\varepsilon k}$ denote the random errors in the measured value of range, azimuth, and elevation, respectively, of the kth sensor. These random errors are assumed to be zero-mean errors with known standard deviations σ_{rk}, $\sigma_{\theta k}$, and $\sigma_{\varepsilon k}$, and they are also assumed to be mutually uncorrelated.

Kalman filtering techniques using (37) can be applied to estimate $\mathbf{b}$. However, simulations have shown that there is a problem in estimating the range alignment parameter Δr using this approach. The problem occurs when $|\Delta r| \ll r_2|\Delta\theta|$, $|\Delta r| \ll r_2|\Delta\varepsilon|$. etc.; that is, when the alignment error due to the range bias is much less than that caused by the angular biases. This problem can be avoided by decoupling the range bias estimation problem from the angular bias estimation problem.

The vector equation presented in (37) can be expressed as the following three scalar equations

$$x_1 = x_{12} + y_{12}(\Delta\theta + e_\theta) - z_{12}\Delta\eta$$
$$- (z_{12}\sin\theta_{12})(\Delta\varepsilon + e_\varepsilon) + \frac{x_{12}}{r_{12}}(\Delta r + e_r) \tag{40}$$

$$y_1 = y_{12} - x_{12}(\Delta\theta + e_\theta) + z_{12}\Delta\psi$$
$$- (z_{12}\cos\theta_{12})(\Delta\varepsilon + e_\varepsilon) + \frac{y_{12}}{r_{12}}(\Delta r + e_r) \tag{41}$$

$$z_1 = z_{12} + x_{12}\Delta\eta - y_{12}\Delta\psi$$
$$+ (r_{12}\cos\varepsilon_{12})(\Delta\varepsilon + e_\varepsilon) + \frac{z_{12}}{r_{12}}(\Delta r + e_r). \tag{42}$$

The decoupling is obtained by simple manipulations of these three equations. First, the range bias is decoupled from the angular biases by multiplying (40) by x_{12}, (41) by y_{12}, (42) by z_{12}, and adding the results. This gives

$$\begin{aligned} \Delta r + e_r &= \frac{x_1 x_{12} + y_1 y_{12} + z_1 z_{12}}{r_{12}} - r_{12} \\ &= \mathbf{r}_1^{\mathrm{T}}\left(\frac{\mathbf{r}_{12}}{r_{12}}\right) - r_{12}. \end{aligned} \tag{43}$$

This result was obtained by noting that $\mathbf{r}_{12}$ is orthogonal to $\mathbf{c}_1$, $\mathbf{c}_2$, $\mathbf{c}_3$, and $\mathbf{c}_4$. Since $\partial\mathbf{r}_{12}/\partial r_{12} = \mathbf{r}_{12}/r_{12}$, $\partial\mathbf{r}_1/\partial r_1 = \mathbf{r}_1/r_1$, and $\partial\mathbf{r}_1/\partial r_1 \approx \partial\mathbf{r}_{12}//\partial r_{12}$, (43) can be approximated by the simpler equation

$$\Delta r + e_r \approx r_1 - r_{12}. \tag{44}$$

The angular biases can also be decoupled from the range bias. Multiplying (40) by $\cos\theta_{12}$, (41) by $-\sin\theta_{12}$, and adding the results give

$$\begin{aligned} &(r_{12}\cos\varepsilon_{12})(\Delta\theta + e_\theta) - (z_{12}\cos\theta_{12})\Delta\eta - (z_{12}\sin\theta_{12})\Delta\psi \\ &\quad = (x_1 - x_{12})\cos\theta_{12} - (y_1 - y_{12})\sin\theta_{12}. \end{aligned} \tag{45}$$

Finally, multiplying (40) by $\sin\varepsilon_{12}\sin\theta_{12}$, (41) by $\sin\varepsilon_{12}\cos\theta_{12}$, (42) by $-\cos\varepsilon_{12}$, and adding the results give

$$\begin{aligned} &-r_{12}(\Delta\varepsilon + e_\varepsilon) - (r_{12}\sin\theta_{12})\Delta\eta + (r_{12}\cos\theta_{12})\Delta\psi \\ &\quad = [(x_1 - x_{12})\sin\theta_{12} + (y_1 - y_{12})\cos\theta_{12}] \\ &\quad \times \sin\varepsilon_{12} - (z_1 - z_{12})\cos\varepsilon_{12}. \end{aligned} \tag{46}$$

These last two results were obtained by noting that the vectors $[\cos\theta_{12} \quad -\sin\theta_{12} \quad 0]^{\mathrm{T}}$ and $[\sin\varepsilon_{12}\sin\theta_{12} \quad \sin\varepsilon_{12}\cos\theta_{12} \quad -\cos\varepsilon_{12}]^{\mathrm{T}}$ are orthogonal to $\mathbf{c}_5$. These equations can be simplified by using (22), which gives

$$
\begin{aligned}
(\Delta\theta + e_\theta) &- (\tan\varepsilon_{12}\cos\theta_{12})\Delta\eta - (\tan\varepsilon_{12}\sin\theta_{12})\Delta\psi \\
&= \frac{r_1\cos\varepsilon_1}{r_{12}\cos\varepsilon_{12}}\sin(\theta_1 - \theta_{12}) \\
&\approx \theta_1 - \theta_{12} \qquad (47)
\end{aligned}
$$

$$
\begin{aligned}
(\Delta\varepsilon + e_\varepsilon) &+ (\sin\theta_{12})\Delta\eta - (\cos\theta_{12})\Delta\psi \\
&= \frac{r_1}{r_{12}}(\sin\varepsilon_1\cos\varepsilon_{12} - \cos\varepsilon_1\sin\varepsilon_{12}\cos(\theta_1 - \theta_{12})) \\
&\approx \varepsilon_1 - \varepsilon_{12}. \qquad (48)
\end{aligned}
$$

The last lines in (47) and (48) were obtained by using the approximations $r_1/r_{12} \approx 1$ and $\cos\varepsilon_1/\cos\varepsilon_{12} \approx 1$, and assuming that $\theta_1 - \theta_{12}$ and $\varepsilon_1 - \varepsilon_{12}$ are small quantities.

In addition to being used in the estimation of the biases, (44), (47), and (48) can also be used to align the spherical measurements from the sensors. In this case, the equations are expressed as

$$r_1 = r_{12} + \Delta r \qquad (49)$$

$$
\begin{aligned}
\theta_1 = \theta_{12} &+ \Delta\theta - (\tan\varepsilon_{12}\cos\theta_{12})\Delta\eta \\
&- (\tan\varepsilon_{12}\sin\theta_{12})\Delta\psi \qquad (50)
\end{aligned}
$$

$$\varepsilon_1 = \varepsilon_{12} + \Delta\varepsilon + (\sin\theta_{12})\Delta\eta - (\cos\theta_{12})\Delta\psi \qquad (51)$$

where the random error terms have been ignored. Assuming the biases are known, the alignment is accomplished by transforming the measured target position $\mathbf{r}_2$ of the second sensor to the frame of the first sensor using (11). This produces $\mathbf{r}_{12}$, which is used to calculate the spherical coordinates r_{12}, θ_{12}, and ε_{12}. Then, (49)–(51) are used to calculate r_1, θ_1, and ε_1, which represent r_{12}, θ_{12}, and ε_{12}, respectively, but aligned to the frame of the first sensor. Equation (22) can then be used to obtain $\mathbf{r}_1$. In simulations, this approach has produced better performance than by directly using the alignment equation in (35).

Equations (44), (47), and (48) are also used as the measurement equations for the biases. The measurement equation for the range bias is given by

$$m_r = \Delta r + e_r \qquad (52)$$

where the measurement m_r is defined by

$$m_r = r_1 - r_{12}. \qquad (53)$$

The error e_r is a zero-mean error with variance σ_r, which is given by

$$\sigma_r^2 = \sigma_{r1}^2 + \sigma_{r2}^2. \qquad (54)$$

The measurement equation for the angular biases is given by

$$z = \mathbf{H}\mathbf{d} + \mathbf{v} \qquad (55)$$

where $\mathbf{d} = [\Delta\theta \quad \Delta\eta \quad \Delta\psi \quad \Delta\varepsilon]^{\mathrm{T}}$ is the angular bias vector, $z = [(\theta_1 - \theta_{12}) \quad (\varepsilon_1 - \varepsilon_{12})]^{\mathrm{T}}$ is the measurement vector, $\mathbf{H}$ is the matrix given by

$$\mathbf{H} = \begin{bmatrix} 1 & -\tan\varepsilon_{12}\cos\theta_{12} & -\tan\varepsilon_{12}\sin\theta_{12} & 0 \\ 0 & \sin\theta_{12} & -\cos\theta_{12} & 1 \end{bmatrix} \qquad (56)$$

and $\mathbf{v} = [e_\theta \quad e_\varepsilon]^{\mathrm{T}}$ is the random measurement error, which is a zero-mean error with covariance

$$\mathbf{W} = \mathrm{cov}(\mathbf{v}) = \begin{bmatrix} \sigma_\theta^2 & 0 \\ 0 & \sigma_\varepsilon^2 \end{bmatrix} \qquad (57)$$

where

$$\sigma_\theta^2 = \sigma_{\theta 1}^2 + \sigma_{\theta 2}^2; \qquad \sigma_\varepsilon^2 = \sigma_{\varepsilon 1}^2 + \sigma_{\varepsilon 2}^2. \qquad (58)$$

Since the biases are assumed to be constants, the dynamics for the biases are modeled as constants that are driven by zero-mean white noise. The dynamical equation for the range bias Δr is given by

$$\Delta r_j = \Delta r_{j-1} + w_{j-1} \qquad (59)$$

and the dynamical equation for the angular bias vector $\mathbf{d}$ is given by

$$\mathbf{d}_j = \mathbf{d}_{j-1} + \mathbf{u}_{j-1}. \qquad (60)$$

The indices $j-1$ and j refer to the times at which the $(j-1)$th and jth samples or measurements of the target occur. The process noise terms w_j and $\mathbf{u}_j$ are assumed to be zero-mean white noise processes with positive-definite covariances s_j and $\mathbf{Q}_j$, respectively.

The range alignment parameter Δr can be estimated using a first-order Kalman filter designed for the system with measurements and dynamics given by (52) and (59). The angular bias vector $\mathbf{d}$ can be estimated using a fourth-order Kalman filter designed using (55) and (60). If two or more common targets are being tracked by the sensors, then the measurement equations describing the range bias and the angular biases can be augmented into two larger measurement equations. Then, the estimates of the biases can be obtained by applying Kalman filtering techniques to the dynamical equations above and the augmented measurement equations.

Inspection of the equations for the range bias shows that Δr is observable. That is, an estimate of the range bias can be obtained. For the angular biases, one needs to consider the observability grammian [13]. For a single target, the system describing the angular bias vector has the following observability grammian

$$\mathbf{M} = \sum_{j=1}^{N} \mathbf{H}_j^{\mathrm{T}}\mathbf{W}_j^{-1}\mathbf{H}_j = \overline{\mathbf{H}}^{\mathrm{T}}\overline{\mathbf{W}}^{-1}\overline{\mathbf{H}} \qquad (61)$$

where $\mathbf{H}_j$ and $\mathbf{W}_j$ are the matrices in (56) and (57) evaluated at the jth sample period, and the augmented matrices $\overline{\mathbf{H}}$ and $\overline{\mathbf{W}}$ are defined by $\overline{\mathbf{H}} = [\mathbf{H}_1^{\mathrm{T}}\mathbf{H}_2^{\mathrm{T}}\ldots\mathbf{H}_N^{\mathrm{T}}]^{\mathrm{T}}$ and $\overline{\mathbf{W}} = \mathrm{diag}[\mathbf{W}_1\mathbf{W}_2\ldots\mathbf{W}_N]$. The angular bias vector is observable provided that $\mathbf{M}$ is positive definite for some finite N. Here, N must be at least two because the measurement equation in (55) represents two equations with four unknowns. That is, for a single target, it takes at least two sample periods to estimate the angular biases.

Since the $\mathbf{W}_j$s are assumed to be positive definite, $\mathbf{M}$ is not positive definite provided that $\overline{\mathbf{H}}$ does not have full column rank (i.e., some of its columns are linearly dependent). Such a situation occurs when the target is radially inbound or radially outbound; that is, θ_{12} = constant. In this case, the second, third, and fourth columns of $\overline{\mathbf{H}}$ are dependent. It can be shown that the only angular parameters that can be estimated are $\Delta\theta$, α, and β, where

$$\begin{aligned}\alpha &= (\cos\theta_{12})\Delta\eta + (\sin\theta_{12})\Delta\psi;\\ \beta &= \Delta\varepsilon + (\sin\theta_{12})\Delta\eta - (\cos\theta_{12})\Delta\psi.\end{aligned} \tag{62}$$

If, in addition to moving along a radial line, the elevation angle of the target is also constant, the first, second, and third columns of $\overline{\mathbf{H}}$ are dependent as are the second, third, and fourth columns. In this case, the only angular parameters that can be estimated are β and γ, where

$$\gamma = \Delta\theta - (\tan\varepsilon_{12})\alpha. \tag{63}$$

The above discussion implies that there should be no problems in estimating the range bias, but potential problems can occur in estimating the angular biases. For a radially inbound or radially outbound target, all four of the angular biases cannot be determined; only linear combinations of the angular biases can be estimated. If the alignment is performed using a target of opportunity, it may not be possible to overcome these difficulties if the target is moving along a radial line. However, if one has control of the target, these problems can be overcome by choosing the trajectory of the target so that the azimuth varies significantly with time. Another possible solution to this problem is to use more than one target to estimate the biases.

IV. SIMULATION RESULTS

The alignment algorithm was tested by generating two common tracks and including alignment and random errors in the track data. One of the common tracks was used to generate the estimates of the five bias parameters, which were then applied to both tracks. The second common track was included to see how well the algorithm performs when another track is used to generate the bias estimates.

Both sensors are stationary and the second sensor is located at $t_x = t_y = 25$ m and $t_z = 10$ m relative to the stabilized frame of the first sensor. The reported values of the yaw, pitch, and roll angles at both sensors were taken to be zero. The standard deviations in the measurements of the sensor are given by

$$\begin{aligned}\sigma_{r1} = \sigma_{r2} &= 10 \text{ m};\\ \sigma_{\theta1} = \sigma_{\theta2} = \sigma_{\varepsilon1} &= \sigma_{\varepsilon2} = 0.1^\circ.\end{aligned} \tag{64}$$

The bias errors at the first sensor were given by

$$\begin{gathered}\Delta r_1 = 25 \text{ m}; \qquad \Delta\theta_1 = -1^\circ; \qquad \Delta\varepsilon_1 = -0.5^\circ;\\ \Delta\phi_1 = \Delta\eta_1 = 1^\circ; \qquad \Delta\psi_1 = -1^\circ\end{gathered} \tag{65}$$

and the second sensor had the following bias errors

$$\begin{gathered}\Delta r_2 = 50 \text{ m}; \qquad \Delta\theta_2 = 1^\circ; \qquad \Delta\varepsilon_2 = 0.5^\circ;\\ \Delta\phi_2 = \Delta\eta_2 = -1^\circ; \qquad \Delta\psi_2 = 1^\circ.\end{gathered} \tag{66}$$

Using these in (21) and (31) gives the following values for the five bias parameters,

$$\begin{gathered}\Delta r = 25 \text{ m}; \qquad \Delta\theta = 4^\circ; \qquad \Delta\varepsilon = 1^\circ;\\ \Delta\eta = 2^\circ; \qquad \Delta\psi = -2^\circ.\end{gathered} \tag{67}$$

The first target is a closing target that is undergoing a simple maneuver in the xy-plane. The true z-coordinate of the target is 3 km and constant. The azimuth and elevation angles vary significantly with time, and this target is used to estimate the five biases. The second target is a closing target that is moving at constant velocity. The true z-coordinate of this target is 1 km and constant. The bias and random errors above were included in the measurements of the sensors. The data are assumed to be time coincident and both sensors are reporting data at regular intervals of $T = 0.5$ s. The Kalman filters described in the previous section were implemented to obtain the estimates of the biases. The first measurement was used to initialize the range bias filter and the first five measurements were used to initialize the angular bias filter. The covariances s_j and $\mathbf{Q}_j$ for the input noises were taken to be constants with $s_j = 10^{-4}T^2$ (m^2) and $\mathbf{Q}_j = \sigma^2\mathbf{I}$, where $\mathbf{I}$ is the 4×4 identity matrix and $\sigma = 10^{-6}T$ (rads).

The rectangular coordinates of the targets in the xy-plane before alignment are presented in Fig. 3(a) and the z-coordinates in Fig. 4(a). The solid lines are the track positions reported by the first sensor and the dashed lines are the corresponding track positions from the second sensor. Here, all of the positional data are expressed in the frame of the first sensor. The asterisk at the origin in Fig. 3(a) denotes the locations of the sensors. The separations between the corresponding tracks positions reported by the two sensors is due to the bias errors in the sensors.

The spherical coordinates of track 1 as reported by the two sensors were input in the Kalman filters to generate the estimates of the biases. That is, track 1 was used to generate the bias estimates, which are then applied to both track 1 and track 2. Specifically, the biases were used in (49)–(51) to align the spherical coordinates. Then, (22) was used to calculate the rectangular coordinates. This alignment was performed in real time (dynamically); that is, as bias estimates were generated at each time point, they were then applied to align the coordinates. Since the estimates of the biases were initially very noisy, the biases

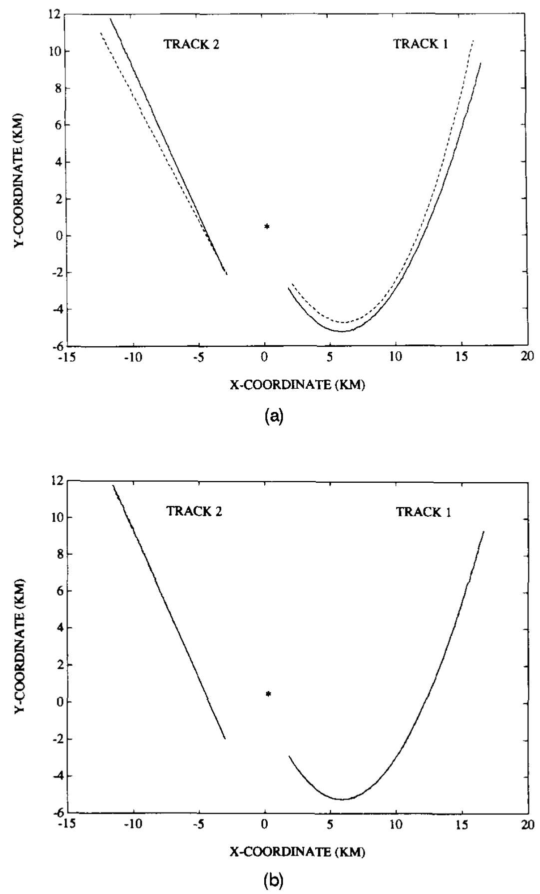

Fig. 3. Track coordinates in xy-plane. (a) Before alignment. (b) After alignment.

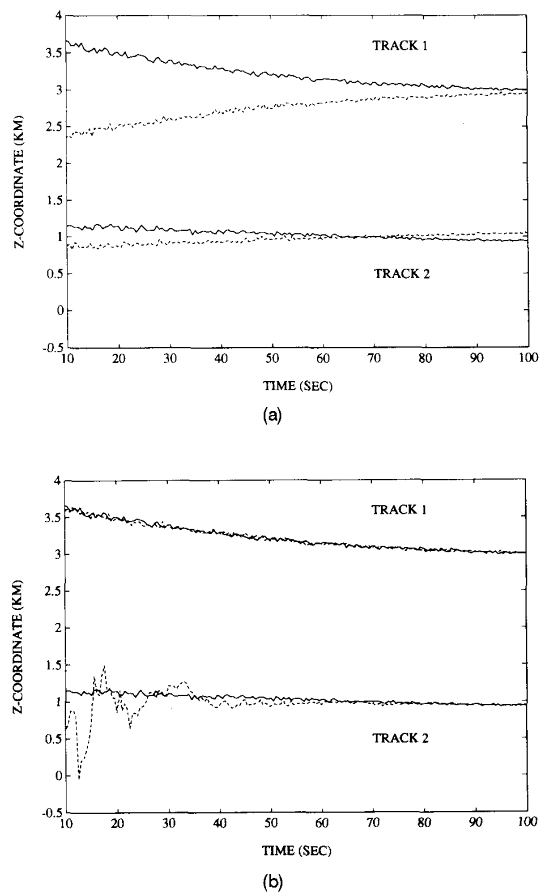

Fig. 4. Vertical coordinates of track. (a) Before alignment. (b) After alignment.

were applied only after they had been estimated for 10 s (20 updates). Fig. 3(b) presents the rectangular coordinates of the targets in the xy-plane after the biases have been applied to the measurement of the second sensor, and Fig. 4(b) presents the z-coordinates. To the scale of the graph in Fig. 3(b), there is little discernible difference between the corresponding tracks reported by the two sensors in the xy-plane. As shown in Fig. 4(b), the alignment of the z-coordinates for track 1 was very good, but the alignment for track 2 was poor for the first 40 s. Of course, track 1 was used to estimate the biases and it is expected that the alignment for track 1 should be good. Note that the z-coordinates reported by the sensors are not constants, but the true z-coordinates are constants. This occurs because of the bias errors in the sensors and the changing geometry due to the motion of each target.

The estimates of the biases from the Kalman filters are presented in Fig. 5. The dashed lines in these figures represent the true values of the biases, which are also given in (67). It takes approximately 30 to 40 s (60 to 80 updates) before the elevation, pitch, and roll biases converge to values near their true values, but the range and azimuth biases converge more quickly. The reason that the range bias converges more quickly is that it is estimated using one equation with one unknown, namely the range bias in (49). Similarly, the azimuth bias is usually the dominant term in (50), at least for targets with low elevations. In this simulation, track 1 started at a fairly low elevation and the azimuth parameter was the dominant term. The elevation, pitch, and roll biases must rely mainly on the single equation in (51) for their estimates, and it is expected that they will converge more slowly than the other two biases.

The alignment error between the corresponding tracks reported by the sensors is presented in Fig. 6, where the total error is shown in Fig. 6(a) and the error in the xy-plane in Fig. 6(b). The total error before the biases are applied is quite large (an average of 980 m for track 1 and 608 m for track 2). After the biases have been applied, the average total error is reduced to 37 m for track 1 and 102 m for track 2; this

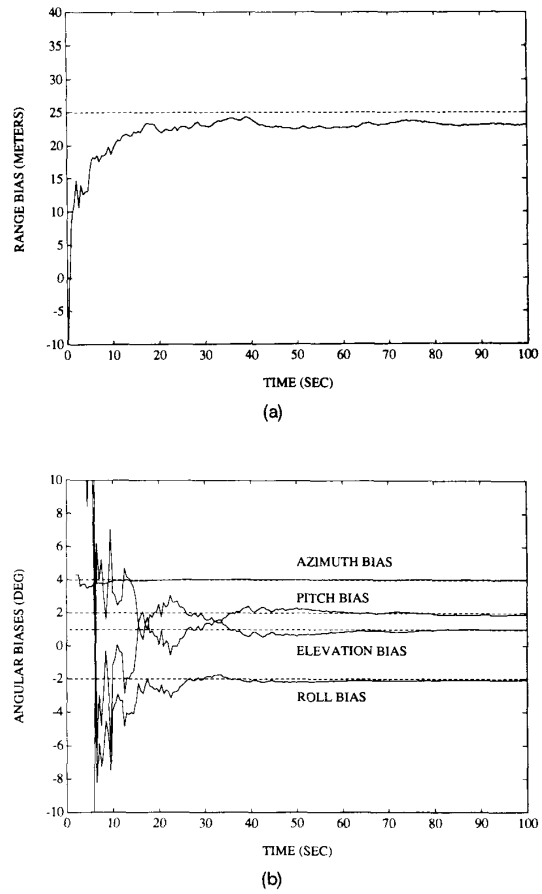

Fig. 5. Estimates of biases from Kalman filters.
(a) Range bias estimates. (b) Angular bias estimates.

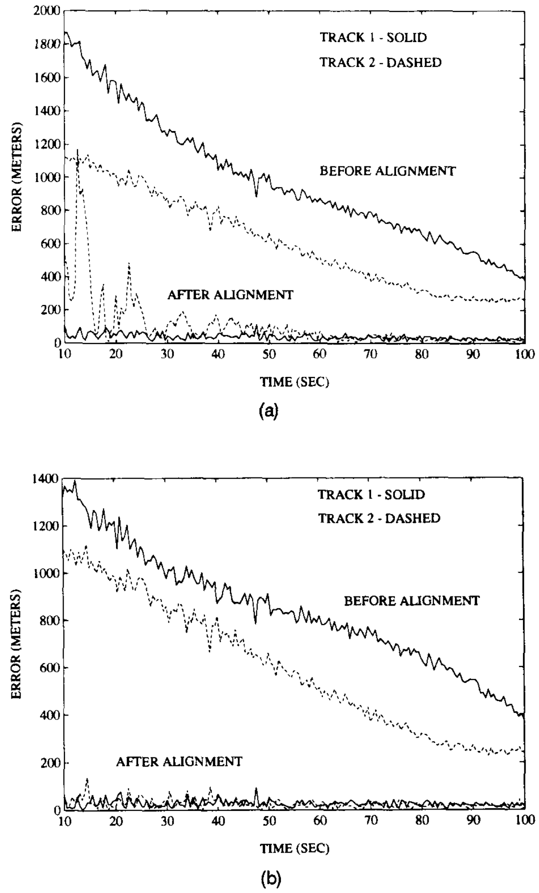

Fig. 6. Alignment errors in tracks before and after alignment.
(a) Total alignment error. (b) Alignment error in xy-plane.

represents a reduction in the total error by a factor of 25 for track 1 and a factor of 6 for track 2. The reason that the error was reduced more for track 1 is that it was used to generate the bias estimates. Most of the error in track 2 after the biases have been applied is due to the poor alignment of the z-coordinates for track 2 in the first 40 s. The alignment in the xy-plane is much better. The average error in the xy-plane before the biases are applied is 841 m for track 1 and 596 m for track 2; but after the biases have been applied it reduces to 26 m for track 1 and 25 m for track 2. This represents a reduction in the error in the xy-plane by a factor of 32 for track 1 and a factor of 24 for track 2. Thus, the proposed algorithm performed very well in reducing the alignment error in the xy-plane, but it required quite a long time before the error in the z-coordinate was reduced. This occurred because of the long time required for the elevation, pitch, and roll bias estimates to converge to their true values. This problem can be reduced by using more than one common target to estimate the biases.

V. SUMMARY AND CONCLUSIONS

The problem of aligning two 3-D sensors using common targets tracked by the sensors was examined, and an alignment algorithm was developed to relatively align the sensors. The alignment is accomplished using five bias parameters: one range bias and four angular biases. Two Kalman filters are used to estimate the biases: a first-order filter for the range bias and a fourth-order filter for the angular biases. An observability analysis shows that there should be no problem in estimating the range bias, but problems do occur in estimating the angular biases when the common target is a radially inbound or radially outbound target.

The alignment algorithm is applicable to those situations where there are no sensor location errors, the distance between the sensors is small (e.g., sensors located on the same platform), the magnitude of the bias errors is small, and these errors do not change with time or vary slowly with time. It is also assumed that the sensors are synchronized in time and have the same update rates. For illustrative purposes, the alignment algorithm was applied to simulated data from two sensors that were tracking two common targets. Only one of the targets was used to generate the estimates of the biases, which were then applied to both tracks. Each of the sensors had realistic values for their measurement errors. The filters converged within 30 to 40 s to values of the range, azimuth, elevation, pitch, and roll biases that were close to their actual values. Utilizing these bias estimates, it was possible to obtain a dramatic 24-fold reduction in the alignment error in the xy-plane, but it required at least 40 s before the error in the z-coordinate was reduced. This occurred because of the long time required for the elevation, pitch, and roll bias estimates to converge to their true values. This problem can be reduced by using more than one common target to estimate the biases.

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Digital Baseband Processor for the GPS Receiver Modeling and Simulations

A Global Positioning System (GPS) receiver has been modeled mathematically and implemented in software. The digital baseband processor of the receiver performs the maximum likelihood estimations of the GPS observables. The following issues are discussed: 1) the fundamentals of the digital GPS receiver, 2) the modeling of the digital baseband processor, and 3) the performance of the modeled static and dynamic receivers. The software-based receiver is more flexible, less expensive and more accurate compared with hardware receivers in receiver designs and GPS system performance analysis.

I. INTRODUCTION

The NAVSTAR (Navigation Satellite Timing and Ranging) Global Positioning System (GPS) is a satellite-based, worldwide, all-weather navigation and timing system [1]. The GPS is designed to provide precise position, velocity, and timing information on a global common grid system to an unlimited number of suitably equipped users. A GPS receiver is the key for a user to access the system and it has undergone extensive development since the GPS concept was initiated in 1973. The GPS signal structure, the fundamental principles and operations of the receivers, the basic technical approaches to high accuracy and low cost hardware receiver designs are discussed in [2–4]. A functional description of signal processing in the Rogue GPS prototype receiver is presented in

Manuscript received November 17, 1992; revised January 26, 1993.

IEEE Log No. T-AES/29/4/10999.