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Renormalization group study
of the standard model and its extensions:
II. The minimal supersymmetric standard model.

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Abstract

In this paper we summarize the minimal supersymmetric standard model as well as the renormalization group equations of its parameters. We proceed to examine the feasability of the model when the breaking of supersymmetry is parametrized by the soft terms suggested by supergravity theories. In such models, the electroweak symmetry is exact at tree level and is broken spontaneously at one loop order. We make the additional assumption that the GUT-inspired relation $m_b = m_\tau$ be valid at the scale where the gauge coupling constants unify, which constrains the value of the top quark mass. For all types of soft breaking terms expected in supergravity theories, we present the results of numerical runs which yield electroweak breaking at the required scale. These yield not only the allowed ranges for the soft supersymmetry breaking parameters, but also the value of the supersymmetric partner' masses. For example in the strict no-scale model, in which global supersymmetry breaking arises solely from soft gaugino masses, we find that M_t can be no heavier than ~ 127 GeV.

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I. INTRODUCTION

In the last few years, it has become apparent, using the ever increasing accuracy in the measurement of the strong coupling, that Supersymmetry (SUSY) affords an elegant means to achieve gauge coupling unification [1–3] at scales consistent with Grand Unified Theories (GUTs) [4–8]. Whereas in the Standard Model (SM) the three gauge couplings unify "two by two" forming the "GUT triangle," in the simplest Minimal SUSY Extension of the SM (MSSM) these gauge couplings spectacularly unify at a point (within the experimental errors in their values). Given that the scale of unification in these models is generally above the lower bound set by proton decay, these supersymmetric grand unified theories (SUSY-GUTs) have regained increasing interest. Constraints from the Yukawa sectors of such models have also yielded interesting predictions for various low energy parameters including the top quark mass [9–12].

The analyses mentioned above have employed the Renormalization Group (RG). In the first part of this two part series [13], we reviewed the use of the RG for the purposes stated above in the framework of the SM. We discussed the aspects of data extraction involved in arriving at boundary conditions for the renormalization group equations (RGEs). In attempting to make the work as complete as possible, we included all RG β-functions to 2-loops making no approximations in the Yukawa sector. Plots of the evolution of all the SM parameters to this order were presented. Furthermore, we discussed threshold effects in the running fermion masses, an analysis which to our knowledge had not been adequately treated in the literature.

In the present paper we generalize this analysis to the minimal supersymmetric extension of the standard model. We include the two loop renormalization group equations for the parameters of the model. We only consider the case where supersymmetry is broken by soft terms in the Lagrangean, and use the renormalization group equations for these soft breaking parameters at the one loop level. One of the purposes of this paper is to determine the range of these parameters which leads to electroweak breaking with the correct value of the Z-mass. We discuss in some detail the aspects of the effective one loop potential which are relevant to this breaking. The mass formulas for the sfermions, Higgses, charginos, and neutralinos are also presented for completeness. Various boundary conditions at the unification scale in these minimal low energy supergravity models are discussed. We then describe our numerical procedure. The treatment of thresholds and the "special" form of the β-functions needed is then discussed . Although similar analyses have appeared in the literature [14–17], we feel it important to present our results in a way that directly compares all allowed sets of values for the soft supersymmetry breaking parameters. Our purpose is to use the results of our analysis to find to which extent the low energy data constrains the types of supersymmetry breakings. Supersymmetry presents us with a way to extrapolate low energy data to the deep ultraviolet, where they may be compared with the underlying theory, be it GUTs and/or Superstrings.

II. MINIMAL SUPERSYMMETRIC STANDARD MODEL

In the minimal supersymmetric extension of the standard model, every particle has a supersymmetric partner, their spins differing by a half [18]. In addition, two Higgs fields with opposite hypercharges are needed. Since the superpotential cannot consist of fields and their complex conjugates, phenomenology requires two Higgs fields to give mass to the charge $+2/3$ and $-1/3$, -1 sectors, respectively. Remarkably, two Higgs fields are also needed for both chiral and $SU(2)$ global anomaly cancellation.

For renormalizable theories, the superpotential can have at most degree three interactions. The superpotential for the MSSM is (suppressing the $SU(2)$ and Weyl metrics)

$$W = \hat{\bar{u}} \mathbf{Y}_u \hat{\Phi}_u \hat{Q} + \hat{\bar{d}} \mathbf{Y}_d \hat{\Phi}_d \hat{Q} + \hat{\bar{e}} \mathbf{Y}_e \hat{\Phi}_d \hat{L} , \tag{2.1}$$

where the hat indicates a chiral superfield and the overline denotes a left handed CP conjugate of a right handed field, $\overline{\psi} = i\sigma_2 \psi_R^*$. The usual Yukawa interactions are accompanied by new Yukawa interactions among the scalar quarks and leptons and the Higgsinos in the supersymmetric Lagrangean. There are also new gauge Yukawa interactions involving the gauginos. The new purely scalar interactions form the scalar potential which is positive definite in supersymmetric theories. The scalar potential will be discussed in a subsequent section. A remarkable aspect of supersymmetry is that all these new interactions require no new couplings. Table I displays the $SU(3) \times SU(2) \times U(1)$ quantum numbers of the chiral (all left handed) and vector superfields of the MSSM.

This superpotential exhibits two anomalous chiral global symmetries. The first, called R-symmetry, is characteristic of supersymmetric theories with cubic superpotentials; it implies massless gauginos, it must be broken; this is simply done by adding gaugino masses, breaking it down to a discrete R-parity. The other is an exact Peccei-Quinn (PQ) symmetry which, with this superpotential, can only be spontaneously broken at the electroweak scale. This is well known to lead to an axion, heavy enough to be ruled out by experiment. Hence the superpotential has to be improved so as to avoid this difficulty. This can be done in several ways.

• The standard way is to break the PQ symmetry explicitly through the addition of the following supersymmetric-invariant term to the superpotential

$$\mu \hat{\Phi}_u \hat{\Phi}_d . \tag{2.2}$$

This represents the simplest way to avoid the phenomenological disaster just described. It is intriguing that, in order to achieve the required electroweak breaking scale, the value of μ turns out to be (as we shall see in the next section)of the same order as the soft breaking parameters. At this level, it is not possible to explain this coincidence, since these terms break different symmetries.

• Alternatively, the μ can be interpreted dynamically as the vacuum expectation value of a singlet chiral superfield, $\hat{N}$, through the following interactions which also preserve supersymmetry and break PQ explicitly in the Lagrangean

$$\lambda \hat{N} \hat{\Phi}_u \hat{\Phi}_d - \lambda_0 \hat{N} \hat{N} \hat{N} . \tag{2.3}$$

TABLE I. Quantum Numbers

	$\hat{Q}$	$\hat{\bar{u}}$	$\hat{\bar{d}}$	$\hat{L}$	$\hat{\bar{e}}$	$\hat{\Phi}_u$	$\hat{\Phi}_d$	$\hat{g}^A$	$\hat{W}^a$	$\hat{B}$
$U(1)$	$+\frac{1}{6}$	$-\frac{2}{3}$	$+\frac{1}{3}$	$-\frac{1}{2}$	$+1$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
$SU(2)$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$
$SU(3)$	$\mathbf{3}$	$\mathbf{\bar{3}}$	$\mathbf{\bar{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{1}$	$\mathbf{1}$

The effective μ can be identified with $\lambda < N >$. This approach provides a natural explanation for $\mu \sim \mathcal{O}(M_W)$, if λ, $\lambda_0 \sim \mathcal{O}(g_2)$ [19].

• A third possibility implements non-electroweak spontaneous PQ symmetry breaking and leads to an invisible axion, but it needs a special scale of the order $\sim 10^{11}$ GeV. This introduces another hierarchy of scales. This model is parametrized by the following addition to the superpotential

$$\lambda \hat{X} \hat{\Phi}_u \hat{\Phi}_d + \lambda_0 \hat{X} \hat{Y} \hat{Y} \ . \tag{2.4}$$

• Finally, one can also break both the PQ symmetry and supersymmetry by adding to the Lagrangean the explicit soft breaking term,

$$m_3^2 \Phi_u \Phi_d \ . \tag{2.5}$$

This term can be put in by hand or generated in the low energy supergravity model for sufficiently general couplings of supergravity to the standard model [20].

If the superpotential contains the μ term, then this soft term will be generated even for the case of minimal coupling. In such a case, it is convenient to set $m_3^2 = B\mu$, where B is the soft bilinear coefficient. In the following we will adopt the simpler scheme of the μ term, leaving the other possibilities to future investigations.

III. MINIMAL LOW ENERGY SUPERGRAVITY MODEL

Since no super particles have been observed experimentally, supersymmetry, if truly present in nature, must be broken. One way to accomplish this breaking is to add to the Lagrangean soft breaking terms.

The general soft symmetry breaking potential for the MSSM can be written (including gaugino mass terms)

$$V_{soft} = m_{\Phi_u}^2 \Phi_u^\dagger \Phi_u + m_{\Phi_d}^2 \Phi_d^\dagger \Phi_d + B\mu(\Phi_u \Phi_d + h.c.)$$
$$+ \sum_i \Big(m_{\tilde{Q}_i}^2 \tilde{Q}_i^\dagger \tilde{Q}_i + m_{\tilde{L}_i}^2 \tilde{L}_i^\dagger \tilde{L}_i + m_{\tilde{\bar{u}}_i}^2 \tilde{\bar{u}}_i^\dagger \tilde{\bar{u}}_i + m_{\tilde{\bar{d}}_i}^2 \tilde{\bar{d}}_i^\dagger \tilde{\bar{d}}_i + m_{\tilde{\bar{e}}_i}^2 \tilde{\bar{e}}_i^\dagger \tilde{\bar{e}}_i \Big)$$
$$+ \sum_{i,j} \Big(A_u^{ij} Y_u^{ij} \tilde{\bar{u}}_i \Phi_u \tilde{Q}_j + A_d^{ij} Y_d^{ij} \tilde{\bar{d}}_i \Phi_d \tilde{Q}_j + A_e^{ij} Y_e^{ij} \tilde{\bar{e}}_i \Phi_d \tilde{L}_j + h.c. \Big) \ , \tag{3.1a}$$

$$V_{gaugino} = \frac{1}{2} \sum_{l=1}^3 M_l \lambda_l \lambda_l + h.c. \ , \tag{3.1b}$$

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where $V_{gaugino}$ is the Majorana mass terms for the gaugino fields, λ_l (suppressing the group index), corresponding to $U(1)$, $SU(2)$, and $SU(3)$, respectively.

We see that there can be sixty-three different soft symmetry breaking parameters. In addition, these could each be introduced in the effective Lagrangean at their own scale! Thus the supersymmetry breaking section runs the risk of being more complicated than the standard model itself, which only has a mere eighteen parameters!

It is clear that we need a further organizing principle to describe supersymmetry breaking. One attractive possibility is suggested by the coupling of the $N = 1$ standard model to $N = 1$ supergravity (SUGRA). The idea is to have supersymmetry broken in a sector that is uncoupled to the fields of the standard model, except through the universal interactions with supergravity. Then the net result of this picture is to produce in the effective Lagrangean of the minimal low energy supergravity model, a specific pattern of induced soft breaking terms of the above form, but with far fewer parameters.

Let us summarize the basic facts of supersymmetry breaking. From the supersymmetry algebra, one deduces that spontaneous symmetry breaking occurs if and only if the vacuum energy is not zero. In global supersymmetric theories, the scalar potential is a sum of F- and D-terms. Supersymmetry is spontaneously broken if either the vacuum values of the F-term [21] or D-term [22] are non-zero. A consequence of the spontaneous supersymmetry breaking is a massless fermion in analogy with the breaking of an ordinary global symmetry. Spontaneous supersymmetry breaking in supergravity occurs via the super-Higgs mechanism. The Nambu-Goldstone fermion, or Goldstino, associated with the breaking of global supersymmetry is eaten by the gravitino thereby providing it with a mass. We will assume that the spontaneous breaking of the local N=1 supersymmetry is communicated to the "visible" sector by weak gravitational interactions from some "hidden" sector. This type of spontaneous breaking of supergravity manifests itself at low energy as explicit soft breaking terms of supersymmetry.

The SUGRA Lagrangean is characterized by two arbitrary functions of the fields, a real function K (the Kähler potential) that determines the kinetic terms of the chiral superfields, and an analytic function $f_{\alpha\beta}$, transforming as the symmetric product of the adjoint representation of the gauge group, that determines the kinetic terms of the gauge fields. In terms of these, the supergravity scalar potential is

$$V_{SUGRA} = e^{K/M^2}\left[\left(\frac{\partial W}{\partial \phi_i} + \frac{1}{M^2}\frac{\partial K}{\partial \phi_i}W\right)\left(\frac{\partial W^*}{\partial \phi_j^*} + \frac{1}{M^2}\frac{\partial K}{\partial \phi_j^*}W^*\right)\frac{\partial^2 K}{\partial\phi^i\partial\phi^{j*}} - 3\frac{|W|^2}{M^2}\right]$$
$$+ \frac{1}{2}f_{\alpha\beta}^{-1}D^\alpha D^\beta \ , \tag{3.2}$$

where $M = M_{Planck}/\sqrt{8\pi}$ and D^α are auxiliary fields. In models with minimal kinetic terms for the chiral superfields (flat Kähler potential, $\partial^2 K/\partial\phi^i\partial\phi^{j*} = \delta_{ij}$), this leads to a common (gravitino) mass, m_0, for all the scalars of the model. The presence of non-minimal gauge kinetic terms ($f_{\alpha\beta} \neq \delta_{\alpha\beta}$) implies non-zero masses, M_l, for the gauginos at the GUT scale, M_X. By further assuming gauge coupling unification, we can take the three gaugino masses to be equal. Furthermore, the trilinear soft couplings A_u^{ij}, A_d^{ij}, and A_e^{ij} are all equal to a common value A_0. With minimal chiral kinetic terms, the bilinear soft coupling B_0 is related to A_0 as $B_0 = A_0 - m_0$. This scenario has obvious, desirable features. First, it is

very predictive since it has a few parameters accounting for thirty-one new masses. Second, the universal nature of the squark and slepton masses at M_X helps to avoid the appearance of unwanted flavor changing neutral current (FCNC) effects. In fact, one could argue that the absence of FCNCs hints at a universal mass for the scalars.

In this paper, we will not concern ourselves with the exact nature of the soft breaking terms. We will leave to others the issue of finding the exact, more fundamental supergravity models which engenders particular sets of values. Our philosophy will be to assume the presence of these terms and to explore their phenomenological consequences. We shall study the effects of varying the values the soft breaking terms over some interesting ranges and for some exceptional cases. The scales at which these soft breaking parameters enter the effective low energy Lagrangean are determined by theoretical prejudices. In view of the unification of gauge couplings with supersymmetry, it seems natural to input the three gaugino masses at the GUT scale. We note that the scales at which the sparticle, Higgs masses and trilinear couplings enter our Lagrangean could in principle be anywhere between the Planck and GUT scales. However, since we are interested in the rough features of the models, we have chosen, in the name of simplicity, to enter all the soft breaking parameters at the same GUT scale.

IV. RADIATIVE ELECTROWEAK BREAKING

The complete scalar tree level potential now appears as

$$V = V_F + V_D + V_{soft} \ , \tag{4.1}$$

where V_F contains the potential contributions from the F-terms

$$
\begin{aligned}
V_F = &\ |\tilde{\bar{u}}\mathbf{Y}_u\tilde{Q} + \mu\Phi_d|^2 + |\tilde{\bar{d}}\mathbf{Y}_d\tilde{Q} + \tilde{\bar{e}}\mathbf{Y}_e\tilde{L} + \mu\Phi_u|^2 \\
&+ |\mathbf{Y}_u\tilde{Q}\Phi_u|^2 + |\mathbf{Y}_d\tilde{Q}\Phi_d + \mathbf{Y}_e\tilde{L}\Phi_d|^2 \\
&+ |\tilde{\bar{u}}\mathbf{Y}_u\Phi_u + \tilde{\bar{d}}\mathbf{Y}_d\Phi_d|^2 + |\tilde{\bar{e}}\mathbf{Y}_e\Phi_d|^2 \ ,
\end{aligned}
\tag{4.2}
$$

and V_D contains the potential contributions from the D-terms

$$
\begin{aligned}
V_D = &\ \frac{g'^2}{2}(\ \frac{1}{6}\ \tilde{Q}_i^\dagger\tilde{Q}_i - \frac{2}{3}\tilde{\bar{u}}_i^\dagger\tilde{\bar{u}}_i + \frac{1}{3}\tilde{\bar{d}}_i^\dagger\tilde{\bar{d}}_i - \frac{1}{2}\tilde{L}_i^\dagger\tilde{L}_i + \tilde{\bar{e}}_i^\dagger\tilde{\bar{e}}_i + \frac{1}{2}\Phi_u^\dagger\Phi_u - \frac{1}{2}\Phi_d^\dagger\Phi_d\ )^2 \\
&+ \frac{g_2^2}{8}(\ \tilde{Q}_i^\dagger\vec{\tau}\tilde{Q}_i + \tilde{L}_i^\dagger\vec{\tau}\tilde{L}_i + \Phi_u^\dagger\vec{\tau}\Phi_u + \Phi_d^\dagger\vec{\tau}\Phi_d\ )^2 \\
&+ \frac{g_3^2}{8}(\ \tilde{Q}_i^\dagger\vec{\lambda}\tilde{Q}_i - \tilde{\bar{u}}_i^\dagger\vec{\lambda}^*\tilde{\bar{u}}_i - \tilde{\bar{d}}_i^\dagger\vec{\lambda}^*\tilde{\bar{d}}_i\ )^2 \ ,
\end{aligned}
\tag{4.3}
$$

where $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ are the $SU(2)$ Pauli matrices and $\vec{\lambda} = (\lambda_1, \ldots, \lambda_8)$ are the Gell-Mann $SU(3)$ matrices. In general, one must impose constraints on the parameters to avoid charge and color breaking minima in the scalar potential. Some *necessary* constraints have been formulated, such as

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$$A_U^2 < 3(m_Q^2 + m_{\tilde{u}}^2 + m_{\Phi_u}^2) \ , \tag{4.4a}$$

$$A_D^2 < 3(m_Q^2 + m_{\tilde{d}}^2 + m_{\Phi_d}^2) \ , \tag{4.4b}$$

$$A_E^2 < 3(m_L^2 + m_{\tilde{e}}^2 + m_{\Phi_d}^2) \ . \tag{4.4c}$$

However, these relations are in general neither *sufficient* nor indeed always *necessary* [23]. Their derivation involves very specific assumptions about the spontaneous symmetry breaking.

An appealing feature of the models we are considering is that they can lead to the breaking of the electroweak symmetry radiatively [24–27]. The one loop effective Higgs potential in these models can be expressed as the sum of the tree level potential plus a correction coming from the sum of all one loop diagrams with external lines having zero momenta

$$V_{1-loop} = V_{tree}(\Lambda) + \Delta V_1(\Lambda) \ . \tag{4.5}$$

The right hand side is Λ-independent up to one loop. The one loop correction is given by

$$\Delta V_1(\Lambda) = \frac{1}{64\pi^2} \text{Str}\{\mathcal{M}^4(\ln \frac{\mathcal{M}^2}{\Lambda^2} - \frac{3}{2})\}$$
$$= \frac{1}{64\pi^2} \sum_p (-1)^{2s_p}(2s_p+1) m_p^4 (\ln \frac{m_p^2}{\Lambda^2} - \frac{3}{2}), \tag{4.6}$$

where $\mathcal{M}^2$ is the field dependent squared mass matrix of the model and m_p is the eigenvalue mass of the pth particle of spin s_p. The tree level part of the potential is

$$V_{tree}(\Lambda) = m_1^2(\Lambda)\Phi_d^\dagger(\Lambda)\Phi_d(\Lambda) + m_2^2(\Lambda)\Phi_u^\dagger(\Lambda)\Phi_u(\Lambda) + m_3^2(\Lambda)(\Phi_u(\Lambda)\Phi_d(\Lambda) + h.c.)$$
$$+ \frac{g'^2(\Lambda)}{8}(\Phi_u^\dagger(\Lambda)\Phi_u(\Lambda) - \Phi_d^\dagger(\Lambda)\Phi_d(\Lambda))^2$$
$$+ \frac{g_2^2(\Lambda)}{8}(\Phi_u^\dagger(\Lambda)\vec{\tau}\Phi_u(\Lambda) + \Phi_d^\dagger(\Lambda)\vec{\tau}\Phi_d(\Lambda))^2 \tag{4.7}$$

where

$$m_1^2(\Lambda) = m_{\Phi_d}^2(\Lambda) + \mu^2(\Lambda) \ , \tag{4.8}$$

$$m_2^2(\Lambda) = m_{\Phi_u}^2(\Lambda) + \mu^2(\Lambda) \ , \tag{4.9}$$

$$m_3^2(\Lambda) = B(\Lambda)\mu(\Lambda) \ . \tag{4.10}$$

The parameters of the potential are taken as running ones, that is, they vary with scale according to the renormalization group. The logarithmic term in the one loop correction is necessary in making $V_{1-loop}(\Lambda)$ independent of Λ to this order (up to non-field dependent terms).

Given the low energy scale of electroweak breaking, we must use the renormalization group to evolve the parameters of the potential to a convenient scale such as M_Z (where the experimental values of the gauge couplings are usually cited) thereby making this leading log approximation valid. The exact scale is not critical as long as it is in the electroweak range. If we define,

$$\overline{m}_i^2 = m_i^2 + \frac{\partial \Delta V_1}{\partial v_i^2} , \tag{4.11}$$

with $v_1 = v_d$, $v_2 = v_u$ and

$$\frac{\partial \Delta V_1}{\partial v_i^2} = \frac{1}{32\pi^2} \sum_p (-1)^{2s_p} (2s_p + 1) m_p^2 (\ln \frac{m_p^2}{\Lambda^2} - 1) \frac{\partial m_p^2}{\partial v_i^2} , \tag{4.12}$$

then minimization of the potential yields the following two conditions among its parameters

$$\frac{1}{2} m_Z^2 = \frac{\overline{m}_1^2 - \overline{m}_2^2 \tan^2 \beta}{\tan^2 \beta - 1}, \tag{4.13}$$

where $m_Z^2 = (g'^2 + g_2^2) v^2 / 2$, $v^2 = v_u^2 + v_d^2$, and

$$B\mu = \frac{1}{2}(\overline{m}_1^2 + \overline{m}_2^2) \sin 2\beta , \tag{4.14}$$

where $\tan \beta = v_u / v_d$.

Although results based on the tree level potential cannot always be trusted, one can still use it to get some idea under what conditions electroweak breaking occurs. The renormalization group evolution of $m_{\Phi_u}^2$ (see Appendix A) can be such that it turns negative at low energies, if the top Yukawa coupling is large enough, whereas $m_{\Phi_d}^2$ runs positive. From Eq. (4.7), the scale at which breaking occurs Λ_b is set by the condition

$$m_1^2(\Lambda_b) \, m_2^2(\Lambda_b) - m_3^4(\Lambda_b) = 0 . \tag{4.15}$$

If the free parameters are adjusted properly, then the correct value of the Z^0 mass ($M_Z = 91.17$ GeV) can be achieved.

In the tree level analysis, there is another critical scale Λ_s that must be considered. It is evident from Eq. (4.7) that the potential becomes unbounded from below along the equal field (neutral component) direction, if

$$m_1^2(\Lambda_s) + m_2^2(\Lambda_s) < 2m_3^2(\Lambda_s) . \tag{4.16}$$

Since $m_1^2 m_2^2 - m_3^4 \geq 0$ implies $m_1^2 + m_2^2 \geq 2m_3^2$, condition (4.16) can only occur at scales lower than condition (4.15), so $\Lambda_s < \Lambda_b$. From this analysis, one concludes that the tree level vacuum expectation values (VEVs) of the scalar fields obtained by minimizing the potential are zero above Λ_b, and grow to infinity as one approaches Λ_s where the potential becomes unbounded from below. It follows that the appropriate scale at which to minimize the tree level potential and evaluate the VEVs is critical. This scale must be such that the one loop corrections of the effective potential may safely be neglected. Only at such a scale can the tree level results be trusted. However, there is more than one scale involved, and therefore, it is difficult if not impossible to find a scale at which all logarithms may be neglected. Indeed, the use of tree level minimization conditions to compute the VEVs at an arbitrary scale (e.g., $\Lambda = M_Z$) leads to incorrect conclusions about the regions of parameter space that yield consistent electroweak breaking scenarios [28]. When ΔV_1 is included, however, the value of Λ is not critical as long as it is in the neighborhood of M_Z.

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Reference [28] gives a prescription for arriving at a scale ($\hat{\Lambda}$) at which the tree level and the one loop effective potential results for the VEVs agree. Three qualitatively different cases are considered. In Ref. [28], M_{SUSY} parametrizes the superparticle thresholds, then the cases can be characterized by the orderings: (a) $M_{SUSY} < \Lambda_s < \Lambda_b$, (b) $\Lambda_s < M_{SUSY} < \Lambda_b$, and (c) $\Lambda_s < \Lambda_b < M_{SUSY}$. In each case, the prescription is to take $\hat{\Lambda} = \max\{M_{SUSY}, \Lambda_s\}$. Two cases deserve special mention. Case (a) cannot be handled using the tree level analysis because $v_u, v_d \to \infty$ near Λ_s. Fortunately, phenomenological bounds rule this case out anyway. In case (c), there is actually no electroweak breaking. For scales below M_{SUSY}, the superparticles have decoupled and the effective theory is not supersymmetric. Therefore, the running mass parameters of the potential freeze into their values at $\Lambda = M_{SUSY}$ at which scale there is no electroweak breaking. Finally, it must be emphasized that the apparent violent behavior of the VEVs with scale in the tree level analysis is an artifact of the approximation. The only physical potential is the full effective potential, and it either breaks electroweak symmetry or not. If it does, then the scalar fields have non-zero VEVs, and these VEVs are non-zero over all scales varying according to the anomalous dimension of their respective scalar fields (in the Landau gauge).

In contrast to the tree level potential, the one loop effective potential is constant against the renormalization group to this order around the electroweak scale. The exact scale at which to minimize is no longer critical. Moreover the assessment of the masses of the Higgs bosons based on the one loop effective potential is more accurate. The tree level restriction $M_h < M_Z$ is known not to be valid when one loop corrections, which are large because M_t is large, are included in the determination of M_h [29–31].

In this paper, we do not rely on the tree level analysis, rather we incorporate the one loop corrections. We include the dominant contributions from the third family, that is, those of the top and stop, bottom and sbottom, and tau and stau [32,33]. We choose the Z^0 mass as the scale at which to evaluate the minimization conditions. Equations (4.13) and (4.14) can be written

$$\mu^2(M_Z) = \frac{\overline{m}_{\Phi_d}^2 - \overline{m}_{\Phi_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} m_Z^2 \ , \tag{4.17a}$$

$$B(M_Z) = \frac{(\overline{m}_1^2 + \overline{m}_2^2) \sin 2\beta}{2\mu(M_Z)} \ , \tag{4.17b}$$

where $\overline{m}_{\Phi_{u,d}}^2 = m_{\Phi_{u,d}}^2 + \partial \Delta V_1 / \partial v_{u,d}^2$ and used to solve for $\mu(M_Z)$ and $B(M_Z)$ given the value of all the relevant parameters at M_Z. We note that the form of Eq. (4.17a) does not fix the sign of μ, and a choice for its sign must be made (μ is multiplicatively renormalized; see Appendix A). The right hand sides of these equations implicitly involve the VEV at M_Z. In a consistent scenario it would have the value of $v(M_Z) = 174.1$ GeV. If the parameters are such that the right hand side of (4.17a) is negative, then the scenario is inconsistent and the electroweak symmetry fails to be broken.

The validity of using (4.5) at M_Z hinges on the assumption that there is spontaneous symmetry breaking and that $M_Z = 91.17$ GeV. Given a set of values for the input parameters (soft terms, etc.) at M_X, we proceed by assuming valid electroweak breaking (*i.e.*, $M_Z = 91.17$ GeV). The one loop effective potential of Eq. (4.5) should then be perturbatively valid at M_Z with all its running parameters evaluated at this scale. In this way, we are

renormalization-group-improving the potential. The validity of this assumption is tested by the consistency of Eqs. (4.17a) and (4.17b). Failure to attain consistency invalidates the initial assumption that the given input values at M_X can accommodate a low energy world as we know it.

V. SPARTICLE MASSES

In the following, we list the tree level mass formulas for the different superpartners.

A. Sfermion masses

The mass matrices for scalar matter are constructed from Eq.(4.1). For example, in the up squark sector the relevant mass matrix appears as

$$\mathcal{M}_{\tilde{u}}^2 = \begin{pmatrix} M_{L_i L_j}^2 & M_{L_i R_j}^2 \\ M_{R_i L_j}^2 & M_{R_i R_j}^2 \end{pmatrix} \tag{5.1}$$

where $i, j = 1, 2, 3$ are flavor indices and

$$M_{L_i L_j}^2 = m_{Q_i}^2 \delta_{ij} + v_u^2 (\mathbf{Y}_u^\dagger \mathbf{Y}_u)_{ij} - \frac{1}{2}(v_d^2 - v_u^2)(Y(u_L)g'^2 - T_3(u_L)g_2^2)\delta_{ij} , \tag{5.2a}$$

$$M_{R_i R_j}^2 = m_{u_i}^2 \delta_{ij} + v_u^2 (\mathbf{Y}_u^\dagger \mathbf{Y}_u)_{ij} - \frac{1}{2}(v_d^2 - v_u^2)(Y(u_R)g'^2)\delta_{ij} , \tag{5.2b}$$

$$M_{R_i L_j}^2 = \mu v_d Y_u^{ij} + v_u A_u^{ij} Y_u^{ij}, \tag{5.2c}$$

$$M_{L_i R_j}^2 = M_{R_j L_i}^{2*} . \tag{5.2d}$$

Note in Table I that $Y = Q - T_3$ in our notation. Similar matrices follow for the other sfermions. These mass formulas as well as the ones to follow are given in terms of running parameters. The domain of validity of these formulas is at low energies ($\sim M_Z$) with the parameters taking on their renormalization group evolved values at this scale.

B. Higgs masses

If we employ the notation

$$\Phi_1 = \begin{pmatrix} \phi_1^0 \\ \phi_1^- \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} , \tag{5.3}$$

then the physical masses of the Higgs at tree level are calculated from the following three matrices

$$\frac{1}{2}\frac{\partial^2 V_{tree}}{\partial(\Im\phi_i^0)\partial(\Im\phi_j^0)} = \frac{1}{2}M_A^2 \sin 2\beta \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} \tag{5.4a}$$

$$\frac{1}{2}\frac{\partial^2 V_{tree}}{\partial(\Re\phi_i^0)\partial(\Re\phi_j^0)} = \frac{1}{2}M_A^2 \sin 2\beta \begin{pmatrix} \tan\beta & -1 \\ -1 & \cot\beta \end{pmatrix} + \frac{1}{2}m_Z^2 \sin 2\beta \begin{pmatrix} \cot\beta & -1 \\ -1 & \tan\beta \end{pmatrix} \tag{5.4b}$$

$$\frac{\partial^2 V_{tree}}{\partial(\phi_i^-)\partial(\phi_j^+)} = \frac{1}{2}M_{H_\pm}^2 \sin 2\beta \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} \tag{5.4c}$$

where $M_A^2 = m_1^2 + m_2^2$, $M_{H_\pm}^2 = M_A^2 + m_W^2$, and $m_W^2 = g_2^2 v^2/2$. The eigenvalues for the first matrix are 0 corresponding to the Goldstone boson and M_A^2 corresponding to the CP odd scalar. The second matrix gives the masses of the light and heavy Higgs bosons

$$M_{H,h}^2 = \frac{1}{2}[(M_A^2 + m_Z^2). \pm \sqrt{(M_A^2 + m_Z^2)^2 - 4M_A^2 m_Z^2 \cos^2 2\beta}] . \tag{5.5}$$

The mixing angle α that diagonalizes the matrix (5.4b) can be expressed

$$\tan 2\alpha = \frac{(M_A^2 + m_Z^2)}{(M_A^2 - m_Z^2)} \tan 2\beta . \tag{5.6}$$

If $M_A^2 \gg m_z^2$ which is the limit where the heavy Higgs is very heavy, this angle coincides with β. This tree level result predicts $M_h < M_Z$. One loop calculations show that this need not be the case [29–31]. The third matrix has eigenvalues 0 and $M_{H_\pm}^2$ corresponding to a massless, charged Goldstone boson and a charged scalar.

Including Eq. (4.6) in the calculations leads to corresponding one loop versions of these masses [32,33].

C. Chargino masses

The following four terms contribute to the chargino masses

$$- i\sqrt{2}g_2 \Phi_u^\dagger \frac{\tau^i}{2}\tilde{\Phi}_u \tilde{W}^i - i\sqrt{2}g_2 \Phi_d^\dagger \frac{\tau^i}{2}\tilde{\Phi}_d \tilde{W}^i - \mu \tilde{\Phi}_u \tilde{\Phi}_d + \frac{1}{2}M_2 \tilde{W}^i \tilde{W}^i + h.c. . \tag{5.7}$$

The first two terms are the supersymmetric Yukawa-gauge terms. Letting $\lambda^\pm = (\tilde{W}_2 \pm i\tilde{W}_1)/\sqrt{2}$, the mass matrix follows

$$\begin{pmatrix} \lambda^+ & \tilde{\phi}_u^+ & \lambda^- & \tilde{\phi}_d^- \end{pmatrix} \frac{1}{2} \begin{pmatrix} 0 & 0 & M_2 & -g_2 v_d \\ 0 & 0 & g_2 v_u & -\mu \\ M_2 & g_2 v_u & 0 & 0 \\ -g_2 v_d & -\mu & 0 & 0 \end{pmatrix} \begin{pmatrix} \lambda^+ \\ \tilde{\phi}_u^+ \\ \lambda^- \\ \tilde{\phi}_d^- \end{pmatrix} . \tag{5.8}$$

Diagonalization yields two charged Dirac fermions, $\tilde{C}_1$, $\tilde{C}_2$, with masses

$$M_{\tilde{C}_{1,2}} = \frac{1}{2}[(M_2^2 + \mu^2 + 2m_W^2) \mp \sqrt{(M_2^2 + \mu^2 + 2m_W^2)^2 - 4(M_2\mu - m_W^2 \sin 2\beta)^2}] . \tag{5.9}$$

D. Neutralino masses

Contributing to the neutralino masses are the terms in Eq. (5.7) and

$$- ig'\sqrt{2}\Phi_u^\dagger(+\frac{1}{2})\tilde{\Phi}_u \tilde{B} - ig'\sqrt{2}\Phi_d^\dagger(-\frac{1}{2})\tilde{\Phi}_d \tilde{B} + \frac{1}{2}M_1 \tilde{B}\tilde{B} . \tag{5.10}$$

The neutralino mass matrix follows

$$\left(\begin{array}{cccc} i\tilde{B} & i\tilde{W}_3 & \tilde{\phi}_d^0 & \tilde{\phi}_u^0 \end{array} \right) \frac{1}{2} \left(\begin{array}{cccc} -M_1 & 0 & g'v_d/\sqrt{2} & -g'v_u/\sqrt{2} \\ 0 & -M_2 & -g_2v_d/\sqrt{2} & g_2v_u/\sqrt{2} \\ g'v_d/\sqrt{2} & -g_2v_d/\sqrt{2} & 0 & \mu \\ -g'v_u/\sqrt{2} & g_2v_u/\sqrt{2} & \mu & 0 \end{array} \right) \left(\begin{array}{c} i\tilde{B} \\ i\tilde{W}_3 \\ \tilde{\phi}_d^0 \\ \tilde{\phi}_u^0 \end{array} \right) . \quad (5.11)$$

Its eigenvalues are the masses of the four neutralinos, $\tilde{N}_1$, $\tilde{N}_2$, $\tilde{N}_3$, $\tilde{N}_4$. Typically not all the eigenvalues of this mass matrix have the same sign. It was noted in [34] that over most of the range of the parameters, one of the eigenvalues is of the opposite sign compared to the others. This sign is important in distinguishing this neutralino (the "flippino") and especially in deriving mass sum rules [34].

VI. BOUNDARY CONDITIONS AT M_X

In this paper, as in the previous one [13], we work in the modified minimal subtraction scheme ($\overline{\rm MS}$) of renormalization. The parameters of the Lagrangean are not in general equal to any corresponding physical constant. For example, in the case of masses, except for those of the bottom and top quark (see [13]), all other physical masses, M, will be determined from their corresponding running masses by the relation

$$M = m(\Lambda)|_{\Lambda=M} . \quad (6.1)$$

This equation is easily solved in the course of an integration of the RGEs for the different masses by noting the scale at which it is valid. We have collected the renormalization group β-functions of the MSSM for the gauge and Yukawa couplings to two loops without making any approximations in the Yukawa sector in Appendix A. Also included are the one loop β-functions for the soft breaking terms.

Because SUGRA models make simplifying predictions about the soft parameters at some large scale, we initiate the evolution of the renormalization group equations at this scale. For simplicity, we take this scale to be the gauge unification scale where we expect the gaugino masses to be equal to some common value. It has been demonstrated [1] that the introduction of supersymmetry leads to gauge coupling unification at approximately $\sim 10^{16}$ GeV. Therefore we take $M_X = 10^{16}$ GeV, and evolve down to 1 GeV, the conventional scale at which the running quark masses are given [35]. Furthermore, as already discussed, the Higgs potential must be analyzed at some low energy scale that we choose to be M_Z.

At the unification scale, M_X, all the scalars will have a common mass, m_0,

$$\begin{aligned} m_{\tilde{Q}_i}(M_X) = m_{\tilde{\bar{u}}_i}(M_X) = m_{\tilde{\bar{d}}_i}(M_X) &= m_{\tilde{L}_i}(M_X) \\ &= m_{\tilde{\bar{e}}_i}(M_X) = m_{\Phi_u}(M_X) = m_{\Phi_d}(M_X) \\ &\equiv m_0 , \end{aligned} \quad (6.2a)$$

the gauginos will also have a common mass, $m_{1/2}$,

$$M_1(M_X) = M_2(M_X) = M_3(M_X) \equiv m_{1/2} . \quad (6.3)$$

The prefactors of the trilinear soft scalar terms are equal to A_0 at M_X

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TABLE II. Models

	A_0	m_0	$m_{1/2}$	B_0
General	any	any	any	any
Strict No-scale	0	0	any	0
No-scale	0	0	any	any
String Inspired	0	any	any	0
Minimal SUGRA	any	any	any	$A_0 - m_0$

$$A_u^{ij}(M_X) = A_d^{ij}(M_X) = A_e^{ij}(M_X) \equiv A_0 \ . \tag{6.4}$$

Also we define the bilinear soft scalar coupling and the mixing mass at M_X by $B(M_X) \equiv B_0$, and $\mu(M_X) \equiv \mu_0$.

Furthermore, to constrain the parameter space, we will take the bottom and tau masses equal at M_X

$$m_b(M_X) = m_\tau(M_X) \ . \tag{6.5}$$

This being the best motivated mass relation in supersymmetric grand unified theories [36].

The purpose of our present analasys is to determine the allowed range of the input parameters, A_0, m_0, $m_{1/2}$, and $\tan\beta$, which reproduce the known low energy physics. Within this general set of solutions, there are four subclasses of soft supersymmetry breaking which are of particular interest. Three of these have various soft parameters equal to zero at M_X. The fourth predicts a definite relationship among three of these parameters. The first class of models follows from the no-scale model [37] and has $A_0 = m_0 = B_0 = 0$ (strict no-scale model). In these models, only gaugino masses provide global supersymmetry breaking. The second class is the less constraining no-scale case that has only $A_0 = m_0 = 0$. A third class we consider with $A_0 = B_0 = 0$ comes from some string derived models. Minimal SUGRA models with canonical kinetic terms for the chiral superfields form another class and have $B_0 = A_0 - m_0$. Table II lists these four possibilities. In both strict no-scale and string inspired cases, we must have $B_0 = 0$. However, since for us B_0 is an output rather than an input variable, $B_0 = 0$ results must be inferred from its behavior upon varying other input parameters.

VII. NUMERICAL PROCEDURE

Our numerical procedure relies on the Runge-Kutta technique of numerical integration. To make the process of initialization, or the determination of all parameters at one scale, more transparent, it is useful to think of the Runge-Kutta integration of the renormalization group equations as a vector valued function of a vector variable. The input vector is the N-plet of values for all the (N) parameters at M_X. The Runge-Kutta function then returns an N-plet result representing the running values of all the parameters at final scales such as M_Z or 1 GeV.

In order to make a run (using Runge-Kutta) starting from M_X, we must have the values of all the parameters at this scale, but this is difficult to achieve. The problem is that the

values of many parameters are known experimentally at low scales. Also, relations (4.17a) and (4.17b) coming from the one loop effective potential hold at such low energies. However, the values of other parameters, such as soft breaking terms, are most easily understood at higher energies where theoretical simplification (*e.g.*, universality) may be invoked. We therefore have no scale at which there is both theoretical simplicity and experimental data.

We can phrase the problem in another way. Choosing M_X (where we have theoretical simplicity) as our starting point, can we find the M_X values of all parameters such that we recover the expected low energy values after renormalization group evolution to experimental scales, and all constraint relations among the parameters are satisfied at the appropriate scales? We distinguish between constrained and free parameters. The former are constained by experiment (*e.g.*, quark masses, gauge couplings, etc.) or relations among themselves (*e.g.*, bottom τ Yukawa equality at M_X, minimization conditions at M_Z, etc.). Given the present experimental data, the latter cannot be constrained by these two criteria and must be viewed as input parameters. After we have run, we find some ranges of their values to be inconsistent with phenomenological considerations such as electroweak symmetry breaking and the resulting sparticle spectrum. The exact values of the constrained parameters at M_X are affected by the choice of values for the free parameters at M_X since the evolution of all parameters are coupled. Therefore, given a choice for the free parameters at M_X, we must find the M_X values of the constrained parameters consistent with all the constraints.

The functional nature of the (Runge-Kutta) integration allows us to define a set of n (where n is the total number of constrained parameters) equations

$$G_k(\mathbf{x}_0) = 0 \ , \tag{7.1}$$

where $k = 1, \ldots, n$, and $\mathbf{x}_0$ represents the values of all constrained parameters at M_X. The solution to the initialization problem, therefore is reduced to solving a system of n simultaneous non-linear equations in n unknowns. In this work as in Ref. [13], we will use routines, based on the "shooting" method, to solve systems of nonlinear equations. The method involves making a guess for the solution, then assessing its merits based on how well the equations are satisfied, given some tolerance. The next guess (or shot) is adjusted according to how accurate the previous one was. The process is optimized and iterated until the routine converges on a solution. In this way, the values of all parameters are ascertained at one common scale which we take to be M_X. When the parameters are evolved to lower scales using these initial values, they attain the experimentally known values and they satisfy any relations amongst themselves that were used as constraints in the shooting procedure.

As discussed previously, we start our runs at M_X at which scale we can make simplifying assumptions about the soft breaking terms based on various SUGRA models. This requires that we use the solution routines to consistently find the M_X values of all known low energy parameters such as lepton and quark masses and mixing angles and gauge couplings. This amounts to solving for sixteen unknowns (nine masses, three angles and a phase, and three gauge couplings). Alternatively, we could start our runs at M_Z or 1 GeV; however, this now requires solving for sixty-three unknowns (the values of the soft breaking terms at low energy) that must evolve to just four different values at M_X. The efficiency of the former method is obvious.

There are ostensibly seven parameters in the models we consider. These are A_0, B_0, m_0, $m_{1/2}$, μ_0, $\tan\beta$, and m_t. The two minimization constraints (4.13) and (4.14) reduce this

set to five, which are taken to be A_0, m_0, $m_{1/2}$, $\tan\beta$, and m_t. In the present framework, B_0 and μ_0 will be determined using the numerical solutions routines in conjunction with the minimization of the one loop effective potential at M_Z in the process of evolving from M_X to 1 GeV. Minimization at M_Z will give $B(M_Z)$ and $\mu(M_Z)$. To arrive at B_0 and μ_0 (their corresponding values at M_X), we employ the solution routine as follows. A guess for B_0 and μ_0 is made at M_X and then the parameters of the model are run to M_Z at which scale the evolved value of B is compared to the minimization output value for B at M_Z. The same is done for μ. If the compared values agree to some set accuracy, then B_0 and μ_0 are the required values. Other analyses that also extract $B(M_Z)$ and $\mu(M_Z)$ simply evolve these two parameters via their renormalization group equations back to M_X to find B_0 and μ_0 relying on their near decoupling from the full set of RGEs. We note that the sign of μ is not determined from the minimization procedure, thus we must make a choice for it. To constrain the parameter space further, the bottom quark and tau lepton masses will be taken equal at M_X. This equality is a characteristic of many SUSY-GUTs. This constrains the model to four free parameters, A_0, m_0, $m_{1/2}$, and $\tan\beta$. Demanding that $m_b(M_X) = m_\tau(M_X)$ and achieving the correct physical masses for the bottom quark and tau lepton fixes the mass of the top quark which affects the evolution of the bottom Yukawa significantly. We shall assume gauge coupling unification, an assumption which appears reasonable when one considers SUSY models with SUSY breaking scales $\lesssim 10$ TeV.

In a complete treatment, the solution routines would be used to find the precise (similar) values of α_1, α_2, and α_3 at M_X that will evolve to the experimentally known values at M_Z, however this increases the CPU time considerably. We shall therefore sacrifice some precision in their M_Z values by taking them exactly equal at M_X. This is already a theoretical oversimplification since one does not expect the gauge couplings to be exactly equal due to threshold effects at the GUT scale [39]. We find that for all cases we have studied, the common value $\alpha_1^{-1}(M_X) = \alpha_2^{-1}(M_X) = \alpha_3^{-1}(M_X) = 25.31$ leads to errors no bigger than 1%, 5%, and 10% in $\alpha_1(M_Z)$, $\alpha_2(M_Z)$, and $\alpha_3(M_Z)$, respectively. This is not so bad considering that the (combined experimental and theoretical) errors on $\alpha_3(M_Z)$ from some processes can be as large as 10% [13].

It is well known that there is a fine tuning problem inherent in the radiatively induced electroweak models. For certain values of the parameters, the top quark mass must be tuned to an "unnaturally" high degree of accuracy to achieve the correct value of M_Z. This problem is generally handled by rejecting models that require "too much" tuning. The amount of tuning is usually defined quite arbitrarily. The usual procedure is to define fine tuning parameters [38]

$$c_i = \left| \frac{x_i^2}{M_Z^2} \frac{\partial M_Z^2}{\partial x_i^2} \right| , \qquad (7.2)$$

where x_i are parameters of the theory such as m_0, $m_{1/2}$, μ, or m_t. One then demands that the c_i be less than some chosen value that is typically taken to be 10.

We have analyzed to some extent the differences in using the tree level *vs.* one loop effective potential. The basis for the "theoretical" fine tuning problem can be seen (for example, assuming the renormalization group equation for the up sector Higgs mass is dominated by the top quark Yukawa coupling) in the dependence of M_W on the top quark Yukawa coupling y_t [40]

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$$M_W \sim M_X e^{-1/y_t^2} \ . \tag{7.3}$$

We remark that this fine tuning problem is exacerbated if one uses only the tree level analysis of the potential. The vacuum expectation value coming from the minimization conditions of the tree level potential changes rapidly from 0 to infinity over the interval (Λ_s, Λ_b). Using the prescription of Ref. [28] for the scale $\hat{\Lambda}$ at which to adequately minimize the tree level potential, to extract $v(\hat{\Lambda})$, and thereby to arrive at a value for M_Z, one finds that although a small variation in $y_t(M_X)$ may lead to a small variation in Λ_b, the steepness in the tree level VEV can lead to a large variation in the value of $v(\hat{\Lambda})$ and therefore in M_Z. Hence, in the tree level analysis, solutions which may be within the bounds of the "theoretical" fine tuning may nevertheless display a fine tuning aspect because of this "tree level" fine tuning of $\hat{\Lambda}$.

However, our use of the one loop effective potential (i.e., including ΔV_1) stabilizes the VEV around the M_Z scale and this particular fine tuning goes away. The true VEVs depend on scale through wave function renormalization effects which are never large as can be seen from the form of the renormalization group equations for the VEVs in the Appendix A.

In this analysis, we shall also reject solutions based on fine tuning considerations; however, our method differs somewhat from the usual one in that it is incorporated in the solution routine described above. The routine is an iterative one which determines the convergence properties of the solution. Very slow convergence reflects an inherent fine tuning. Therefore, if the convergence is too slow, we will reject the solution. Effectively we are rejecting any solution which the computer cannot pinpoint within an allotted number of iterations.

Given values for A_0, m_0, $m_{1/2}$, $\tan\beta$, and $\mathrm{sign}(\mu)$, the solution routines search for the values of $v(M_X)$, $m_{b,\tau}(M_X)$, $m_t(M_X)$, B_0, and μ_0. The process by which B_0 and μ_0 are found was described above. The remaining three parameters are determined similarly. The routine makes a guess for $v(M_X)$, $m_{b,\tau}(M_X)$, and $m_t(M_X)$, then the full renormalization group equations are evolved to 1 GeV calculating superparticle threshold masses in the process and minimizing the one loop effective potential at M_Z. The merits of the guess for $v(M_X)$, $m_{b,\tau}(M_X)$, and $m_t(M_X)$ are assessed by comparing the resulting values of M_Z, $m_\tau(1 \text{ GeV})$, and $m_b(1 \text{ GeV})$ with the expected ones. The process is iterated until the correct values are achieved to within a tolerance of 1%.

By a "solution," we will mean that a choice for the four free parameters (plus a choice for the sign of μ) at M_X results in a complete set of all parameters at M_X (using the shooting routines) consistent with electroweak breaking ($M_Z = 91.17$ GeV), with equality of the bottom and τ masses at M_X, and with low energy experiment.

Such a solution then yields a precise spectrum of sparticle masses. This spectrum can be further used to limit the allowable free parameter space by subjecting the spectrum to experimental restrictions.

VIII. THRESHOLDS

In the minimal low energy supergravity model being considered, the super particle spectrum is no longer degenerate as in the simple global supersymmetry model in which all the

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super particles are given a common mass, M_{SUSY}. In this simple case, one makes one course correction in the renormalization group evolution at M_{SUSY}. In the model with soft symmetry breaking, however, the nondegenerate spectrum should lead to various course corrections each occuring at the super particle mass thresholds. To this end, the renormalization group β-functions must be cast in a new form which makes the implementation of the thresholds effects (albeit naive) evident (see Appendix B). Since the $\overline{\text{MS}}$ renormalization group equations are mass independent, particle thresholds must be handled using the decoupling theorem [41], and each super particle mass has associated with it a boundary between two effective theories. Above a particular mass threshold the associated particle is present in the effective theory, below the threshold the particle is absent.

The simplest way to incorporate this is to (naively) treat the thresholds as steps in the particle content of the renormalization group β-functions [42]. This method is not always entirely adequate. For example, in the case of the $SU(2)$ gauge coupling there will be scales in the integration process at which there are effectively a half integer number of doublets using this method. For example, in the region $M_b < \Lambda < M_t$ between the mass of the bottom and top quarks, the number of quark doublets is taken to be two and a half using this method. A similar situation occurs with sparticle doublets. We believe, nevertheless, that this method does yield the correct, general behavior of the evolution. It is a simple means of implementing the smearing effects of the non-degenerate super particle spectrum. The determination of the spectrum of masses is done without iteration as is common in other analyses. Our method deduces the physical masses by solving the equation $m(\Lambda) = \Lambda$ for each superparticle in the process of evolving from M_X to 1 GeV. The usual iterative method requires several runs to find a consistent solution.

IX. ANALYSIS

The tremendous computing task involved in analyzing the full parameter space of the soft symmetry breaking models, using the methods described as designed, would be far too time consuming given the computing facilities available to us. Therefore, in the following analysis, some simplifications will be made in the procedural method. First, only the heaviest family of quarks and leptons will have non-zero mass. This will cut down on the CPU time required for the solution routines to consistently find the running mass values at M_X since there are three instead of nine masses to determine after this simplification. Second, as stated previously, the value of the strong coupling at M_Z will be allowed to vary from its central value of .113 by at most 10%. This translates into a similar error in the bottom quark mass. Third, the allotted number of Runge-Kutta steps, involved in numerically integrating the renormalization group equations, will be cut down to ~ 100.

Our method involves four input parameters A_0, m_0, $m_{1/2}$, $\tan\beta$ (and the sign(μ)). The output is B_0, μ_0, M_t, M_h, and all the masses of the extra particles associated with the MSSM. Efficient use of CPU time required that we proceed as follows. For a given model, our initial exploration of the parameter space was performed in a coarse grained fashion. $\tan\beta$ was most commonly coarse grained as 2, 5, and 10, with only some runs involving higher values (e.g., 15, 20). The other three input parameters were varied in steps of 50 and 100 GeV. Values larger than ~ 500 GeV were rarely ever used. We subsequently narrowed

down on the allowed hyperenvelope by fine graining around the edges of the expected region (based on the coarse graining results).

Our raw data consists of those runs which satisfy the following two criteria. The first is consistency with electroweak breaking; that is, the correct value of M_Z is achieved from the minimization of the one loop effective potential with $\mu^2(M_Z) > 0$. The second criterion is no fine tuning, as implemented in our method (see Section VII). The solution routines employed return a numbered code representing the convergence properties of the solution which we use to screen the runs.

The raw data can then be progressively filtered based on at least three physical constraints. The first is cosmological. Conserved R-parity ($+1$ for particles and -1 for superparticles) requires the existence of a stable lightest supersymmetric particle (LSP). Astrophysical considerations indicate that the LSP must be neutral and colorless. Cosmological considerations based on the LSPs contribution to the density of the universe indicate that it must have a mass less than ~ 200 GeV [43–45]. Therefore, points in parameter space that lead to LSPs other than neutralinos with masses less than ~ 200 GeV are cut. Second, flavor changing neutral current bounds can be used to reject runs in which interfamily splitting of squark and slepton masses is too large, although this is seldom the case in the type of low energy supergravity models with soft breaking universality that we consider. Third, experimental limits on the masses of the superparticles can also be used as another criterion to reject some runs.

X. RESULTS

In the following, we discuss the results of our analysis of the four classes of models listed in Table II and of a general, non-constrained class. In the general case, we discuss results based on all our runs with no constraints on the soft parameters. Because we have performed a coarse grained study, using fine graining only for selected regions, our results should only be considered qualitatively valid. Based on these, we hope to be able to ascertain the general trends in the data, and make some general predictions about the feasibility of the models considered.

Because the GUT inspired constraint Eq. (6.5) is enforced in this analysis, the results will depend on the mass of the bottom quark. Namely, lower bottom quark masses require larger values of the top quark mass to satisfy this relation, and higher bottom quark masses require smaller values of the top quark mass. For simplicity, most results will be reported for the case $m_b(1 \text{ GeV}) = 6.00$ GeV, but lower mass (5.70 GeV) and higher mass (6.33 GeV) cases were also studied. These lower and higher values are within experimental uncertainty. The running value of 6.00 GeV for $m_b(1 \text{ GeV})$ corresponds to a physical bottom mass $M_b = 4.85 \pm .15$ GeV, with the uncertainty coming from the error in the strong coupling, as discussed above.

In Table III, we present the allowable ranges of the soft breaking parameters for the general unrestricted case and for each of the four subcases considered. Since B_0 and μ_0 are not input parameters in our method, we must infer the results for the strict no-scale, string inspired, and minimal SUGRA cases. We do this by setting a small tolerance on $|B_0|$ and $|\chi|$ of 25 GeV. In compiling this table, we have discarded all runs with $M_t < 120$ GeV

TABLE III. Allowed Soft Parameter Ranges (in units of GeV).

Case	A_0	m_0	$m_{1/2}$	μ_0	B_0
General	$(0, 500)$	$(0, 800)$	$(190, 400)$	$(33, 817)$	$(-56, 1012)$
No-Scale	0	0	$(190, 262)$	$(40, 230)$	$(-60, 130)$
Strict No-Scale	0	0	$(210, 240)$	$(159, 188)$	$|B_0| < 25$
String Inspired	0	$(0, 200)$	$(200, 250)$	$(88, 188)$	$|B_0| < 25$
Minimal SUGRA	$(0, 500)$	$(0, 300)$	$(200, 300)$	$(159, 467)$	$(-12, 407)$

based on what we feel is a reasonable experimental lower bound for the top quark mass. We note from the table that μ_0 is less than ~ 200 GeV in the first three cases, and only in the minimal SUGRA case does it reach larger values. The largest value of μ_0 was 817 GeV and occured for a run that does not correspond to any of the above cases. The values of the other soft parameters in this run and the resulting spectrum are displayed in Table IV.

A. General Case

Based on all runs and imposing no particular constraints on any soft parameters, we find that $86 < M_t < 185$ GeV for the top quark and $35 < M_h < 141$ GeV for Higgs boson. Table IV contains three scenarios with input parameters that do not conform to any of the four aforementioned cases. Scenario (a) has a very large value of m_0. The sparticle masses are the largest we have encounterd so far in our discussion. One interesting feature of this scenario is that the lighter stop is lighter than any of the sleptons. Scenario (b) has a gluino of mass 908 GeV and a top quark of mass 184 GeV. The Higgs boson mass is one of the heaviest of all the scenarios discussed at 132 GeV. Both (a) and (b) have very large neutralino masses ~ 170 GeV. The spectrum in (c) represents another scenario in the general case, but with a larger value of $\tan\beta$.

B. No-scale Case

In all cases considered, the mass of the LSP, when it is a neutralino, is observed to be correlated with the value of $m_{1/2}$. Therefore, we find that $m_{1/2}$ cannot be taken too large ($\lesssim 400$ GeV). In the no-scale case, we present plots of M_t vs. $m_{1/2}$ for three values of $m_b(1\ \text{GeV})$ and containing all points satisfying the various criteria outlined in Section IX. Figure 1 is such a plot for $m_b(1\ \text{GeV}) = 6.00$ GeV. The "right edge" of the envelope is defined by points whose neutralino LSPs are just slightly heavier than the lightest charged superparticle (usually a $\tilde{\tau}_R$ for us). The "left edge" defines the threshold of consistent electroweak breaking. The top and bottom edges are set by the requirement that Eq. (6.5) hold. The range of $\tan\beta$ considered leads to definite lower and upper bounds on M_t [9–12]. In Figs. 2 and 3, we display similar plots with lower ($m_b(1\ \text{GeV}) = 5.70$ GeV) and higher ($m_b(1\ \text{GeV}) = 6.33$ GeV) bottom quark masses. The previously noted dependence of the data on M_b is evident from these figures.

TABLE IV. Sample Particle Spectra for General Models (in units of GeV).

	(a)	(b)	(c)
A_0	0	200	0
m_0	800	500	100
$m_{1/2}$	400	400	300
$\tan\beta$	2	5	15
$\text{sign}(\mu)$	$+$	$+$	$+$
B_0	638	326	44
μ_0	817	687	301
M_t	175	184	155
$\tilde{g}$	913	913	692
$\tilde{u}_1, \tilde{u}_2$	1112, 1137	926, 956	604, 629
$\tilde{t}_1, \tilde{t}_2$	696, 993	626, 869	476, 636
$\tilde{d}_1, \tilde{d}_2$	1109, 1139	922, 959	602, 634
$\tilde{b}_1, \tilde{b}_2$	966, 1109	831, 923	573, 611
$\tilde{e}_1, \tilde{e}_2$	815, 847	525, 573	159, 236
$\tilde{\tau}_1, \tilde{\tau}_2$	815, 847	521, 573	143, 241
$\tilde{\nu}_e$	844	568	223
$\tilde{\nu}_\tau$	844	567	221
$\tilde{C}_1$	324	323	222
$\tilde{C}_2$	724	604	358
$\tilde{N}_1$	171	171	125
$\tilde{N}_2$	324	324	223
$\tilde{N}_3$	-712	-590	-326
$\tilde{N}_4$	726	604	358
h	118	132	112
H	1149	755	178
A	1147	755	177
$H^\pm$	1150	759	194

From these, we note that the available range of $m_{1/2}$ decreases with increasing M_b. Fig. 1 indicates that 190 GeV $\lesssim m_{1/2} \lesssim$ 265 GeV. We can draw no conclusions, however, about the value of $\tan\beta$. Although, we found that values of $\tan\beta$ larger than 18 never led to solutions in the no-scale case. We can conclude from the $m_b(1 \text{ GeV}) = 6.00$ GeV case that $85 \lesssim M_t \lesssim 132$ GeV. From similar plots involving M_h, we conclude that $35 \lesssim M_h \lesssim 110$ GeV. In Fig. 4, we display the familiar dependence of M_t on $\tan\beta$ for a particular value of $m_{1/2}$ in the allowed range.

The official experimental lower bound on the top quark mass is 108 GeV. The figures indicate that the top quark cannot have a mass greater than $\sim$ 132 GeV in this model, if $m_b(1 \text{ GeV}) = 6.00$ GeV. This upper bound is raised to $\sim$ 160 GeV, if $m_b(1 \text{ GeV}) = 5.70$ GeV, and the model is ruled out, if $m_b(1 \text{ GeV}) = 6.33$ GeV.

Figure 5 is similar to Fig. 1, but we have chosen the sign of μ negative. The allowed region is displaced down with respect to the positive μ case with the upper bound on M_t now 113 GeV and very close to the experimental limit.

Table V displays various spectra of superparticle masses in the no-scale case. In spectrum (a), we present a scenario in which $m_{1/2} = 254$ GeV and $\tan\beta = 5$. In this particular scenario, the top quark mass is 131 GeV and slightly below is the mass of the light Higgs boson at 98 GeV. A qualitative feature of most spectra is that the sleptons are lighter than the squarks. Another feature is that one of the stops is lighter than all other squarks. In this table, as in similar ones to follow, we do not include the second family sfermion masses since they are generally degenerate with those of the first family. Also, we have singled out the flippino by associating with it a negative mass. The input parameters of spectra (b) and (c) differ only in the respective values of $\tan\beta$, 3 and 10. The spectra are identical in almost every respect except for the splitting in the stop masses. The stop mass splitting in spectrum (b) is 154 GeV and 140 GeV in spectrum (c) as expected since the product $\mu \cot\beta$ is bigger in case (b). Spectrum (a) represents a shifted version of (b) or (c).

C. Strict No-scale Case

The results of the strict no-scale case must be interpolated from the no-scale results, because B_0 is not an input parameter in our procedural method. Therefore, we plot in Fig. 6 M_t vs. B_0 and deduce the M_t bounds from slicing the data along $B_0 = 0$. Admittedly, the relatively small number of points makes the perimeter of the region unclear in some areas. We get approximately $90 \lesssim M_t \lesssim 127$ GeV. For M_h, we find $57 \lesssim M_h \lesssim 110$ GeV with an uncertainty in the lower bound of $\sim$ 10 GeV due to lack of definition in the lower end of the envelope. Inspection of the data indicates that $3.5 \lesssim \tan\beta \lesssim 9$. Thus, it appears that $\tan\beta$ cannot be too small or too large to accommodate the strict no-scale case. This case is a special case of both the no-scale and string inspired cases.

Finally, in Table VI, we display spectra of superparticle masses for three representative strict no-scale scenarios with various $\tan\beta$ and $m_{1/2}$. Although, as evident from the table, none of these scenarios have B_0 strictly equal to zero, they do satisfy $|B_0| < 25$ GeV, which is the tolerance we have settled upon. The LSPs are neutralinos in these three scenarios and have masses that increase with increasing $m_{1/2}$. The top quark masses are comparable at $\sim$ 126 GeV and just above the experimental limit, and the Higgs bosons masses are also

TABLE V. Sample Particle Spectra for No-Scale Models (in units of GeV).

	(a)	(b)	(c)
A_0	0	0	0
m_0	0	0	0
$m_{1/2}$	254	240	240
$\tan \beta$	5	3	10
$\text{sign}(\mu)$	$+$	$+$	$+$
B_0	29	66	-12
μ_0	203	198	166
M_t	131	126	124
$\tilde{g}$	589	558	558
$\tilde{u}_1, \tilde{u}_2$	508, 530	482, 502	482, 501
$\tilde{t}_1, \tilde{t}_2$	408, 559	381, 535	392, 532
$\tilde{d}_1, \tilde{d}_2$	507, 535	481, 507	481, 507
$\tilde{b}_1, \tilde{b}_2$	507, 508	476, 486	472, 491
$\tilde{e}_1, \tilde{e}_2$	107, 183	101, 173	102, 174
$\tilde{\tau}_1, \tilde{\tau}_2$	105, 184	100, 173	97, 176
$\tilde{\nu}_e$	166	157	155
$\tilde{\nu}_\tau$	166	157	155
$\tilde{C}_1$	164	150	145
$\tilde{C}_2$	296	293	267
$\tilde{N}_1$	100	90	92
$\tilde{N}_2$	170	157	152
$\tilde{N}_3$	-246	-240	-208
$\tilde{N}_4$	298	296	268
h	98	88	99
H	278	290	203
A	276	285	202
$H^{\pm}$	287	296	217

approximately equal ~ 99 GeV. As in the no-scale case, we observe that increasing the value of $m_{1/2}$ has the effect of shifting the squark and slepton spectra up.

D. String Inspired Case

As in the strict no-scale, our results for the string inspired case necessitates interpolating $A_0 = 0$ data to $B_0 = 0$. Once again we find the perimeter of the allowed region is not well defined everywhere. Hence, our results are only qualitative. Figure 7 is similar to Fig. 6, but in this case we only fix $A_0 = 0$. Since the strict no-scale case is a special case of the string inspired one, the 127 GeV top quark mass upper bound is not expected to decrease but rather to increase in this case. Slicing along $B_0 = 0$ yields $85 \lesssim M_t \lesssim 140$ GeV. Similarly, for the light Higgs we get $57 \lesssim M_h \lesssim 113$ GeV. The data indicates in this case, as in the strict no-scale case, that there is a lower bound on $\tan \beta$ of ~ 3.

In Table VII, we collect some spectra in the string inspired case. As in the strict no-scale case, we have settled on a tolerance of $|B_0| < 25$ GeV for the scenarios considered. Spectrum (a) with input parameters $A_0 = 0$, $m_0 = 100$ GeV, $m_{1/2} = 200$ GeV, and $\tan \beta = 10$ has $B_0 = -4$ GeV. The strict $B_0 = 0$ run has a value of $\tan \beta$ between 5 and 10, but the qualitativee features should be very similar to the scenario we present with $\tan \beta = 10$. Scenario (b) is similar to scenario (a) except it has a larger value of $m_{1/2} = 250$ GeV. As noted before, this leads to an upward shift in the squark and slepton spectrum of (b). The upward shift is 23% in the squark masses and 15% in the slepton masses. Scenario (c) is similar to scenario (a) except it has a larger value of $m_0 = 200$ GeV. This shifts up the squark and slepton masses of (c) with respect to (a). In this case, the squarks shift up by only 7% and the sleptons shift up by 49%. Therefore, spectra (b) and (c) have the curious property that the squarks of (b) are heavier than those of (c), but the sleptons of (b) are lighter than those of (c). Lastly, we observe that the lighter selectron and stau of all spectra discussed so far display a comparatively large mass gap from the rest of their respective slepton partners, however in scenario (c) this mass gap is comparatively small; and all sleptons are almost degenerate.

E. Minimal SUGRA Case

We now consider models for which the relation $B_0 = A_0 - m_0$ among the soft couplings holds. We find it convenient to define $\chi \equiv A_0 - B_0 - m_0$, which should equal zero in this case. In Fig. 8, we plot M_t vs. χ. Taking a slice along $\chi = 0$ of the region depicted in this plot, we find that $82 \lesssim M_t \lesssim 185$ GeV. A similar plot was used to arrive at bounds for the light Higgs boson $57 \lesssim M_h \lesssim 139$ GeV. Our analysis indicates that there is no preferred values of $\tan \beta$ in this case.

Table VIII includes the spectra for some particular scenarios in the minimal SUGRA case. The three spectra appearing in this table have the interesting feature that the slepton masses are larger than they have been in all cases discussed so far. In fact the slepton masses have values comparable to the lighter stop in (a), (b), and (c). These three scenarios each have relatively large, non-zero values of A_0. As expected, scenario (b) has the larger value of $\mu \cot \beta$ and therefore has the larger splitting in stop masses when compared to (a). Indeed,

TABLE VI. Sample Particle Spectra for Strict No-Scale Models (in units of GeV).

	(a)	(b)	(c)
A_0	0	0	0
m_0	0	0	0
$m_{1/2}$	240	230	210
$\tan\beta$	8.3	10	5
$\text{sign}(\mu)$	$+$	$+$	$+$
B_0	-4	-11	18
μ_0	171	159	159
M_t	126	124	128
$\tilde{g}$	558	536	491
$\tilde{u}_1, \tilde{u}_2$	482, 501	462, 481	424, 441
$\tilde{t}_1, \tilde{t}_2$	390, 533	373, 514	331, 480
$\tilde{d}_1, \tilde{d}_2$	481, 507	462, 487	423, 447
$\tilde{b}_1, \tilde{b}_2$	473, 490	453, 472	418, 430
$\tilde{e}_1, \tilde{e}_2$	102, 174	99, 167	92, 154
$\tilde{\tau}_1, \tilde{\tau}_2$	98, 175	93, 169	90, 154
$\tilde{\nu}_e$	156	148	133
$\tilde{\nu}_\tau$	155	148	133
$\tilde{C}_1$	147	137	122
$\tilde{C}_2$	270	259	253
$\tilde{N}_1$	92	87	76
$\tilde{N}_2$	153	144	131
$\tilde{N}_3$	-213	-200	-197
$\tilde{N}_4$	271	260	255
h	99	99	96
H	222	194	223
A	221	193	221
$H^{\pm}$	235	209	234

TABLE VII. Sample Particle Spectra for String Inspired Models (in units of GeV).

	(a)	(b)	(c)
A_0	0	0	0
m_0	100	100	200
$m_{1/2}$	200	250	200
$\tan\beta$	10	10	10
$\text{sign}(\mu)$	$+$	$+$	$+$
B_0	-4	-6	20
μ_0	132	168	88
M_t	124	125	123
$\tilde{g}$	469	580	469
$\tilde{u}_1, \tilde{u}_2$	416, 431	510, 530	449, 464
$\tilde{t}_1, \tilde{t}_2$	330, 467	417, 556	361, 489
$\tilde{d}_1, \tilde{d}_2$	416, 438	509, 536	449, 470
$\tilde{b}_1, \tilde{b}_2$	407, 424	509, 509	440, 453
$\tilde{e}_1, \tilde{e}_2$	134, 178	145, 206	219, 248
$\tilde{\tau}_1, \tilde{\tau}_2$	129, 179	141, 208	216, 248
$\tilde{\nu}_e$	160	191	235
$\tilde{\nu}_\tau$	159	190	234
$\tilde{C}_1$	109	150	77
$\tilde{C}_2$	231	272	214
$\tilde{N}_1$	71	96	53
$\tilde{N}_2$	119	157	105
$\tilde{N}_3$	-170	-209	-120
$\tilde{N}_4$	232	273	214
h	98	100	98
H	189	226	230
A	188	225	229
$H^\pm$	204	238	242

TABLE VIII. Sample Particle Spectra for Minimal SUGRA Models (in units of GeV).

	(a)	(b)	(c)
A_0	400	500	400
m_0	200	200	300
$m_{1/2}$	300	300	200
$\tan\beta$	10	5	10
$\mathrm{sign}(\mu)$	$+$	$+$	$+$
B_0	212	288	115
μ_0	404	436	158
M_t	164	163	124
$\tilde{g}$	692	692	469
$\tilde{u}_1, \tilde{u}_2$	627, 652	627, 652	500, 513
$\tilde{t}_1, \tilde{t}_2$	415, 643	389, 644	330, 529
$\tilde{d}_1, \tilde{d}_2$	625, 656	625, 656	500, 519
$\tilde{b}_1, \tilde{b}_2$	573, 630	574, 627	463, 502
$\tilde{e}_1, \tilde{e}_2$	235, 293	235, 292	313, 334
$\tilde{\tau}_1, \tilde{\tau}_2$	221, 295	230, 294	306, 333
$\tilde{\nu}_e$	282	282	324
$\tilde{\nu}_\tau$	279	281	322
$\tilde{C}_1$	236	236	123
$\tilde{C}_2$	438	471	247
$\tilde{N}_1$	127	126	76
$\tilde{N}_2$	236	236	129
$\tilde{N}_3$	-421	-454	-200
$\tilde{N}_4$	438	472	248
h	119	118	101
H	371	491	325
A	371	490	325
$H^\pm$	379	497	334

this is the only significant difference in these two scenarios. Scenario (c) in the minimal SUGRA case has a fairly degenerate slepton sector like that of (c) in Table VII. Once again we observe the curious feature when comparing (a) and (c) that the squarks of (a) are heavier than those of (c), but the sleptons of (a) are lighter than those of (c). It appears that the run with the larger value of m_0 has the heavier sleptons and that with the larger value of $m_{1/2}$ has the heavier squarks. Scenarios (a) and (b) are interesting because they have large values of $\mu \tan \beta$ which leads to large splittings in the bottom squark and stau masses. This represents an example where neglecting the bottom or τ Yukawa can radically alter the spectrum. The value of M_t in (a) and (b) is ~ 163 GeV, however in (c) it is 124 GeV. We find based on studying other similar runs that the input parameter accounting mostly for this large difference is the value of $m_{1/2}$. Both (a) and (b) have $m_{1/2} = 300$ GeV whereas (c) has $m_{1/2} = 200$ GeV. We observe also that (a) and (b) have larger (comparable) values of μ_0 and B_0 whereas (c) has a $\sim 50\%$ smaller values of these input parameters.

XI. CONCLUSIONS

Minimal low energy supergravity models were considered. They have the appealing feature that the electro-weak symmetry is radiatively broken for certain ranges of the soft breaking parameters and of the top quark mass. The study of specific models, with some soft parameter fixed or related, resulted in upper bounds for the top quark mass. No-scale models in which only gaugino masses provide global supersymmetry breaking yield top quarks with masses less than ~ 127 GeV. The results are sensitive to the value of the bottom quark mass. Lower bottom quark masses, within the experimental uncertainty, lead to higher top quark upper bounds. In these models, the ratio of vacuum expectation values of the two Higgs fields is expected to be larger than $\sim 70°$.

Although the perimeter of the allowed regions were often fuzzy, we could, nevertheless, draw some general conclusions from our results. For all our runs, with no restrictions on the soft terms, we find for the top quark $M_t \lesssim 185$ GeV and for the light Higgs boson $M_h \lesssim 141$ GeV .

APPENDIX A: THE SUSY β-FUNCTIONS

Using some of the notation of Falck [46], the superpotential and soft symmetry breaking potential are as follows:

$$W = \hat{\bar{u}}\mathbf{Y}_u\hat{\Phi}_u\hat{Q} + \hat{\bar{d}}\mathbf{Y}_d\hat{\Phi}_d\hat{Q} + \hat{\bar{e}}\mathbf{Y}_e\hat{\Phi}_d\hat{L} + \mu\hat{\Phi}_u\hat{\Phi}_d + h.c. \ , \tag{A1}$$

$$V_{soft} = m_{\Phi_u}^2\Phi_u^\dagger\Phi_u + m_{\Phi_d}^2\Phi_d^\dagger\Phi_d + B\mu(\Phi_u\Phi_d + h.c.)$$

$$+ \sum_i (\ m_{\tilde{Q}_i}^2\tilde{Q}_i^\dagger\tilde{Q}_i + m_{\tilde{L}_i}^2\tilde{L}_i^\dagger\tilde{L}_i + m_{\tilde{\bar{u}}_i}^2\tilde{\bar{u}}_i^\dagger\tilde{\bar{u}}_i + m_{\tilde{\bar{d}}_i}^2\tilde{\bar{d}}_i^\dagger\tilde{\bar{d}}_i + m_{\tilde{\bar{e}}_i}^2\tilde{\bar{e}}_i^\dagger\tilde{\bar{e}}_i \ )$$

$$+ \sum_{i,j}\Big(\ A_u^{ij}Y_u^{ij}\tilde{\bar{u}}_i\Phi_u\tilde{Q}_j + A_d^{ij}Y_d^{ij}\tilde{\bar{d}}_i\Phi_d\tilde{Q}_j + A_e^{ij}Y_e^{ij}\tilde{\bar{e}}_i\Phi_d\tilde{L}_j + h.c. \ \Big)\ , \tag{A2}$$

$$V_{gaugino} = \frac{1}{2}\sum_{l=1}^3 M_l\lambda_l\lambda_l + h.c. \ . \tag{A3}$$

Various σ_2's have been omitted and a sum over the number of families is implied in the squark and slepton mass terms. Also, hats imply superfields and tildes the superpartners of the given fields.

We start with the gauge couplings

$$\frac{dg_l}{dt} = -\frac{1}{16\pi^2}b_l g_l^3 + \frac{g_l^3}{(16\pi^2)^2}\Big[\sum_k b_{lk}g_k^2 - \mathrm{Tr}\{C_{lu}\mathbf{Y}_u^\dagger\mathbf{Y}_u + C_{ld}\mathbf{Y}_d^\dagger\mathbf{Y}_d + C_{le}\mathbf{Y}_e^\dagger\mathbf{Y}_e\}\Big] \tag{A4}$$

where $t = \ln\Lambda$ and $l = 1,\ 2,\ 3$, corresponding to gauge group $\mathrm{SU}(3)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}$ of the Standard Model.

In the Yukawa sector the β-functions are

$$\frac{d\mathbf{Y}_{u,d,e}}{dt} = \mathbf{Y}_{u,d,e}\Big(\frac{1}{16\pi^2}\beta_{u,d,e}^{(1)} + \frac{1}{(16\pi^2)^2}\beta_{u,d,e}^{(2)}\Big)\ . \tag{A5}$$

The evolution of the vacuum expectation values of the Higgs's is given by

$$\frac{d\ln v_{\Phi_u,\Phi_d}}{dt} = \frac{1}{16\pi^2}\gamma_{\Phi_u,\Phi_d}^{(1)} + \frac{1}{(16\pi^2)^2}\gamma_{\Phi_u,\Phi_d}^{(2)}\ . \tag{A6}$$

1. The one loop SUSY β-functions

The various one loop coefficients for the gauge couplings are defined to be

$$\begin{cases} b_1 = -\frac{3}{5} - 2n_g\ , \\ b_2 = \ \ 5 - 2n_g\ , \\ b_3 = \ \ 9 - 2n_g\ , \end{cases} \tag{A7}$$

with $n_g = \frac{1}{2}n_{fl}$.

In the following, we list the one loop contributions for the parameters of the superpotential.

$$\frac{d\ln\mu}{dt} = \frac{1}{16\pi^2}\big[\ \mathrm{Tr}\{3\mathbf{Y}_u^\dagger\mathbf{Y}_u + 3\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_e^\dagger\mathbf{Y}_e\} - 3(\frac{1}{5}g_1^2 + g_2^2)\ \big]\ . \tag{A8}$$

The one-loop contributions for the Yukwas are given by

$$\beta_u^{(1)} = 3\mathbf{Y}_u^\dagger\mathbf{Y}_u + \mathbf{Y}_d^\dagger\mathbf{Y}_d + 3\mathrm{Tr}\{\mathbf{Y}_u^\dagger\mathbf{Y}_u\} - (\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2) \ , \tag{A9}$$

$$\beta_d^{(1)} = 3\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_u^\dagger\mathbf{Y}_u + \mathrm{Tr}\{3\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_e^\dagger\mathbf{Y}_e\} - (\frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2) \ , \tag{A10}$$

$$\beta_e^{(1)} = 3\mathbf{Y}_e^\dagger\mathbf{Y}_e + \mathrm{Tr}\{3\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_e^\dagger\mathbf{Y}_e\} - (\frac{9}{5}g_1^2 + 3g_2^2) \ . \tag{A11}$$

The one-loop contributions for the running VEVs are

$$\gamma_{\Phi_u}^{(1)} = \frac{3}{4}(\frac{1}{5}g_1^2 + g_2^2) - 3\mathrm{Tr}\{\mathbf{Y}_u^\dagger\mathbf{Y}_u\} \ , \tag{A12}$$

$$\gamma_{\Phi_d}^{(1)} = \frac{3}{4}(\frac{1}{5}g_1^2 + g_2^2) - 3\mathrm{Tr}\{\mathbf{Y}_d^\dagger\mathbf{Y}_d\} - \mathrm{Tr}\{\mathbf{Y}_e^\dagger\mathbf{Y}_e\} \ . \tag{A13}$$

The soft symmetry breaking terms are known to us only to one loop

$$\frac{dA_e^{ij}}{dt} = \frac{1}{16\pi^2}\Big[\ 4(\mathbf{Y}_e\mathbf{Y}_e^\dagger)^{ik}A_e^{kj}\frac{Y_e^{kj}}{Y_e^{ij}} + 5A_e^{ik}\frac{Y_e^{ik}}{Y_e^{ij}}(\mathbf{Y}_e^\dagger\mathbf{Y}_e)^{kj} - 3\frac{A_e^{ij}}{Y_e^{ij}}(\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e)^{ij}$$
$$+ 2(A_e^{km}|Y_e^{km}|^2 + 3A_d^{km}|Y_d^{km}|^2) - 6(\frac{3}{5}g_1^2 M_1 + g_2^2 M_2)\ \Big] \ , \tag{A14}$$

$$\frac{dA_d^{ij}}{dt} = \frac{1}{16\pi^2}\Big[\ 4(\mathbf{Y}_d\mathbf{Y}_d^\dagger)^{ik}A_d^{kj}\frac{Y_d^{kj}}{Y_d^{ij}} + 5A_d^{ik}\frac{Y_d^{ik}}{Y_d^{ij}}(\mathbf{Y}_d^\dagger\mathbf{Y}_d)^{kj} - 3\frac{A_d^{ij}}{Y_d^{ij}}(\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d)^{ij}$$
$$+ (A_d^{ik} - A_d^{ij})(\mathbf{Y}_u^\dagger\mathbf{Y}_u)^{kj}\frac{Y_d^{ik}}{Y_d^{ij}} + 2(\mathbf{Y}_d\mathbf{Y}_u^\dagger)^{ik}A_u^{kj}\frac{Y_u^{kj}}{Y_d^{ij}} + 2(A_e^{km}|Y_e^{km}|^2$$
$$+ 3A_d^{km}|Y_d^{km}|^2) - \frac{14}{15}g_1^2 M_1 - 6g_2^2 M_2 - \frac{32}{3}g_3^2 M_3\ \Big] \ , \tag{A15}$$

$$\frac{dA_u^{ij}}{dt} = \frac{1}{16\pi^2}\Big[\ 4(\mathbf{Y}_u\mathbf{Y}_u^\dagger)^{ik}A_u^{kj}\frac{Y_u^{kj}}{Y_u^{ij}} + 5A_u^{ik}\frac{Y_u^{ik}}{Y_u^{ij}}(\mathbf{Y}_u^\dagger\mathbf{Y}_u)^{kj} - 3\frac{A_u^{ij}}{Y_u^{ij}}(\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u)^{ij}$$
$$+ (A_u^{ik} - A_u^{ij})(\mathbf{Y}_d^\dagger\mathbf{Y}_d)^{kj}\frac{Y_u^{ik}}{Y_u^{ij}} + 2(\mathbf{Y}_u\mathbf{Y}_d^\dagger)^{ik}A_d^{kj}\frac{Y_d^{kj}}{Y_u^{ij}} + 6A_u^{km}|Y_u^{km}|^2$$
$$- \frac{26}{15}g_1^2 M_1 - 6g_2^2 M_2 - \frac{32}{3}g_3^2 M_3\ \Big] \ , \tag{A16}$$

$$\frac{dm_{\Phi_u}^2}{dt} = \frac{1}{8\pi^2}\Big[\ \sum_{i,j}3|Y_u^{ji}|^2(m_{\Phi_u}^2 + m_{Q_i}^2 + m_{u_j}^2 + |A_u^{ji}|^2) + \frac{3}{10}g_1^2\mathrm{Tr}\{Ym^2\} - \frac{3}{5}g_1^2 M_1^2$$
$$- 3g_2^2 M_2^2\ \Big] \ , \tag{A17}$$

$$\frac{dm_{\Phi_d}^2}{dt} = \frac{1}{8\pi^2}\Big[\ \sum_{i,j}\Big(|Y_e^{ji}|^2(m_{\Phi_d}^2 + m_{L_i}^2 + m_{e_j}^2 + |A_e^{ji}|^2) + 3|Y_d^{ji}|^2(m_{\Phi_d}^2 + m_{Q_i}^2 + m_{d_j}^2$$
$$+ |A_d^{ji}|^2)\Big) - \frac{3}{10}g_1^2\mathrm{Tr}\{Ym^2\} - \frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2\ \Big] \ , \tag{A18}$$

$$\frac{dm_{e_i}^2}{dt} = \frac{1}{8\pi^2}\Big[\ \sum_j 2|Y_e^{ij}|^2(m_{\Phi_d}^2 + m_{e_i}^2 + m_{L_j}^2 + |A_e^{ij}|^2) + \frac{3}{5}g_1^2\mathrm{Tr}\{Ym^2\} - \frac{12}{5}g_1^2 M_1^2\ \Big] \ , \tag{A19}$$

$$\frac{dm_{L_i}^2}{dt} = \frac{1}{8\pi^2}\Big[\ \sum_j |Y_e^{ji}|^2(m_{\Phi_d}^2 + m_{L_i}^2 + m_{e_j}^2 + |A_e^{ji}|^2) - \frac{3}{10}g_1^2\mathrm{Tr}\{Ym^2\}$$

$$- \frac{3}{5} g_1^2 M_1^2 - 3 g_2^2 M_2^2 \ ] \ , \tag{A20}$$

$$\frac{dm_{d_i}^2}{dt} = \frac{1}{8\pi^2} [\ \sum_j 2 |Y_d^{ij}|^2 (m_{\Phi_d}^2 + m_{d_i}^2 + m_{Q_j}^2 + |A_d^{ij}|^2) + \frac{1}{5} g_1^2 \mathrm{Tr}\{Y m^2\}$$

$$- \frac{4}{15} g_1^2 M_1^2 - \frac{16}{3} g_3^2 M_3^2 \ ] \ , \tag{A21}$$

$$\frac{dm_{u_i}^2}{dt} = \frac{1}{8\pi^2} [\ \sum_j 2 |Y_u^{ij}|^2 (m_{\Phi_u}^2 + m_{u_i}^2 + m_{Q_j}^2 + |A_u^{ij}|^2) - \frac{2}{5} g_1^2 \mathrm{Tr}\{Y m^2\}$$

$$- \frac{16}{15} g_1^2 M_1^2 - \frac{16}{3} g_3^2 M_3^2 \ ] \ , \tag{A22}$$

$$\frac{dm_{Q_i}^2}{dt} = \frac{1}{8\pi^2} [\ \sum_{i,j} \Big(|Y_u^{ji}|^2 (m_{\Phi_u}^2 + m_{Q_i}^2 + m_{u_j}^2 + |A_u^{ji}|^2) + |Y_d^{ji}|^2 (m_{\Phi_d}^2 + m_{Q_i}^2 + m_{d_j}^2$$

$$+ |A_d^{ji}|^2) \Big) + \frac{1}{10} g_1^2 \mathrm{Tr}\{Y m^2\} - \frac{1}{15} g_1^2 M_1^2 - 3 g_2^2 M_2^2 - \frac{16}{3} g_3^2 M_3^2 \ ] \ , \tag{A23}$$

$$\frac{dB}{dt} = \frac{1}{8\pi^2} [\ 3 A_u^{ij} |Y_u^{ij}|^2 + 3 A_d^{ij} |Y_d^{ij}|^2 + A_e^{ij} |Y_e^{ij}|^2 - \frac{3}{5} g_1^2 M_1 - 3 g_2^2 M_2 \ ] \ . \tag{A24}$$

where, as in Falck, sums are implied over all indices not appearing on the left hand side and where

$$\mathrm{Tr}\{Y m^2\} = \sum_{i=1}^{n_g} (m_{Q_i}^2 - 2 m_{u_i}^2 + m_{d_i}^2 - m_{L_i}^2 + m_{e_i}^2) + m_{\Phi_u}^2 - m_{\Phi_d}^2 \ . \tag{A25}$$

Note that in an anomaly free theory, this term is zero if all the masses are equal at some scale. That it is zero at such a scale can be seen by using the fact that there is no gravitational anomaly. In such a case, $\mathrm{Tr}\{Y m^2\} = m^2 \mathrm{Tr}\{Y\} = 0$. To show that it remains true at all scales, one also needs the cancellation of the other triangle anomalies. For example, in Eqs. (A17-A23), the $g_1^2 M_1^2$ terms come from one loop mass corrections involving two bino-particle-sparticle vertices and are therefore proportional to Y^2. Thus one needs to have the $U(1)$ anomaly cancellation condition $\mathrm{Tr}\{Y^3\} = 0$ in order to show $d \mathrm{Tr}\{Y m^2\}/dt = 0$. The gaugino masses evolve as follows

$$\frac{d \ln M_l}{dt} = - \frac{1}{8\pi^2} b_l g_l^2 \ . \tag{A26}$$

2. The two loop SUSY β-functions

The two loop contributions to the gauge couplings are

$$(b_{lk}) = \begin{pmatrix} \frac{38}{15} & \frac{6}{5} & \frac{88}{15} \\ \frac{2}{5} & 14 & 8 \\ \frac{11}{15} & 3 & \frac{68}{3} \end{pmatrix} n_g + \begin{pmatrix} \frac{9}{25} & \frac{9}{5} & 0 \\ \frac{3}{5} & -17 & 0 \\ 0 & 0 & -54 \end{pmatrix} \ , \tag{A27}$$

and

$$(C_{lf}) = \begin{pmatrix} \frac{26}{5} & \frac{14}{5} & \frac{18}{5} \\ 6 & 6 & 2 \\ 4 & 4 & 0 \end{pmatrix} \ , \ \text{with } f = u \ , \ d \ , \ e \ , \tag{A28}$$

The two-loop contributions to the Yukawa couplings are given by

$$
\begin{aligned}
\beta_u^{(2)} = & -4(\mathbf{Y}_u^\dagger \mathbf{Y}_u)^2 - 2(\mathbf{Y}_d^\dagger \mathbf{Y}_d)^2 - 2\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u - 9\text{Tr}\{\mathbf{Y}_u^\dagger \mathbf{Y}_u\}\mathbf{Y}_u^\dagger \mathbf{Y}_u \\
& - \text{Tr}\{3\mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_e^\dagger \mathbf{Y}_e\}\mathbf{Y}_d^\dagger \mathbf{Y}_d - 3\text{Tr}\{3(\mathbf{Y}_u^\dagger \mathbf{Y}_u)^2 + \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u\} \\
& + (\tfrac{2}{5}g_1^2 + 6g_2^2)\mathbf{Y}_u^\dagger \mathbf{Y}_u + (\tfrac{2}{5}g_1^2)\mathbf{Y}_d^\dagger \mathbf{Y}_d + (\tfrac{4}{5}g_1^2 + 16g_3^2)\text{Tr}\{\mathbf{Y}_u^\dagger \mathbf{Y}_u\} \\
& + (\tfrac{26}{15}n_g + \tfrac{403}{450})g_1^4 + (6n_g - \tfrac{21}{2})g_2^4 + (\tfrac{32}{3}n_g - \tfrac{304}{9})g_3^4 \\
& + g_1^2 g_2^2 + \tfrac{136}{15}g_1^2 g_3^2 + 8g_2^2 g_3^2
\end{aligned}
\tag{A29}
$$

$$
\begin{aligned}
\beta_d^{(2)} = & -4(\mathbf{Y}_d^\dagger \mathbf{Y}_d)^2 - 2(\mathbf{Y}_u^\dagger \mathbf{Y}_u)^2 - 2\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d - 3\text{Tr}\{\mathbf{Y}_u^\dagger \mathbf{Y}_u\}\mathbf{Y}_u^\dagger \mathbf{Y}_u \\
& - 3\text{Tr}\{3\mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_e^\dagger \mathbf{Y}_e\}\mathbf{Y}_d^\dagger \mathbf{Y}_d - 3\text{Tr}\{3(\mathbf{Y}_d^\dagger \mathbf{Y}_d)^2 + (\mathbf{Y}_e^\dagger \mathbf{Y}_e)^2 + \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u\} \\
& + (\tfrac{4}{5}g_1^2)\mathbf{Y}_u^\dagger \mathbf{Y}_u + (\tfrac{4}{5}g_1^2 + 6g_2^2)\mathbf{Y}_d^\dagger \mathbf{Y}_d + (-\tfrac{2}{5}g_1^2 + 16g_3^2)\text{Tr}\{\mathbf{Y}_d^\dagger \mathbf{Y}_d\} + (\tfrac{6}{5}g_1^2)\text{Tr}\{\mathbf{Y}_e^\dagger \mathbf{Y}_e\} \\
& + (\tfrac{14}{15}n_g + \tfrac{7}{18})g_1^4 + (6n_g - \tfrac{21}{2})g_2^4 + (\tfrac{32}{3}n_g - \tfrac{304}{9})g_3^4 + g_1^2 g_2^2 + \tfrac{8}{9}g_1^2 g_3^2 + 8g_2^2 g_3^2
\end{aligned}
\tag{A30}
$$

$$
\begin{aligned}
\beta_e^{(2)} = & -4(\mathbf{Y}_e^\dagger \mathbf{Y}_e)^2 - 3\text{Tr}\{3\mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_e^\dagger \mathbf{Y}_e\}\mathbf{Y}_e^\dagger \mathbf{Y}_e - 3\text{Tr}\{3(\mathbf{Y}_d^\dagger \mathbf{Y}_d)^2 + (\mathbf{Y}_e^\dagger \mathbf{Y}_e)^2 \\
& + \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u\} + (6g_2^2)\mathbf{Y}_e^\dagger \mathbf{Y}_e + (\tfrac{6}{5}g_1^2)\text{Tr}\{\mathbf{Y}_e^\dagger \mathbf{Y}_e\} + (-\tfrac{2}{5}g_1^2 + 16g_3^2)\text{Tr}\{\mathbf{Y}_d^\dagger \mathbf{Y}_d\} \\
& + (\tfrac{18}{5}n_g + \tfrac{27}{10})g_1^4 + (6n_g - \tfrac{21}{2})g_2^4 + \tfrac{9}{5}g_1^2 g_2^2
\end{aligned}
\tag{A31}
$$

The two-loop contributions to the anomalous dimension of the scalars are

$$
\begin{aligned}
\gamma_{\Phi_u}^{(2)} = & \frac{3}{4}\text{Tr}\{3(\mathbf{Y}_u^\dagger \mathbf{Y}_u)^2 + 3\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d\} - (\tfrac{19}{10}g_1^2 + \tfrac{9}{2}g_2^2 + 20g_3^2)\text{Tr}\{\mathbf{Y}_u^\dagger \mathbf{Y}_u\} \\
& - (\tfrac{279}{800} + \tfrac{1803}{1600}n_g)g_1^4 - (\tfrac{207}{32} + \tfrac{357}{64}n_g)g_2^4 - (\tfrac{27}{80} + \tfrac{9}{80}n_g)g_1^2 g_2^2 \ ,
\end{aligned}
\tag{A32}
$$

$$
\begin{aligned}
\gamma_{\Phi_d}^{(2)} = & \frac{3}{4}\text{Tr}\{3(\mathbf{Y}_d^\dagger \mathbf{Y}_d)^2 + 3\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u + (\mathbf{Y}_e^\dagger \mathbf{Y}_e)^2\} - (\tfrac{2}{5}g_1^2 + \tfrac{9}{2}g_2^2 + 20g_3^2)\text{Tr}\{\mathbf{Y}_d^\dagger \mathbf{Y}_d\} \\
& - (\tfrac{9}{5}g_1^2 + \tfrac{3}{2}g_2^2)\text{Tr}\{\mathbf{Y}_e^\dagger \mathbf{Y}_e\} - (\tfrac{279}{800} + \tfrac{1803}{1600}n_g)g_1^4 - (\tfrac{207}{32} + \tfrac{357}{64}n_g)g_2^4 \\
& - (\tfrac{27}{80} + \tfrac{9}{80}n_g)g_1^2 g_2^2 \ .
\end{aligned}
\tag{A33}
$$

APPENDIX B: THRESHOLDS

Ref. [47] gives formulas valid to two loops to compute the β-functions of gauge, Yukawa, and scalar self-quartic couplings in a general gauge theory. These will be useful in obtaining the required form of these β-functions for the purpose of including thresholds.

To implement the super particle thresholds in the minimal low energy super gravity model, the renormalization group β-function must be calculated in a form that allows every particle to be counted in the simplest possible way. This, of course, implies that we will have to make allowances for effective theories with half odd integer doublets, as discussed in Section VIII. We will, therefore, implement particle thresholds as steps in the particle content of the model. In the following, the one loop β-functions of the gauge couplings and Yukawa couplings are considered.

1. Gauge Couplings

In the general case (but for a single, simple gauge group G), at one loop

$$\frac{dg}{dt} = \frac{1}{(4\pi)^2} b^{(1)} g^3 , \tag{B1}$$

where

$$b^{(1)} = \frac{2}{3} T_2(F) + \frac{1}{3} T_2(S) - \frac{11}{3} C_2(G) , \tag{B2}$$

and where F, S, and G stand for the fermion, scalar, and adjoint representations, respectively. Using the fact that gauginos are in the adjoint representation gives in the $SU(3)$ case

$$b_3^{(1)} = \frac{2}{3} T_2(F_3) + \frac{1}{3} T_2(S_3) + \frac{2}{3} C_2(G_3) \theta_{\tilde{g}} - \frac{11}{3} C_2(G_3) , \tag{B3}$$

where F_3, S_3 refer to the fermion and scalar representations, and $G_3 = SU(3)$ and where

$$\theta_{\tilde{g}} = \begin{cases} 1 , & \Lambda > M_g ; \\ 0 , & \Lambda < M_g . \end{cases} \tag{B4}$$

M_g is the mass of the gluino. In the $SU(3)$ case, one has

$$T_2(R_3) = 2(\frac{1}{2}) N_Q + (\frac{1}{2}) N_{\overline{u}} + (\frac{1}{2}) N_{\overline{d}} , \tag{B5}$$

where N_p equals the number of families of particle p. This result is valid for both fermion ($R = F$) and scalar ($R = S$) representations. Equations (B3) and (B5) lead to

$$b_3^{(1)} = \frac{2}{3}(N_u + N_d) + \frac{1}{3} N_{\tilde{Q}} + \frac{1}{6} N_{\tilde{\overline{u}}} + \frac{1}{6} N_{\tilde{\overline{d}}} + 2\theta_{\tilde{g}} - 11 , \tag{B6}$$

where we have assumed that $N_Q = (N_u + N_d)/2$. Also the fact that left and right handed quarks of a given flavor have the same mass allowed us to use $N_{\overline{u}} = N_u$ and $N_{\overline{d}} = N_d$. Note that Eq. (B6) reduces to the right standard model result when $N_{\tilde{Q}} = N_{\tilde{\overline{u}}} = N_{\tilde{\overline{d}}} = \theta_{\tilde{g}} = 0$, and to the right supersymmetric result ($\Lambda > M_{SUSY}$) when $N_{\tilde{Q}} = N_{\tilde{\overline{u}}} = N_{\tilde{\overline{d}}} = 3$ and $\theta_{\tilde{g}} = 1$. Similar formulas are calculated for g_1 and g_2

$$b_1^{(1)} = \frac{2}{5}\left(\frac{17}{12}N_u + \frac{5}{12}N_d + \frac{5}{4}N_e + \frac{1}{4}N_\nu\right) + \frac{1}{30}N_{\tilde{Q}} + \frac{4}{15}N_{\tilde{u}} + \frac{1}{15}N_{\tilde{d}}$$

$$+ \frac{1}{10}N_{\tilde{L}} + \frac{1}{5}N_{\tilde{e}} + \frac{1}{5}(N_{\tilde{\Phi}_u} + N_{\tilde{\Phi}_d}) + \frac{1}{10}(N_{\Phi_u} + N_{\Phi_d}) \ , \tag{B7a}$$

$$b_2^{(1)} = -\frac{22}{3} + \frac{1}{2}(N_u + N_d) + \frac{1}{6}(N_e + N_\nu) + \frac{1}{2}N_{\tilde{Q}} + \frac{1}{6}N_{\tilde{L}}$$

$$+ \frac{1}{3}(N_{\tilde{\Phi}_u} + N_{\tilde{\Phi}_d}) + \frac{1}{6}(N_{\Phi_u} + N_{\Phi_d}) + \frac{4}{3}\theta_{\tilde{W}} \ . \tag{B7b}$$

The two loop contributions may also be calculated in this manner.

2. Yukawa Couplings

We find it useful to define new doublet fields

$$\Phi_h = \sin\beta\tilde{\Phi}_u + \cos\beta\Phi_d \ , \tag{B8a}$$

$$\Phi_H = -\cos\beta\Phi_u - \sin\beta\tilde{\Phi}_d \ , \tag{B8b}$$

where $\tan\beta = v_u/v_d$ and $\tilde{\Phi} = i\tau_2\Phi^*$. The VEVs of these two new fields are $<\Phi_h^0> = v$ and $<\Phi_H^0> = 0$. We can now rewrite the Yukawa interaction Lagrangean of the MSSM in terms of these fields. The β-function for the Yukawa couplings can be computed from formulas in Ref. [47]. At one loop, the renormalization group equation for a general Yukawa coupling is given by

$$\frac{d\mathbf{Y}}{dt} = \frac{1}{(4\pi)^2}\mathbf{Y}\beta^{(1)} \ . \tag{B9}$$

After replacing

$$\Phi_u = \sin\beta\tilde{\Phi}_h - \cos\beta\Phi_H \ , \tag{B10a}$$

$$\Phi_d = \cos\beta\Phi_h - \sin\beta\tilde{\Phi}_H \ , \tag{B10b}$$

in the up sector Yukawa interaction Lagragian, Eq. (B9) yields

$$\frac{d(\mathrm{s}\mathbf{Y}_u)}{dt} = \frac{1}{(4\pi)^2}(\mathrm{s}\mathbf{Y}_u)\beta_{(\mathrm{s}\mathbf{Y}_u)} \ , \tag{B11}$$

where we will use the notation $\mathrm{s} \equiv \sin\beta$ and $\mathrm{c} \equiv \cos\beta$. From Eq. (B11), we get

$$\frac{d\mathbf{Y}_u}{dt} = \frac{1}{(4\pi)^2}(\mathbf{Y}_u\beta_{(\mathrm{s}\mathbf{Y}_u)}) - \frac{d\ln\mathrm{s}}{dt}\mathbf{Y}_u \ . \tag{B12}$$

Using $\sin\beta = v_u/v$ and Eq. (A6) to one loop, we can write

$$\frac{d\ln\mathrm{s}}{dt} = \mathrm{c}^2\left(\frac{\dot{v}_u}{v_u} - \frac{\dot{v}_d}{v_d}\right)$$

$$= \frac{1}{(4\pi)^2}\mathrm{c}^2(\gamma_{\Phi_u} - \gamma_{\Phi_d}) \ . \tag{B13}$$

Putting Eqs. (B11) and (B13) together yields

$$\frac{d\mathbf{Y}_u}{dt} = \frac{1}{(4\pi)^2}[\mathbf{Y}_u\beta_{(s\mathbf{Y}_u)} - c^2(\gamma_{\Phi_u} - \gamma_{\Phi_d})\mathbf{Y}_u] \ . \tag{B14}$$

Similarly for the down and lepton sectors, we get

$$\frac{d\mathbf{Y}_d}{dt} = \frac{1}{(4\pi)^2}[\mathbf{Y}_d\beta_{(c\mathbf{Y}_d)} + s^2(\gamma_{\Phi_u} - \gamma_{\Phi_d})\mathbf{Y}_u] \ , \tag{B15a}$$

$$\frac{d\mathbf{Y}_e}{dt} = \frac{1}{(4\pi)^2}[\mathbf{Y}_e\beta_{(c\mathbf{Y}_e)} + s^2(\gamma_{\Phi_u} - \gamma_{\Phi_d})\mathbf{Y}_u] \ . \tag{B15b}$$

Now we list the forms of the β-functions appearing in Eqs. (B14), (B15a), and (B15b)

$$\begin{aligned}
\left[\mathbf{Y}_u\beta_{(s\mathbf{Y}_u)}\right]_{ij} = \Big[&\frac{3}{2}(s^2\theta_h + c^2\theta_H)\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u + \frac{1}{2}(s^2\theta_{\tilde{h}} + c^2\theta_{\tilde{H}})(2\{\sum_{k=1}^{N_{\tilde{Q}}}\mathbf{Y}_u\mathbf{Y}_u^\dagger\}\mathbf{Y}_u \\
&+ \mathbf{Y}_u\{\sum_{k=1}^{N_{\tilde{u}}}\mathbf{Y}_u^\dagger\mathbf{Y}_u\}) + \frac{1}{2}(c^2\theta_h + s^2\theta_H - 4c^2(\theta_h - \theta_H))\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d \\
&+ \frac{1}{2}(c^2\theta_{\tilde{h}} + s^2\theta_{\tilde{H}})\mathbf{Y}_u\{\sum_{k=1}^{N_{\tilde{d}}}\mathbf{Y}_d^\dagger\mathbf{Y}_d\} + \mathbf{Y}_u[(s^2\theta_h + c^2\theta_H)\ \mathrm{Tr}\{3\mathbf{Y}_u^\dagger\mathbf{Y}_u\} \\
&+ c^2(\theta_h - \theta_H)\ \mathrm{Tr}\{3\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_e^\dagger\mathbf{Y}_e\}] - \{ \frac{3}{5}g_1^2[\ \frac{17}{12} + \frac{3}{4}\theta_{\tilde{h}} \\
&- (\frac{1}{36}\theta_{\tilde{Q}_j} + \frac{4}{9}\theta_{\tilde{u}_i} + \frac{1}{4}\theta_{\tilde{h}})\theta_{\tilde{B}}] + g_2^2[\ \frac{9}{4} + \frac{9}{4}\theta_{\tilde{h}} - \frac{3}{4}(\theta_{\tilde{Q}_j} + \theta_{\tilde{h}})\theta_{\tilde{W}}] \\
&+ g_3^2[\ 8 - \frac{4}{3}(\theta_{\tilde{Q}_j} + \theta_{\tilde{u}_i})\theta_{\tilde{g}}] \} \Big]_{ij} \ ,
\end{aligned} \tag{B16a}$$

$$\begin{aligned}
\left[\mathbf{Y}_d\beta_{(c\mathbf{Y}_d)}\right]_{ij} = \Big[&\frac{3}{2}(c^2\theta_h + s^2\theta_H)\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d + \frac{1}{2}(c^2\theta_{\tilde{h}} + s^2\theta_{\tilde{H}})(2\{\sum_{k=1}^{N_{\tilde{Q}}}\mathbf{Y}_d\mathbf{Y}_d^\dagger\}\mathbf{Y}_d \\
&+ \mathbf{Y}_d\{\sum_{k=1}^{N_{\tilde{d}}}\mathbf{Y}_d^\dagger\mathbf{Y}_d\}) + \frac{1}{2}(s^2\theta_h + c^2\theta_H - 4s^2(\theta_h - \theta_H))\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u \\
&+ \frac{1}{2}(s^2\theta_{\tilde{h}} + c^2\theta_{\tilde{H}})\mathbf{Y}_d\{\sum_{k=1}^{N_{\tilde{u}}}\mathbf{Y}_u^\dagger\mathbf{Y}_u\} + \mathbf{Y}_d[(s^2(\theta_h - \theta_H)\ \mathrm{Tr}\{3\mathbf{Y}_u^\dagger\mathbf{Y}_u\} \\
&+ (c^2\theta_h + s^2\theta_H)\ \mathrm{Tr}\{3\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_e^\dagger\mathbf{Y}_e\}] - \{ \frac{3}{5}g_1^2[\ \frac{5}{12} + \frac{3}{4}\theta_{\tilde{h}} \\
&- (\frac{1}{36}\theta_{\tilde{Q}_j} + \frac{1}{9}\theta_{\tilde{d}_i} + \frac{1}{4}\theta_{\tilde{h}})\theta_{\tilde{B}}] + g_2^2[\ \frac{9}{4} + \frac{9}{4}\theta_{\tilde{h}} - \frac{3}{4}(\theta_{\tilde{Q}_j} + \theta_{\tilde{h}})\theta_{\tilde{W}}] \\
&+ g_3^2[\ 8 - \frac{4}{3}(\theta_{\tilde{Q}_j} + \theta_{\tilde{d}_i})\theta_{\tilde{g}}] \} \Big]_{ij} \ ,
\end{aligned} \tag{B16b}$$

$$\begin{aligned}
\left[\mathbf{Y}_e\beta_{(c\mathbf{Y}_e)}\right]_{ij} = \Big[&\frac{3}{2}(c^2\theta_h + s^2\theta_H)\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e + \frac{1}{2}(c^2\theta_{\tilde{h}} + s^2\theta_{\tilde{H}})(2\{\sum_{k=1}^{N_{\tilde{L}}}\mathbf{Y}_e\mathbf{Y}_e^\dagger\}\mathbf{Y}_e \\
&+ \mathbf{Y}_e\{\sum_{k=1}^{N_{\tilde{e}}}\mathbf{Y}_e^\dagger\mathbf{Y}_e\}) + \mathbf{Y}_e[(s^2(\theta_h - \theta_H)\ \mathrm{Tr}\{3\mathbf{Y}_u^\dagger\mathbf{Y}_u\}
\end{aligned}$$

$$+ (c^2\theta_h + s^2\theta_H) \ \text{Tr}\{3\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_e^\dagger\mathbf{Y}_e\} \ ] - \{ \ \frac{3}{5}g_1^2[\ \frac{15}{4} + \frac{3}{4}\theta_{\tilde{h}}$$

$$- (\frac{1}{4}\theta_{\tilde{L}_j} + \theta_{\tilde{e}_i} + \frac{1}{4}\theta_{\tilde{h}})\theta_{\tilde{B}} \ ] + g_2^2[\ \frac{9}{4} + \frac{9}{4}\theta_{\tilde{h}} - \frac{3}{4}(\theta_{\tilde{L}_j} + \theta_{\tilde{h}})\theta_{\tilde{W}} \ ] \ \} \ \Big]_{ij} \ , \quad \text{(B16c)}$$

where the various θs equal zero below the mass threshold of the respective particle and equal one above it. Note that Eqs. (B16a), (B16b), and (B16c) reduce to Eqs. (A9), (A10), and (A11), respectively, in the supersymmetric limit in which all θs are equal to one. When $\theta_h = 1$ and all other θs in the above equations are equal to zero, we recover the standard model result after identifying the standard model Yukawas as $\mathbf{Y}_u^h = s\mathbf{Y}_u$, $\mathbf{Y}_d^h = c\mathbf{Y}_d$, and $\mathbf{Y}_e^h = c\mathbf{Y}_e$.

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