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**Renegotiation Design with  
Unverifiable Information**

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## ABSTRACT

This paper considers a buyer-seller relationship with observable but unverifiable investments and/or random utility parameters. In such situations, it is known that contract renegotiation may prevent the implementation of first-best outcomes. In this paper, we show however that efficient investments and optimal risk-sharing can typically be achieved provided the initial contract is able to monitor the ex-post renegotiation process. Specifically, we focus on the following two features of renegotiation design. First, default options in case renegotiation breaks down ; second, the allocation of all bargaining power to either contracting party. Moreover, we show that these two features can be obtained in standard Rubinstein bargaining games through contractual provisions, such as specific-performance clauses and penalties for delay (or, equivalently, financial "hostages" refundable without interest).

## KEY WORDS

INVESTMENT - INCOMPLETE CONTRACTS - RENEGOTIATION - UNVERIFIABLE INFORMATION -  
RISK-SHARING - OPPORTUNISM - HOSTAGES

## RESUME

Ce papier considère des relations entre un acheteur et un revendeur en présence d'investissements initiaux et de facteurs aléatoires observables mais non vérifiables. Dans de telles situations, il a été montré que la possibilité de renégocier les contrats initiaux peut empêcher d'atteindre l'efficacité de premier rang. Dans ce papier nous montrons toutefois qu'il est possible d'inciter à investir correctement et de partager les risques de manière efficace. Dès lors que le contrat initial peut affecter le processus utilisé à ce poste pour renégocier plus précisément, nous nous intéressons à deux régles de renégociation. La première consiste en des options par défaut, utilisées en cas de rupture de négociation. La deuxième consiste à allouer le pouvoir de marchandage à l'un ou l'autre des contractants. De plus, nous montrons que ces deux règles de renégociation peuvent être obtenues via des causes raisonnables dans des modèles standards de marchandage à la Rubinstein.

## I. INTRODUCTION

Contracts play a crucial role in situations involving important investments in relationship-specific capital. Once such investments have been sunk, each party is to some extent "locked-in", and therefore vulnerable to opportunistic behavior from the other parties. Williamson (1979) and Klein-Crawford-Alchian (1978) have already argued that the risk of *ex post* breach or renegotiation, when an unspecified event occurs, can lead to underinvestment in transaction-specific capital. Holmstrom (1982) has formalized this argument as a "moral hazard in teams" problem, where the (unobservable and therefore non-contractible) investments of several parties contribute to the total surplus. Underinvestment then follows from the fact that at least one party does not capture the full benefits of an increase in his investment.

More recently, Hart-Moore (1988) have obtained a similar underinvestment result, assuming that investments and future contingencies, although *ex post* observable, are unverifiable by third parties.

In this paper, we argue that unverifiability is not sufficient to explain underinvestment. More specifically, we show that the underinvestment problem can often be overcome by contractual renegotiation design, that is, by the design of rules that govern the process of renegotiation.

Such rules, while overlooked in contract theory, are sometimes observed in practice. For instance, joint-venture contracts, which typically require specific investments, outline in certain cases a procedure to be followed in case one of the parent firms wants to quit the joint venture.<sup>(1)</sup> Construction

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(1) One such contract between the French car manufacturer Renault and an American subcontractor specified for instance that the party willing to quit had to set a price, at which the other party could choose between buying his partner's shares or selling his own to his partner. The provisions also stipulated deadlines for each step.

contracts sometimes include *per diem* penalties when some pre-specified deadlines are not met. For office building projects, some contracts stress that only the architect can make new price proposals in case a contracting party wishes to modify the initial project. If a base-ball player wants to quit his team before the end of his contract, he must abide by bargaining rules which, although not formally written, are thoroughly followed.<sup>(2)</sup>

In this paper we show that renegotiation design can be quite powerful. Efficient investments and risk-sharing can be achieved in a variety of situations, assuming the initial contract can: (1) specify a default option in case renegotiation fails or is unnecessary; and (2) assign all bargaining power to one or the other party. In general, bargaining powers and default options have to be made contingent on signals sent by the parties regarding the state of nature. If parties are risk-neutral, efficiency can however be achieved by simply choosing a non-contingent price-quantity pair as default option and by assigning all bargaining power to one party.

Our analysis stands in contrast with the underinvestment result of Hart and Moore, although their informational framework is similar to ours. The difference comes from their assumption that trade is voluntary and that courts cannot enforce contractually specified levels of trade, even upon request by one party. As a consequence, no trade is the only possible default option for renegotiation. In U.S. legal language, Hart and Moore focus on "at-will contracts", while we allow "specific performance contracts".

The anecdotal evidence mentioned earlier suggests that renegotiation design may take various forms in practice. We focus here on only two instruments, which we show are sufficient to support the above contracting assumptions (1) and (2). We model renegotiation as a standard infinite horizon bargaining model à la Rubinstein, and assume that the initial contract

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(2) We thank Peter Diamond for having introduced us to some of the intricacies of the American and National leagues.

can introduce default options and specify either initial transfers ("hostages") refundable without interest upon agreement or, equivalently, *per diem* penalties to be paid by one party to the other until an agreement is reached. Both instruments influence the parties' relative degree of impatience, and thus their bargaining powers. For large enough hostages or penalties, one party ends up as residual claimant. In other words, the contracting features (1) and (2) are reasonable in the sense that they can be achieved by simple, commonly observed instruments (default options, hostages or penalties).

Our paper also contributes to the implementation literature, which has stressed that unverifiability problems can be circumvented by revelation mechanisms (Maskin (1977), (1985), and Moore-Repullo (1988)). These mechanisms typically use the threat of inefficient punishments to deter "deviations". Allowing for mutually profitable renegotiation *a priori* restricts what can be achieved through revelation mechanisms. This paper shows that renegotiation design can circumvent this restriction.

The outline of the paper is as follows. Section II introduces and motivates our three assumptions on renegotiation design: the requirement of *ex post* efficiency, the ability of the parties to impose default options, and the ability to allocate all bargaining power to either party in the *ex post* renegotiation stage. Section III analyzes investment incentives under risk-neutrality; Section IV focuses on efficient risk sharing in the absence of investment decisions; Section V investigates the possibility of simultaneously achieving efficient risk sharing and *ex ante* efficient investments through appropriate renegotiation design; finally, Section VI discusses our results in relation to the underinvestment and implementation literatures. (3)

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(3) Renegotiation in the context of observable but unverifiable information has already been analyzed by Hart-Moore (1988), Green-Laffont (1988),

## II. EX POST RENEGOTIATION DESIGN

### 1. Basic framework

We consider a bilateral trading relationship with specific investments. At date  $t_1$ , the buyer (B) and the seller (S) choose investment levels  $i \in I$  and  $j \in J$ . Together with investment choices, the realization of a random variable  $\theta \in \Theta$  at date  $t_2$  affects the future benefits from trade. Finally, trade occurs only once and takes place no sooner than  $t_3$ , with  $t_3 > t_2 > t_1$ . A trade  $r \in R = R_+^2$  is summarized by a price  $p \in R_+$  and a quantity  $q \in R_+$ . Von Neumann-Morgenstern utilities are given by  $U_S(r, i, \theta)$  and  $U_B(r, j, \theta)$ .

We are interested in the implementation of a first-best allocation, consisting in *ex ante* efficient levels of investment  $(i^*, j^*)$  and *ex post* contingent trades  $(r_\theta^*)_{\theta \in \Theta}$ . In order to implement this allocation, the two parties engage in a contractual relationship at an initial date  $t_0 < t_1$ . Contractual possibilities are limited by the assumption that the random variable  $\theta$  and the investments  $(i, j)$ , although observable by the two agents, are not verifiable by any third party. Thus, a contract cannot directly specify investment targets, nor can it make trade contractually contingent upon the realization of the random variable  $\theta$ .

To circumvent this unverifiability problem, the parties could include in the initial contract a "revelation mechanism" in the spirit of Maskin (1977) or Moore-Repullo (1988). That is, the contract could specify trades contingent upon messages sent by the parties after investments have been sunk

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Maskin-Moore (1988), Rubinstein-Wolinsky (1992) and Hermalin-Katz (1991). In Hart-Moore, Rubinstein-Wolinsky and Hermalin-Katz, renegotiation is modeled as a specific bargaining game, which occurs before any irreversible decision is taken. In Green-Laffont and Maskin-Moore renegotiation is instead treated as an exogenous "black box" which describes how an inefficient agreement is replaced *ex post* by an efficient one. MacLeod-Malcomson (1989) also focus on (mainly unilateral) investment, but in a world of simple contracts.

and the random variable  $\theta$  has been realized (see Figure II.1). In the trading situations considered here, there exists an adequate revelation mechanism which induces the parties to choose first-best investments, credibly reveal the true realization of  $\theta$  and finally implement first-best contingent trades.

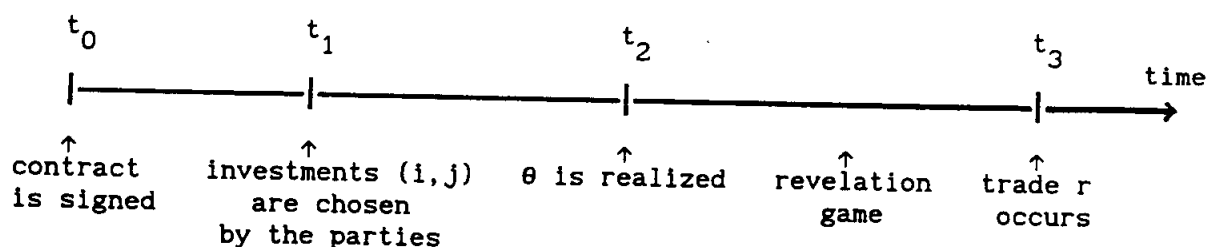


Figure II.1

Such mechanisms may however not be robust to the possibility of contract renegotiation. For example, if messages have been sent which induce an *ex post* inefficient trade given  $i$ ,  $j$  and  $\theta$ , the parties have an incentive to renegotiate the initial agreement. In turn, the prospect of such renegotiation affects the parties' investment and revelation behaviors and may prevent the implementation of the first-best allocation (Hart-Moore (1988)).

In this paper, we are concerned with the possible influence of the initial contract on future renegotiation. Through most of the paper only ex post renegotiation, taking place after  $i$ ,  $j$  have been chosen and  $\theta$  has been realized, is relevant, so that we rule out for the moment any earlier renegotiation. Subsection II.4 briefly addresses other types of renegotiation, which may become relevant when, as in Section V, investment and risk-sharing issues are simultaneously at stake.

More specifically, we assume that renegotiation occurs at date  $t_3$ , just before trade can take place. We model this *ex post* renegotiation as an exogenous mapping  $h$ , parameterized by a vector of contractible variables, which for each state of nature  $(i, j, \theta)$  specifies the trade that actually takes place. This mapping characterizes the scope of renegotiation design; the



richer the set of contractible parameters, the more powerful renegotiation design will be.

We assume that the initial contract can only influence the starting point of this renegotiation,  $r \in R_+^2$ , which we call a default option, and the allocation of the bargaining power between the two parties,  $\alpha \in A$  (defined precisely below). Neither  $\alpha$  nor  $r$  can be made directly contingent upon  $\theta$ ,  $i$  or  $j$ , which are not verifiable, and we call a simple contract a pair  $(\alpha, r) \in A \times R_+^2$ . However, these can be made contingent indirectly upon  $\theta$ ,  $i$  and  $j$ , using a revelation game between  $t_2$  and  $t_3$ . We restrict attention to direct revelation games, in which the two parties are asked to simultaneously announce the state of nature, and call a complex contract a mapping from  $(I \times J \times \Theta)^2$  into  $A \times R_+^2$  which stipulates, for each pair  $((i_B, j_B, \theta_B), (i_S, j_S, \theta_S))$  of announcements of states of nature by the buyer and the seller, a "simple contract"  $(\alpha, r)$ .<sup>(4)</sup>

Whatever the initial (simple or complex) contract, *ex post* renegotiation actually "starts" from some  $(\alpha, r)$ ; we thus describe the mapping  $h$  as:

$$\begin{aligned} h(\alpha, r; \cdot): I \times J \times \Theta &\rightarrow R_+^2 & \text{(II.1)} \\ (i, j, \theta) &\rightarrow \hat{r} = h(\alpha, r; i, j, \theta) \end{aligned}$$

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(4) One could imagine more sophisticated contracts. According to the revelation principle, however, if there exists a revelation mechanism and an equilibrium of this game leading to the desired outcome, then there exists a direct mechanism for which truthful announcements constitute an equilibrium leading to the same outcome. Unfortunately, this direct mechanism may also generate other, undesired equilibria. Here, however, renegotiation leads to *ex post* efficient trades and all revelation games are sort of constant-sum games, in which all pure strategy equilibria are payoff-equivalent. Moreover, following Maskin-Moore (1988), Aghion-Dewatripont-Rey (1989) shows that undesired mixed-strategy equilibria can be eliminated by asking sequential rather than simultaneous announcements. Altogether, this implies that restricting attention to direct mechanisms involves a loss of efficiency only in peculiar situations where the desired allocation could be implemented only via mixed strategy equilibria.

As in bargaining theory, we can develop our ideas on renegotiation design by describing this mapping  $h$  either in an axiomatic way or as the reduced form of a non-cooperative bargaining game. Each approach has its own merits: the axiomatic approach (subsection II.2) provides a more transparent presentation of our assumptions, but a convincing defense of our axioms is best provided using a standard non-cooperative bargaining game to model contract renegotiation (subsection II.3).

## 2. Contracts and renegotiation: an axiomatic treatment

Here we consider the renegotiation mapping  $h$  as a "black box". Our assumptions about renegotiation design translate into three axioms. The first one reflects the parties' inability to prevent renegotiation, whereas the other two describe the parties' ability to monitor renegotiation.

Assumption 0: *Ex post* renegotiation leads to efficient trade:

$$\forall (\alpha, r) \in A \times R_+^2, \forall (i, j, \theta) \in I \times J \times \Theta,$$

$$h(\alpha, r; i, j, \theta) \text{ is } \textit{ex post} \text{ efficient given } (i, j, \theta).$$

Assumption 0 says that the parties cannot precommit at  $t_0$  not to take advantage of *ex post* renegotiation; moreover, since this renegotiation takes place under full information, its result is *ex post* efficient.

Assumption 1: Renegotiation leads to Pareto improvements:

$$\forall (\alpha, r) \in A \times R_+^2, \forall (i, j, \theta) \in I \times J \times \Theta:$$

$$U_B(h(\alpha, r; i, j, \theta), j, \theta) \geq U_B(r, j, \theta),$$

$$U_S(h(\alpha, r; i, j, \theta), i, \theta) \geq U_S(r, i, \theta).$$

This assumption means that the starting point,  $r$ , constitutes a default option which can only be voluntarily renegotiated; each party is protected against unilateral violations of the initial agreement by the other party.

Assumption 2: The contract can allocate bargaining power either entirely to the buyer or entirely to the seller:

$$A = \{B, S\}, \text{ and } \forall r \in R_+^2, \forall (i, j, \theta) \in I \times J \times \Theta:$$

$$U_S(h(B, r; i, j, \theta), i, \theta) = U_S(r, i, \theta).$$

$$U_B(h(S, r; i, j, \theta), j, \theta) = U_B(r, j, \theta),$$

Figure II.2 visualizes our three assumptions. It represents the *ex post* Pareto frontier, for a given state  $(i, j, \theta)$ , as well as the starting point of the renegotiation,  $R = (U_B(r, j, \theta), U_S(r, i, \theta))$ . Assumption 0 asserts that the outcome lies on the Pareto frontier. Assumption 1 states that the outcome lies between S (all bargaining power to the seller) and B (all bargaining power to the buyer). Assumption 2 finally says that the initial contract can make sure the outcome is S or B. (5)

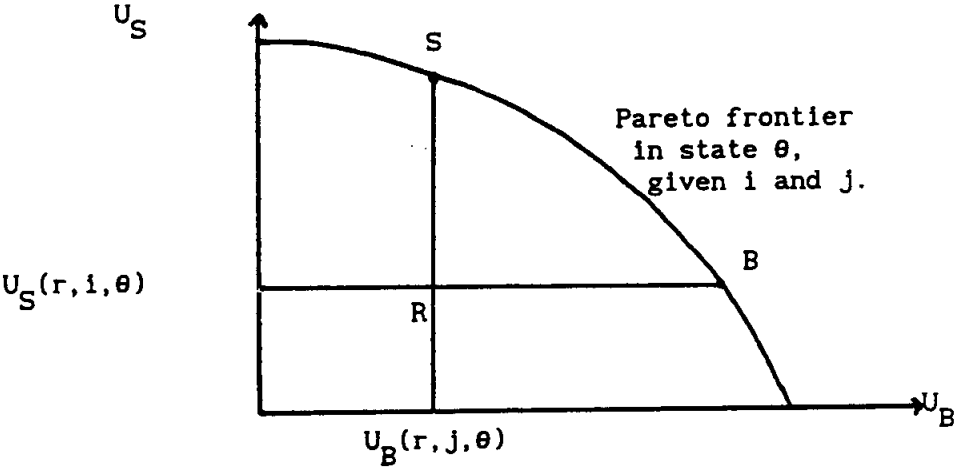


Figure II.2

(5) Allowing  $A \supset \{B, S\}$  would only enlarge implementation possibilities.

We can now define precisely the contract set which corresponds to the above assumptions. A simple contract is from now on a pair  $(\alpha, r) \in \Gamma_s \equiv A \times R_+^2$ , where  $A = \{B, S\}$ , and a complex contract  $\gamma \in \Gamma$  is a mapping from  $(I \times J \times \Theta)^2$  into  $\Gamma_s$ , which maps each pair of announcements of states of nature by the buyer and the seller into a simple contract. We identify the contract set with  $\Gamma$ , which of course formally includes  $\Gamma_s$ . Sections III to V ask whether simple or complex contracts can implement the first-best, that is, achieve optimal investments, risk-sharing and trades (see the beginning of each of these Sections for the appropriate definition of implementation).

### 3. Contracts and renegotiation as a non-cooperative game

In this subsection we show how contractual provisions can affect *ex post* renegotiation and support the axioms presented in the previous subsection. We first model the *ex post* renegotiation process which would take place in the absence of any contract as a standard bargaining model with discounting à la Rubinstein. Complete information and sequential bargaining ensure *ex post* efficiency (Rubinstein (1982)) as in Assumption 0. We then introduce two contractual provisions. The first one, which allows the two parties to impose a default option, yields Assumption 1 (Binmore-Rubinstein-Wolinsky (1986)). The second one yields Assumption 2 through the use of deadlines and penalties. <sup>(6)</sup>

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(6) The bargaining power could alternatively be monitored by specifying who can make successive offers (Rubinstein-Wolinsky (1992)); this however would suppose the ability to control the timing of the renegotiation process, which raises two types of issues. First, the renegotiation process used by the parties to rescind the initial contract can use various channels, some of them being impossible or prohibitively costly to control. Second, if indeed the renegotiation process can completely be controlled, the parties could as well commit to never renegotiate. In Aghion-Dewatripont-Rey (1989), we showed that monitoring the bargaining power could be achieved by superposing a contractual

a) *The ex post renegotiation process*

In keeping with most of the non-cooperative bargaining literature, we model renegotiation as a discrete time, infinite-horizon bargaining process with alternating offers. Renegotiation rounds take place at dates  $t_3 + n\Delta$ , for  $n \in \mathbb{N}$ . Trade still takes place only once, but can occur in any of the bargaining rounds. Because of discounting, *ex post* efficiency requires that trade take place at  $t_3$ . In each round, one party can make an offer  $r$ , which the other party can either accept or reject and wait  $\Delta$  to make a counter-offer. The equilibrium of such a game is unique and *ex post* efficient (Rubinstein (1982), Binmore (1982)), so that Assumption 0 is satisfied.

b) *Default options*

Let us assume that the initial contract allows each of the two parties to (unilaterally) impose a given trade  $r$ . This modifies the bargaining game, since the players can now stop it by imposing this default option. The precise timing of the modified game is determined by the bargaining technology. We assume, as in Binmore-Rubinstein-Wolinsky (1986), that the default option specified in the contract is available to the player not making the offer (see Figure II.3).<sup>(7)</sup>

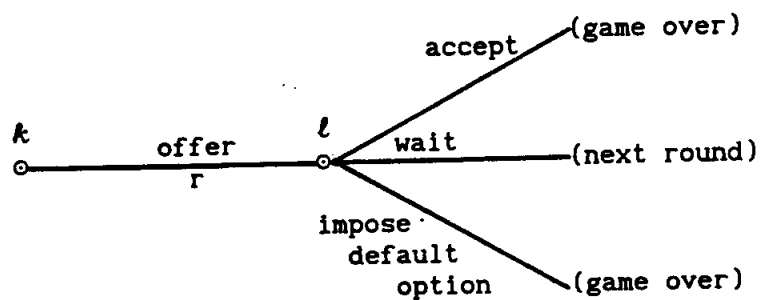


Figure II.3

sequential bargaining process upon the existing one, and making sure that the contractual process would be the dominant one.

(7) Our results do not depend crucially on the precise timing. Alternate timings are discussed in footnote 8.

In each round, one player ( $k$ ) makes an offer. The other player ( $l$ ) can either choose the default option, accept the offer (which is then executed), or wait  $\Delta$  and make a counter-offer in the next round. The game continues only in this last case. Binmore-Rubinstein-Wolinsky (1986) have shown that the outcome of such an infinite-horizon game with default option  $r$  is unique and as pictured in Figure II.4. Call  $E$  the unique outcome of the game without default option, which lies on the *ex post* Pareto-frontier,  $R$  the point corresponding to the default option  $r$ , and  $D$  the outcome of the game with this default option. If both parties prefer  $E$  to  $R$ ,  $D=E$ . If instead the seller (respectively, the buyer) prefers  $R$  to  $E$ ,  $D=B$  (respectively,  $D=S$ ). The renegotiation game with default option thus satisfies Assumption 1.

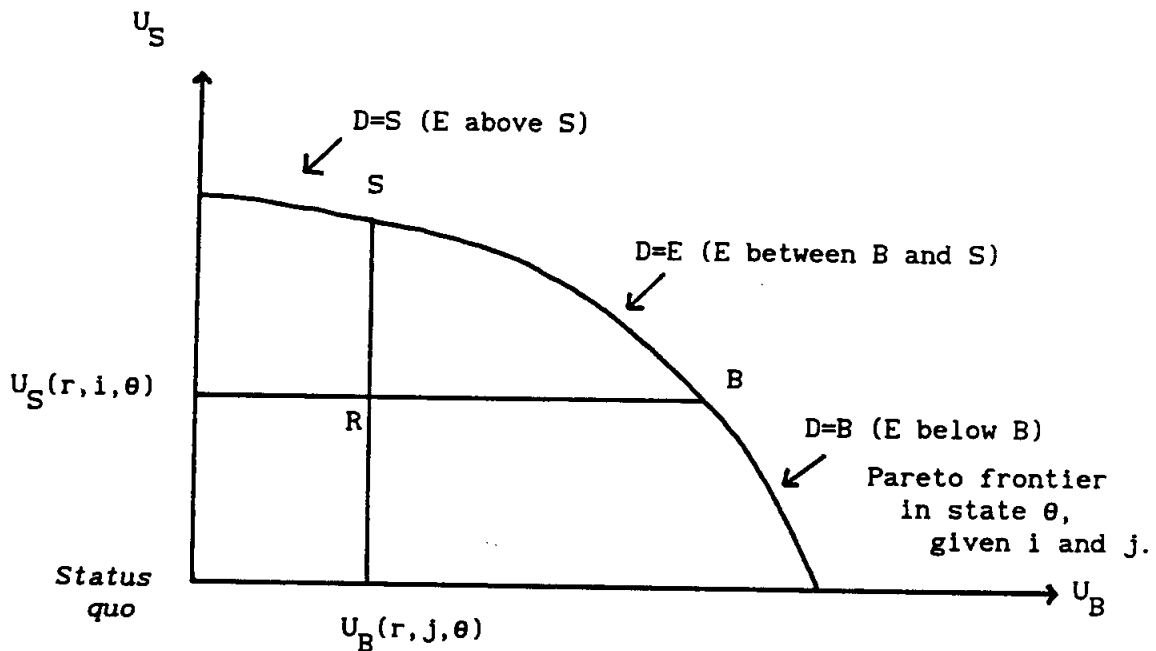


Figure II.4

c) *deadlines and penalties*

Allocating full bargaining power to one party (Assumption 2) can be achieved by making the default option sufficiently attractive to the other

party for all  $i, j$  and  $\theta$ , for example by penalizing this party if trade is delayed. Such a scheme satisfies Assumption 2 under the following conditions:

Condition C: The sets  $I, J$ , and  $\theta$  are compact and utility functions  $U_B$  and  $U_S$  are continuous.

Condition U:  $\forall q \in \mathbb{R}_+, \forall (i, j, \theta) \in I \times J \times \theta,$

$$\lim_{p \rightarrow +\infty} U_S(p, q, i, \theta) = +\infty,$$

$$\lim_{p \rightarrow +\infty} U_B(p, q, j, \theta) = -\infty.$$

Condition C gathers usual compactness and continuity assumptions, whereas condition U requires unboundedness of utilities with respect to prices.

Proposition II.1: Assume that at date  $t_3$ :

- both parties can impose trade  $(p_0, q_0)$  as a default option; (8)
- at some date  $t^* \geq t_3 + 2\Delta$ , party 1 must pay party 2 a monetary penalty if no trade has occurred until then.

Then under conditions C and U there exists  $\bar{z} \in \mathbb{R}$  such that for any penalty  $z \geq \bar{z}$  the unique subgame perfect equilibrium outcome of the above bargaining game is *ex post* efficient for all  $i, j$  and  $\theta$ , and gives party 1 a utility level which, when  $\Delta \rightarrow 0$  and  $t^* \rightarrow t_3$ , converges towards the reservation utility associated with  $(p_0, q_0)$ .

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(8) One could object that the result of the Proposition relies on specific assumptions about the timing of moves in the bargaining game (Figure II.3): Shaked (1987) has shown that there may be very different equilibria (an infinity of them, including inefficient ones) when one player can impose the default option just after the other player's refusal (i.e., without waiting for the next round). The idea is that this player can extract surplus from the other player by threatening to immediately impose the inefficient default option in case of refusal. This problem can however be circumvented as follows. Assume for instance that the seller has full bargaining power, and introduce a contractual provision according to which, before bargaining takes place, he can give the buyer a second default option (i.e., the buyer, and only this player, can choose between two trades whenever he decides to impose a default option). A good strategy for the seller is to give the buyer a

The proof is relegated in the Appendix. For high enough a penalty, the agent having to pay the penalty at  $t^*$  would thus prefer to impose  $(p_0, q_0)$  (or, rather, an efficient trade which gives him slightly more) instead of paying the penalty, even if he had full bargaining power afterwards. This party must thus have the possibility of imposing the default option (so that  $t^*$  has to be at least equal to  $t+2\Delta$ ). On the other hand, for  $\Delta$  small enough, the deadline  $t^*$  can be chosen arbitrarily close to  $t_3$ , so that extreme allocations of bargaining power can be approximated. (9)

Proposition II.1 imposes few restrictions on utilities, but may be quite demanding as for the required levels of monetary penalties (this "once-and-for-all" penalty could however be replaced with a sequence of small penalties, to be paid as long as trade is delayed). Another interpretation of this penalty scheme is that, at the beginning of the relationship, one party grants the other a financial "hostage" which is given back, without interest, when trade takes place. All these contractual provisions enable the two players to precommit themselves to giving all bargaining power to one party, starting from any arbitrary trade  $(p_0, q_0)$ , as assumed in Section II.3.

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second default option which is *ex post* efficient and gives the buyer (slightly more than) what he would get with the initial default option (point S in Figure II.4): then, whenever the buyer is about to impose a default option, he is induced to ignore the initial one and to give (almost) all the surplus to the seller. One can check that the equilibrium of this new game is unique and converges to S, whatever the precise timing of the game, when the delay between offers goes to zero. Therefore, the possibility of adding default options introduces enough contract flexibility to overrule in our framework Shaked's objection to the "outside option principle" of Binmore-Rubinstein-Wolinsky (1986).

(9) It is not necessary to set the deadline precisely at  $t_3+2\Delta$ ; the threat of penalty can be used without knowing exactly  $\Delta$ ; one could think for example of  $t-t_3$  as "one week", while  $\Delta$  could be "one day". Moreover, additional bargaining rounds could be contracted upon in order to speed up the process (that is, to generate smaller  $\Delta$ 's and allow a deadline  $t^*$  closer from  $t_3$ ).



#### 4. Other renegotiation opportunities

So far, we have implicitly limited renegotiation opportunities in two ways: (a) from  $t_3$  on, the parties are allowed to negotiate trades, but not bargaining rules (default options, deadlines, new contractual provisions, etc.); (b) renegotiation before  $t_3$  has not been considered.

The first issue is not a problem. At  $t_3$ , possibly after some revelation game, the parties are endowed with a simple contract. This contract generates a renegotiation game, which at any time has an *ex post* efficient continuation equilibrium. There are therefore no Pareto-improving bargaining rules, and we can assume, without loss of generality, that the two parties stick to the bargaining rules dictated by the simple contract.

Concerning renegotiation before  $t_3$ , two time intervals have to be distinguished:  $[t_2, t_3]$ , after  $\theta$  has been realized, and  $[t_1, t_2]$ , after  $i$  and  $j$  have been chosen but before  $\theta$  has been realized.<sup>(10)</sup>

Interval  $[t_2, t_3]$  is not relevant for the same reasons as above. Once investment levels have been chosen and  $\theta$  has been realized, the parties can predict the specific simple contract which will be the outcome of the revelation game and the *ex post* efficient trade which it will induce. Once again, Pareto-improving renegotiation is impossible.

The same is not necessarily true for interval  $[t_1, t_2]$ , at least if the parties are risk-averse (this issue is relevant only in Section V). For some investment choices, the revelation game prescribed by the initial contract may not induce optimal risk-sharing. There may thus be scope for Pareto-improving *interim* renegotiation, provided that investment choices are observed before the realization of  $\theta$ . We stress however in section V that even in that case

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<sup>(10)</sup>  $[t_0, t_1]$  is not relevant since no information has arrived and no action has been taken since the signature of the initial contract.

optimal contracts can be signed which are robust to any such interim renegotiation.

### III. INVESTMENT UNDER RISK-NEUTRALITY

This section analyzes how renegotiation design can provide correct incentives to solve Williamson's classical problem of underinvestment. Agents are assumed to be risk-neutral (section V adds risk-sharing concerns to this problem). We show that simple contracts achieve first-best efficiency.

Investment is assumed to be specific to the relationship and its cost has to be sunk before the realization of the state of nature. We moreover assume that the two parties are only interested in monetary payoffs:

$$U_S(p, q, i, \theta) = p - k(q, i, \theta) - \varphi(i) \quad (III.1)$$

$$U_B(p, q, i, \theta) = v(q, j, \theta) - p - \psi(j) \quad (III.2)$$

where  $k(q, i, \theta)$  is the seller's production cost and  $v(q, j, \theta)$  is the buyer's valuation;  $\varphi(i)$  and  $\psi(j)$  are the seller's and buyer's investment costs. Both objective functions are assumed to be twice continuously differentiable and concave with respect to trade and investment levels. The economically interesting case is one where incentives to invest are sensitive to expected levels of trade:  $v_{qj} \geq 0$  and  $k_{qi} \leq 0$ .<sup>(11)</sup>

The first-best levels of investment and trade,  $(i^*, j^*)$  and  $(q_\theta^*)_{\theta \in \Theta}$ , are characterized by the first-order conditions:<sup>(12)</sup>

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(11) When  $v_{qj} \equiv 0$  and  $k_{qi} \equiv 0$ , the level of trade does not affect the returns from investment and Williamsonian underinvestment is not an issue.

(12) For the sake of simplicity we concentrate on interior solutions.

$$\forall \theta \in \Theta, \quad k_q(q_\theta^*, i^*, \theta) = v_q(q_\theta^*, j^*, \theta) \quad (\text{III.3})$$

$$E_\theta [k_i(q_\theta^*, i^*, \theta)] + \varphi'(i^*) = 0 \quad (\text{III.4})$$

$$E_\theta [v_j(q_\theta^*, j^*, \theta)] - \psi'(j^*) = 0 \quad (\text{III.5})$$

Risk-neutrality makes the price structure irrelevant: only the average price matters, which determines how the surplus is divided between the parties. Moreover, for given investment levels, the state of nature uniquely determines the *ex post* efficient output level, which is therefore achieved through *ex post* renegotiation, whatever the starting point of the bargaining process and the relative bargaining power of the parties. Thus, if the parties choose efficient levels of investment, renegotiation in itself guarantees the implementation of the first-best allocation.<sup>(13)</sup>

We can now be more precise about the implementation problem. We focus in this Section on simple contracts, which turn out to be rich enough for our purpose. A simple contract  $(\alpha, r)$  generates a game, which can be described as follows: in the first stage, both parties simultaneously choose their investment levels  $i$  and  $j$ ; then Nature chooses the realization of the random variable  $\theta$ ; finally, given the state of nature  $(i, j, \theta)$ , the actual payoffs are  $U_B(h(\alpha, r; i, j, \theta), j, \theta)$  and  $U_S(h(\alpha, r; i, j, \theta), i, \theta)$ .

A simple contract is said to implement the first-best allocation if for all perfect equilibria of the corresponding game, the (expected) utilities coincide with the first-best levels of utility. Proposition III.1 shows that simple contracts can indeed implement the first-best in the current context:

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(13) More precisely, it guarantees that, given investment levels, the *ex ante* surplus (considered at some date before the realization of the state of nature), is equal to the optimal one. This surplus can then be correctly divided through adequate lump sum transfers.

Proposition III.1: <sup>(14)</sup> The following simple contract implements the first-best allocation: give all renegotiation power to the seller and take as a default option  $(p_0, q_0)$ , where  $q_0$  is defined by:

$$E_{\theta} [v_j(q_0, j^*, \theta)] = \psi'(j^*), \quad (\text{III.6})$$

and  $p_0$  gives the buyer his first-best expected level of utility.

Proof: Let  $\bar{q} = \max_{\theta \in \Theta} q_{\theta}^*$  and  $g = \min_{\theta \in \Theta} q_{\theta}^*$ . Since  $v_{qj} \geq 0$ , we have:

$$E_{\theta} [v_j(\bar{q}, j^*, \theta)] \geq \psi'(j^*) \geq E_{\theta} [v_j(g, j^*, \theta)] \quad (\text{III.7})$$

The continuity of  $v_j$  guarantees the existence of the desired  $q_0$ . Having no bargaining power, the buyer obtains  $v(q_0, j, \theta) - p_0$  if he initially invests  $j$ . By the definition of  $q_0$ ,  $j=j^*$  is a dominant choice for the buyer: the concavity of the buyer's objective function implies that, whatever the investment level chosen by the seller, the buyer chooses  $j=j^*$ . Furthermore, the mechanism defined above makes the seller residual claimant, thereby inducing him to choose  $i=i^*$  when the buyer chooses  $j=j^*$ .

Both parties thus choose correct levels of investments. *Ex post* renegotiation ensures that efficient trades take place, and  $p_0$  is chosen so as to yield the desired division of total surplus.

Q.E.D.

Two contractual instruments are used to reach the investment targets: the allocation of all bargaining power to one party, who becomes residual claimant, and the adequate choice of the default option, which gives

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(14) After writing an early version of this paper, we learned that Chung (1991) had independently provided a similar result.

appropriate investment incentives to the other party.<sup>(15)</sup> First, given the chosen levels of investment, *ex post* renegotiation yields the corresponding efficient surplus. Second, the party who has full bargaining power therefore gets all returns from his own investment, minus a constant sum (the other party's surplus), which is independent of his own investment. Thus, if the party without bargaining power correctly chooses his investment, the other party is induced to also correctly invest. It is at first more surprising that the party without bargaining power also invests optimally. This is because, thanks to the default option  $q_0$ , he can "count" on a trade level which is "equivalent" to the expected first-best trade level, from his own point of view. In other words, this party is locally residual claimant: by cutting  $j$  from  $j^*$ , the party's loss of utility from the outside option  $(p_0, q_0)$  equals the loss in total expected quasi-rents.

Since choosing a higher default level of trade would give the buyer incentives to overinvest, it is possible to deal with one-sided direct externalities as well. Let us for example replace (III.1) by:

$$U_S(p, q, i, \theta) = p - k(q, i, j, \theta) - \varphi(i) \quad (\text{III.1}')$$

In the characterization of the first-best allocation,  $j$  appears in the cost components of (III.3) and (III.4), while (III.5) becomes:

$$E_\theta [v_j(q_\theta^*, j^*, \theta)] - k_j(q_\theta^*, i^*, j^*, \theta) = \psi'(j^*) \quad (\text{III.5}')$$

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(15) Only one party needs to be able to impose the default option, and the first-best allocation could as well be implemented by allocating the bargaining power to the buyer; the quantity  $q_0$  defined in the Proposition must in that case satisfy  $E_\theta [k_i(q_0, i^*, \theta)] = -\varphi'(i^*)$ , whereas the price  $p_0$  must guarantee the seller his first-best level of expected utility.

The optimal mechanism remains exactly as in Proposition III.1, with  $j^*$  correctly redefined (which amounts to raising the default level of trade). Note that bilateral direct externalities cannot be dealt with by our mechanisms: the party with full bargaining power does not internalize any direct externality.

#### IV. RISK-SHARING WITHOUT INVESTMENT

This section addresses risk-sharing issues, ruling out investment decisions. The first-best allocation is now characterized by an efficient risk-sharing rule  $r^*$ , which associates a price/quantity pair with each possible realization of uncertainty.

In contrast with the risk-neutrality case analyzed above, implementing the first-best requires the introduction of complex contracts, making bargaining powers and default options contingent on announcements about the state of nature. The timing of the relationship is thus as in Figure IV.1:

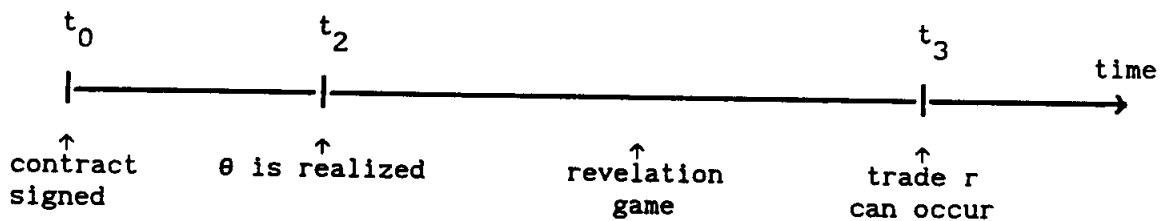


Figure IV.1

Each complex contract  $\gamma$  (here, a mapping from  $\Theta^2$  into  $A \times R_+^2$ ) generates a game, described as follows: first, Nature chooses the realization of the random variable  $\theta$ ; then the two parties simultaneously send messages  $\theta_S$  and  $\theta_B$ ; given the state of nature  $\theta$ , the actual payoffs are  $U_B(h(\gamma(\theta_S, \theta_B)); \theta), \theta$  and  $U_S(h(\gamma(\theta_S, \theta_B)); \theta), \theta$ .

A complex contract is said to implement the first-best if for all perfect equilibria of the corresponding game, the (expected) utilities coincide with the first-best utility levels.

Risk-aversion can be introduced in various ways: it may for example affect total surplus (Hart-Moore (1988)) or monetary transfers only (Green-Laffont (1988)). We follow the first route and moreover assume that uncertainty is solely generated by trade. Specifically we assume that the two parties' utility functions are given by:

$$U_B(p, q, \theta) = u_B(v(q, \theta) - p) \quad (IV.1)$$

$$U_S(p, q, \theta) = u_S(p - k(q, \theta)), \quad (IV.2)$$

where  $u_B(\cdot)$  and  $u_S(\cdot)$  are increasing concave functions and  $\forall \theta \in \Theta$ ,  $v(0, \theta) = k(0, \theta) = 0$ . We know from Borch (1962) that the first-best *ex post* utility levels vary co-monotonically,<sup>(16)</sup> that is:

$$\begin{aligned} \forall (\theta_1, \theta_2) \in \Theta^2, & \quad ( U_B(p_{\theta_1}^*, q_{\theta_1}^*, \theta_1) < U_B(p_{\theta_2}^*, q_{\theta_2}^*, \theta_2) ) \\ \Leftrightarrow & \quad ( U_S(p_{\theta_1}^*, q_{\theta_1}^*, \theta_1) < U_S(p_{\theta_2}^*, q_{\theta_2}^*, \theta_2) ). \end{aligned} \quad (IV.3)$$

We can then say that a state  $\theta_2$  is "better" than another state  $\theta_1$ . The following proposition uses this co-monotonicity property to implement the first-best:

Proposition IV.1: Under the above assumptions on the utility functions, the following complex contract implements the first-best: ask the two

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(16) For an interior solution, for example, first-order conditions imply  $u_B'(v(q_\theta, \theta) - p_\theta) = \lambda u_S'(p_\theta - k(q_\theta, \theta))$  for some multiplier  $\lambda$ ; co-monotonicity then follows from the concavity of  $u_B$  and  $u_S$ .

parties to announce the state of nature, and:

- if the parties announce the same state  $\theta$ , renegotiation starts from the corresponding first-best trade  $(p_{\theta}^*, q_{\theta}^*)$ ;
- if one party announces a "better" state than the other, full bargaining power is granted to the "optimistic" party and renegotiation starts from  $(p, q)$ , where  $q=0$  and  $p$  gives the "pessimistic" party the first-best utility level corresponding to a state of nature which can be either of the announced ones or, more generally, any arbitrarily chosen state between these two ones, according to the co-monotonicity order.

Proof: It suffices to show that truthtelling is a Nash equilibrium. (As stressed in Section II, equilibrium payoffs are unique, using a constant-sum game argument.) Consider a unilateral deviation from truthtelling, say by the buyer. If he lies by being "pessimistic", all bargaining power is given to the seller and renegotiation starts from  $(p, q) = (p_{\theta}^* - v(q_{\theta}^*, \theta), 0)$ , where the state  $\theta$  is at best as good as the true state. Therefore the buyer ends up with a lower utility level. If the buyer lies by being "optimistic", he has all bargaining power and renegotiation starts from  $(p, q) = (p_{\theta}^*, -k(q_{\theta}^*, \theta'), 0)$ , where the state  $\theta'$  is least as good as the true state of nature. Thus the seller gets the first-best utility level corresponding to a better state, implying that the buyer is not better off deviating.

Q.E.D.

In contrast with Section III, this proposition uses a revelation game. This revelation game avoids renegotiation in equilibrium, at the cost of contractual complexity.



Note that co-monotonicity is satisfied when parties are only interested in their monetary payoffs, but obtains in other cases as well.<sup>(17)</sup> When co-monotonicity fails the contracts described in the previous proposition do not work any more. Other mechanisms may still implement first-best efficiency, using different default options in case of disagreement. An example is provided in the Appendix.

## V. INVESTMENT AND RISK AVERSION

Let us now introduce risk-sharing as an additional concern in the investment problem. The first-best allocation is now characterized not only by the efficient levels of investment  $i^*$  and  $j^*$  and by the "average" sharing of the surplus, but also by an efficient risk-sharing rule  $r^*$ , which associates a price/quantity pair with each possible realization of uncertainty.

As stressed in subsection II.4, renegotiation is here potentially relevant not only at the *ex post* stage, but also at the *interim* stage, once investment levels have been chosen but the realization of  $\theta$  is yet unknown. At that stage, *interim* efficiency entails both *ex post* efficiency and efficient risk-sharing, given the chosen investment levels. As before, *ex post* renegotiation guarantees the implementation of efficient contingent trades. Moreover, the parties can agree on a revelation game which, for given  $i$  and  $j$ , implement efficient risk-sharing. However, since the revelation game cannot be made contingent on actual, unverifiable investment levels, there is

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(17) For example, suppose that the two parties' objectives are given by  $U_B(p, q, \theta) = u(q, \theta) - v(p)$  and  $U_S(p, q, \theta) = w(p) - k(q, \theta)$ , with  $w' > 0$ ,  $v' > 0$ ,  $w'' < 0$  and  $v'' > 0$ , and  $k(0, \theta) = v(0, \theta) = 0$ ,  $u_\theta \geq 0$ ,  $u_{\theta q} \geq 0$ ,  $k_\theta \leq 0$  and  $k_{\theta q} \leq 0$ . Then first-best payoff co-monotonicity obtains provided  $\theta' < \theta''$  implies  $k(q_\theta^*, \theta') > k(q_\theta^*, \theta'')$ , which is satisfied if the effect of  $\theta$  on the buyer's valuation is not too large in comparison with its effect on the seller's cost.

scope for *interim* renegotiation whenever the initially agreed revelation game is not adapted to the investment levels that are actually chosen.

Whether such scope exists much depends on when investment choices are observed. If no party observes the other party's investment before the realization of the variable  $\theta$ , then there is no room for *interim* renegotiation. Otherwise, implementation possibilities *a priori* depends on the extent to which an initial contract can influence renegotiation at that stage. We will first suppose that investment levels are not observed before the realization of  $\theta$ , and thus assume away *interim* renegotiation. In that context, we construct a contract which implements the first-best through *ex post* renegotiation design. We then show that this contract is robust to the introduction of any Pareto-improving *interim* renegotiation, even if investment levels are observed before the realization of  $\theta$ . In other words, *ex post* renegotiation design can be sufficient for achieving the first-best, even in the presence of *interim* renegotiation, and whatever the particular form of this renegotiation.

As in the previous Section, a complex contract  $\gamma \in \Gamma$  makes the bargaining power and default options for the *ex post* renegotiation stage contingent on announcements about the state of nature; the state of nature, however, now includes  $i$  and  $j$  as well as  $\theta$ .

The game generated by such a contract  $\gamma$  is as follows: first, the two parties choose their own investment levels; then Nature chooses  $\theta$  and the two parties observe  $i$ ,  $j$  and  $\theta$ ; finally, both parties announce a "state" of nature  $(i, j, \theta)$ , leading to a default option  $r$  and an allocation of bargaining powers  $\alpha$  for the *ex post* negotiation stage; this pair  $(\alpha, r)$  determines the trade that actually takes place after *ex post* renegotiation,  $h(\alpha, r; i, j, \theta)$ . We say that the contract  $\gamma$  weakly implements the first-best allocation if for at least one perfect equilibrium of the corresponding game, the (expected) utilities coincide with the first-best levels of utility. Note that, in

contrast with Sections III and IV, we do not require here the uniqueness of equilibrium payoffs.

We study the case where the two parties are only interested in monetary payoffs (we briefly discuss at the end of this section how the introduction of wealth effects complicates the analysis). We also assume that  $\theta$  is discrete and for notational simplicity we pose  $\theta=(\beta,\sigma)$ , where  $\beta\in\{\underline{\beta},\bar{\beta}\}$  and  $\sigma\in\{\underline{\sigma},\bar{\sigma}\}$  are scalars, and write utilities as:

$$U_S(p,q,i,\theta) = u_S(p-k(q,i,\sigma)-\varphi(i)), \quad (V.1)$$

$$U_B(p,q,j,\theta) = u_B(v(q,j,\beta)-p-\psi(j)). \quad (V.2)$$

with the usual assumptions  $v(0,j,\beta)=k(0,i,\sigma)\equiv 0$ ,  $v_\beta \geq 0$ ,  $k_\sigma \leq 0$ ,  $v_{jj} \leq 0$  and  $k_{ii} \geq 0$ .

The first-best allocation consists of: (i) *ex ante* efficient investment levels,  $i^*$  and  $j^*$ ; (ii) *ex post* efficient levels of trade,  $q_\theta^*$ , characterized by  $v_q(q_\theta^*,j^*,\beta) = k_q(q_\theta^*,i^*,\sigma)$ , for  $\theta=(\beta,\sigma)$ ; (iii) prices  $p_\theta^*$  determining the relative shares of quasi-rents,  $B_\theta^* = v(q_\theta^*,j^*,\beta)-p_\theta^*$  and  $S_\theta^* = p_\theta^*-k(q_\theta^*,i^*,\sigma)$ , so as to yield efficient risk-sharing, which implies co-monotonicity:

$$\forall (\theta_1, \theta_2) \in \Theta^2, B_{\theta_1}^* < B_{\theta_2}^* \Leftrightarrow S_{\theta_1}^* < S_{\theta_2}^* \Leftrightarrow W_{\theta_1}^* \equiv B_{\theta_1}^* + S_{\theta_1}^* < W_{\theta_2}^* \equiv B_{\theta_2}^* + S_{\theta_2}^*. \quad (V.3)$$

It can thus be said that a state  $\theta_2$  is better than another state  $\theta_1$ , which is denoted by  $\theta_2 > \theta_1$ . From now on, define  $\theta' \equiv (\bar{\beta}, \underline{\sigma})$  and  $\theta'' \equiv (\underline{\beta}, \bar{\sigma})$ . That is, cost and benefit are highest in state  $\theta'$  and lowest in state  $\theta''$ . (Note that  $\theta'$  can be either better or worse than  $\theta''$ ).<sup>(18)</sup> Lastly, for any pair of quantities  $(q', q'')$ , define the contract  $\gamma_{q', q''} \in \Gamma$  as follows :

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(18) The analysis which follows implicitly assumes that states  $(\bar{\beta}, \underline{\sigma})$  and  $(\underline{\beta}, \bar{\sigma})$  do occur. Otherwise,  $\theta'$  should be defined as the state in which the buyer's valuation is maximal, among those in which the seller's cost is the highest, and  $\theta''$  as the state which minimizes the seller's cost among those in which the buyer's valuation is the lowest.

$$\begin{aligned} \gamma_{q', q''}(i_B, j_B, \theta_B, i_S, j_S, \theta_S) = & ((q', p' = k(q', i^*, \sigma) + S_{\theta}^*), B) \\ & \text{if } (i_B, j_B) \neq (i^*, j^*). \\ & ((q'', p'' = v(q'', j^*, \beta) - B_{\theta''}^*), S) \\ & \text{if } (i_B, j_B) = (i^*, j^*) \\ & \text{and } (i_S, j_S) \neq (i^*, j^*), \\ g^*(\theta_B, \theta_S) & \text{ if } (i_B, j_B) = (i_S, j_S) = (i^*, j^*) \end{aligned}$$

$$\begin{aligned} \text{where } g^*(\theta_B, \theta_S) = & ((0, S_{\theta_B}^*), B) & \text{if } \theta_B > \theta_S, \\ & ((0, -B_{\theta_S}^*), S) & \text{if } \theta_B < \theta_S, \\ & ((q_{\theta}^*, p_{\theta}^*), B \text{ or } S) & \text{if } \theta_B = \theta_S = \theta. \end{aligned} \quad (V.4)$$

This contract can be described as follows : each party can agree or disagree on the fact that efficient investment levels have been chosen. If none of them disagree, a game  $g^*$  similar to the contracts used in Section IV is played. If only the seller disagrees, he obtains full bargaining power for the *ex post* renegotiation stage, with a default option  $q''$  and a price  $p''$ ;  $p''$  is computed so as to leave quasi-rents of  $B_{\theta''}^*$  to the buyer if  $\theta = \theta''$ , provided his investment level is efficient. If the buyer disagrees, he obtains full bargaining power, with a starting point for the renegotiation,  $(q', p')$ , which for  $\theta = \theta'$  would give the seller  $S_{\theta}^*$ , if his level of investment was efficient.

**Proposition V.1:** Suppose that utilities are given by (V.1) and (V.2), and that  $v_j, v_{\beta}, k_i, k_{\sigma}, u_B$  and  $u_S$  are continuous and unbounded when  $q$  increases. Then there exist  $q'$  and  $q''$  such that the contract  $\gamma_{q', q''} \in \Gamma$  defined by (V.4) weakly implements the first-best allocation.

The detailed proof is presented in the Appendix, where it is shown that the contract  $\gamma_{q',q''}$  generates a game, for which investing correctly and announcing truthfully the state of nature is an equilibrium, which implements the first-best. We sketch here the main steps.

If  $(i,j) = (i^*,j^*)$ , the game  $g^*$ , similar to those used in section IV, induces efficient risk sharing and, through *ex post* renegotiation, efficient trades. The only problem is thus to avoid (unilateral) deviations from efficient investment levels. Since such deviations are *ex ante* Pareto-inefficient, it suffices to show that they cannot reduce the other party's payoff.

When the game  $g^*$  is played *ex post*, announcing the true  $\theta$  ensures whoever has invested correctly to get at least his first-best payoff if the other party either has invested correctly or has overinvested. For example, by announcing *ex post* the true  $\theta$  the buyer gets quasi-rents of either  $B_\theta^*$  or  $\hat{W}_\theta^*(i,j,\theta) - S_\theta^*$ , where  $\hat{W}_\theta^*(i,j,\theta)$  is the maximal sum of quasi-rents that can be generated by  $(i,j)$  given  $\theta$ ; but if the seller has overinvested ( $i \geq i^*$ ) and the buyer has chosen  $j=j^*$ ,  $\hat{W}_\theta^*(i,j^*,\theta) - S_\theta^* \geq W_\theta^* - S_\theta^* = B_\theta^*$ , and thus being truthful gives the buyer at least  $B_\theta^*$ . Thus playing  $g^*$  *ex post* would suffice to deter (unilateral) overinvestment.

However, underinvestment does reduce the other party's payoff when  $g^*$  is played *ex post*. In order to prevent such a deviation, the contract  $\gamma_{q',q''}$  gives each party the option to denounce the deviator and start bargaining from a very high level of trade, with full bargaining power, making the deviator's default payoff very sensitive to his investment level. Note that for  $q'$  and  $q''$  large enough these options ( $((q',p'),B)$  for the buyer and  $((q'',p''),S)$  for the seller) cannot be used to reduce the other party's payoff if he has invested correctly.<sup>(19)</sup> In that case, playing  $\gamma_{q',q''}$  still prevents unilateral

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(19) If for example the seller invests  $i=i^*$ , the buyer's option  $((q',p'),B)$

overinvestment; moreover, the only way to reduce the other party's payoff is to underinvest and play  $g^*$  *ex post*.

Suppose then that the seller, say, underinvests while the buyer invests correctly. By so doing, the seller can indeed reduce the buyer's payoff when  $g^*$  is played *ex post*. This reduction, however, is independent from either  $q'$  or  $q''$ . Moreover, exerting his option when  $\theta=\theta'$  gives the buyer  $\hat{W}^*(i, j^*, \theta') - [p' - k(q', i, \sigma)] = [\hat{W}^*(i, j^*, \theta') - S_\theta^*] + [k(q', i, \sigma) - k(q', i^*, \sigma)]$ , which increases unboundedly with  $q'$ . Since  $\theta'$  has positive probability, for  $q'$  large enough the option  $((q', p'), B)$  guarantees that the buyer can get more than his first-best expected level of utility whenever the seller underinvests. Similarly, for  $q''$  high enough, exerting the option  $((q'', p''), S)$  when  $\theta=\theta''$  suffices to give to the seller more than his first-best expected level of utility. It remains to check that for  $q'$  large enough, the seller still gets more than his first-best expected level of utility even if the buyer exerts his own option  $((q', p'), B)$  (which has priority over  $((q'', p''), S)$ ) when  $\theta=\theta''$ .

Let us end this Section with three remarks. First, assume that parties observe their investment choices before the realization of  $\theta$ . Of course, this would only enlarge the implementation possibilities in the absence of *interim* renegotiation. (For instance, a revelation game relative to  $i$  and  $j$  could take place before the realization of  $\theta$ .) Note however that the contract used in Proposition V.1 has the following property: it still implements the first best when the parties observe their investment choices before they observe  $\theta$ , for any Pareto-improving renegotiation which takes place at the *interim* stage. Indeed, a unilateral deviation from efficient investment cannot be profitable for any party because the other party can prevent a fall in his own expected

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gives the seller quasi-rents of  $S_\theta^* + k(q', i^*, \sigma) - k(q', i^*, \sigma)$  in state  $\theta=(\beta, \sigma)$ , and these quasi-rents are higher than  $S_\theta^*$  for  $q'$  high enough.

surplus. But this surplus, computed in the absence of *interim* renegotiation, can only increase through Pareto-improving *interim* renegotiation; the argument of the proof is thus still valid when *interim* renegotiation is introduced. (20)

Second, Proposition V.1 relies on  $\Theta$  being discrete. (21) We conjecture that contracts similar to  $\gamma_{q',q''}$  may implement "almost first-best" allocations when  $\Theta$  is a continuum; this deserves further research.

Finally, Proposition V.1 also relies on the assumed form of risk-aversion. The absence of wealth effects allows to have only zero-output levels as starting points off-the-diagonal in matrix  $g^*$ . This turns out to deter overinvestment, so that it only remains to prevent underinvestment through adequate threats  $(q', p')$  and  $(q'', p'')$ . In the presence of wealth effects, achieving optimal risk-sharing may instead require very high output levels as starting points of future renegotiation (see the Appendix for an example), in which case contracts such as  $\gamma_{q',q''}$  may not allow to implement the first-best allocation. An analysis of this case is also an interesting avenue for further research.

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(20) *Interim* renegotiation design would also help implement the first-best. If for example Assumptions 0-1-2 apply to *interim* renegotiation as well, then as in Section III a simple contract giving all bargaining power for both stages of renegotiation to the seller, say, and stipulating a default option  $(\hat{q}, \hat{p})$  such that  $E_{\theta} [u_B(v_j(\hat{q}, j^*, \beta) - \hat{p} - \psi'(j^*))] = 0$  and  $E_{\theta} [u_B(v_j(\hat{q}, j^*, \beta) - \hat{p} - \psi(j^*))] = U_B^*$ , implements the first-best. (The buyer, who expects  $E_{\theta} [u_B(v_j(\hat{q}, j, \beta) - \hat{p} - \psi(j))]$ , chooses  $j=j^*$  and obtains  $U_B^*$ ; efficient renegotiation make the seller *ex ante* residual claimant, which concludes the argument.) Hermalin-Katz (1991) use *interim* renegotiation design to achieve first-best efficiency in a standard moral hazard problem with only one investment (the agent's action) the first-best can then be achieved without monitoring bargaining powers. As discussed in Aghion-Dewatripont-Rey (1989), *interim* renegotiation design may however be more difficult to implement than *ex post* renegotiation design.

(21) More precisely, it relies on the fact that states  $\theta'$  and  $\theta''$  occur with some positive (mass) probability.

## VI. COMPARISON WITH THE LITERATURE

Our analysis contributes to both the underinvestment and the implementation literatures, which we address in turn.

### 1. The underinvestment problem

As argued in the previous Sections, underinvestment can be avoided in a wide range of situations. Our results are in sharp contrast with the prediction of the incomplete contract literature. In order to understand such a difference, let us first compare our framework with that of Hart-Moore (1988). Their assumption on the enforcement technology allows the initial contract to allocate full bargaining power to one party,<sup>(22)</sup> but the only possible default option is no-trade. Specifically, they assume trade to be voluntary: it takes place only if both parties agree to trade *ex post*, and courts cannot observe who did not want to trade when trade did not take place. This leads to underinvestment, unless the contract is renegotiation-proof in all states of nature or unless one party's investment is irrelevant (so that it suffices to make the other one residual claimant).<sup>(23)</sup>

In contrast, our scheme circumvents the underinvestment problem by setting the default option at an "average" trade level. The key difference is the ability of the court to verify that each party did his part of the trade (that is, the seller provided  $q$  and the buyer paid  $p$ ).

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(22) In their extensive form, a high enough price differential allocates full bargaining power to the seller whenever renegotiation takes place, whereas a low enough price differential gives full bargaining power to the buyer.

(23) Hart and Moore only have two levels of trade: 0 and 1. However, this creates no problem *per se*. Indeed, if the parties could set other default options than no-trade, the appropriate  $q_0$  of our Proposition III.1 could be achieved through randomization. (See Aghion-Dewatripont-Rey (1989) for a simple example comparing our mechanism with that of Hart and Moore.)



Hart and Moore are in keeping with the literature on underinvestment where the key factor is the "no-trade" threat. With "no-trade" as the unique default option, both parties' *ex post* payoffs (weakly) increase with the total surplus; therefore, the expected return from a rise in investment cannot be higher than the corresponding expected increase in total surplus, and it has to be smaller for at least one party: hence the underinvestment result, as in the classic "moral hazard in team" problem (Holmstrom (1982)). When the default option can be adjusted, the return from investment can instead be made higher than the total increase in surplus: if one party has no bargaining power, his payoff increases less or more than the total surplus depending on whether the default level of trade is lower or higher than the efficient one. (24)

While being of some relevance for inter-firm relationships, our analysis might also generate insights for firm-union relationships. (25) As emphasized by Grout (1984), the lack of wage commitment by labor unions also creates an underinvestment problem. In Grout's model, only the firm invests, and renegotiation is formalized as a generalized Nash bargaining solution with exogenous bargaining powers. In our framework, when investment is unilateral, one can choose zero output or employment as the default option, while giving full bargaining power to the firm can be interpreted as penalizing the union for striking. The default payoff for the union is then given by the opportunity wage when quitting plus the contractually determined severance

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(24) Tirole (1986) and Rogerson (1992) have analyzed investment problems in the presence of asymmetric information. Tirole focuses on *interim* individual rationality constraints, as in Myerson-Satterthwaite (1983), and shows that inefficient trade and underinvestment are the only possible equilibrium outcomes. By contrast, Rogerson considers *ex ante* individual rationality constraints, as in d'Aspremont-Gerard-Varet (1979), and shows that first-best investment decisions can be implemented when parties are risk-neutral.

(25) Having a multi-worker union raises the issue of the correct specification of the union's objective function. This problem has not yet been fully resolved by the theoretical literature on labor unions.

payment. Our analysis goes beyond Grout's concerns by allowing for specific investment on the union side as well. In this case, the default option must involve a positive employment level, that is, employment guarantees. Our model then predicts wage and employment guarantees, in exchange for no-strike provisions.

We are aware of the difficulties involved in the legal enforceability of the above instruments of renegotiation design (specific performance contracts for firm-firm relationships, and no-strike provisions and employment guarantees for firm-union relationships). This paper can be seen as pointing out sufficient conditions for efficient bilateral investment in the presence of renegotiation, and clarifying the reasons why they might fail in some real-world situations.

## 2. Links with the implementation literature

The implementation literature (Maskin (1977,1985), Moore-Repullo (1988)) has emphasized that unverifiability problems could be circumvented by revelation mechanisms. These mechanisms however use the threat of inefficient punishments to deter "deviations", which in turn requires a strong form of commitment: the parties should commit themselves never to renegotiate, even when both could benefit from rescinding the initial agreement.

More recently, Maskin-Moore (1988) and Rubinstein-Wolinsky (1992) have removed this requirement by allowing for *ex post* renegotiation of inefficient outcomes, that is, by introducing the equivalent of our Assumption 0.<sup>(26)</sup> Maskin and Moore formalize this renegotiation process as a black box (a function  $h$  which, conditional on the state of nature, replaces any inefficient

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(26) See Holmstrom-Myerson (1983) for a first attempt at introducing renegotiation in the context of Bayesian implementation.

outcome with an efficient one), whereas Rubinstein and Wolinsky use an explicit extensive form to model renegotiation. (27)

Maskin and Moore address the general problem of implementing a given allocation via revelation games, given that inefficient outcomes are "replaced" by efficient ones through a given *ex post* renegotiation process. Our assumption 1 on the role of default options is implicit in their approach, since they assume that both parties must be better off after renegotiation. They derive additional conditions on the pairs {allocation, renegotiation process} under which implementation is indeed feasible. Our approach can be reinterpreted in this light: we make an additional assumption on the renegotiation process (concerning the monitoring of bargaining powers), and then show that first-best allocations can be implemented in a large variety of situations.

As shown by the extensive form developed in Section II, Assumption 1 may also require the ability for the parties to unilaterally enforce the default option if they want to, although this default option is generally inefficient. This, in turn, suggests that both agents have access to some form of commitment by taking irreversible technological decisions leading to inefficient outcomes.

This could at first glance seem inconsistent with Assumption 0, according to which the parties can always renegotiate away inefficient trades specified in the initial contract. There is no inconsistency, however, since Assumption 0 only asserts that the parties cannot be forced by a court to execute a contractual trade when there is still time to renegotiate, that is, before irreversible technological decisions have been taken. On the other hand, Assumption 1 can be interpreted as follows: suppose that the court cannot

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(27) Hart-Moore (1988), although focusing on the underinvestment problem, can also be seen as a contribution to the implementation literature. As stressed above, they make Assumptions 0 and 2, but not Assumption 1.

force the agents to trade, but can observe the agents' choices of  $p$  and  $q$  (a rather weak assumption; one may even suppose that the court observes  $p$  and  $q$  only after some delay, without altering the argument). Suppose moreover that each party can take an action such that an *ex post* inefficient default option becomes efficient given that action.<sup>(28)</sup> In this case, in order to enforce the default option, one party only has to take this specific action and threaten the other party to alert the court: since no other level of trade is then mutually preferable, the other agent has no choice left but complying with the default option.

Rubinstein and Wolinsky instead explicitly assume away production irreversibility; in their framework, time is the only irreversible element and inefficiencies can only arise from delays. On the other hand, they use a stronger version of Assumption 2 on the allocation of bargaining powers, by assuming that the initial contract can fully design the extensive form. This allows them to achieve first-best efficiency in specific contexts.

The above discussion highlights the sensitivity of implementation results to precise assumptions on renegotiation design. Evaluating the range of applicability of the various assumptions is an important topic for future research. Also, it may be worthwhile to explore the implementation possibilities opened by a more sophisticated design of default options and bargaining powers.<sup>(29)</sup>

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(28) This is quite natural for the seller, who could just decide to produce  $q$ . For the buyer, one could assume that he has to combine the good produced by the seller with other inputs to make a product to be sold in turn; buying specific inputs may then make  $q$  *ex post* efficient.

(29) For example, Aghion-Dewatripont-Rey (1989) analyzes the possibility of influencing the sequence of renegotiation offers, a possibility also explored by Rubinstein-Wolinsky (1992). This could in particular allow to allocate "intermediate" bargaining powers. In the line of our first version. Another extension, suggested by a referee, would be to give different default options to the two parties.

## APPENDIX

### I. PROOF OF PROPOSITION II.1

Fix  $i, j$  and  $\theta$  and let player 1 be the buyer. Let  $(p'_0, q'_0)$  denote the *ex post* efficient trade which would give the buyer exactly  $U_B(p_0, q_0, j, \theta)$ , and choose a penalty  $z$  such that  $U_S(p_0 + z(i, j, \theta), q_0, i, \theta) = U_S(p'_0, q'_0, i, \theta)$ . Such a  $z(i, j, \theta)$ , which depends upon  $j$  through the definition of  $(p'_0, q'_0)$ , exists from the unboundedness of  $U_S$  with respect to  $p_0 + z$  (condition U).

If the game goes on until  $t^*$ , the seller can impose  $(p_0, q_0)$  as a default option and obtain  $U_S(p_0 + z(i, j, \theta), q_0, i, \theta) = U_S(p'_0, q'_0, i, \theta)$ ; since  $(p'_0, q'_0)$  is *ex post* efficient, the buyer then obtains less than  $U_B(p_0, q_0, j, \theta)$ . Therefore, when his last opportunity to impose  $(p_0, q_0)$  before  $t^*$  arrives, the buyer will either do it or accept any better offer in order to avoid paying  $z(i, j, \theta)$ . The bargaining game is thus equivalent to its finite-horizon truncation after the last opportunity prior to  $t^*$ , for the buyer to impose  $(p_0, q_0)$ . The outcome of this truncated game is *ex post* efficient and gives the buyer a utility level at  $t_3$  which converges to  $U_B(p_0, q_0, j, \theta)$  when  $\Delta \rightarrow 0$  and  $t^* \rightarrow t_3$ . Using the continuity and compactness condition C, it remains to choose  $\bar{z} = \max_{(i, j, \theta) \in I \times J \times \Theta} z(i, j, \theta)$  to complete the proof.

Q.E.D.

### II. PROOF OF PROPOSITION V.1

We assume throughout this proof that the conditions of Proposition V.1 are satisfied. Moreover, since we study mainly the *ex post* outcome of the game  $\gamma_{q', q''}$ , except indicated otherwise the gains will be expressed in terms of

quasi-rents,  $S = p - k(q, i, \sigma)$  and  $B = v(q, j, \beta) - p$ , and  $\hat{W}^*(i, j, \theta)$  will denote the maximal quasi-rents generated by investments  $(i, j)$  given  $\theta$ .

Lemma 1:  $\exists (\tilde{q}', \tilde{q}'')$  such that:

- i) If  $i = i^*$ ,  $\forall \theta \in \Theta$ ,  $\forall q' > \tilde{q}'$ , in state  $\theta$  the price-quantity pair  $(q', p' = k(q', i^*, \underline{\sigma}) + S_{\theta}^*)$  gives the seller at least  $S_{\theta}^*$ .
- ii) If  $j = j^*$ ,  $\forall \theta \in \Theta$ ,  $\forall q'' > \tilde{q}''$ , in state  $\theta$  the price-quantity pair  $(q'', p'' = v(q'', j^*, \underline{\beta}) - B_{\theta}^*)$  gives the buyer at least  $B_{\theta}^*$ .

Proof of Lemma 1:

We prove i). The proof of ii) is similar.

If  $i = i^*$ , in state  $\theta$  the pair  $(q', p' = k(q', i^*, \underline{\sigma}) + S_{\theta}^*)$  gives the seller  $p' - k(q', i^*, \sigma) = S_{\theta}^* + k(q', i^*, \underline{\sigma}) - k(q', i^*, \sigma)$ . This clearly is at least equal to  $S_{\theta}^*$  in all states  $\theta$  in which  $\sigma = \underline{\sigma}$ , since then  $\theta \in \theta' = (\bar{\beta}, \underline{\sigma})$ . In all other states, this is larger than  $S_{\theta}^*$  for  $q'$  large enough, since  $\sigma > \underline{\sigma}$  and  $k_{\sigma}$  is unbounded.

Q.E.D.

Lemma 2: When the revelation game  $\gamma_{q', q''}$  is played *ex post*, for a given  $(i, j, \theta)$ , and for  $q' > \tilde{q}'$  and  $q'' > \tilde{q}''$ :

- i) if  $i = i^*$  and  $j = j^*$ , all (pure-strategy) equilibria yield  $(B_{\theta}^*, S_{\theta}^*)$ ;
  - ii) if  $i = i^*$  and  $j > j^*$ , no equilibrium gives the seller less than  $S_{\theta}^*$ ;
- if  $i > i^*$  and  $j = j^*$ , no equilibrium gives the buyer less than  $B_{\theta}^*$ .

Proof of the lemma 2:

i) *Ex post* the revelation game is a constant-sum game; all (pure-strategy) equilibria thus are payoff-equivalent, and it suffices to show that no party can decrease the other's payoff by departing from  $(i_B, j_B, \theta_B) = (i_S, j_S, \theta_S) = (i^*, j^*, \theta)$ . We show that for  $q'' \geq \tilde{q}''$ , the buyer obtains at least  $B_{\theta}^*$  if he announces  $(i^*, j^*, \theta)$ :

- If  $(i_S, j_S, \theta_S) = (i^*, j^*, \theta_S \geq \theta)$ , the buyer obtains  $B_\theta^*$ .

- If  $(i_S, j_S, \theta_S) = (i^*, j^*, \theta_S < \theta)$ , he obtains  $W_\theta^* - S_\theta^* > B_\theta^*$ .

- If  $(i_S, j_S) \neq (i^*, j^*)$ , he obtains  $B_{\theta''}^* + v(q'', j^*, \beta) - v(q'', j^*, \underline{\beta})$ , which from Lemma 1 is larger than  $B_\theta^*$  for  $q'' \geq \tilde{q}''$ .

Similarly, for  $q' \geq \tilde{q}'$   $(i_S, j_S, \theta_S) = (i^*, j^*, \theta)$  gives the seller at least  $S_\theta^*$ .

ii) We consider the case  $(i=i^*, j > j^*)$ , and show that the seller can guarantee himself at least  $S_\theta^*$  by announcing  $(i_S, j_S, \theta_S) = (i^*, j^*, \theta)$ ; the analysis is identical when it is the seller who overinvests.

If  $(i_B, j_B) \neq (i^*, j^*)$ , the seller obtains  $S_\theta^* + k(q', i^*, \underline{\sigma}) - k(q', i^*, \sigma)$ , which from Lemma 1 is larger than  $S_\theta^*$  for  $q' \geq \tilde{q}'$ .

If  $(i_B, j_B) = (i^*, j^*)$  and  $\theta_B \geq \theta$ , the seller gets  $S_{\theta_B}^* \geq S_\theta^*$ .

If  $(i_B, j_B) = (i^*, j^*)$  and  $\theta_B < \theta$ , the seller gets full bargaining power and obtains  $\hat{W}^*(i^*, j, \theta) - B_\theta^*$ ;  $j > j^*$  implies  $\hat{W}^*(i^*, j, \theta) > \hat{W}^*(i^*, j^*, \theta) = W_\theta^*$ , and thus the seller obtains more than  $S_\theta^*$ .

Lemma 3:  $\exists q' \geq \tilde{q}'$  and  $q'' \geq \tilde{q}''$  such that for  $q' > q''$  and  $q'' > \underline{q}$ , if the revelation game  $\gamma_{q', q''}$  is played *ex post*, then for a given  $(i, j)$ :

i) If  $i < i^*$  and  $j = j^*$ , in no equilibrium the buyer's expected utility is lower than  $E_\theta [u_B(B_\theta^* - \psi(j^*))]$ .

ii) If  $i = i^*$  and  $j < j^*$ , in no equilibrium the seller's expected utility is lower than  $E_\theta [u_S(S_\theta^* - \varphi(i^*))]$ ;

Proof of the lemma 3:

1) We show that if  $i < i^*$  and  $q'$  is high enough, then by announcing *ex post*  $(i_B, j_B) \neq (i, j^*)$  whenever  $\theta = \theta'$  and  $(i_B, j_B, \theta_B) = (i^*, j^*, \theta)$  otherwise, the buyer is sure to get at least his first-best expected utility.

If  $\theta = \theta'$ , this announcement strategy yields full bargaining power to the buyer, with a final payoff equal to:

$$\begin{aligned}
& \hat{W}^*(i, j^*, \theta') - [p' - k(q', i, \underline{\sigma})] \\
&= \hat{W}^*(i, j^*, \theta') - [k(q', i^*, \underline{\sigma}) + S_{\theta}^* - k(q', i, \underline{\sigma})] \\
&= [\hat{W}^*(i, j^*, \theta') - S_{\theta}^*] + [k(q', i, \underline{\sigma}) - k(q', i^*, \underline{\sigma})] \tag{A.1}
\end{aligned}$$

which can be made as high as desired by increasing  $q'$ .

Consider now  $\theta \neq \theta'$ , and suppose the buyer announces  $(i_B, j_B, \theta_B) = (i^*, j^*, \theta)$ .

- If  $(i_S, j_S) \neq (i^*, j^*)$ , from Lemma 1 the buyer obtains at least  $B_{\theta}^*$ .

- If  $(i_S, j_S) = (i^*, j^*)$  and  $\theta_S \geq \theta$ , the buyer obtains  $B_{\theta_S}^* \geq B_{\theta}^*$ .

- If  $(i_S, j_S) = (i^*, j^*)$  and  $\theta_S < \theta$ , however, the buyer obtains less than  $B_{\theta}^*$ ;

more precisely, he obtains:

$$\hat{W}^*(i, j^*, \theta) - S_{\theta}^* = B_{\theta}^* - [W_{\theta}^* - \hat{W}^*(i, j^*, \theta)] \tag{A.2}$$

Therefore, if  $i < i^*$ , the above strategy gives the buyer at least his first-best expected utility, provided that  $q'$  satisfies:

$$\begin{aligned}
& \text{Prob}(\theta = \theta') \cdot \{u_B(\hat{W}^*(i, j^*, \theta') - S_{\theta}^* - \psi(j^*) + [k(q', i, \underline{\sigma}) - k(q', i^*, \underline{\sigma})]) - u_B(B_{\theta}^* - \psi(j^*))\} \\
& \geq \text{Prob}(\theta \neq \theta') \cdot E_{\theta \neq \theta'} [u_B(B_{\theta}^* - \psi(j^*)) - u_B(\hat{W}^*(i, j^*, \theta) - S_{\theta}^* - \psi(j^*))] \tag{A.3}
\end{aligned}$$

For any  $i < i^*$ , the RHS of (A.3) is independent of  $q'$  and its LHS tends to  $+\infty$  when  $q'$  tends to  $+\infty$ , since  $k_1$  and  $u_B$  are unbounded. Thus, for any  $i \in [0, i^*[$ , there exists a level of trade  $\hat{q}'(i)$  which satisfies exactly (A.3), and which varies continuously with respect to  $i$  on  $i \in [0, i^*[$ . Moreover, since  $k_1$  is unbounded, there exists  $\hat{q}'$  such that (the absolute value of) the derivative of the LHS of (A.3) with respect to  $i$ , evaluated for  $i = i^*$ , is strictly greater than that of the RHS for  $q \geq \hat{q}'$ . Since the LHS and RHS of (A.3) are equal for  $i = i^*$  (whatever  $q'$ ),  $\hat{q}'(i)$  cannot be higher than  $\hat{q}'$  for  $i$



close enough to  $i^*$ . Therefore  $\sup_{i \in [0, i^*]} \{\hat{q}'(i)\} < +\infty$  and for  $q'$  larger than

$\max\{\sup_{i \in [0, i^*]} \hat{q}'(i), \tilde{q}'\}$  and  $(i < i^*, j = j^*)$ , no equilibrium of the game generated by  $\gamma_{q', q''}$  can give the buyer an expected utility lower than his first-best one.

ii) Similarly, we exhibit conditions under which, if  $j < j^*$ , the seller is sure to get at least his first-best expected utility by announcing  $(i_S, j_S) \neq (i, j^*)$  when  $\theta = \theta''$  and  $(i_S, j_S, \theta_S) = (i^*, j^*, \theta)$  otherwise. We have to consider two cases, however, since for  $\theta = \theta''$  the buyer's "option"  $((p', q'), B)$  may preempt the seller's "option"  $((p'', q''), S)$ . If the buyer does not exert his option, it suffices to choose  $q''$  larger than  $\max\{\tilde{q}'', \hat{q}''(j)\}$ , where  $\hat{q}''(j)$  is defined by:

$\text{Prob}(\theta = \theta'')$ .

$$\begin{aligned} & \{u_S(\hat{W}^*(i^*, j, \theta'') - B_{\theta''}^* - \varphi(i^*)) + [v(\hat{q}''(j), j^*, \underline{\beta}) - v(\hat{q}''(j), j, \underline{\beta})] - u_S(S_{\theta}^* - \varphi(i^*))\} \\ & = \text{Prob}(\theta \neq \theta') \cdot E_{\theta \neq \theta'} [u_S(S_{\theta}^* - \varphi(i^*)) - u_S(\hat{W}^*(i^*, j, \theta) - B_{\theta}^* - \varphi(i^*))] \quad (\text{A.3}') \end{aligned}$$

If instead the buyer exerts his option in state  $\theta''$ , the seller obtains:

$$S_{\theta}^* + [k(q', i^*, \underline{\sigma}) - k(q', i^*, \bar{\sigma})] \quad (\text{A.4})$$

This expression again goes to  $+\infty$  when  $q'$  goes to  $+\infty$ . For any  $j < j^*$  we can thus find a level of trade  $\check{q}'(j)$  such that:

$$\begin{aligned} & \text{Prob}(\theta = \theta'') \cdot \{u_S(S_{\theta}^* + [k(\check{q}'(j), i^*, \underline{\sigma}) - k(\check{q}'(j), i^*, \bar{\sigma})] - \varphi(i^*))\} - u_S(S_{\theta}^* - \varphi(i^*))\} \\ & = \text{Prob}(\theta \neq \theta') \cdot E_{\theta \neq \theta'} [u_S(S_{\theta}^* - \varphi(i^*)) - u_S(\hat{W}^*(i^*, j, \theta) - B_{\theta}^* - \varphi(i^*))] \quad (\text{A.3}'') \end{aligned}$$

It thus suffices to choose  $q''$  larger than  $\max\{\tilde{q}'', \sup_{j \in [0, j^*]} \hat{q}''(j)\}$  and  $q'$  larger than  $\max_{j \in [0, j^*]} \check{q}'(j)$  to make sure that the above strategy gives the

seller at least his first-best level of utility.

Finally, choosing  $q''$  larger than  $\max\{\tilde{q}'', \sup_{j \in \{0, j^*\}} \hat{q}''(j)\}$  and  $q'$  larger than  $\max\{\tilde{q}', \sup_{i \in \{0, i^*\}} \hat{q}'(i), \max_{j \in \{0, j^*\}} \check{q}'(j)\}$  concludes the argument.

Q.E.D.

The proof of Proposition V.1 can now be completed as follows. From Lemmas 2 and 3 we know that for trade levels  $q' > \underline{q}'$  and  $q'' > \underline{q}''$ : (i) if investments are correct, optimal risk-sharing is achieved, thus implementing the first-best; (ii) whoever invests correctly can guarantee himself at least his first-best expected utility, whatever the other party's investment. Since deviating from the correct investment decreases the total surplus (in *ex ante* terms), such deviation cannot be profitable, which completes the argument.

Q.E.D.

### III. AN EXAMPLE WHERE PAYOFFS ARE NOT CO-MONOTONIC

Let us for instance assume that  $\theta$  only affects the buyer's valuation, and that utility functions are given by<sup>(30)</sup>:

$$U_B(p, q, \theta) = u(q, \theta) - v(p) \quad (\text{A.5})$$

$$U_S(p, q) = w(p) - k(q) \quad (\text{A.6})$$

where:  $u(q, \theta) - k(q)$  is concave and unbounded above in  $q$ ;  $v(p)$  and  $w(p)$  are increasing, resp. convex and concave, and unbounded above; and  $u_\theta$  is non-negative and unbounded above in  $q$ . Payoffs are not co-monotonic: if

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(30) These utility functions satisfy condition U, so that Proposition II.1 applies.

$\theta_1 < \theta_2$ , *ex post* efficiency implies  $q_1 < q_2$  but efficient risk-sharing implies  $p_1 = p_2$ ; thus, the buyer's payoff is higher in state  $\theta_2$  and the seller's payoff is higher in state  $\theta_1$ .

Proposition A.1: Under the above assumptions on the buyer's and seller's utility functions, the first-best allocation can be implemented.

Proof: For the sake of simplicity, we consider here only two states of nature,  $\theta_1$  and  $\theta_2$ , and exhibit a "direct revelation" game for which truthtelling is an equilibrium. This game can then be used as a building block for the case where there are more than two states (see Aghion-Dewatripont-Rey (1989)). The "constant-sum" property of this game, due to the *ex post* efficiency of the renegotiation process, then ensures that all equilibrium payoffs are the same.

Without loss of generality, we suppose  $\theta_1 < \theta_2$ . The first-best allocation entails  $p_1^* = p_2^* = p^*$  and  $q_1^* < q_2^*$ , and can be implemented using the following revelation game:

		$\theta_1$	Seller	$\theta_2$
	$\theta_1$	$(p^*, q_1^*)$		$(p^*, q_2^*)$ $\alpha = S$
Buyer	$\theta_2$	$(p_0, q_0)$ $\alpha = S$		$(p^*, q_2^*)$

Figure A.4

By construction, since  $q_1^* < q_2^*$ , there is no incentive for the seller to unilaterally deviate from truthtelling when  $\theta = \theta_1$  and there is no incentive for the buyer to unilaterally deviate from truthtelling when  $\theta = \theta_2$ . Using the *ex post* efficiency of the renegotiation process, the default option  $(p_0, q_0)$  must satisfy:

$$u(q_2^*, \theta_2) - u(q_0, \theta_2) \leq v(p^*) - v(p_0) \leq u(q_1^*, \theta_1) - u(q_0, \theta_1) \quad (\text{A.7})$$

$\theta_2 > \theta_1$  and the assumption that  $u_\theta(q, \theta)$  is unbounded above in  $q$  guarantee that for  $q_0$  large enough the LHS of the above inequality is strictly lower than the RHS. It then suffices to choose  $p_0$  appropriately, which can be done since  $v$  is unbounded above.

Q.E.D.

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