

Renormalization group equations of Higgs- R^2 inflation

Yohei Ema, Kyohei Mukaida and Jorinde van de Vis

DESY,

Notkestraße 85, D-22607 Hamburg, Germany

E-mail: yohei.ema@desy.de, kyohei.mukaida@cern.ch,
jorinde.van.de.vis@desy.de

ABSTRACT: We derive one- and two-loop renormalization group equations (RGEs) of Higgs- R^2 inflation. This model has a non-minimal coupling between the Higgs and the Ricci scalar and a Ricci scalar squared term on top of the standard model. The RGEs derived in this paper are valid as long as the energy scale of interest (in the Einstein frame) is below the Planck scale. We also discuss implications to the inflationary predictions and the electroweak vacuum metastability.

KEYWORDS: Cosmology of Theories beyond the SM, Renormalization Group, Sigma Models

ARXIV EPRINT: [2008.01096](https://arxiv.org/abs/2008.01096)

Contents

1	Introduction	1
2	Renormalization group equation	3
2.1	Scaloron and conformal mode	3
2.2	Graviton, gauge fixing and decoupling of Faddeev-Popov ghost	6
2.3	One-loop RGE	8
2.4	Two-loop RGE	9
3	Inflation with massive parameters	11
3.1	Background evolution	12
3.2	Inflationary prediction	14
4	Electroweak vacuum metastability	16
4.1	Threshold correction	17
4.2	RG evolution	17
5	Conclusion and discussion	19
A	General coordinate transformation	21
B	Einstein and conformal frame	23
C	Computational details of one-loop RGEs	24
C.1	Feynman rules	24
C.2	Divergent part of four-point functions	25
C.3	Counterterms	26
D	Computational details of two-loop RGEs	26
E	Renormalizability and residual gauge symmetry	29
E.1	Residual gauge symmetry and conformal weight	29
E.2	Possible interactions	31
E.3	A few remarks	31

1 Introduction

Higgs inflation [1–3] stands out among many other models of inflation as it is minimal and consistent with the cosmic microwave background (CMB) observation [4]. It introduces a non-minimal coupling to gravity,

$$\mathcal{L}_\xi = \xi R |H|^2, \tag{1.1}$$

with H the standard model (SM) Higgs doublet, R the Ricci scalar, and ξ the non-minimal coupling, so that the Higgs potential becomes flat in the Einstein frame for $|H| \gtrsim M_P/\xi$. The CMB normalization requires $\xi^2 \sim 2 \times 10^9 \lambda$, indicating a large non-minimal coupling, $\xi \gg 1$, unless the Higgs quartic coupling λ is tiny. However, such a large non-minimal coupling causes a strong coupling problem, where the model loses perturbative unitarity at M_P/ξ , well below the Planck scale [5–8]. While this problem does not necessarily ruin the model’s validity during inflation [9], during preheating NG bosons (or equivalently longitudinal gauge bosons) with energy exceeding M_P/ξ are produced, threatening perturbative unitarity [10–12].¹ It is thus desirable to push up the scale M_P/ξ where the theory becomes strongly coupled.

Higgs- R^2 inflation is a natural solution to the strong coupling problem of Higgs inflation [14, 15]. In addition to the non-minimal couplings term, it also includes the R^2 -term

$$\mathcal{L}_\alpha = \alpha R^2. \tag{1.2}$$

The R^2 -term is not only required for the renormalizability of Higgs inflation [16], but is enhanced for a large ξ because of quantum corrections [14, 17–21]. The renormalization group equation (RGE) implies a natural value of the coupling to be $\alpha \sim \xi^2$, which gives rise to a scalaron degree of freedom [22–25] with $m_\sigma \sim M_P/\sqrt{\alpha} \sim M_P/\xi$ right at the strong coupling scale of Higgs inflation. In the same way as the SM Higgs unitarizes the Fermi theory at the electroweak (EW) scale, the scalaron unitarizes Higgs inflation at $m_\sigma \sim M_P/\xi$, as shown in refs. [20, 21]. Remarkably, the Higgs- R^2 model is unitary up to M_P and its quantum corrections never induce other operators (such as the R^n -terms with $n \geq 3$) below M_P [21].² Therefore, by computing the beta functions, we can connect the observational input at the EW scale to the UV parameters without ambiguities as long as the scale of our interest is below M_P .

In this paper, the beta functions of the Higgs- R^2 inflation up to two-loop which are applicable below the Planck scale are presented for the first time. While the spin-2 part of the metric, i.e., the graviton, only couples to the other fields with Planck-suppressed interactions, the large value of ξ enhances the coupling of matter fields to the conformal mode of the metric φ defined by

$$g_{\mu\nu} = e^{2\varphi} \tilde{g}_{\mu\nu}, \quad \text{Det}[\tilde{g}_{\mu\nu}] = -1. \tag{1.3}$$

Hence, as long as we are interested in the beta functions below M_P , we may extract the conformal mode of the metric and neglect the contributions from the graviton. By doing so, we show that the Higgs- R^2 model below M_P can be written as a linear sigma model (LSM) of the Higgs, the scalaron, and the conformal mode of the metric with interaction terms which are renormalizable. This observation dramatically simplifies the calculations. Moreover, the general RGEs up to two-loop for the scalar/gauge/fermion system with renormalizable interactions are already known in the literature [31–34]. We provide a systematic approach to obtain the RGEs of the Higgs- R^2 model from these existing studies.

¹The NG boson production itself may be affected by higher dimensional operators [13], but the conclusion that we need knowledge of the UV completion to understand preheating is anyway unchanged.

²This fact is related to the renormalizability of Quadratic Gravity [26–30].

We then study phenomenological consequences of the obtained RGEs. Above the scalaron mass scale, the presence of the scalaron gives the threshold correction to the Higgs four-point interaction [14] and induces the Higgs mass term and the cosmological constant of the order of the scalaron mass via the RGEs. We discuss its implications for the inflationary predictions and the stability of the EW vacuum.

The organization of this paper is as follows. Section 2 is the main part of this paper. In section 2.1, we extract the conformal mode from the rest of the metric. In section 2.2, we discuss our gauge fixing condition, and show that we can ignore the spin-2 part of the gravity, i.e., the graviton, below the Planck scale. The full one- and two-loop beta functions of the Higgs- R^2 theory are then shown in sections 2.3 and 2.4, with computational details given in appendix C and D, respectively. Phenomenological implications to the inflationary prediction and the EW vacuum metastability are discussed in sections 3 and 4, respectively. Finally section 5 is devoted to conclusion and discussion. We follow the convention of ref. [21] throughout this paper.

2 Renormalization group equation

In this section, we derive the one- and two-loop RGEs of Higgs- R^2 inflation that are valid below the Planck scale. The action in the Jordan frame is given by

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R \left(1 + \frac{2\xi |H|^2}{M_P^2} \right) + g^{\mu\nu} D_\mu H^\dagger D_\nu H - m^2 |H|^2 - \lambda |H|^4 + \alpha R^2 - \Lambda \right] + S_{\psi+A}[g_{\mu\nu}], \tag{2.1}$$

where $g_{\mu\nu}$ is the spacetime metric with g its determinant, R is the Ricci scalar, M_P is the (reduced) Planck mass, H is the Higgs doublet, D_μ is its covariant derivative, m is the Higgs mass, and Λ is the cosmological constant. The action for the SM fermions and gauge bosons $S_{\psi+A}[g_{\mu\nu}]$ includes the Yukawa interactions with its dependence on the metric made explicit. We have included the Higgs mass and the cosmological constant since they are required as counterterms to absorb divergences, as we will see below. We denote the beta function of a given coupling g up to two-loop as

$$\frac{dg}{d \ln \mu} \equiv \beta_g = \frac{1}{(4\pi)^2} \beta_g^{(1)} + \frac{1}{(4\pi)^4} \beta_g^{(2)}, \tag{2.2}$$

where μ is the renormalization scale, and $\beta_g^{(1)}$ and $\beta_g^{(2)}$ are the one- and two-loop contributions to the beta function, respectively.

2.1 Scalaron and conformal mode

To see how the R^2 -term gives rise to the scalaron degree of freedom, we first extract the conformal mode of the metric and rewrite the action in the Jordan frame (2.1). Without loss of generality, the metric can be decomposed as

$$g_{\mu\nu} = e^{2\varphi} \tilde{g}_{\mu\nu}, \quad \text{Det}[\tilde{g}_{\mu\nu}] = -1. \tag{2.3}$$

We call the scalar mode φ the conformal mode of the metric; it consists of the determinant part of the metric. The Ricci scalar is decomposed as

$$R = e^{-2\varphi} \tilde{R} + 6e^{-3\varphi} \tilde{\square} e^\varphi, \quad (2.4)$$

where the quantities with the tilde are constructed from $\tilde{g}_{\mu\nu}$. We redefine the fields as

$$H \rightarrow e^{-\varphi} H, \quad \psi \rightarrow e^{-3\varphi/2} \psi, \quad (2.5)$$

and define

$$\Phi \equiv \sqrt{6} M_P e^\varphi. \quad (2.6)$$

We also refer to Φ as the conformal mode. Notice that the conformal mode Φ can be regarded as a *measure* of dimensional quantities in units of the Planck scale. For instance, the ratio $H/(\sqrt{6}M_P)$ is mapped to H/Φ by the rescaling of eq. (2.5). Therefore, when we neglect Planck-suppressed operators in the original action, corresponding operators suppressed by Φ should be dropped after the rescaling.

By using the rescalings defined above, we may rewrite the action (2.1) as

$$\begin{aligned} S = \int d^4x \left\{ \frac{\tilde{R}}{12} (\Phi^2 + 12\xi |H|^2) - \frac{1}{2} \tilde{g}_{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \tilde{g}_{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda |H|^4 \right. \\ \left. - \frac{\lambda_m}{2} \Phi^2 |H|^2 - \frac{\lambda_\Lambda}{4} \Phi^4 + [(6\xi + 1) |H|^2 + 12\alpha \tilde{R}] \frac{\square \Phi}{\Phi} + \alpha \tilde{R}^2 + 36\alpha \left(\frac{\square \Phi}{\Phi} \right)^2 \right\} \\ + S_{\psi+A} [\tilde{g}_{\mu\nu}], \end{aligned} \quad (2.7)$$

where we have defined

$$\lambda_m \equiv \frac{m^2}{3M_P^2}, \quad \lambda_\Lambda \equiv \frac{\Lambda}{9M_P^4}. \quad (2.8)$$

An important feature is that the last term in the curly brackets involves higher derivatives, $(\square \Phi)^2/\Phi^2$. Since the conformal mode Φ has a kinetic term with the wrong sign,³ this higher-derivative term implies the existence of an additional *physical* degree of freedom which has a kinetic term with the correct sign.

To extract this physical degree of freedom as a fundamental field, we introduce an auxiliary field σ as

$$\begin{aligned} S = \int d^4x \left\{ \frac{\tilde{R}}{12} (\Phi^2 + 12\xi |H|^2) - \frac{1}{2} \tilde{g}_{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \tilde{g}_{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda |H|^4 \right. \\ \left. - \frac{\lambda_m}{2} \Phi^2 |H|^2 - \frac{\lambda_\Lambda}{4} \Phi^4 + [(6\xi + 1) |H|^2 + 12\alpha \tilde{R}] \frac{\square \Phi}{\Phi} + \alpha \tilde{R}^2 \right. \\ \left. + 36\alpha \left[\left(\frac{\square \Phi}{\Phi} \right)^2 - \left(\frac{\Phi \sigma}{72\alpha} + \frac{\square \Phi}{\Phi} \right)^2 \right] \right\} + S_{\psi+A} [\tilde{g}_{\mu\nu}]. \end{aligned} \quad (2.9)$$

³Note that the wrong-sign kinetic term of Φ is harmless because of a residual gauge symmetry [21], which is analogous to the Coulomb potential in U(1) gauge theory.

By substituting the solution of the constraint equation $\delta S/\delta\sigma = 0$, namely $\sigma = -72\alpha\Box\Phi/\Phi^2$, one recovers the original action (2.7). Now we can remove all the terms involving $\Box\Phi/\Phi$ by shifting the auxiliary field σ as

$$\sigma \rightarrow \sigma + (6\xi + 1) \frac{|H|^2}{\Phi} + 12\alpha \frac{\tilde{R}}{\Phi}. \quad (2.10)$$

We then remove the kinetic mixing between Φ and σ by the shift

$$\Phi \rightarrow \Phi + \sigma, \quad (2.11)$$

and we arrive at

$$\begin{aligned} S = \int d^4x \left\{ \frac{\tilde{R}}{12} (\Phi^2 - 2|H|^2 - \sigma^2) - \frac{1}{2} \tilde{g}^{\mu\nu} \partial^\mu \Phi \partial^\nu \Phi + \tilde{g}^{\mu\nu} D_\mu H^\dagger D_\nu H + \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma \right. \\ \left. - \lambda |H|^4 - \frac{\lambda_m}{2} (\Phi + \sigma)^2 |H|^2 - \frac{\lambda_\Lambda}{4} (\Phi + \sigma)^4 - \frac{\lambda_\alpha}{4} [\sigma (\Phi + \sigma) + 2\bar{\xi} |H|^2]^2 \right\} \\ + S_{\psi+A} [\tilde{g}_{\mu\nu}], \end{aligned} \quad (2.12)$$

where we define

$$\lambda_\alpha \equiv \frac{1}{36\alpha}, \quad \bar{\xi} \equiv \frac{6\xi + 1}{2}. \quad (2.13)$$

This action is the starting point of our computation. Note that the kinetic terms of the scalar fields and the scalaron σ are canonical and the potential contains only renormalizable terms. This feature is related to the fact that this theory is unitary up to infinite energy if we ignore the spin-2 part, or more generally the renormalizability of the quadratic gravity [26–30], as emphasized in ref. [21]. In the next subsection 2.2, we discuss the interactions of the graviton, the gauge fixing condition and the resultant Faddeev-Popov ghost. There we confirm that the contributions from the graviton can be safely neglected below the Planck scale even after introducing the Faddeev-Popov ghost by properly choosing the gauge fixing condition.

Before closing this section, let us illustrate how one may write down eq. (2.12) in a more familiar form. By using the redefinitions

$$H \rightarrow e^{\varphi_C} H, \quad \psi \rightarrow e^{3\varphi_C/2} \psi, \quad \sigma \rightarrow e^{\varphi_C} \sigma, \quad \Phi \rightarrow \sqrt{6} M_P e^{\varphi_C}, \quad \tilde{g}_{\mu\nu} \rightarrow e^{-\varphi_C} g_{\mu\nu}, \quad (2.14)$$

one finds

$$\begin{aligned} S = \int d^4x \sqrt{-g} \left\{ \frac{M_P^2}{2} R \left(1 - \frac{2|H|^2 + \sigma^2}{6M_P^2} \right) + g^{\mu\nu} D_\mu H^\dagger D_\nu H + \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma \right. \\ \left. - m^2 \left(1 + \frac{\sigma}{\sqrt{6}M_P} \right)^2 |H|^2 - \lambda |H|^4 - \Lambda \left(1 + \frac{\sigma}{\sqrt{6}M_P} \right)^4 \right. \\ \left. - \frac{1}{144\alpha} \left[\frac{3M_P^2}{2} - \left(\sigma + \frac{\sqrt{6}M_P}{2} \right)^2 - (6\xi + 1) |H|^2 \right]^2 \right\} + S_{\psi+A} [g_{\mu\nu}]. \end{aligned} \quad (2.15)$$

This is nothing but the action of the Higgs- R^2 theory in the conformal frame, where the Higgs and the scalaron have the conformal coupling to gravity. Note that the rescaling factor φ_C is different from φ in eq. (2.3).

2.2 Graviton, gauge fixing and decoupling of Faddeev-Popov ghost

In this section, we discuss the gauge fixing condition used in this paper. We expand the fields around the flat spacetime metric $\eta_{\mu\nu}$ as

$$\tilde{g}_{\mu\nu} = [e^h]_{\mu\nu} = \eta_{\mu\rho} \left(\delta_{\nu}^{\rho} + h^{\rho}_{\nu} + \frac{1}{2} h^{\rho}_{\alpha} h^{\alpha}_{\nu} + \dots \right), \quad (2.16)$$

$$h^{\mu}_{\mu} = 0, \quad (2.17)$$

where the contractions are taken with respect to $\eta_{\mu\nu}$.⁴ The second equation follows from $\text{Det}[\tilde{g}_{\mu\nu}] = -1$.

Under the general coordinate transformation $x^{\mu} \rightarrow x^{\mu} - \xi^{\mu}$ at the first order in ξ^{μ} , the fields φ and $h_{\mu\nu}$ transform as follows (see appendix A for the derivation):⁵

$$\varphi \rightarrow \varphi + \xi^{\alpha} \partial_{\alpha} \varphi + \frac{1}{4} \partial_{\alpha} \xi^{\alpha}, \quad (2.18)$$

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \left[\left(\frac{\text{ad}_h}{e^{\text{ad}_h} - 1} \right) \partial \xi \right]_{\mu\nu} + \left[\left(\frac{\text{ad}_h}{e^{\text{ad}_h} - 1} \right) \partial \xi \right]_{\nu\mu} - \frac{1}{2} (\partial_{\alpha} \xi^{\alpha}) \eta_{\mu\nu} + \xi^{\alpha} \partial_{\alpha} h_{\mu\nu}, \quad (2.19)$$

where $[\partial \xi]_{\mu\nu} \equiv \partial_{\mu} \xi_{\nu}$ and

$$\left[\left(\frac{\text{ad}_h}{e^{\text{ad}_h} - 1} \right) X \right]_{\mu\nu} \equiv X_{\mu\nu} - \frac{1}{2} [h, X]_{\mu\nu} + \frac{1}{12} [h, [h, X]]_{\mu\nu} + \dots, \quad (2.20)$$

with the indices raised/lowered/contracted by $\eta_{\mu\nu}$. Here the “ad” is an abbreviation of the adjoint action, and higher order terms in the right-hand-side of eq. (2.20) should be derived from the Taylor expansion. Note that we have *not* performed any expansion with respect to φ nor $h_{\mu\nu}$ here.

We fix this gauge degree of freedom as

$$\partial^{\mu} h_{\mu\nu} = 0, \quad (2.21)$$

in this paper. According to the Faddeev-Popov procedure, we have to introduce ghost fields associated with this gauge fixing condition. Here, a crucial property of eq. (2.19) is that it does not depend on φ . It follows that the Faddeev-Popov ghost associated with the gauge fixing condition (2.21) does not directly couple to the conformal mode φ . Thus, if we can neglect contributions of the graviton $h_{\mu\nu}$, we can also neglect contributions of the Faddeev-Popov ghost.

Now we discuss the condition under which we can neglect contributions of $h_{\mu\nu}$ to the RGEs. One may already infer from eq. (2.15) that $h_{\mu\nu}$ couples to the matter fields only via Planck-suppressed operators and hence can be ignored below M_P , in particular if H and σ do not develop any vacuum expectation values (VEVs), which we assume to be the case in our computation. On the contrary, the conformal mode of the metric and the scalaron couple to the matter fields via, e.g., λ_{α} and $\tilde{\xi}$ that are not suppressed by M_P , and thus

⁴Remember that the beta functions do not depend on the choice of the background around which one expands the fields, since they are related to the UV properties of the theory.

⁵This ξ^{μ} should not be confused with the non-minimal coupling ξ .

cannot be ignored especially for $\xi \gg 1$. In the following, we show explicitly that this expectation is indeed the case by clarifying the meaning of physical scales in eq. (2.12). We also allow both H and σ to have finite VEVs, as they do during inflation.

By expanding the metric $\tilde{g}_{\mu\nu}$ as in eq. (2.16) and applying the gauge fixing condition of eq. (2.21), we obtain the kinetic term of the graviton:

$$\int d^4x \frac{\tilde{R}}{12} (\Phi^2 - 2|H|^2 - \sigma^2) = \int d^4x \frac{1}{48} (\Phi^2 - 2|H|^2 - \sigma^2) \partial_\rho h_{\mu\nu} \partial^\rho h^{\mu\nu} + \mathcal{O}(h^3). \quad (2.22)$$

The graviton couples to other fields through their kinetic terms as

$$\begin{aligned} \tilde{g}^{\mu\nu} \left(-\frac{1}{2} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{2} \partial_\mu \sigma \partial_\nu \sigma + D_\mu H^\dagger D_\nu H \right) \\ \supset -h^{\mu\nu} \left(-\frac{1}{2} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{2} \partial_\mu \sigma \partial_\nu \sigma + D_\mu H^\dagger D_\nu H \right) + \mathcal{O}(h^2). \end{aligned} \quad (2.23)$$

After canonically normalizing the graviton, one may see that the coupling of the graviton to the matter fields and the conformal mode is suppressed by $\sqrt{\Phi^2 - 2|H|^2 - \sigma^2}$. Hence, at a given energy scale μ , we can safely neglect the coupling to $h_{\mu\nu}$ as long as

$$\frac{\mu}{\sqrt{\Phi^2 - 2|H|^2 - \sigma^2}} \ll 1, \quad (2.24)$$

which we require throughout this paper. The physical meaning of this condition is as follows. If the scalaron σ and the Higgs field H do not develop VEVs comparable to Φ , one can simplify this condition as $\mu/\Phi \ll 1$. This implies that a typical scale in the conformal frame (2.15), $\mu_C \equiv e^{-\varphi_C} \mu$,⁶ should be below the Planck scale $\mu_C/M_P \ll 1$. However, during inflation, σ and H develop their VEVs, and in particular the VEV of σ becomes comparable to Φ , which enhances the coupling to the graviton. To understand the physical meaning of the condition in this case, let us recall how the action in eq. (2.12) is related to that in the Einstein frame. The action in the Einstein frame is obtained by the following redefinitions

$$\Phi_E^2 = \Phi^2 - 2|H|^2 - \sigma^2, \quad \Phi_E = \sqrt{6} M_P e^{\varphi_E}, \quad g_{\mu\nu} = e^{2\varphi_E} \tilde{g}_{\mu\nu}, \quad (2.25)$$

and an appropriate rescaling of the fields (see appendix B for details). Now the condition (2.24) can be written as

$$\frac{\mu_E}{M_P} \ll 1, \quad (2.26)$$

with $\mu_E \equiv e^{\varphi_E} \mu$ being a typical scale in the Einstein frame.

From these observations, we can simplify our computation of the RGEs as follows.

1. We can ignore $h_{\mu\nu}$ as long as the condition (2.24) is met for a given scale μ . This condition implies that a typical energy scale in the Einstein frame should be below the Planck scale, i.e., $\mu_E/M_P \ll 1$.

⁶For a given metric $g_{\mu\nu}$, its typical scale μ is defined as $\mu = (g_{\mu\nu} \Delta x^\mu \Delta x^\nu)^{-1/2}$ with Δx^μ being a typical length scale. Let the typical scale in $\tilde{g}_{\mu\nu}$ be μ . Then, the typical scale μ_\bullet in a metric of $g_{\mu\nu} = e^{\varphi_\bullet} \tilde{g}_{\mu\nu}$ can be expressed as $\mu_\bullet = e^{-\varphi_\bullet} \mu$.

2. We do not have to take into account any Faddeev-Popov ghosts, with the assumption that our gauge fixing condition is eq. (2.21), once we can ignore $h_{\mu\nu}$.

Thus, we take the metric as $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$ to compute the RGEs below M_P in the following. The action then reduces to

$$\begin{aligned}
 S = \int d^4x & \left[-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \eta^{\mu\nu}D_\mu H^\dagger D_\nu H + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \lambda|H|^4 \right. \\
 & \left. - \frac{\lambda_m}{2}(\Phi + \sigma)^2|H|^2 - \frac{\lambda_\Lambda}{4}(\Phi + \sigma)^4 - \frac{\lambda_\alpha}{4}\left(\sigma(\Phi + \sigma) + 2\bar{\xi}|H|^2\right)^2 \right] \\
 & + S_{\psi+A}[\eta_{\mu\nu}].
 \end{aligned} \tag{2.27}$$

It can be viewed as a LSM composed of Φ , σ and H coupled to the SM fermions and gauge bosons, as studied in detail in ref. [21].⁷ This LSM contains only two additional scalar modes, the conformal mode Φ and the scalaron, on top of the SM particles, which greatly simplifies our computation of the RGEs. An important feature is that the kinetic term of the conformal mode Φ has a wrong sign and hence this field is ghost-like. It is still harmless thanks to a residual gauge symmetry (see ref. [21] for more details), playing a role that is similar to the Coulomb potential of U(1) gauge theory. In the following, we derive the one- and two-loop RGEs of the Higgs- R^2 model from this action in sections 2.3 and 2.4, respectively. Of the SM parameters we include only the top Yukawa y_t and the gauge couplings g_3, g_2 and g_1 in the RGEs, since the other couplings are all much smaller than unity. See appendices C and D for details of the derivation.

2.3 One-loop RGE

In this subsection, we give the one-loop RGEs of the Higgs- R^2 theory that are valid up to M_P . At the one-loop level, the RGEs are given by

$$\beta_{g_1}^{(1)} = \frac{41}{10}g_1^3, \quad \beta_{g_2}^{(1)} = -\frac{19}{6}g_2^3, \quad \beta_{g_3}^{(1)} = -7g_3^3, \tag{2.28}$$

$$\beta_{y_t}^{(1)} = y_t \left[\frac{9y_t^2}{2} - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 \right], \tag{2.29}$$

$$\begin{aligned}
 \beta_\lambda^{(1)} = & \left(8\bar{\xi}^2 - 8\bar{\xi} + 2 \right) \bar{\xi}^2 \lambda_\alpha^2 + 24\bar{\xi}^2 \lambda \lambda_\alpha + 24\lambda^2 - 6y_t^4 + \frac{27g_1^4}{200} \\
 & + \frac{9g_2^4}{8} + \frac{9}{20}g_1^2 g_2^2 + \left[12y_t^2 - \frac{9g_1^2}{5} - 9g_2^2 \right] \lambda,
 \end{aligned} \tag{2.30}$$

$$\beta_{\lambda_m}^{(1)} = 2\bar{\xi} \left(2\bar{\xi} - 1 \right) \lambda_\alpha^2 - 8\bar{\xi} \lambda_m^2 + \lambda_m \left[4\bar{\xi}^2 \lambda_\alpha + 8\bar{\xi} \lambda_\alpha - 3\lambda_\alpha + 12\lambda + 6y_t^2 - \frac{9g_1^2}{10} - \frac{9g_2^2}{2} \right], \tag{2.31}$$

$$\beta_{\bar{\xi}}^{(1)} = \bar{\xi} \left[\left(4\bar{\xi}^2 + 4\bar{\xi} - 3 \right) \lambda_\alpha + 12\lambda + 6y_t^2 - \frac{9}{10}g_1^2 - \frac{9}{2}g_2^2 \right], \tag{2.32}$$

$$\beta_{\lambda_\alpha}^{(1)} = \left(8\bar{\xi}^2 + 5 \right) \lambda_\alpha^2, \tag{2.33}$$

$$\beta_{\lambda_\Lambda}^{(1)} = \frac{\lambda_\alpha^2}{2} - 2\lambda_\alpha \lambda_\Lambda - 16\bar{\xi} \lambda_\Lambda \lambda_m + 2\lambda_m^2. \tag{2.34}$$

⁷The inclusion of Φ makes this definition of the LSM invariant under a frame transformation, i.e., the Weyl transformation.

In appendix C, we have derived these RGEs by computing scalar four-point functions. In the course of the derivation, we have explicitly checked that all the divergences that appear at the one-loop level are indeed renormalized by the operators within the action (2.27). It is a non-trivial verification of our formulation based on the decomposition (2.3) and the gauge fixing condition (2.21). We have also checked that these RGEs agree with the results obtained with the help of SARAH [35] outlined in appendix D. Moreover, these RGEs coincide with refs. [30, 36, 37] if we ignore the contributions from the spin-2 particles in the latter.⁸

Several comments are in order. First, it can be seen that the Higgs mass and the cosmological constant emerge due to the RG running even if they are highly suppressed at a lower energy scale. This is simply an example of the infamous fine tuning problem; the scalaron introduces an additional scale, the scalaron mass scale. The Higgs mass and the cosmological constant are sensitive to this additional scale since they are not protected by any symmetry. Indeed, in order to realize the current universe with the electroweak (EW) scale and the almost vanishing cosmological constant, we have to tune the UV boundary conditions such that

$$\lambda_m(\mu = m_\sigma) \sim \frac{v_{\text{EW}}^2}{m_\sigma^2} \lambda_\alpha \ll \lambda_\alpha, \tag{2.35}$$

and a similar condition for the cosmological constant,⁹ where $m_\sigma^2 = M_P^2/12\alpha$ is the scalaron mass scale and $v_{\text{EW}} = 246$ GeV. It is indeed a tuning since the natural size of λ_m inferred from the RGEs is $\lambda_m \sim \lambda_\alpha$ for $\xi \gg 1$. These massive parameters were not included in the previous studies on the inflationary prediction of this model. Thus, we study their effects on the inflationary prediction in section 3.

Second, our method allows us to compute the beta functions of only the ratios of the massive parameters, not the massive parameters themselves. This is, however, not a limitation of our method, since only the ratios are important for physics.¹⁰ This point is also emphasized in refs. [36, 37].

Finally, it may be interesting to observe that a parameter with a higher mass dimension does not contribute to the beta functions of those with a lower mass dimension. For instance, λ_Λ does not contribute to β_{λ_α} , $\beta_{\bar{\xi}}$, β_λ and β_{λ_m} , while λ_m does not contribute to β_{λ_α} , $\beta_{\bar{\xi}}$ and β_λ . This is trivial in the original Jordan frame language, but it is non-trivial once we map the theory to the LSM with the renormalizable potential (2.27). From this point of view, all the couplings λ_α , λ_Λ , $\bar{\xi}$, λ_m and λ are equally dimensionless, and hence it is not entirely obvious (at least to the present authors) that the above hierarchy indeed holds.

2.4 Two-loop RGE

In this subsection, we show the two-loop RGEs of the Higgs- R^2 theory valid below the Planck scale obtained with the help of SARAH [35]. It is non-trivial to translate the outputs

⁸Some pieces of the RGEs shown here have also been derived, e.g., in refs. [38–44]. However, refs. [39–42] obtained the opposite sign for β_α .

⁹The cosmological constant may be affected by the EW scale below the scalaron mass scale, and hence the condition to realize the current tiny cosmological constant may need to take this into account.

¹⁰One can see this fact, e.g., by remembering that one can compute the spectral index and the tensor-to-scalar ratio without any problem even in Planck units $M_P = 1$.

of SARAH (or public codes in general) to the RGEs of the Higgs- R^2 theory since we have to take into account the ghost-like property of Φ . We only show the final results in this subsection, and explain how this translation can be done in appendix D. We note that, during the course of deriving the results, we have performed several non-trivial checks, which strongly supports the validity of our formalism. We again refer to appendix D on this point.

The RGEs of the gauge couplings are not affected by Φ nor σ up to two-loop, and are given by

$$\beta_{g_1}^{(2)} = g_1^3 \left[\frac{199}{50} g_1^2 + \frac{27}{10} g_2^2 + \frac{44}{5} g_3^2 - \frac{17}{10} y_t^2 \right], \quad (2.36)$$

$$\beta_{g_2}^{(2)} = g_2^3 \left[\frac{9}{10} g_1^2 + \frac{35}{6} g_2^2 + 12 g_3^2 - \frac{3}{2} y_t^2 \right], \quad (2.37)$$

$$\beta_{g_3}^{(2)} = g_3^3 \left[\frac{11}{10} g_1^2 + \frac{9}{2} g_2^2 - 26 g_3^2 - 2 y_t^2 \right]. \quad (2.38)$$

The RGE of the top Yukawa coupling is affected by Φ and σ and becomes

$$\begin{aligned} \beta_{y_t}^{(2)} = & -12 y_t^5 + y_t^3 \left[-12 \bar{\xi}^2 \lambda_\alpha + \frac{393 g_1^2}{80} + \frac{225 g_2^2}{16} + 36 g_3^2 - 12 \lambda \right] \\ & + y_t \left[6 \bar{\xi}^4 \lambda_\alpha^2 + \frac{1}{2} \bar{\xi}^2 \lambda_\alpha^2 + 12 \bar{\xi}^2 \lambda \lambda_\alpha + \frac{1187 g_1^4}{600} - \frac{23 g_2^4}{4} - 108 g_3^4 \right. \\ & \left. - \frac{9}{20} g_1^2 g_2^2 + \frac{19}{15} g_1^2 g_3^2 + 9 g_2^2 g_3^2 + 6 \lambda^2 \right]. \end{aligned} \quad (2.39)$$

The RGE of the Higgs quartic coupling is given by

$$\begin{aligned} \beta_\lambda^{(2)} = & -312 \lambda^3 + \lambda^2 \left[-816 \bar{\xi}^2 \lambda_\alpha + \frac{108 g_1^2}{5} + 108 g_2^2 - 144 y_t^2 \right] \\ & + \lambda y_t^2 \left[-144 \bar{\xi}^2 \lambda_\alpha + \frac{17 g_1^2}{2} + \frac{45 g_2^2}{2} + 80 g_3^2 \right] \\ & + \lambda \left[-696 \bar{\xi}^4 \lambda_\alpha^2 + 144 \bar{\xi}^3 \lambda_\alpha^2 - 10 \bar{\xi}^2 \lambda_\alpha^2 + \frac{72}{5} g_1^2 \bar{\xi}^2 \lambda_\alpha + 72 g_2^2 \bar{\xi}^2 \lambda_\alpha \right. \\ & \left. + \frac{1887 g_1^4}{200} - \frac{73 g_2^4}{8} + \frac{117}{20} g_1^2 g_2^2 \right] + y_t^4 \left[24 \bar{\xi}^2 \lambda_\alpha - \frac{8 g_1^2}{5} - 32 g_3^2 \right] \\ & + y_t^2 \left[-48 \bar{\xi}^4 \lambda_\alpha^2 + 24 \bar{\xi}^3 \lambda_\alpha^2 - \frac{171 g_1^4}{100} - \frac{9 g_2^4}{4} + \frac{63}{10} g_1^2 g_2^2 \right] - 192 \bar{\xi}^6 \lambda_\alpha^3 + 112 \bar{\xi}^5 \lambda_\alpha^3 \\ & - 8 \bar{\xi}^4 \lambda_\alpha^3 + 20 \bar{\xi}^3 \lambda_\alpha^3 - 10 \bar{\xi}^2 \lambda_\alpha^3 + \frac{12}{5} g_1^2 \bar{\xi}^4 \lambda_\alpha^2 + 12 g_2^2 \bar{\xi}^4 \lambda_\alpha^2 - \frac{6}{5} g_1^2 \bar{\xi}^3 \lambda_\alpha^2 - 6 g_2^2 \bar{\xi}^3 \lambda_\alpha^2 \\ & + \frac{27}{25} g_1^4 \bar{\xi}^2 \lambda_\alpha + 9 g_2^4 \bar{\xi}^2 \lambda_\alpha + \frac{18}{5} g_1^2 g_2^2 \bar{\xi}^2 \lambda_\alpha - \frac{3411 g_1^6}{2000} + \frac{305 g_2^6}{16} - \frac{289}{80} g_1^2 g_2^4 - \frac{1677}{400} g_1^4 g_2^2. \end{aligned} \quad (2.40)$$

The RGEs of the Higgs mass term and the non-minimal coupling are given by

$$\begin{aligned}
\beta_{\lambda_m}^{(2)} = & \lambda_m^2 \left[48\bar{\xi}^2 \lambda_\alpha + 20\bar{\xi} \lambda_\alpha - \frac{48}{5} g_1^2 \bar{\xi} - 48 g_2^2 \bar{\xi} + 48 \bar{\xi} y_t^2 \right] \\
& + \lambda_m \lambda_\alpha^2 \left[-60\bar{\xi}^4 - 80\bar{\xi}^3 + 9\bar{\xi}^2 - 16\bar{\xi} + 23 \right] \\
& + \lambda_m \lambda_\alpha \left[\frac{24}{5} g_1^2 \bar{\xi}^2 + 24 g_2^2 \bar{\xi}^2 + \frac{6}{5} g_1^2 \bar{\xi} + 6 g_2^2 \bar{\xi} - 120 \lambda \bar{\xi}^2 - 144 \lambda \bar{\xi} - 24 \bar{\xi}^2 y_t^2 - 24 \bar{\xi} y_t^2 \right] \\
& + \lambda_m \left[\frac{72 g_1^2 \lambda}{5} + 72 g_2^2 \lambda + \frac{17}{4} g_1^2 y_t^2 + \frac{45}{4} g_2^2 y_t^2 + 40 g_3^2 y_t^2 + \frac{1671 g_1^4}{400} + \frac{9}{8} g_2^2 g_1^2 \right. \\
& \left. - \frac{145 g_2^4}{16} - 60 \lambda^2 - 72 \lambda y_t^2 - \frac{27 y_t^4}{2} \right] + \lambda_\alpha^3 \left[-56 \bar{\xi}^4 - 22 \bar{\xi}^2 + 18 \bar{\xi} \right] \\
& + \lambda_\alpha^2 \left[\frac{3}{5} g_1^2 \bar{\xi}^2 + 3 g_2^2 \bar{\xi}^2 - 72 \lambda \bar{\xi}^2 - 12 \bar{\xi}^2 y_t^2 \right], \tag{2.41}
\end{aligned}$$

and

$$\begin{aligned}
\beta_{\bar{\xi}}^{(2)} = & -60 \lambda_\alpha^2 \bar{\xi}^5 - 40 \lambda_\alpha^2 \bar{\xi}^4 + \bar{\xi}^3 \left[9 \lambda_\alpha^2 - 120 \lambda \lambda_\alpha + \frac{24}{5} g_1^2 \lambda_\alpha + 24 g_2^2 \lambda_\alpha - 24 \lambda_\alpha y_t^2 \right] \\
& + \bar{\xi}^2 \left[-12 \lambda_\alpha^2 - 72 \lambda \lambda_\alpha + \frac{3}{5} g_1^2 \lambda_\alpha + 3 g_2^2 \lambda_\alpha - 12 \lambda_\alpha y_t^2 \right] \\
& + \bar{\xi} \left[\frac{25 \lambda_\alpha^2}{2} + \frac{72 g_1^2 \lambda}{5} + 72 g_2^2 \lambda + \frac{17}{4} g_1^2 y_t^2 + \frac{45}{4} g_2^2 y_t^2 + 40 g_3^2 y_t^2 \right. \\
& \left. + \frac{1671 g_1^4}{400} + \frac{9}{8} g_2^2 g_1^2 - \frac{145 g_2^4}{16} - 60 \lambda^2 - 72 \lambda y_t^2 - \frac{27 y_t^4}{2} \right]. \tag{2.42}
\end{aligned}$$

Finally, the RGEs of λ_α and λ_Λ are given by

$$\beta_{\lambda_\alpha}^{(2)} = \frac{1}{5} \lambda_\alpha^2 \left[48 \bar{\xi}^2 (g_1^2 + 5 g_2^2 - 5 y_t^2) - 5 (32 \bar{\xi}^3 + 20 \bar{\xi}^2 + 15) \lambda_\alpha \right], \tag{2.43}$$

and

$$\begin{aligned}
\beta_{\lambda_\Lambda}^{(2)} = & \lambda_\alpha^3 \left[-4 \bar{\xi}^2 - 7 \right] + \lambda_\alpha^2 \left[32 \bar{\xi}^3 \lambda_\Lambda + 20 \bar{\xi}^2 \lambda_\Lambda - 8 \bar{\xi}^2 \lambda_m - 8 \bar{\xi} \lambda_m + 26 \lambda_\Lambda \right] \\
& + \lambda_\alpha \left[96 \bar{\xi}^2 \lambda_\Lambda \lambda_m + 40 \bar{\xi} \lambda_\Lambda \lambda_m - 16 \bar{\xi} \lambda_m^2 - 4 \lambda_m^2 \right] \\
& - \frac{96}{5} g_1^2 \bar{\xi} \lambda_\Lambda \lambda_m - 96 g_2^2 \bar{\xi} \lambda_\Lambda \lambda_m + 96 \bar{\xi} \lambda_\Lambda \lambda_m y_t^2 + \frac{12}{5} g_1^2 \lambda_m^2 + 12 g_2^2 \lambda_m^2 - 12 \lambda_m^2 y_t^2. \tag{2.44}
\end{aligned}$$

These are the full two-loop RGEs of the Higgs- R^2 theory that are valid below the Planck scale. To our knowledge, it is this paper that has derived them for the first time.

3 Inflation with massive parameters

In section 2, we have seen that the Higgs mass and the cosmological constant arise due to the RG running above the scalaron mass scale even if they are tiny at low energies. In this section, we study whether these massive parameters change the inflationary prediction of

the Higgs- R^2 model or not. We assume here that the Higgs quartic coupling λ is positive due to either an uncertainty in the value of the top quark mass or some new physics. In section 4, we study effects of the scalaron on the sign of λ at high energy in detail. We assume that the values of α , m and Λ are consistent with the values naturally inferred from the RGE's, i.e.

$$\alpha \sim \xi^2, \quad \frac{m}{M_P} \sim \frac{1}{\xi}, \quad \frac{\Lambda}{M_P^4} \sim \frac{1}{\xi^4}. \quad (3.1)$$

We also assume that $\xi \gg 1$ in our analysis, as in the typical case of Higgs- R^2 inflation.

3.1 Background evolution

The inflationary trajectory is most easily studied in the Einstein frame. Starting with the action of eq. (2.1), we introduce an auxiliary field χ to rewrite it as

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R \left(1 + \frac{2\xi |H|^2 + 4\chi}{M_P^2} \right) + g^{\mu\nu} D_\mu H^\dagger D_\nu H - m^2 |H|^2 - \lambda |H|^4 - \frac{\chi^2}{\alpha} - \Lambda \right], \quad (3.2)$$

where we neglected the interactions with the SM fermions and gauge fields, as they do not affect the inflationary trajectory. We make the following Weyl transformation

$$g_{\mu\nu} \rightarrow \Omega_E^{-2} g_{\mu\nu}, \quad \Omega_E^2 = 1 + \frac{2\xi |H|^2 + 4\chi}{M_P^2}. \quad (3.3)$$

Note that we only rescale the metric, but the other fields are unaffected. Upon defining the field

$$\frac{\sigma_E}{M_P} = \sqrt{\frac{3}{2}} \log \Omega_E^2, \quad (3.4)$$

the action in the Einstein frame is given by

$$S_E = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma_E \partial_\nu \sigma_E + \frac{1}{2} e^{-\sqrt{\frac{2}{3}} \frac{\sigma_E}{M_P}} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - e^{-2\sqrt{\frac{2}{3}} \frac{\sigma_E}{M_P}} \left(\frac{m^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 + \frac{M_P^4}{16\alpha} \left(e^{\sqrt{\frac{2}{3}} \frac{\sigma_E}{M_P}} - 1 - \frac{\xi \phi^2}{M_P^2} \right)^2 + \Lambda \right) \right]. \quad (3.5)$$

We are using unitary gauge, in which the radial direction of the Higgs field ϕ is defined as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}. \quad (3.6)$$

The inflationary trajectory of this theory in the limit $m, \Lambda = 0$ was studied in refs. [14, 45–48]. We will now study the effect of the mass term and cosmological constant on the background evolution. Our analysis is similar to the one in ref. [47]. In the following we

omit the subscript E from σ_E for notational simplicity, but it should be noticed that it is not exactly the σ -meson of the LSM (2.27) contrary to σ in the previous section.

We will use Φ^I as shorthand notation for the fields

$$\Phi^I = \begin{pmatrix} \sigma \\ \phi \end{pmatrix}. \quad (3.7)$$

It should not be confused with the conformal mode Φ since the former always carries an index I . In our analysis in the Einstein frame, the ϕ -field has a non-canonical kinetic term, requiring a covariant treatment of the field space [49–52]. Here let us emphasize that there are two different definitions of the “target space” (or “field space”) in the literature. In section 2, we have identified the Higgs- R^2 theory as a LSM. In this identification, we have defined the target space by including not only the scalar fields but also the conformal mode of the metric Φ , following ref. [21]. The inclusion of Φ makes this definition of the target space frame independent, and hence this definition is useful to discuss the unitarity structure and the cut-off scale of the theory, which are of course frame independent. On the other hand, the target space is often defined just by the kinetic terms of the scalar fields, without including Φ , in the context of inflationary perturbations as in refs. [49–52]. This definition of the target space is frame dependent, nevertheless it is useful to compute inflationary perturbations. The frame dependence does not cause any problem as long as one works only in the Einstein frame and never moves to other frames. In order to distinguish between the two definitions of the target space, and in order to emphasize that the second target space is understood to be defined only in the Einstein frame, we refer to the latter as the “Einstein frame target space” in the following.

The Einstein frame target space metric is given by

$$G_{IJ} = \begin{pmatrix} 1 & 0 \\ 0 & e^{-\sqrt{\frac{2}{3}}\frac{\sigma}{M_P}} \end{pmatrix}. \quad (3.8)$$

It induces a covariant derivative \mathcal{D}_I , which operates on a vector in the field space V^J as

$$\mathcal{D}_I V^J = \frac{\partial V^J}{\partial \Phi^I} + \Gamma^J{}_{IK} V^K, \quad (3.9)$$

with $\Gamma^J{}_{IK}$ the Christoffel symbols of the Einstein frame target space. The background equation of motion of the fields Φ^I becomes

$$\mathcal{D}_t \dot{\Phi}^I = -3\mathcal{H}\dot{\Phi}^I - G^{IJ}V_{,J}, \quad (3.10)$$

where V is the potential and $\mathcal{D}_t = \dot{\Phi}^I \mathcal{D}_I$. The Hubble parameter \mathcal{H} and its evolution are given by

$$\mathcal{H}^2 = \frac{1}{3M_P^2} \left(\frac{1}{2} G_{IJ} \dot{\Phi}^I \dot{\Phi}^J + V(\Phi) \right), \quad \dot{\mathcal{H}} = \frac{1}{2M_P^2} G_{IJ} \dot{\Phi}^I \dot{\Phi}^J. \quad (3.11)$$

For our Einstein frame target space of eq. (3.8) the nonzero Christoffel symbols are

$$\Gamma^\sigma{}_{\phi\phi} = \frac{e^{-\sqrt{\frac{2}{3}}\frac{\sigma}{M_P}}}{\sqrt{6}M_P}, \quad \Gamma^\phi{}_{\sigma\phi} = \Gamma^\phi{}_{\phi\sigma} = -\frac{1}{\sqrt{6}M_P}, \quad (3.12)$$

and the background equations of motion are

$$\ddot{\sigma} = -3\mathcal{H}\dot{\sigma} - \frac{e^{-\sqrt{\frac{2}{3}}\frac{\sigma}{M_P}}}{\sqrt{6}M_P}\dot{\phi}^2 - V_{,\sigma}, \quad (3.13)$$

$$\ddot{\phi} = -3\mathcal{H}\dot{\phi} + \sqrt{\frac{2}{3}}\frac{1}{M_P}\dot{\phi}\dot{\sigma} - e^{\sqrt{\frac{2}{3}}\frac{\sigma}{M_P}}V_{,\phi}. \quad (3.14)$$

We will study the inflationary prediction of this model in the following.

3.2 Inflationary prediction

The potential V has two valleys that are symmetrically aligned with respect to the ϕ -axis. For our parameter choice we expect inflation to take place along this valley [14, 47]. Setting $V_{,\phi} = 0$ gives the σ -dependence of the ϕ -field in the valley

$$\phi_v^2(\sigma) = \frac{\xi M_P^2 \left(e^{\sqrt{\frac{2}{3}}\frac{\sigma}{M_P}} - 1 \right) - 4\alpha m^2}{\xi^2 + 4\alpha\lambda}. \quad (3.15)$$

We assume that during inflation $e^{\sqrt{\frac{2}{3}}\frac{\sigma}{M_P}} \gg 1$ (and we will see that this indeed gives us a consistent solution with at least 60 e-folds of inflation). During inflation, the contribution from the mass term to $\phi_v(\sigma)$ is thus negligible for $\xi \gg 1$.

Inserting eq. (3.15) into the kinetic term for ϕ we can express it in terms of σ

$$\frac{1}{2}e^{-\sqrt{\frac{2}{3}}\frac{\sigma}{M_P}}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \sim \frac{\xi}{12(4\alpha\lambda + \xi^2)}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma, \quad (3.16)$$

where we used $e^{\sqrt{\frac{2}{3}}\frac{\sigma}{M_P}} \gg 1$ to obtain the r.h.s. . The kinetic term of ϕ is thus suppressed by $\mathcal{O}(1/\xi)$ with respect to the kinetic term of σ and we will neglect it in our analytical approximation. Inserting (3.15) into the potential gives:

$$V(\sigma) = \frac{e^{-2\sqrt{\frac{2}{3}}\frac{\sigma}{M_P}}}{4(4\alpha\lambda + \xi^2)} \left[-4m^4\alpha + M_P^4\lambda \left(e^{\sqrt{\frac{2}{3}}\frac{\sigma}{M_P}} - 1 \right)^2 + 2m^2M_P^2\xi \left(e^{\sqrt{\frac{2}{3}}\frac{\sigma}{M_P}} - 1 \right) + 4\Lambda(4\alpha\lambda + \xi^2) \right]. \quad (3.17)$$

From this potential we can compute the inflationary observables in the slow-roll approximation. Note that in the regime $e^{\sqrt{\frac{2}{3}}\frac{\sigma}{M_P}} \gg 1$ and for the parameter values of eq. (3.1) the expression between square brackets is dominated by the term $M_P^4\lambda e^{2\sqrt{\frac{2}{3}}\frac{\sigma}{M_P}}$.

We first determine the value of σ at the moment when the observable modes left the horizon, σ_* , using:

$$N_* - N_{\text{end}} \sim \frac{1}{M_P^2} \int_{\sigma_{\text{end}}}^{\sigma_*} d\sigma \frac{V}{V_{,\sigma}} \sim \frac{1}{M_P^2} \int_{\sigma_{\text{end}}}^{\sigma_*} d\sigma \frac{\sqrt{\frac{3}{2}}M_P^3\lambda e^{\sqrt{\frac{2}{3}}\frac{\sigma}{M_P}}}{2M_P^2\lambda - 2m^2\xi}, \quad (3.18)$$

and we set $N_{\text{end}} = 0$. We neglect the contribution from the lower bound of the integral in (3.18) and find

$$N_* = \frac{\frac{3}{4}M_P^2\lambda e^{\sqrt{\frac{2}{3}}\frac{\sigma_*}{M_P}}}{M_P^2\lambda - m^2\xi} \rightarrow \sigma_* = \frac{3}{2}M_P \log \left[\frac{4}{3} \left(1 - \frac{m^2\xi}{M_P^2\lambda} \right) N_* \right], \quad (3.19)$$

where we have assumed $\xi m^2 < \lambda M_P^2$, which is the case for the typical value of m as long as $\lambda\xi \gg 1$. Plugging this value into the slow-roll parameters ϵ_V and η_V gives:

$$\epsilon_V = \frac{M_P^2}{2} \left(\frac{V_{,\sigma}}{V} \right)^2 \simeq \frac{3}{4N_*^2}, \quad (3.20)$$

$$\eta_V = M_P^2 \frac{V_{,\sigma\sigma}}{V} \simeq -\frac{1}{N_*}, \quad (3.21)$$

in the limit of large N_* . The scalar spectral index $n_{\mathcal{R}}^*$ and the tensor to scalar ratio r^* have the usual values of the R^2 -like inflationary models

$$n_{\mathcal{R}}^* = 1 - \frac{2}{N_*}, \quad r^* = \frac{12}{N_*^2}. \quad (3.22)$$

The scalar amplitude is

$$A_{\mathcal{R}}^* = \frac{V^*}{24\pi^2 M_P^4 \epsilon_V^*} = \frac{\lambda N_*^2}{72\pi^2 (4\alpha\lambda + \xi^2)}. \quad (3.23)$$

Choosing $N_* \sim 50 - 60$, measurement of the power spectrum fixes the relation

$$\frac{\xi^2}{\lambda} + 4\alpha \sim 2 \times 10^9. \quad (3.24)$$

Our analytical analysis above shows that none of the inflationary observables is affected by the presence of the nonzero Higgs mass term and cosmological constant. In other words, these massive parameters, inevitably induced due to the RGEs, do not spoil the consistency of Higgs- R^2 inflation with the CMB observation.

As a consistency check we will compute the turn rate in the valley, which ought to remain small for the single-field approximation to be valid. We introduce unit vectors \hat{T} and \hat{N} that are covariant with respect to coordinate transformations in the Einstein frame target space [49–52]. The basis vector \hat{T}^I is tangent to the inflationary trajectory, and \hat{N}^I is orthogonal to it:

$$\hat{T}^I = \frac{\dot{\Phi}^I}{\sqrt{G_{IJ}\dot{\Phi}^I\dot{\Phi}^J}}, \quad \hat{N}^I = \frac{\omega^I}{\omega}, \quad \omega^I = D_t \hat{T}^I, \quad (3.25)$$

where ω is the norm of the turn rate vector with

$$\omega = -\frac{V_{,N}}{\sqrt{G_{IJ}\dot{\Phi}^I\dot{\Phi}^J}}, \quad V_{,N} = \hat{N}^I \frac{\partial V}{\partial \Phi^I}. \quad (3.26)$$

Unit length and orthogonality imply

$$G_{IJ}\hat{T}^I\hat{T}^J = 1, \quad G_{IJ}\hat{N}^I\hat{N}^J = 1, \quad G_{IJ}\hat{N}^I\hat{T}^J = 0. \quad (3.27)$$

The components of \hat{T} and \hat{N} along the inflationary trajectory are given by

$$\hat{T} \simeq \left(\frac{1}{\sqrt{\frac{\xi}{6(4\lambda\alpha + \xi^2)}} e^{\frac{\sigma}{\sqrt{6}M_P}}}, \quad \hat{N} \simeq \pm \left(-\sqrt{\frac{\xi}{6(4\lambda\alpha + \xi^2)}} e^{\frac{\sigma}{\sqrt{6}M_P}} \right). \quad (3.28)$$

At first sight, it might look as if the second component of \hat{T} is large, but it is actually subdominant for $\xi \gg 1$ since it is always contracted with $G_{\sigma\sigma}$. The turn rate inside the valley is then given by

$$\frac{|\omega|}{\mathcal{H}} \simeq \sqrt{\frac{3\xi}{2(4\lambda\alpha + \xi^2)}} \sim \mathcal{O}(\xi^{-1/2}) \ll 1, \quad (3.29)$$

for $\xi \gg 1$, confirming that the single-field approximation was reasonable. The smallness of the turn rate guarantees that isocurvature modes would not modify the curvature power spectrum [52].

We can also show that the mass of the isocurvature mode is large, again consistent with the valley approximation (3.15). The isocurvature mass is found by projecting the mass matrix

$$M_{IJ} = \nabla_I \nabla_J V + R_{IKJL} \dot{\Phi}^K \dot{\Phi}^L, \quad (3.30)$$

onto the \hat{N} -direction:

$$m_N^2 = \hat{N}^I \hat{N}^J (\nabla_I \nabla_J V + R_{IKJL} \dot{\Phi}^K \dot{\Phi}^L), \quad (3.31)$$

all evaluated at $\phi = \phi_v$. A straightforward computation shows that the ratio of the isocurvature mass over the Hubble parameter is

$$\frac{m_N^2}{3\mathcal{H}^2} \simeq -\frac{\epsilon_T}{9} - \sqrt{\frac{\epsilon_T}{3}} + \frac{2\xi(4\alpha\lambda + \xi^2)}{\alpha\lambda}, \quad (3.32)$$

where ϵ_T is the slow roll parameter defined along the direction \hat{T}

$$\epsilon_T = \frac{M_P^2}{2} \left(\frac{V_{;T}}{V} \right)^2, \quad (3.33)$$

and in our case $\epsilon_T \sim \epsilon_V$ with ϵ_V defined in eq. (3.20). The only significant contribution to the isocurvature mass is the third term, implying that the isocurvature mass is positive and large compared to the Hubble parameter for $\xi \gg 1$. Thus the isocurvature power spectrum is suppressed, in agreement with observations [4].

4 Electroweak vacuum metastability

In section 3, we have assumed that the Higgs quartic coupling λ is positive at the inflationary scale. It is well-known, however, that λ becomes negative at the renormalization scale $\mu = 10^9$ - 10^{10} GeV for the current central value of the top quark mass within the SM [53, 54]. This feature strongly depends on the top quark mass, and the absolute stability of the EW vacuum is still allowed given an uncertainty in the determination of the top quark mass. In this section, we study how the scalaron affects this picture.

First of all, the scalaron cannot make λ positive for all the energy scales if we assume that inflation occurs within the Higgs- R^2 sector. As we have seen in section 3, assuming that $\lambda > 0$, the CMB normalization requires

$$\frac{\xi^2}{\lambda} + 4\alpha \simeq 2 \times 10^9. \quad (4.1)$$

It implies that the scalaron mass is bounded from below as¹¹

$$m_\sigma = \frac{M_P}{2\sqrt{3}\alpha} \gtrsim 3 \times 10^{13} \text{ GeV}. \quad (4.2)$$

The scalaron does not affect the RG flow of λ below this scale. Since λ already turns negative below this scale for the central value of the top quark mass, the scalaron cannot make $\lambda > 0$ for all energy scales. Nevertheless, the scalaron may affect the positivity of λ above the scalaron mass scale, which is the main topic of this section.

We separate our discussion into two parts. The first part focuses on the threshold corrections that arise by matching parameters between low and high energy theories. The second part is on the RG evolution of the parameters. We note that the same topic is analyzed in ref. [15]. A difference is that we have included the full RGEs up to two-loop, while (it seems that) they have included only the one-loop correction from ξ and α to the running of λ . Thus, our analysis here is more precise.

4.1 Threshold correction

In this subsection, we discuss the tree-level threshold correction. Our conclusion is that there is no tree-level threshold correction that makes λ positive.

First, it is important to notice that, although we refer to λ as the Higgs quartic coupling, it is not the coefficient of the Higgs quartic term above the scalaron mass scale, the UV theory. We refer to this tree-level quartic Higgs coupling as λ_{UV} . It can be obtained from the potential in eq. (2.12)

$$\lambda_{UV} = \lambda + \frac{(\xi + 1/6)^2}{\alpha}. \quad (4.3)$$

The tree-level matching at the scalaron mass scale is demonstrated in appendix B of ref. [14].¹² There, λ_m and λ_Λ were not included, but this is consistent with the very small values of eq. (2.35), so we will also take $\lambda_m(\mu = m_\sigma) = \lambda_\Lambda(\mu = m_\sigma) = 0$. The threshold correction is then obtained by comparing the 4-point interaction in the IR (which only includes the four-point vertex) and the UV (which includes a four-point vertex and diagrams with scalaron exchange). One finds that the parameter λ does not receive a threshold correction at tree level. This can also be seen from the fact that the λ_α -term vanishes by minimizing the scalar potential with respect to σ for $\lambda_m = \lambda_\Lambda = 0$, thus only the λ -term is left. The EW vacuum stability is therefore not affected by the threshold correction, at least in the case $\xi^2 \sim \alpha \gg 1$. Note that it is the positivity of λ , not λ_{UV} , that is crucial for the Higgs to play (a part of) the inflaton in Higgs- R^2 inflation.

4.2 RG evolution

Although the threshold correction does not affect λ , the scalaron can make λ positive through the RG evolution, as pointed out in ref. [15]. The reason is that scalar fields in

¹¹Strictly speaking, we here assume that the Higgs mass term and the cosmological constant are negligible at the scalaron mass scale, which should be true to be consistent with the current universe.

¹²Ref. [14] ignores kinetic mixings among the scalar fields in the Einstein frame and hence the sub-leading terms suppressed by ξ , or the “1/6” in eq. (4.3).

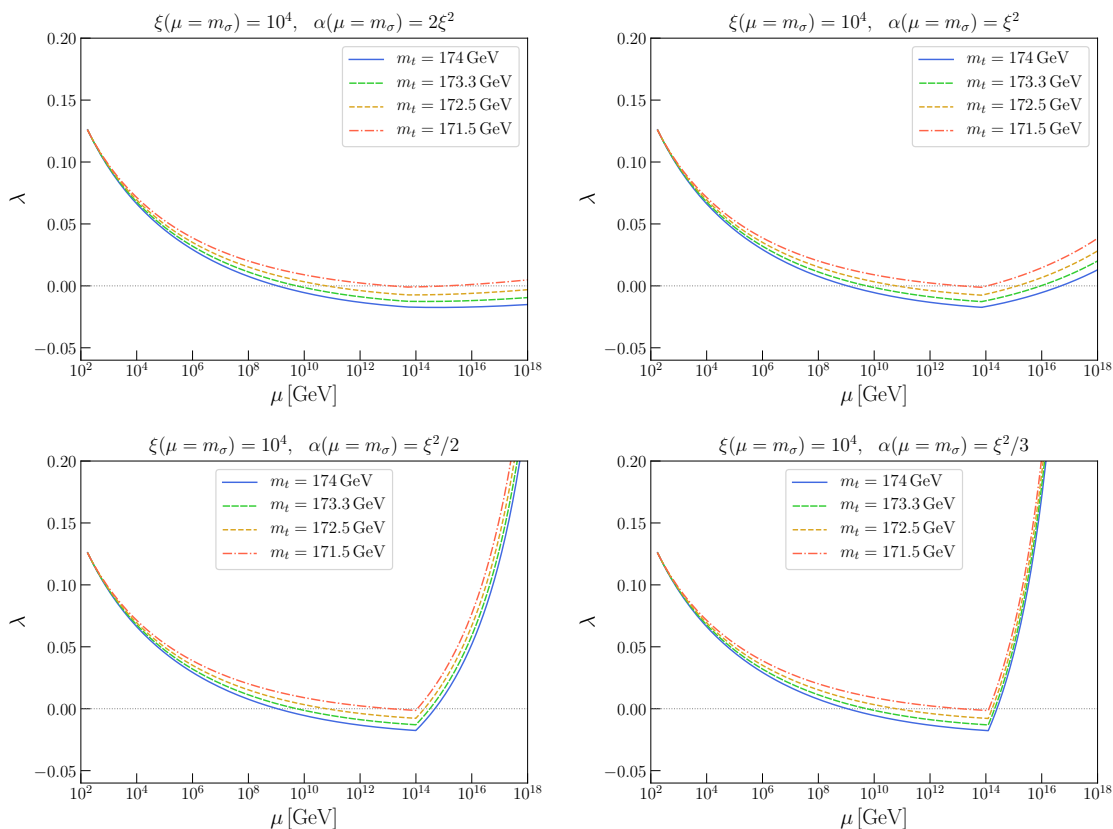


Figure 1. The RG running of λ for different values of the ratio ξ^2/α with $\xi(\mu = m_\sigma) = 10^4$. The additional contribution from the scalaron makes λ positive if ξ^2/α is large enough. A too large value of ξ^2/α however results in a Landau pole below the Planck scale. The Higgs mass is fixed as $m_h = 125.15$ GeV, and the other SM parameters are fixed according to ref. [54]. We have used the two-loop SM beta functions for $m_t < \mu < m_\sigma$, and the beta functions of sections 2.3 and 2.4 for $m_\sigma < \mu < M_P$.

general give positive contributions to the beta function of λ . In figure 1, we show the RG running of λ for several parameter sets. We have used the two-loop SM RGEs below the scalaron mass scale $m_\sigma = M_P/12\alpha$, and the full RGEs up to two-loop given in sections 2.3 and 2.4 above m_σ . We have used, however, only the tree-level matching conditions at m_σ , and hence figure 1 should be understood as a demonstration, not having the full two-loop precision. It is beyond the scope of this paper to compute the one-loop matching conditions, and we leave it for future work. The boundary conditions for the Higgs mass and the cosmological constant are $\lambda_m(\mu = m_\sigma) = \lambda_\Lambda(\mu = m_\sigma) = 0$. We have fixed $\xi = 10^4$ and varied the ratio ξ^2/α as $\xi^2/\alpha = 1/2, 1, 2, 3$ at $\mu = m_\sigma$ since the additional contribution to β_λ is controlled by this ratio. One can see that λ turns positive at high energy if the ratio ξ^2/α is large enough. A too large ξ^2/α however makes λ too large so that the theory hits a Landau pole and loses perturbativity below the Planck scale.

As we have seen, the scalaron cannot make λ positive for all energy scales, yet it can push λ to be positive at high energy for a relatively large value of ξ^2/α . As a result, the

scalar potential may develop a local minimum around m_σ . It would be interesting to discuss the cosmological implication of this local minimum, especially during and after inflation. Cosmological implications of EW vacuum metastability have been studied in detail in the literature. For instance, if the inflationary scale is too high, it triggers EW vacuum decay during inflation, resulting in an upper bound on the inflationary scale [55–67]. In addition, the inflaton-Higgs couplings or the non-minimal coupling between the Higgs and the Ricci scalar can produce a large amount of Higgs quanta during the preheating epoch, causing EW vacuum decay after inflation and hence putting upper bounds on these couplings [68–77]. These works usually assume that the Higgs potential is negative (at least) up to the Planck scale, and hence it is difficult to recover the EW vacuum once the Higgs field rolls down to the negative region. In the current case, the situation can be different, as the Higgs potential may be again positive at high energy. Hence, even if the Higgs is trapped at the local minimum at some epoch in the early universe, it may be easier to restore the EW vacuum by, e.g., thermal effects. We leave a detailed study for future work.

5 Conclusion and discussion

Higgs- R^2 inflation is an interesting model, as it gives inflationary predictions consistent with the Planck data [4]. It includes a new scalaron degree of freedom at the inflationary scale on top of Higgs inflation, which makes the theory unitary and renormalizable up to the Planck scale (in contrast with Higgs inflation without a scalaron). In the present work, we have presented the one- and two-loop Renormalization Group Equations for this system and studied their phenomenological consequences. The RGEs are valid up to the Planck scale in the large ξ regime that we studied (which is the most relevant for inflation). Below we summarize each section of this paper and list possible future directions.

One- and two-loop RGEs. In section 2.1, to derive the RGEs, we decomposed the metric into the determinant part, which we call the conformal mode, and a spin-2 part. We show in section 2.2 that the spin-2 part couples to the matter fields only via Planck-suppressed operators and we can thus neglect $h_{\mu\nu}$ below the Planck scale. Furthermore, for our gauge fixing condition, eq. (2.21), we can also ignore the Faddeev-Popov ghost field. These properties greatly simplify our computations.

We thus derived the RGEs from eq. (2.27), which contains only the conformal mode, the scalaron, the Higgs field and the other SM fields. The one-loop RGEs are presented in section 2.3. These were obtained by computing four-point functions explicitly. We cross-checked the RGEs with the help of **SARAH** and confirmed that the RGEs match the results of refs. [30, 36, 37] if we ignore the spin-2 contribution. The details of the computation are given in appendix C. Using **SARAH**, we also obtained the two-loop RGEs in section 2.4. Appendix D contains the computational details. To our knowledge, the two-loop RGEs of the Higgs- R^2 theory are presented for the first time in this present work.

Consequences for inflation and reheating. The interactions corresponding to the couplings λ_m , which generates a Higgs mass, and λ_Λ , the cosmological constant, have to be included to renormalize the theory. These terms were not taken into account in

previous studies of the Higgs- R^2 -model. Therefore, in section 3, we analytically study the inflationary prediction with a nonzero λ_m and λ_Λ . The predictions for the single-field inflationary trajectory along the valley are not affected by the nonzero values of m and Λ , and are thus consistent with the Planck data. Although the inflationary predictions are unaffected, the presence of the mass term and the cosmological constant might affect the particle production stage at the end of inflation. Reheating in the Higgs- R^2 system was studied in refs. [78–81], but these studies did not include nonzero m and Λ . In ref. [82] reheating was studied in a similar model, with a nonzero mass term for the field that resembles our Higgs field. The study suggests that the nonzero mass term leads to a smaller variance of the Higgs-like fields in the rescattering regime and also affects the evolution of the equation of state.

Electroweak stability. In section 4 we studied another phenomenological consequence of the RGEs: the running of λ , which is relevant for the stability of the Higgs potential. The inclusion of the scalaron can not make λ positive for all energy scales, since the scalaron mass scale is above the scale where λ typically runs negative (this scale depends on the value of the top quark mass), assuming that inflation happens in the Higgs- R^2 sector. Nevertheless, the Higgs potential can be stabilized at larger energy scales. This stabilization mechanism depends on the ratio ξ^2/α . If the ratio is too small, the stabilizing effect is also small. However, if the ratio is too large, λ encounters a Landau pole below the Planck scale. Let us point out that we used only tree-level matching at the scalaron mass scale in this paper. In this case λ receives no stabilizing threshold correction. We leave the computation of the threshold correction in a one-loop matching procedure for future work.

Residual gauge symmetry. The fact that there are only six independent parameters in eq. (2.27) and the theory can thus be renormalized by only six counterterms, instead of possibly nine, suggests that the shape of the potential is restricted by some symmetry. In appendix E we show that the requirement that the action is invariant under (the scalar part of) the residual gauge symmetry $x^\mu \rightarrow x^\mu - \partial^\mu \xi$, restricts the allowed interactions. The residual gauge symmetry can explain the shape of the potential, but only if we do not include the scalaron from the beginning, but obtain it from the $(\square\Phi/\Phi)^2$ -term. To us, this approach seems somewhat ad hoc and a better understanding is lacking. The physical reason for including the operators $(\square\Phi/\Phi)^2$ and $|H|^2\square\Phi/\Phi$, but neglecting all other higher derivative operators is also not clear. We leave a better understanding of these points for future work.

Generalization to other theories/asymptotic safety. In this paper we have focused on the Higgs- R^2 theory, i.e., the theory with the R^2 -term and the SM. Our method can be, however, applied to more general theories which contain an R^2 -term and arbitrary numbers of fields with generic Yukawa, gauge and scalar quartic interactions. Indeed, it is expected to be straightforward to obtain the RGEs of such a general theory with the help of, e.g., refs. [31–34] and the procedure outlined in appendix D. These RGEs may be useful beyond the context of inflation, e.g., for the asymptotic safety program [83, 84] (see ref. [85] for a recent review).

An ultimate goal of asymptotic safety is to understand the UV structure of quantum gravity. Our method cannot be directly connected to this ultimate goal since we ignore the spin-2 particles. Our method also assumes that the theory is perturbative, and is useless in a strong coupling regime. Nevertheless it correctly captures the effect of the scalar sector of the gravity, and hence may serve as a small step toward the ultimate goal of the asymptotic safety program. For instance, a UV fixed point is guaranteed to exist in a general renormalizable theory without gravity in a perturbative regime in the Veneziano limit [86]. We can see how this conclusion is affected by an inclusion of the R^2 -term and hence the conformal mode of the metric and the scalaron. It might also be interesting to study the LSM (2.27) by the functional RG approach [87–90] since it allows us to go beyond the perturbative regime.^{13,14} Finally, it might be interesting by itself that the LSM (2.27) without any matter fields has a fixed point $\lambda_\alpha = 0$ for an arbitrary value of λ_Λ , which is also discussed in ref. [92].

Acknowledgments

This work was funded by the Deutsche Forschungsgemeinschaft under Germany’s Excellence Strategy - EXC 2121 “Quantum Universe” - 390833306. The Feynman diagrams are drawn by TikZ-Feynman [98].

A General coordinate transformation

In this appendix, we derive eqs. (2.18) and (2.19). Under the general coordinate transformation, $x^\mu \rightarrow x^\mu - \xi^\mu$, the spacetime metric transforms as

$$g_{\mu\nu} \rightarrow g'_{\mu\nu} = g_{\mu\nu} + (\partial_\mu \xi^\alpha) g_{\alpha\nu} + g_{\mu\alpha} (\partial_\nu \xi^\alpha) + \xi^\alpha \partial_\alpha g_{\mu\nu}, \quad (\text{A.1})$$

up to the first order in ξ^μ . By substituting the decomposition (2.3), it reads

$$e^{2\varphi'} [e^{h'}]_{\mu\nu} = e^{2\varphi} \left\{ [e^h]_{\mu\nu} + (\partial_\mu \xi^\alpha) [e^h]_{\alpha\nu} + [e^h]_{\mu\alpha} (\partial_\nu \xi^\alpha) + 2\xi^\alpha (\partial_\alpha \varphi) [e^h]_{\mu\nu} + \xi^\alpha \partial_\alpha [e^h]_{\mu\nu} \right\} \quad (\text{A.2})$$

where the indices are raised and lowered by the flat spacetime metric $\eta_{\mu\nu}$ here and hereafter. It can be rewritten as

$$e^{2(\varphi' - \varphi)} [e^{h'}]_{\mu\nu} = [e^{h/2}]_\mu^\rho \left\{ (1 + 2\xi^\alpha \partial_\alpha \varphi) \delta_\rho^\sigma + [e^{-h/2}]_\rho^\beta (\partial_\beta \xi^\alpha) [e^{h/2}]_\alpha^\sigma + [e^{h/2}]_{\rho\alpha} (\partial_\beta \xi^\alpha) [e^{-h/2}]^{\beta\sigma} + [e^{-h/2}]_\rho^\beta (\xi^\alpha \partial_\alpha [e^h]_{\beta\gamma}) [e^{-h/2}]^{\gamma\sigma} \right\} [e^{h/2}]_{\sigma\nu}. \quad (\text{A.3})$$

¹³The functional RG approach is also applied to Higgs inflation [91] and R^2 inflation [92].

¹⁴We believe that our method is similar in spirit as refs. [93–97] since they only focus on the R^n -terms that affect the scalar sector of the theory.

By taking the determinant of both sides, we obtain

$$e^{8(\varphi' - \varphi)} = 1 + 8\xi^\alpha \partial_\alpha \varphi + 2\partial_\alpha \xi^\alpha + \xi^\alpha \text{tr} \left[e^{-h} \partial_\alpha e^h \right] + \mathcal{O}(\xi^2), \quad (\text{A.4})$$

where we have used the relation

$$\text{Det} [\delta + \epsilon A] = 1 + \epsilon \text{Tr} A + \mathcal{O}(\epsilon^2), \quad (\text{A.5})$$

with δ_μ^ν being the identity matrix. We can use Jacobi's formula to show that

$$\text{Tr} \left[e^{-h} \partial_\alpha e^h \right] = \partial_\alpha \ln \text{Det} \left[e^h \right] = 0, \quad (\text{A.6})$$

where we have used the definition of the decomposition (2.3) in the second equality. Thus, we obtain the transformation law of the conformal mode of the metric as

$$\varphi' = \varphi + \xi^\alpha \partial_\alpha \varphi + \frac{1}{4} \partial_\alpha \xi^\alpha, \quad (\text{A.7})$$

up to first order in ξ^μ . By substituting into eq. (A.3), we obtain

$$\begin{aligned} \left[e^{-h/2} e^{h'} e^{-h/2} \right]_{\mu\nu} &= \left(1 - \frac{1}{2} (\partial_\alpha \xi^\alpha) \right) \eta_{\mu\nu} + \left[e^{-h/2} (\partial_\xi) e^{h/2} \right]_{\mu\nu} + \left[e^{-h/2} (\partial_\xi) e^{h/2} \right]_{\nu\mu} \\ &\quad + \left[e^{-h/2} (\xi^\alpha \partial_\alpha e^h) e^{-h/2} \right]_{\mu\nu}, \end{aligned} \quad (\text{A.8})$$

where we have relied on the matrix notation, and $[\partial_\xi]_{\mu\nu} \equiv \partial_\mu \xi_\nu$. We now simplify this equation further. For this purpose, it is useful to define the adjoint action as

$$\text{ad}_X Y \equiv [X, Y] = XY - YX, \quad (\text{A.9})$$

for arbitrary matrices X and Y . Then the following relation holds:

$$e^X Y e^{-X} = e^{\text{ad}_X} Y, \quad (\text{A.10})$$

where the right-hand-side should be understood by the Taylor expansion as

$$e^{\text{ad}_X} Y = Y + [X, Y] + \frac{1}{2!} [X, [X, Y]] + \frac{1}{3!} [X, [X, [X, Y]]] + \dots \quad (\text{A.11})$$

One can easily see that the adjoint action is linear, i.e.,

$$\text{ad}_{cX} = c \text{ad}_X, \quad \text{ad}_{X+Y} = \text{ad}_X + \text{ad}_Y, \quad (\text{A.12})$$

for an arbitrary complex number c . The variation of an exponentiated matrix is expressed with the help of the adjoint action as

$$e^{X+\Delta X} - e^X = e^X \frac{1 - e^{-\text{ad}_X}}{\text{ad}_X} \Delta X. \quad (\text{A.13})$$

Equipped with these expressions, eq. (A.8) is simplified as

$$\begin{aligned} \left[\left(\frac{e^{\text{ad}_h/2} - e^{-\text{ad}_h/2}}{\text{ad}_h} \right) (h' - h) \right]_{\mu\nu} &= -\frac{1}{2} (\partial_\alpha \xi^\alpha) \eta_{\mu\nu} + \left[e^{-\text{ad}_h/2} \partial_\xi \right]_{\mu\nu} + \left[e^{-\text{ad}_h/2} \partial_\xi \right]_{\nu\mu} \\ &\quad + \left[\left(\frac{e^{\text{ad}_h/2} - e^{-\text{ad}_h/2}}{\text{ad}_h} \right) (\xi^\alpha \partial_\alpha h) \right]_{\mu\nu}, \end{aligned} \quad (\text{A.14})$$

to first order in ξ^μ . The adjoint action for a symmetric tensor X (i.e. $X^T = X$) satisfies

$$\text{ad}_X Y^T = -[\text{ad}_X Y]^T, \quad (\text{A.15})$$

where the superscript T indicates the transpose. Therefore, by acting with the inverse operator from the left on eq. (A.14), we finally obtain the transformation law of $h_{\mu\nu}$ as

$$h'_{\mu\nu} = h_{\mu\nu} + \left[\left(\frac{\text{ad}_h}{e^{\text{ad}_h} - 1} \right) \partial\xi \right]_{\mu\nu} + \left[\left(\frac{\text{ad}_h}{e^{\text{ad}_h} - 1} \right) \partial\xi \right]_{\nu\mu} - \frac{1}{2} (\partial_\alpha \xi^\alpha) \eta_{\mu\nu} + \xi^\alpha \partial_\alpha h_{\mu\nu}, \quad (\text{A.16})$$

up to first order in ξ^μ . It completes our derivation of eqs. (2.18) and (2.19). Note that the traceless property of $h_{\mu\nu}$ is indeed preserved since

$$\text{Tr} \left[\left(\frac{\text{ad}_h}{e^{\text{ad}_h} - 1} \right) \partial\xi \right] = \partial_\alpha \xi^\alpha. \quad (\text{A.17})$$

B Einstein and conformal frame

In this appendix, we clarify the relation between the action given in eq. (2.12) and that in the Einstein frame (3.5). Let us start from eq. (3.5). The metric in the Einstein frame can be decomposed in the same way

$$g_{\mu\nu} = e^{2\varphi_E} \tilde{g}_{\mu\nu}, \quad \text{Det} [\tilde{g}_{\mu\nu}] = -1. \quad (\text{B.1})$$

The conformal mode in the Einstein frame is defined as

$$\Phi_E = \sqrt{6} M_P e^{\varphi_E}. \quad (\text{B.2})$$

Here we use the subscript E to clarify that Φ_E is different from Φ . We also rescale the fields as

$$\sigma_E \rightarrow e^{-\varphi_E} \sigma_E, \quad H \rightarrow e^{\sigma_E/\Phi_E - \varphi_E} H, \quad \psi \rightarrow e^{-3\varphi_E/2} \psi. \quad (\text{B.3})$$

The scalaron field σ given in eq. (2.12) is different from σ_E . The relation between them is given by

$$\frac{\sigma_E}{\Phi_E} = -\ln \left(\sqrt{\Phi_E^2 + 2|H|^2 + \sigma^2} + \sigma \right) + \ln \Phi_E. \quad (\text{B.4})$$

Inserting these equations into eq. (3.5), we obtain

$$\begin{aligned} S = \int d^4x & \left[\frac{\tilde{R}}{12} \Phi_E^2 + \tilde{g}^{\mu\nu} D_\mu H^\dagger D_\nu H + \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma \right. \\ & - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \sqrt{\Phi_E^2 + 2|H|^2 + \sigma^2} \partial_\nu \sqrt{\Phi_E^2 + 2|H|^2 + \sigma^2} \\ & \left. - V \left(\sqrt{\Phi_E^2 + 2|H|^2 + \sigma^2}, H, \sigma \right) \right] + S_{\psi+A} [\tilde{g}_{\mu\nu}], \end{aligned} \quad (\text{B.5})$$

where

$$V(\Phi, H, \sigma) = \lambda |H|^4 + \frac{\lambda_m}{2} (\Phi + \sigma)^2 |H|^2 + \frac{\lambda_\Lambda}{4} (\Phi + \sigma)^4 + \frac{\lambda_\alpha}{4} \left[\sigma (\Phi + \sigma) + 2\bar{\xi} |H|^2 \right]^2. \quad (\text{B.6})$$

Therefore we reproduce eq. (2.12) by the following relation between Φ_E and Φ :

$$\Phi_E^2 = \Phi^2 - 2|H|^2 - \sigma^2. \quad (\text{B.7})$$

C Computational details of one-loop RGEs

In this appendix, we provide some intermediate steps to derive the one-loop RGEs. It is shown in the main text that the SM fermions and gauge bosons do not couple to the conformal mode Φ nor the scalaron σ thanks to the Weyl invariance. Therefore, their effects on the RGEs except for the Higgs quartic coupling λ are only through the Higgs anomalous dimension, which is easy to reproduce. For this reason, we ignore the SM fermions and gauge bosons and focus on the scalar sector of the Higgs- R^2 theory in this appendix.

C.1 Feynman rules

As we have explained in the main text, we can simply take $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$ without worrying about any complications associated with the ghost fields below M_P . Thus, the action from which we compute the RGEs is

$$S = \int d^4x \left[-\frac{1}{2} \eta^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i + \frac{1}{2} \eta^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{\lambda}{4} \phi_i^4 - \frac{\lambda_m}{4} (\Phi + \sigma)^2 \phi_i^2 - \frac{\lambda_\Lambda}{4} (\Phi + \sigma)^4 - \frac{\lambda_\alpha}{4} \left(\sigma (\Phi + \sigma) + \xi \bar{\phi}_i^2 \right)^2 \right], \quad (\text{C.1})$$

where we have slightly generalized the action by allowing the index i to run from $i = 1$ to $i = N$, where N is the number of the real scalar fields ($N = 4$ for the SM Higgs). Note that we do not even have to expand Φ around $\sqrt{6}M_P$ to compute the RGEs. The Feynman rules are readily derived from this action. The propagators are given by

$$\Phi : \quad \text{~~~~~} = -\frac{i}{q^2}, \quad (\text{C.2})$$

$$\sigma : \quad \text{————} = \frac{i}{q^2}, \quad (\text{C.3})$$

$$\phi_i : \quad \text{-----}^i_j = \frac{i}{q^2} \delta_{ij}. \quad (\text{C.4})$$

where we have used the wavy line for Φ and the solid line for σ since we do not consider the fermions and gauge bosons in this appendix. They should not be confused with the SM fermions and gauge bosons. Here it is important to notice the additional minus sign in the propagator of the conformal mode Φ . It originates from the ghost-like property of Φ . Although ghost-like, it does not spoil the theory thanks to a residual gauge symmetry, as discussed in detail in ref. [21]. The Feynman rules for the interactions are given by

$$\begin{array}{cc} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} = -6i(\lambda_\alpha + \lambda_\Lambda), & \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \text{~~~~~} \end{array} = -3i(\lambda_\alpha + 2\lambda_\Lambda), \end{array} \quad (\text{C.5})$$

$$\begin{array}{ccc} \begin{array}{c} \diagup \quad \text{~~~~~} \\ \text{~~~~~} \quad \diagdown \end{array} = -i(\lambda_\alpha + 6\lambda_\Lambda), & \begin{array}{c} \diagup \quad \text{~~~~~} \\ \text{~~~~~} \quad \text{~~~~~} \end{array} = -6i\lambda_\Lambda, & \begin{array}{c} \text{~~~~~} \quad \text{~~~~~} \\ \text{~~~~~} \quad \text{~~~~~} \end{array} = -6i\lambda_\Lambda, \end{array} \quad (\text{C.6})$$

$$= -i \left(2\bar{\xi}\lambda_\alpha + \lambda_m \right) \delta_{ij}, \quad = -i \left(\bar{\xi}\lambda_\alpha + \lambda_m \right) \delta_{ij}, \quad (\text{C.7})$$

$$= -i\lambda_m\delta_{ij}, \quad = -2i \left(\bar{\xi}^2\lambda_\alpha + \lambda \right) \left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} \right), \quad (\text{C.8})$$

where the combinatory factors from the external states are taken into account.

C.2 Divergent part of four-point functions

We compute the scalar four-point functions at one-loop level. Below we list the divergent parts of the four-point functions before renormalization, where we have performed dimensional regularization with $d = 4 - 2\epsilon$. The divergent parts are

$$iA_{\sigma^4}|_{\text{div.}} = \frac{3i}{32\pi^2} \frac{1}{\epsilon} \left[\lambda_\alpha (19\lambda_\alpha + 12\lambda_\Lambda) + N \left(2\bar{\xi}\lambda_\alpha + \lambda_m \right)^2 \right], \quad (\text{C.9})$$

$$iA_{\sigma^3\Phi}|_{\text{div.}} = \frac{3i}{32\pi^2} \frac{1}{\epsilon} \left[12\lambda_\alpha (\lambda_\alpha + \lambda_\Lambda) + N \left(2\bar{\xi}\lambda_\alpha + \lambda_m \right) \left(\bar{\xi}\lambda_\alpha + \lambda_m \right) \right], \quad (\text{C.10})$$

$$iA_{\sigma^2\Phi^2}|_{\text{div.}} = \frac{i}{32\pi^2} \frac{1}{\epsilon} \left[4\lambda_\alpha (5\lambda_\alpha + 9\lambda_\Lambda) + N \left(2 \left(\bar{\xi}\lambda_\alpha + \lambda_m \right)^2 + \lambda_m \left(2\bar{\xi}\lambda_\alpha + \lambda_m \right) \right) \right], \quad (\text{C.11})$$

$$iA_{\sigma\Phi^3}|_{\text{div.}} = \frac{3i}{32\pi^2} \frac{1}{\epsilon} \left[3\lambda_\alpha (\lambda_\alpha + 4\lambda_\Lambda) + N \left(\bar{\xi}\lambda_\alpha + \lambda_m \right) \lambda_m \right], \quad (\text{C.12})$$

$$iA_{\Phi^4}|_{\text{div.}} = \frac{3i}{32\pi^2} \frac{1}{\epsilon} \left[\lambda_\alpha (\lambda_\alpha + 12\lambda_\Lambda) + N\lambda_m^2 \right], \quad (\text{C.13})$$

for the four-point functions involving only the conformal mode and the scalaron, and

$$iA_{\sigma^2\phi_i\phi_j}|_{\text{div.}} = \frac{i}{32\pi^2} \frac{\delta_{ij}}{\epsilon} \left[6\bar{\xi} \left(2\bar{\xi} + 1 \right) \lambda_\alpha^2 + \lambda_\alpha \lambda_m \left(1 + 8\bar{\xi} \right) + 2 \left(N + 2 \right) \left(2\bar{\xi}\lambda_\alpha + \lambda_m \right) \left(\bar{\xi}^2\lambda_\alpha + \lambda \right) \right], \quad (\text{C.14})$$

$$iA_{\sigma\Phi\phi_i\phi_j}|_{\text{div.}} = \frac{i}{32\pi^2} \frac{\delta_{ij}}{\epsilon} \left[4\bar{\xi}\lambda_\alpha^2 \left(2\bar{\xi} + 1 \right) + \left(1 + 8\bar{\xi} \right) \lambda_\alpha \lambda_m + 2 \left(N + 2 \right) \left(\bar{\xi}\lambda_\alpha + \lambda_m \right) \left(\bar{\xi}^2\lambda_\alpha + \lambda \right) \right], \quad (\text{C.15})$$

$$iA_{\Phi^2\phi_i\phi_j}|_{\text{div.}} = \frac{i}{32\pi^2} \frac{\delta_{ij}}{\epsilon} \left[2\bar{\xi} \left(2\bar{\xi} + 1 \right) \lambda_\alpha^2 + \left(1 + 8\bar{\xi} \right) \lambda_\alpha \lambda_m + 2 \left(N + 2 \right) \lambda_m \left(\bar{\xi}^2\lambda_\alpha + \lambda \right) \right], \quad (\text{C.16})$$

$$iA_{\phi_i\phi_j\phi_k\phi_l}|_{\text{div.}} = \frac{i}{16\pi^2} \frac{1}{\epsilon} \left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} \right) \left[\bar{\xi}^2\lambda_\alpha^2 + 2 \left(N + 8 \right) \left(\bar{\xi}^2\lambda_\alpha + \lambda \right)^2 \right], \quad (\text{C.17})$$

for the four-point functions involving the scalar fields ϕ_i .

C.3 Counterterms

Now we list the divergent parts of the counterterms. It is important to notice that we have to make use of an $\text{SO}(1,1)$ redundancy to renormalize the theory. We introduce the counterterms to the scalar potential V as

$$V = \frac{\lambda_\alpha + \delta\lambda_\alpha}{4} \left[(1 + \theta) (\sigma + \Phi) (\sigma + \theta\Phi) + (\bar{\xi} + \delta\bar{\xi}) \phi_i^2 \right]^2 + \frac{\lambda_\Lambda + \delta\lambda_\Lambda}{4} (1 + 4\theta) (\sigma + \Phi)^4 + \frac{\lambda_m + \delta\lambda_m}{4} (1 + 2\theta) (\sigma + \Phi)^2 \phi_i^2 + \frac{\lambda + \delta\lambda}{4} \phi_i^4, \quad (\text{C.18})$$

where θ reflects the $\text{SO}(1,1)$ redundancy that corresponds to the shift

$$\sigma \rightarrow \sigma \cosh \theta + \Phi \sinh \theta, \quad \Phi \rightarrow \Phi \cosh \theta + \sigma \sinh \theta. \quad (\text{C.19})$$

Here we expand the fields with respect to θ around $\theta = 0$ to first order. Note that there are only six counterterms, $\delta\lambda_\alpha, \theta, \delta\lambda_\Lambda, \delta\lambda_m, \delta\bar{\xi}, \delta\lambda$, while there are nine divergent four-point functions. Hence it is a non-trivial check of our computation that we can indeed renormalize the theory. After renormalization, the divergent parts of the counterterms are given by

$$\delta\lambda_\alpha = \frac{1}{32\pi^2} \frac{\lambda_\alpha^2}{\epsilon} \left[5 + 2N\bar{\xi}^2 \right], \quad (\text{C.20})$$

$$\theta = \frac{1}{32\pi^2} \frac{1}{\epsilon} \left[2\lambda_\alpha + N\bar{\xi}\lambda_m \right], \quad (\text{C.21})$$

$$\delta\lambda_\Lambda = \frac{1}{64\pi^2} \frac{1}{\epsilon} \left[\lambda_\alpha (\lambda_\alpha - 4\lambda_\Lambda) + N\lambda_m (\lambda_m - 8\bar{\xi}\lambda_\Lambda) \right], \quad (\text{C.22})$$

$$\delta\lambda_m = \frac{1}{32\pi^2} \frac{1}{\epsilon} \left[2\bar{\xi} (2\bar{\xi} - 1) \lambda_\alpha^2 + (-3 + 8\bar{\xi} + 4\bar{\xi}^2) \lambda_\alpha \lambda_m - 2N\bar{\xi}\lambda_m^2 + 2(N+2) \lambda_m \lambda \right], \quad (\text{C.23})$$

$$\delta\bar{\xi} = \frac{1}{32\pi^2} \frac{1}{\epsilon} \left[\bar{\xi}\lambda_\alpha (-3 + 4\bar{\xi} + 4\bar{\xi}^2) + 2(N+2) \bar{\xi}\lambda \right], \quad (\text{C.24})$$

$$\delta\lambda = \frac{1}{16\pi^2} \frac{1}{\epsilon} \left[(2\bar{\xi} - 1)^2 \bar{\xi}^2 \lambda_\alpha^2 + 12\bar{\xi}^2 \lambda_\alpha \lambda + (N+8) \lambda^2 \right]. \quad (\text{C.25})$$

One can readily derive the one-loop RGEs from these expressions.

D Computational details of two-loop RGEs

In this appendix, we explain how we obtain the full two-loop RGEs of the Higgs- R^2 theory shown in section 2.4 with the help of SARAH. Our main purpose in this appendix is to explain how to take into account the minus sign in front of the kinetic term of the conformal mode Φ without modifying the public code itself. For this purpose, it is important to notice that once we replace

$$\Phi = i\phi, \quad (\text{D.1})$$

the kinetic term of ϕ has the conventional sign. After this replacement, the coupling between ϕ and the other fields is multiplied by a power of the imaginary unit i . It suggests that we can take into account the negative sign of the kinetic term of Φ by putting factors

of the imaginary unit i in front of the couplings. One may feel uncomfortable with the imaginary couplings, but they should be understood merely as a mathematical trick to translate an output of public codes/literature (which usually assume the positive sign for the scalar kinetic terms) to our theory with the ghost-like mode Φ .¹⁵ We again emphasize that, although ghost-like, Φ does not cause any issue thanks to the residual gauge symmetry. In the following, we explain how the above idea is implemented in practice. We divide the procedure into five steps. Although we focus on the Higgs- R^2 theory, our procedure can be equally applied to other models.

Step 1: compute the RGEs of a “usual” theory. As a first step, we compute the RGEs of a “usual” theory, i.e., the theory that contains two singlet scalar fields ϕ and σ in addition to the SM, which are later mapped to the conformal mode of the metric Φ , and the scalaron σ , respectively. We take the scalar potential to be generic, or

$$V = \frac{\lambda_1}{4}\phi^4 + \frac{\lambda_2}{4}\phi^3\sigma + \frac{\lambda_3}{4}\phi^2\sigma^2 + \frac{\lambda_4}{4}\phi\sigma^3 + \frac{\lambda_5}{4}\sigma^4 + \left(\frac{\kappa_1}{2}\phi^2 + \kappa_2\phi\sigma + \frac{\kappa_3}{2}\sigma^2\right)|H|^2 + \lambda_{UV}|H|^4, \quad (D.2)$$

where λ_i , κ_i and λ_{UV} are the scalar quartic couplings. At this moment, both singlets ϕ and σ are assumed to have positive kinetic terms, and hence we can easily compute the RGEs of this system with the help of public codes such as **SARAH**, or literature on the general RGEs such as refs. [31–34].

Step 2: map to the Higgs- R^2 theory. After computing the RGEs of the above theory, we map it to the Higgs- R^2 theory. The scalar potential of the Higgs- R^2 theory is given by

$$V = \lambda|H|^4 + \frac{\lambda_m}{2}e^{2\theta}(\Phi + \sigma)^2|H|^2 + \frac{\lambda_\Lambda}{4}e^{4\theta}(\Phi + \sigma)^4 + \frac{\lambda_\alpha}{4}\left[(\sigma \cosh \theta + \Phi \sinh \theta)e^\theta(\Phi + \sigma) + 2\bar{\xi}|H|^2\right]^2, \quad (D.3)$$

where we have also included the $SO(1,1)$ redundancy θ . Note that θ is unphysical since we can always replace

$$\sigma \cosh \theta + \Phi \sinh \theta \rightarrow \sigma, \quad \Phi \cosh \theta + \sigma \sinh \theta \rightarrow \Phi, \quad (D.4)$$

without affecting the kinetic terms of Φ and σ . The inclusion of this redundancy θ is nevertheless important for the renormalization, as we have already seen in appendix C. We replace the scalar modes of the Higgs- R^2 theory as

$$\Phi \rightarrow i\phi, \quad \sigma \rightarrow \sigma. \quad (D.5)$$

As a result, the two theories are related via

$$\begin{aligned} \lambda_1 &= \frac{\lambda_\alpha}{4}\left(1 - e^{2\theta}\right)^2 + e^{4\theta}\lambda_\Lambda, & \lambda_2 &= ie^{2\theta}\left(\lambda_\alpha\left(1 - e^{2\theta}\right) - 4e^{2\theta}\lambda_\Lambda\right), \\ \lambda_3 &= \frac{\lambda_\alpha}{2}\left(1 - 3e^{4\theta}\right) - 6e^{4\theta}\lambda_\Lambda, & & \end{aligned} \quad (D.6)$$

¹⁵At a deeper level, it might be related to the PT symmetry.

$$\lambda_4 = ie^{2\theta} \left(\lambda_\alpha \left(1 + e^{2\theta} \right) + 4e^{2\theta} \lambda_\Lambda \right), \quad \lambda_5 = \frac{\lambda_\alpha}{4} \left(1 + e^{2\theta} \right)^2 + e^{4\theta} \lambda_\Lambda,$$

$$\lambda_{UV} = \lambda + \bar{\xi}^2 \lambda_\alpha, \tag{D.7}$$

$$\kappa_1 = -e^{2\theta} \lambda_m + \left(1 - e^{2\theta} \right) \bar{\xi} \lambda_\alpha, \quad \kappa_2 = ie^{2\theta} \left(\lambda_m + \bar{\xi} \lambda_\alpha \right),$$

$$\kappa_3 = e^{2\theta} \lambda_m + \left(1 + e^{2\theta} \right) \bar{\xi} \lambda_\alpha. \tag{D.8}$$

By inserting these mappings, we can compute the RGEs of the Higgs- R^2 theory from the RGEs computed in the step 1.

Step 3: consistency check I. Before computing the RGEs of the Higgs- R^2 theory, we have to perform a consistency check of our procedure. The “usual” theory contains nine scalar quartic couplings λ_i , κ_i and λ_{UV} , while the corresponding Higgs- R^2 theory has only six parameter in the scalar potential, λ , λ_m , λ_Λ , λ_α , $\bar{\xi}$ and the redundancy θ . Thus, there are three redundancies among λ_i , κ_i and λ_{UV} after the mapping, given by

$$0 = -2\lambda_1 + i\lambda_2 + i\lambda_4 + 2\lambda_5, \tag{D.9}$$

$$0 = -\lambda_1 + i\lambda_2 + \lambda_3 - i\lambda_4 - \lambda_5, \tag{D.10}$$

$$0 = -\kappa_1 + 2i\kappa_2 + \kappa_3. \tag{D.11}$$

We have to check that these redundancies are maintained by the beta functions after the mapping, i.e.,

$$0 = -2\beta_{\lambda_1} + i\beta_{\lambda_2} + i\beta_{\lambda_4} + 2\beta_{\lambda_5}, \tag{D.12}$$

$$0 = -\beta_{\lambda_1} + i\beta_{\lambda_2} + \beta_{\lambda_3} - i\beta_{\lambda_4} - \beta_{\lambda_5}, \tag{D.13}$$

$$0 = -\beta_{\kappa_1} + 2i\beta_{\kappa_2} + \beta_{\kappa_3}, \tag{D.14}$$

after replacing the couplings following eqs. (D.6)–(D.8). These conditions, once satisfied, guarantee that the Higgs- R^2 theory is renormalizable. We have explicitly checked that these conditions are indeed satisfied up to two-loop in the Higgs- R^2 theory.

Step 4: compute the RGEs of the Higgs- R^2 theory. Once the above consistency check is done, we are ready to compute the RGEs of the Higgs- R^2 theory. From the mappings (D.6)–(D.8), we can obtain the RGEs as

$$\beta_{\lambda_\alpha} = \frac{1}{2} (3\beta_{\lambda_1} + \beta_{\lambda_3} + 3\beta_{\lambda_5}), \tag{D.15}$$

$$\beta_\theta = \frac{e^{2\theta} \beta_{\lambda_\alpha}}{2(\lambda_1 - \lambda_5)} + \frac{\beta_{\lambda_1} - \beta_{\lambda_5}}{2(\lambda_1 - \lambda_5)}, \tag{D.16}$$

$$\beta_{\lambda_\Lambda} = -e^{-4\theta} \beta_\theta (2\lambda_1 + 2\lambda_5 - \lambda_\alpha) + \frac{e^{-4\theta}}{2} (\beta_{\lambda_1} + \beta_{\lambda_5}) - \frac{\beta_{\lambda_\alpha}}{4} (1 + e^{-4\theta}), \tag{D.17}$$

$$\beta_{\bar{\xi}} = \frac{(\beta_{\kappa_1} + \beta_{\kappa_3})}{2\lambda_\alpha} - \frac{\bar{\xi}}{\lambda_\alpha} \beta_{\lambda_\alpha}, \tag{D.18}$$

$$\beta_{\lambda_m} = -\frac{1}{2} (\beta_{\kappa_1} + \beta_{\kappa_3}) - ie^{-2\theta} (\beta_{\kappa_2} - 2\kappa_2 \beta_\theta), \tag{D.19}$$

$$\beta_\lambda = \beta_{\lambda_{UV}} - 2\bar{\xi} \lambda_\alpha \beta_{\bar{\xi}} - \bar{\xi}^2 \beta_{\lambda_\alpha}. \tag{D.20}$$

It is convenient to compute the beta functions in this ordering, as the beta functions given in the later equations are expressed by those computed earlier. The RGEs of the other SM parameters, such as the Yukawa and gauge couplings, are also obtained by replacing the parameters following the mappings (D.6)–(D.8).

Step 5: consistency check II. Finally we have two additional consistency checks of our procedure. First, as we have explained above, the parameter θ is unphysical and hence the beta functions should not depend on it. Second, the beta functions β_{λ_α} , $\beta_{\bar{\xi}}$ and β_λ should be independent of λ_m and λ_Λ , and β_{λ_m} should be independent of λ_Λ . This property is easily understood in the Jordan frame from the mass dimension of the couplings. It is, however, a non-trivial check that it is satisfied in our formalism since all the parameters are treated as scalar quartic couplings once we map the Higgs- R^2 theory to the LSM (2.27). We have checked that these properties are indeed satisfied by the RGEs computed by our procedure.

The two-loop RGEs in section 2.4 are derived following the above steps 1-5. We have also checked that the one-loop RGEs computed by this procedure agree with those derived from the direct computation in appendix C.

E Renormalizability and residual gauge symmetry

In this paper, we have confirmed by explicit computation that the LSM (2.27) is renormalizable (at least) up to two-loop. In other words, all the divergences that appear in the theory can be cancelled by the operators within the LSM and hence one does not have to introduce additional operators as counter terms.

We emphasize that the renormalizability of the LSM (2.27) is non-trivial if we regard Φ and σ as usual scalar fields. This is because the scalar potential of the LSM (2.27) does not exhaust all the possible scalar quartic interactions. If one regards Φ and σ as usual scalar fields, one would expect that there are nine operators, Φ^4 , $\Phi^3\sigma$, $\Phi^2\sigma^2$, $\Phi\sigma^3$, σ^4 , $\Phi^2\phi_i^2$, $\Phi\sigma\phi_i^2$, $\sigma^2\phi_i^2$, and ϕ_i^4 , with independent coefficients. Instead, the coefficients of these operators are related in the LSM (2.27) such that there are only six independent parameters (including the SO(1,1) redundancy). It is nevertheless renormalizable thanks to the ghost-like property of Φ , as one can see by explicit computation. In this appendix, we describe our attempt to understand this renormalizability from the residual gauge symmetry. In particular, we see that the residual gauge symmetry restricts possible forms of the scalar quartic interactions of the LSM (2.27).

E.1 Residual gauge symmetry and conformal weight

We first explain the residual gauge symmetry of our gauge fixing condition (2.21). Here we focus on the scalar part of the residual gauge symmetry, which is given by

$$x^\mu \rightarrow x^\mu - \partial^\mu \xi, \tag{E.1}$$

where the index is raised by $\eta^{\mu\nu}$, and ξ satisfies

$$\partial_\mu \partial_\nu \xi = \frac{1}{4} \eta_{\mu\nu} \square \xi. \quad (\text{E.2})$$

It follows that $\xi_\mu \equiv \partial_\mu \xi$ satisfies the conformal killing equation (of flat spacetime),

$$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \frac{2}{d} \eta_{\mu\nu} \partial_\alpha \xi^\alpha, \quad (\text{E.3})$$

where $d = 4$ is the spacetime dimension, and hence this residual gauge symmetry is (one form of) the conformal symmetry, or the dilatation symmetry. Here one should not confuse the conformal transformation with the Weyl transformation. The former is a general coordinate transformation under which a given metric ($\eta_{\mu\nu}$ in our case) does not transform up to the overall normalization,¹⁶ while the latter is a field redefinition of the metric and is unrelated to the general coordinate transformation.¹⁷ Note that the above conformal killing equation in particular means that

$$\partial_\mu \square \xi = \square^2 \xi = 0. \quad (\text{E.4})$$

For our purpose, it is useful to define the conformal weight. We assign the conformal weight n_α to an operator O_α if it transforms as

$$O_\alpha \rightarrow \left[1 + \frac{n_\alpha}{4} \square \xi + (\partial^\mu \xi) \partial_\mu \right] O_\alpha, \quad (\text{E.5})$$

under the residual gauge symmetry. Note that the conformal weight is additive, that is, if operators O_α and O_β have conformal weights n_α and n_β , respectively, the composite operator $O_\alpha O_\beta$ transforms as

$$O_\alpha O_\beta \rightarrow \left[1 + \frac{n_\alpha + n_\beta}{4} \square \xi + (\partial^\mu \xi) \partial_\mu \right] O_\alpha O_\beta. \quad (\text{E.6})$$

The derivative ∂_μ raises the conformal weight of a given operator by unity, or

$$\partial_\mu O_\alpha \rightarrow \left[1 + \frac{n_\alpha + 1}{4} \square \xi + (\partial^\nu \xi) \partial_\nu \right] \partial_\mu O_\alpha, \quad (\text{E.7})$$

as one can show by using the conformal killing equation. If an operator O has conformal weight four, one can show that

$$\int d^4 x O \rightarrow \int d^4 x [1 + \square \xi + (\partial^\nu \xi) \partial_\nu] O = \int d^4 x O, \quad (\text{E.8})$$

i.e., its integrand is invariant under the residual gauge symmetry. It follows that only operators with conformal weight four can show up in the Lagrangian due to the requirement of invariance under the residual gauge transformation.

¹⁶Or equivalently, a general coordinate transformation times a Weyl transformation that keeps a given metric unchanged.

¹⁷In the literature, the word ‘‘conformal transformation’’ is sometimes used instead of the Weyl transformation in the latter meaning. In this paper we follow the terminology we have defined above in order to avoid possible confusion.

E.2 Possible interactions

Now we write down possible interactions of the LSM that are invariant under the residual gauge symmetry. We start with only Φ and the SM particles as the particle content. In other words, we do not include the scalaron as a fundamental degree of freedom at the beginning.

The conformal mode of the metric transforms under the residual gauge symmetry as

$$\Phi \rightarrow \left[1 + \frac{1}{4} \square \xi + (\partial^\mu \xi) \partial_\mu \right] \Phi, \tag{E.9}$$

and hence it has conformal weight one. Since we rescaled the SM fields by an appropriate factor of e^φ , the SM fields (in the “comoving frame”) transform under the residual gauge symmetry as

$$H \rightarrow \left[1 + \frac{1}{4} \square \xi + (\partial^\mu \xi) \partial_\mu \right] H, \tag{E.10}$$

$$\psi \rightarrow \left[1 + \frac{3}{8} \square \xi + (\partial^\mu \xi) \partial_\mu \right] \psi, \tag{E.11}$$

$$A_\nu \rightarrow \left[1 + \frac{1}{4} \square \xi + (\partial^\mu \xi) \partial_\mu \right] A_\nu, \tag{E.12}$$

where H is the Higgs doublet, ψ represents the SM fermions and A_ν the SM gauge bosons.

Since we start from the theory which *does not have the scalaron σ explicitly*, the operators with conformal weight four that are written solely in terms of Φ are

$$(\partial\Phi)^2, \quad \left(\frac{\square\Phi}{\Phi} \right)^2, \quad \Phi^4. \tag{E.13}$$

They correspond to the operators R , R^2 and the cosmological constant, respectively. Of course one can write down more terms by using more derivatives, but they are higher-dimensional operators and hence we ignore them below. In the purely SM sector, all the usual SM interactions are allowed. For the SM- Φ mixed sector, the following are the leading order terms that respect both the residual gauge symmetry and the SM gauge symmetry:

$$|H|^2 \Phi^2, \quad |H|^2 \frac{\square\Phi}{\Phi}, \tag{E.14}$$

where the first one corresponds to the Higgs mass term and the second one to the non-minimal coupling to gravity ξ (or more precisely $\tilde{\xi}$). Again we can write down other operators but we omit them due to their higher-dimensional nature. It follows that the scalar quartic interactions of the LSM (2.27) are determined by the residual gauge symmetry.

E.3 A few remarks

We have seen above that the structure of the theory and hence the scalar potential are controlled by the residual gauge symmetry. We still feel that there are unsatisfactory points in this argument. In this subsection we point out some of them in order to motivate possible future work on the theoretical structure of the Higgs- R^2 theory.

The first unsatisfactory point is that we had to assume that the scalaron is not a fundamental degree of freedom at the beginning in our discussion above. The reason is that if we include the scalaron σ from the beginning, the scalar part of the residual gauge symmetry does not distinguish σ from usual singlet scalar fields, and hence it does not prohibit generic scalar quartic interactions involving σ . Moreover, we can still write down $(\square\Phi/\Phi)^2$ which would even introduce an additional “scalaron.” We think that there should be a property of σ that distinguishes it from other usual scalar fields, whose study we leave as a future work.

The second unsatisfactory point is the criteria of higher dimensional operators. In the discussion above, we have included only the SM operators and the operators in eqs. (E.13) and (E.14) by arguing that others are higher dimensional operators. A rule of thumb is that we count the mass dimensions of the SM particles and the derivative as usual, but we count that of Φ as zero. We also allow only Φ to appear in the denominator of operators. A given operator is then regarded as higher dimensional if the total mass dimensions exceeds four. We know that this criteria practically works, but we do not know a physical reason behind it. We again leave this point for future work.

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY 4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] T. Futamase and K.-i. Maeda, *Chaotic inflationary scenario in models having nonminimal coupling with curvature*, *Phys. Rev. D* **39** (1989) 399 [[INSPIRE](#)].
- [2] J.L. Cervantes-Cota and H. Dehnen, *Induced gravity inflation in the standard model of particle physics*, *Nucl. Phys. B* **442** (1995) 391 [[astro-ph/9505069](#)] [[INSPIRE](#)].
- [3] F.L. Bezrukov and M. Shaposhnikov, *The standard model Higgs boson as the inflaton*, *Phys. Lett. B* **659** (2008) 703 [[arXiv:0710.3755](#)] [[INSPIRE](#)].
- [4] PLANCK collaboration, *Planck 2018 results. X. Constraints on inflation*, *Astron. Astrophys.* **641** (2020) A10 [[arXiv:1807.06211](#)] [[INSPIRE](#)].
- [5] C.P. Burgess, H.M. Lee and M. Trott, *Power-counting and the validity of the classical approximation during inflation*, *JHEP* **09** (2009) 103 [[arXiv:0902.4465](#)] [[INSPIRE](#)].
- [6] J.L.F. Barbón and J.R. Espinosa, *On the naturalness of Higgs inflation*, *Phys. Rev. D* **79** (2009) 081302 [[arXiv:0903.0355](#)] [[INSPIRE](#)].
- [7] C.P. Burgess, H.M. Lee and M. Trott, *Comment on Higgs inflation and naturalness*, *JHEP* **07** (2010) 007 [[arXiv:1002.2730](#)] [[INSPIRE](#)].
- [8] M.P. Hertzberg, *On inflation with non-minimal coupling*, *JHEP* **11** (2010) 023 [[arXiv:1002.2995](#)] [[INSPIRE](#)].
- [9] F. Bezrukov, A. Magnin, M. Shaposhnikov and S. Sibiryakov, *Higgs inflation: consistency and generalisations*, *JHEP* **01** (2011) 016 [[arXiv:1008.5157](#)] [[INSPIRE](#)].
- [10] M.P. DeCross, D.I. Kaiser, A. Prabhu, C. Prescod-WEinstein and E.I. Sfakianakis, *Preheating after multifield inflation with nonminimal couplings, I: covariant formalism and attractor behavior*, *Phys. Rev. D* **97** (2018) 023526 [[arXiv:1510.08553](#)] [[INSPIRE](#)].

- [11] Y. Ema, R. Jinno, K. Mukaida and K. Nakayama, *Violent preheating in inflation with nonminimal coupling*, *JCAP* **02** (2017) 045 [[arXiv:1609.05209](#)] [[INSPIRE](#)].
- [12] E.I. Sfakianakis and J. van de Vis, *Preheating after Higgs inflation: self-resonance and gauge boson production*, *Phys. Rev. D* **99** (2019) 083519 [[arXiv:1810.01304](#)] [[INSPIRE](#)].
- [13] Y. Hamada, K. Kawana and A. Scherlis, *On preheating in hIggs inflation*, [arXiv:2007.04701](#) [[INSPIRE](#)].
- [14] Y. Ema, *Higgs scalaron mixed inflation*, *Phys. Lett. B* **770** (2017) 403 [[arXiv:1701.07665](#)] [[INSPIRE](#)].
- [15] D. Gorbunov and A. Tokareva, *Scalaron the healer: removing the strong-coupling in the Higgs- and Higgs-dilaton inflations*, *Phys. Lett. B* **788** (2019) 37 [[arXiv:1807.02392](#)] [[INSPIRE](#)].
- [16] G. 't Hooft and M. Veltman, *One loop divergencies in the theory of gravitation*, *Ann. Inst. H. Poincaré Phys. Theor. A* **20** (1974) 69.
- [17] A. Salvio and A. Mazumdar, *Classical and quantum initial conditions for Higgs inflation*, *Phys. Lett. B* **750** (2015) 194 [[arXiv:1506.07520](#)] [[INSPIRE](#)].
- [18] X. Calmet and I. Kuntz, *Higgs Starobinsky inflation*, *Eur. Phys. J. C* **76** (2016) 289 [[arXiv:1605.02236](#)] [[INSPIRE](#)].
- [19] D.M. Ghilencea, *Two-loop corrections to Starobinsky-Higgs inflation*, *Phys. Rev. D* **98** (2018) 103524 [[arXiv:1807.06900](#)] [[INSPIRE](#)].
- [20] Y. Ema, *Dynamical emergence of scalaron in Higgs inflation*, *JCAP* **09** (2019) 027 [[arXiv:1907.00993](#)] [[INSPIRE](#)].
- [21] Y. Ema, K. Mukaida and J. van de Vis, *Higgs inflation as nonlinear σ -model and scalaron as its σ -meson*, *JHEP* **11** (2020) 011 [[arXiv:2002.11739](#)] [[INSPIRE](#)].
- [22] A.A. Starobinsky, *A new type of isotropic cosmological models without singularity*, *Adv. Ser. Astrophys. Cosmol.* **3** (1987) 130.
- [23] J.D. Barrow and A.C. Ottewill, *The stability of general relativistic cosmological theory*, *J. Phys. A* **16** (1983) 2757 [[INSPIRE](#)].
- [24] B. Whitt, *Fourth order gravity as general relativity plus matter*, *Phys. Lett. B* **145** (1984) 176 [[INSPIRE](#)].
- [25] J.D. Barrow and S. Cotsakis, *Inflation and the conformal structure of higher order gravity theories*, *Phys. Lett. B* **214** (1988) 515 [[INSPIRE](#)].
- [26] S. Weinberg, *Problems in gauge field theories*, in the proceedings of the 17th *International Conference on High-Energy Physics (ICHEP74)*, July 1–10, London, U.K. (1974).
- [27] S. Deser, *The state of quantum gravity*, *Conf. Proc. C* **750926** (1975) 229 [[INSPIRE](#)].
- [28] K.S. Stelle, *Renormalization of higher derivative quantum gravity*, *Phys. Rev. D* **16** (1977) 953 [[INSPIRE](#)].
- [29] A.O. Bärvinisky, D. Blas, M. Herrero-Valea, S.M. Sibiryakov and C.F. Steinwachs, *Renormalization of gauge theories in the background-field approach*, *JHEP* **07** (2018) 035 [[arXiv:1705.03480](#)] [[INSPIRE](#)].
- [30] A. Salvio, *Quadratic gravity*, *Front. in Phys.* **6** (2018) 77 [[arXiv:1804.09944](#)] [[INSPIRE](#)].
- [31] M.E. Machacek and M.T. Vaughn, *Two loop renormalization group equations in a general quantum field theory. 1. Wave function renormalization*, *Nucl. Phys. B* **222** (1983) 83 [[INSPIRE](#)].

- [32] M.E. Machacek and M.T. Vaughn, *Two loop renormalization group equations in a general quantum field theory. 2. Yukawa couplings*, *Nucl. Phys. B* **236** (1984) 221 [INSPIRE].
- [33] M.E. Machacek and M.T. Vaughn, *Two loop renormalization group equations in a general quantum field theory. 3. Scalar quartic couplings*, *Nucl. Phys. B* **249** (1985) 70 [INSPIRE].
- [34] M.-x. Luo, H.-w. Wang and Y. Xiao, *Two loop renormalization group equations in general gauge field theories*, *Phys. Rev. D* **67** (2003) 065019 [hep-ph/0211440] [INSPIRE].
- [35] F. Staub, *SARAH 4: A tool for (not only SUSY) model builders*, *Comput. Phys. Commun.* **185** (2014) 1773 [arXiv:1309.7223] [INSPIRE].
- [36] A. Salvio and A. Strumia, *Agravity*, *JHEP* **06** (2014) 080 [arXiv:1403.4226] [INSPIRE].
- [37] A. Salvio and A. Strumia, *Agravity up to infinite energy*, *Eur. Phys. J. C* **78** (2018) 124 [arXiv:1705.03896] [INSPIRE].
- [38] E.S. Fradkin and A.A. Tseytlin, *Renormalizable asymptotically free quantum theory of gravity*, *Nucl. Phys. B* **201** (1982) 469 [INSPIRE].
- [39] I. Buchbinder, S. Odintsov, and I. Shapiro, *Effective action in quantum gravity*, CRC press, U.S.A. (1992).
- [40] E. Elizalde and S.D. Odintsov, *Renormalization group improved effective potential for gauge theories in curved space-time*, *Russ. Phys. J.* **37** (1994) 25 [hep-th/9302074] [INSPIRE].
- [41] E. Elizalde and S.D. Odintsov, *Renormalization group improved effective Lagrangian for interacting theories in curved space-time*, *Phys. Lett. B* **321** (1994) 199 [hep-th/9311087] [INSPIRE].
- [42] A. Codello and R.K. Jain, *On the covariant formalism of the effective field theory of gravity and leading order corrections*, *Class. Quant. Grav.* **33** (2016) 225006 [arXiv:1507.06308] [INSPIRE].
- [43] T. Markkanen, S. Nurmi, A. Rajantie and S. Stopyra, *The 1-loop effective potential for the standard model in curved spacetime*, *JHEP* **06** (2018) 040 [arXiv:1804.02020] [INSPIRE].
- [44] J. Kubo, M. Lindner, K. Schmitz and M. Yamada, *Planck mass and inflation as consequences of dynamically broken scale invariance*, *Phys. Rev. D* **100** (2019) 015037 [arXiv:1811.05950] [INSPIRE].
- [45] Y.-C. Wang and T. Wang, *Primordial perturbations generated by Higgs field and R^2 operator*, *Phys. Rev. D* **96** (2017) 123506 [arXiv:1701.06636] [INSPIRE].
- [46] M. He, A.A. Starobinsky and J. Yokoyama, *Inflation in the mixed Higgs- R^2 model*, *JCAP* **05** (2018) 064 [arXiv:1804.00409] [INSPIRE].
- [47] A. Gundhi and C.F. Steinwachs, *Scalaron-Higgs inflation*, *Nucl. Phys. B* **954** (2020) 114989 [arXiv:1810.10546] [INSPIRE].
- [48] V.-M. Enckell, K. Enqvist, S. Rasanen and L.-P. Wahlman, *Higgs- R^2 inflation — Full slow-roll study at tree-level*, *JCAP* **01** (2020) 041 [arXiv:1812.08754] [INSPIRE].
- [49] M. Sasaki and E.D. Stewart, *A general analytic formula for the spectral index of the density perturbations produced during inflation*, *Prog. Theor. Phys.* **95** (1996) 71 [astro-ph/9507001] [INSPIRE].
- [50] C.M. Peterson and M. Tegmark, *Testing two-field inflation*, *Phys. Rev. D* **83** (2011) 023522 [arXiv:1005.4056] [INSPIRE].
- [51] J.-O. Gong and T. Tanaka, *A covariant approach to general field space metric in multi-field inflation*, *JCAP* **03** (2011) 015 [Erratum *ibid.* **02** (2012) E01] [arXiv:1101.4809] [INSPIRE].

- [52] D.I. Kaiser, E.A. Mazenc and E.I. Sfakianakis, *Primordial Bispectrum from Multifield Inflation with Nonminimal Couplings*, *Phys. Rev. D* **87** (2013) 064004 [[arXiv:1210.7487](#)] [[INSPIRE](#)].
- [53] G. Degrassi et al., *Higgs mass and vacuum stability in the standard model at NNLO*, *JHEP* **08** (2012) 098 [[arXiv:1205.6497](#)] [[INSPIRE](#)].
- [54] D. Buttazzo et al., *Investigating the near-criticality of the Higgs boson*, *JHEP* **12** (2013) 089 [[arXiv:1307.3536](#)] [[INSPIRE](#)].
- [55] J.R. Espinosa, G.F. Giudice and A. Riotto, *Cosmological implications of the Higgs mass measurement*, *JCAP* **05** (2008) 002 [[arXiv:0710.2484](#)] [[INSPIRE](#)].
- [56] O. Lebedev and A. Westphal, *Metastable electroweak vacuum: implications for inflation*, *Phys. Lett. B* **719** (2013) 415 [[arXiv:1210.6987](#)] [[INSPIRE](#)].
- [57] A. Kobakhidze and A. Spencer-Smith, *Electroweak vacuum (in)stability in an inflationary universe*, *Phys. Lett. B* **722** (2013) 130 [[arXiv:1301.2846](#)] [[INSPIRE](#)].
- [58] M. Fairbairn and R. Hogan, *Electroweak vacuum stability in light of BICEP2*, *Phys. Rev. Lett.* **112** (2014) 201801 [[arXiv:1403.6786](#)] [[INSPIRE](#)].
- [59] A. Hook, J. Kearney, B. Shakya and K.M. Zurek, *Probable or improbable universe? Correlating electroweak vacuum instability with the scale of inflation*, *JHEP* **01** (2015) 061 [[arXiv:1404.5953](#)] [[INSPIRE](#)].
- [60] K. Kamada, *Inflationary cosmology and the standard model Higgs with a small Hubble induced mass*, *Phys. Lett. B* **742** (2015) 126 [[arXiv:1409.5078](#)] [[INSPIRE](#)].
- [61] M. Herranen, T. Markkanen, S. Nurmi and A. Rajantie, *Spacetime curvature and the Higgs stability during inflation*, *Phys. Rev. Lett.* **113** (2014) 211102 [[arXiv:1407.3141](#)] [[INSPIRE](#)].
- [62] J. Kearney, H. Yoo and K.M. Zurek, *Is a Higgs vacuum instability fatal for high-scale inflation?*, *Phys. Rev. D* **91** (2015) 123537 [[arXiv:1503.05193](#)] [[INSPIRE](#)].
- [63] J.R. Espinosa et al., *The cosmological Higgstory of the vacuum instability*, *JHEP* **09** (2015) 174 [[arXiv:1505.04825](#)] [[INSPIRE](#)].
- [64] W.E. East, J. Kearney, B. Shakya, H. Yoo and K.M. Zurek, *Spacetime Dynamics of a Higgs Vacuum Instability During Inflation*, *Phys. Rev. D* **95** (2017) 023526 [[arXiv:1607.00381](#)] [[INSPIRE](#)].
- [65] A. Jot et al., *(Higgs) vacuum decay during inflation*, *JHEP* **07** (2017) 058 [[arXiv:1706.00792](#)] [[INSPIRE](#)].
- [66] A. Rajantie and S. Stopyra, *Standard model vacuum decay in a de Sitter background*, *Phys. Rev. D* **97** (2018) 025012 [[arXiv:1707.09175](#)] [[INSPIRE](#)].
- [67] J. Fumagalli, S. Renaux-Petel and J.W. Ronayne, *Higgs vacuum (in)stability during inflation: the dangerous relevance of de Sitter departure and Planck-suppressed operators*, *JHEP* **02** (2020) 142 [[arXiv:1910.13430](#)] [[INSPIRE](#)].
- [68] M. Herranen, T. Markkanen, S. Nurmi and A. Rajantie, *Spacetime curvature and Higgs stability after inflation*, *Phys. Rev. Lett.* **115** (2015) 241301 [[arXiv:1506.04065](#)] [[INSPIRE](#)].
- [69] Y. Ema, K. Mukaida and K. Nakayama, *Fate of electroweak vacuum during preheating*, *JCAP* **10** (2016) 043 [[arXiv:1602.00483](#)] [[INSPIRE](#)].
- [70] K. Kohri and H. Matsui, *Higgs vacuum metastability in primordial inflation, preheating, and reheating*, *Phys. Rev. D* **94** (2016) 103509 [[arXiv:1602.02100](#)] [[INSPIRE](#)].
- [71] K. Enqvist, M. Karciauskas, O. Lebedev, S. Rusak and M. Zatta, *Postinflationary vacuum instability and Higgs-inflaton couplings*, *JCAP* **11** (2016) 025 [[arXiv:1608.08848](#)] [[INSPIRE](#)].

- [72] M. Postma and J. van de Vis, *Electroweak stability and non-minimal coupling*, *JCAP* **05** (2017) 004 [[arXiv:1702.07636](#)] [[INSPIRE](#)].
- [73] Y. Ema, M. Karciauskas, O. Lebedev and M. Zatta, *Early universe Higgs dynamics in the presence of the Higgs-inflaton and non-minimal Higgs-gravity couplings*, *JCAP* **06** (2017) 054 [[arXiv:1703.04681](#)] [[INSPIRE](#)].
- [74] Y. Ema, K. Mukaida and K. Nakayama, *Electroweak vacuum metastability and low-scale inflation*, *JCAP* **12** (2017) 030 [[arXiv:1706.08920](#)] [[INSPIRE](#)].
- [75] D.G. Figueroa, A. Rajantie and F. Torrenti, *Higgs field-curvature coupling and postinflationary vacuum instability*, *Phys. Rev. D* **98** (2018) 023532 [[arXiv:1709.00398](#)] [[INSPIRE](#)].
- [76] S. Rusak, *Destabilization of the EW vacuum in non-minimally coupled inflation*, *JCAP* **05** (2020) 020 [[arXiv:1811.10569](#)] [[INSPIRE](#)].
- [77] D. Croon, N. Fernandez, D. McKeen and G. White, *Stability, reheating and leptogenesis*, *JHEP* **06** (2019) 098 [[arXiv:1903.08658](#)] [[INSPIRE](#)].
- [78] M. He, R. Jinno, K. Kamada, S.C. Park, A.A. Starobinsky and J. Yokoyama, *On the violent preheating in the mixed Higgs- R^2 inflationary model*, *Phys. Lett. B* **791** (2019) 36 [[arXiv:1812.10099](#)] [[INSPIRE](#)].
- [79] F. Bezrukov, D. Gorbunov, C. Shepherd and A. Tokareva, *Some like it hot: R^2 heals Higgs inflation, but does not cool it*, *Phys. Lett. B* **795** (2019) 657 [[arXiv:1904.04737](#)] [[INSPIRE](#)].
- [80] M. He, R. Jinno, K. Kamada, A.A. Starobinsky and J. Yokoyama, *Occurrence of tachyonic preheating in the mixed Higgs- R^2 model*, *JCAP* **01** (2021) 066 [[arXiv:2007.10369](#)] [[INSPIRE](#)].
- [81] F. Bezrukov and C. Shepherd, *A heatwave affair: mixed Higgs- R^2 preheating on the lattice*, *JCAP* **12** (2020) 028 [[arXiv:2007.10978](#)] [[INSPIRE](#)].
- [82] C. Fu, P. Wu and H. Yu, *Nonlinear preheating with nonminimally coupled scalar fields in the Starobinsky model*, *Phys. Rev. D* **99** (2019) 123526 [[arXiv:1906.00557](#)] [[INSPIRE](#)].
- [83] S. Weinberg, *Critical Phenomena for Field Theorists*, in the proceedings of the 14th International School of Subnuclear Physics: Understanding the Fundamental Constituents of Matter, July 23–August 8, Erice, Italy (1976) [[INSPIRE](#)].
- [84] S. Weinberg, *Ultraviolet divergences in quantum theories of gravitation*, in *General Relativity: An Einstein Centenary Survey*, S.W. Hawking and W. Israel eds., Cambridge University Press, Cambridge U.K. (1980).
- [85] A. Eichhorn, *An asymptotically safe guide to quantum gravity and matter*, *Front. Astron. Space Sci.* **5** (2019) 47 [[arXiv:1810.07615](#)] [[INSPIRE](#)].
- [86] D.F. Litim and F. Sannino, *Asymptotic safety guaranteed*, *JHEP* **12** (2014) 178 [[arXiv:1406.2337](#)] [[INSPIRE](#)].
- [87] K.G. Wilson and J.B. Kogut, *The renormalization group and the ϵ -expansion*, *Phys. Rept.* **12** (1974) 75 [[INSPIRE](#)].
- [88] C. Wetterich, *Exact evolution equation for the effective potential*, *Phys. Lett. B* **301** (1993) 90 [[arXiv:1710.05815](#)] [[INSPIRE](#)].
- [89] T.R. Morris, *The exact renormalization group and approximate solutions*, *Int. J. Mod. Phys. A* **9** (1994) 2411 [[hep-ph/9308265](#)] [[INSPIRE](#)].
- [90] M. Reuter, *Nonperturbative evolution equation for quantum gravity*, *Phys. Rev. D* **57** (1998) 971 [[hep-th/9605030](#)] [[INSPIRE](#)].

- [91] I.D. Saltas, *Higgs inflation and quantum gravity: an exact renormalisation group approach*, *JCAP* **02** (2016) 048 [[arXiv:1512.06134](#)] [[INSPIRE](#)].
- [92] E.J. Copeland, C. Rahmede and I.D. Saltas, *Asymptotically safe Starobinsky inflation*, *Phys. Rev. D* **91** (2015) 103530 [[arXiv:1311.0881](#)] [[INSPIRE](#)].
- [93] O. Lauscher and M. Reuter, *Flow equation of quantum Einstein gravity in a higher derivative truncation*, *Phys. Rev. D* **66** (2002) 025026 [[hep-th/0205062](#)] [[INSPIRE](#)].
- [94] P.F. Machado and F. Saueressig, *On the renormalization group flow of $f(R)$ -gravity*, *Phys. Rev. D* **77** (2008) 124045 [[arXiv:0712.0445](#)] [[INSPIRE](#)].
- [95] A. Codello, R. Percacci and C. Rahmede, *Investigating the ultraviolet properties of gravity with a Wilsonian renormalization group equation*, *Annals Phys.* **324** (2009) 414 [[arXiv:0805.2909](#)] [[INSPIRE](#)].
- [96] K. Falls, D.F. Litim, K. Nikolakopoulos and C. Rahmede, *A bootstrap towards asymptotic safety*, [arXiv:1301.4191](#) [[INSPIRE](#)].
- [97] K. Falls, D.F. Litim, K. Nikolakopoulos and C. Rahmede, *Further evidence for asymptotic safety of quantum gravity*, *Phys. Rev. D* **93** (2016) 104022 [[arXiv:1410.4815](#)] [[INSPIRE](#)].
- [98] J. Ellis, *TikZ-Feynman: Feynman diagrams with TikZ*, *Comput. Phys. Commun.* **210** (2017) 103 [[arXiv:1601.05437](#)] [[INSPIRE](#)].