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Renormalization of Supersymmetry Breaking Parameters Revisited

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The full renormalization group equations in the minimal $SU(3)\times SU(2)\times U(1)$ gauge model with softly broken supersymmetry, which are originally given in the paper [Prog. Theor. Phys. **68** (1982), 927] are re-examined and corrected.

Supersymmetry has recently attracted much attention as it provides a viable principle to solve the hierarchy problem in grand unified theories.[1),2)] It is well known that a scheme with spontaneously broken supersymmetry at low energies ($\sim O(100\text{ GeV})$) is plagued with difficulties owing to the famous mass sum rule.[3)] Thus the supersymmetry breaking terms in the effective low energy theory should be explicit and soft,[2)] though they may be messages from a hidden sector in which supersymmetry is spontaneously broken at a large mass scale ($\sim O(10^{10}\text{ GeV})$).[4)]

In a previous paper we have studied the renormalization of these soft breaking parameters in the supersymmetric $SU(3)\times SU(2)\times U(1)$ gauge model with the minimal set of chiral matter-Higgs multiplets.[5)] We obtained an important result that the weak gauge symmetry breaking can be induced by the renormalization effects on these soft breaking parameters, provided that the top quark is sufficiently heavy.[5)]

On the other hand, it has recently been recognized that the explicit and soft supersymmetry breaking terms in the effective lagrangian naturally appear in the spontaneously broken $N=1$ supergravity theory.[6),7)] Furthermore it has also been pointed out that the light singlet, which is necessary in order to break the weak gauge symmetry at the tree level, is dangerous because it destabilizes the light-heavy hierarchy.[8)] Thus there is growing interest in the set of renormalization group equations for the breaking parameters in the minimal model.[9),10)] Unfortunately, the original results given in the previous paper contain several errors (i.e., equations for $m_6, \cdots, m_{10}$).[5)] There are already independent calculations for the breaking parameters which originate from the spontaneous breakdown of the $N=1$ supergravity.[10)] In this short note, we shall give corrected set of renormalization group equations for parameters defined in the previous paper for the sake of completeness, since our breaking parameters are somewhat more general than those obtained from the $N=1$ supergravity.

The minimal $SU(3)\times SU(2)\times U(1)$ gauge model consists of vector multiplets V_1, V_2 and V_3, corresponding respectively to $U(1)$, $SU(2)$ and $SU(3)$, three generations of quark-lepton chiral multiplets and two Higgs chiral multiplets. Their representation contents and component fields are shown in Tables I, II and III. We neglect Yukawa couplings except those for the third generation:

$$\mathcal{L}_{\text{Yukawa}}=-\int d^2\theta\{fl_{3i}\varepsilon_{ij}H_{1j}\bar{e}_3+hq^{\rho}_{3i}\varepsilon_{ij}H_{1j}\bar{n}_{3\rho}+\tilde{h}q^{\rho}_{3i}\varepsilon_{ij}H_{2j}\bar{p}_{3\rho}\}+\text{h.c.},$$

where i and $j(=1, 2)$ are the $SU(2)$ indices and $\rho(=1, 2, 3)$ the $SU(3)$ index. The Grassman integration measure and the component expansion of a left-chiral superfield ϕ are defined as $\int d^2\theta\theta^2=1$ and $\phi=\exp(-i\bar{\theta}^{\dot{\alpha}}\partial_{\dot{\alpha}\beta}\theta^{\beta})(A+\sqrt{2}\theta\psi+\theta^2F)$. The supersymmetry breaking masses of matter scalars and of gauge fermions are shown in Table IV. For the Higgs multiplets, we consider the following gauge invariant mass term:

$$\mathcal{L}_{\text{Higgs mass}}=-m_1{}^2A_1{}^{\dagger}A_1-m_2{}^2A_2{}^{\dagger}A_2$$
$$+m_3{}^2\varepsilon_{ij}(A_{1i}A_{2j}+A^*_{1i}A^*_{2j})-m_4\varepsilon_{ij}(\psi_{1i}\psi_{2j}+\bar{\psi}_{1i}\bar{\psi}_{2j}).$$

The cubic scalar couplings, which must be introduced to retain the renormalizability, are defined as

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Table I. Vector multiplets in the minimal model.

V_1	V_1^μ, λ_1, D_1	(1, 1, 0)
V_2	V_2^μ, λ_2, D_2	(1, 3, 0)
V_3	V_3^μ, λ_3, D_3	(8, 1, 0)

Table II. Left-handed Higgs multiplets in the minimal model.

H_1	A_1, ϕ_1, F_1	(1, 2, −1/2)
H_2	A_2, ϕ_2, F_2	(1, 2, 1/2)

Table III. Left-handed matter multiplets in the minimal model. Index r distinguishes generation and runs over 1, 2 and 3.

l_r	$A(l_r)$, $\phi(l_r)$, $F(l_r)$	(1,2, −1/2)
$\bar{e}_r$	$A(e_r)$, $\phi(e_r)$, $F(e_r)$	(1,1, 1)
q_r	$A(q_r)$, $\phi(q_r)$, $F(q_r)$	(3,2, 1/6)
$\bar{p}_r$	$A(p_r)$, $\phi(p_r)$, $F(p_r)$	(3*,1, −2/3)
$\bar{n}_r$	$A(n_r)$, $\phi(n_r)$, $F(n_r)$	(3*,1, 1/3)

Table IV. Supersymmetry-breaking mass parameters of matter scalars and of gauge fermions.

λ_1	M_1	$A(e_r)$	$m(e_r)$
λ_2	M_2	$A(q_r)$	$m(q_r)$
λ_3	M_3	$A(p_r)$	$m(p_r)$
$A(l_r)$	$m(l_r)$	$A(n_r)$	$m(n_r)$

$$\begin{aligned}\mathcal{L}'=&-fm_5A_{2i}^*A(l_3)_iA(e_3)+fm_6\varepsilon_{ij}A_{1i}A(l_3)_jA(e_3)\\&-hm_7A_{2i}^*A(q_3)_i{}^\rho A(n_3)_\rho+hm_8\varepsilon_{ij}A_{1i}A(q_3)_j{}^\rho A(n_3)_\rho\\&+\tilde{h}m_9A_{1i}^*A(q_3)_i{}^\rho A(p_3)_\rho+\tilde{h}m_{10}\varepsilon_{ij}A_{2i}A(q_3)_j{}^\rho A(p_3)_\rho+\text{h.c.}\end{aligned}$$

In the case of the supergravity-induced breaking, m_4, m_5, m_7 and m_9 remain equal and emerge from a supersymmetric mass term $m_4\int d^2\theta\varepsilon_{ij}H_{1i}H_{2j}+\text{h.c.}$ Thus they are not the supersymmetry breaking terms in this case. We denote gauge coupling constants for $SU(3)$, SU(2) and $U(1)$ as g_c, g and g', respectively. The renormalization group equations for all free parameters in our system in the one-loop approximation are as follows:

$$\mu\frac{dg_c^2}{d\mu}=-\frac{1}{(4\pi)^2}6g_c^4, \tag{1}$$

$$\mu\frac{dg^2}{d\mu}=\frac{1}{(4\pi)^2}2g^4, \tag{2}$$

$$\mu\frac{dg'^2}{d\mu}=\frac{1}{(4\pi)^2}22g'^4, \tag{3}$$

$$\mu\frac{df}{d\mu}=\frac{f}{(4\pi)^2}(-3g^2-3g'^2+4ff^*+3hh^*), \tag{4}$$

$$\mu\frac{dh}{d\mu}=\frac{h}{(4\pi)^2}\left(-\frac{16}{3}g_c^2-3g^2-\frac{7}{9}g'^2+ff^*+6hh^*+\tilde{h}\tilde{h}^*\right), \tag{5}$$

$$\mu\frac{d\tilde{h}}{d\mu}=\frac{\tilde{h}}{(4\pi)^2}\left(-\frac{16}{3}g_c^2-3g^2-\frac{13}{9}g'^2+hh^*+6\tilde{h}\tilde{h}^*\right), \tag{6}$$

$$\mu\frac{dM_1}{d\mu}=\frac{1}{(4\pi)^2}22g'^2M_1, \tag{7}$$

$$\mu\frac{dM_2}{d\mu}=\frac{1}{(4\pi)^2}2g^2M_2, \tag{8}$$

$$\mu\frac{dM_3}{d\mu}=-\frac{1}{(4\pi)^2}6g_c^2M_3, \tag{9}$$

$$\begin{aligned}\mu\frac{dm_1^2}{d\mu}=\frac{1}{(4\pi)^2}\{&-(6g^2+2g'^2)m_4^2-6g^2M_2^2-2g'^2M_1^2+2ff^*[m^2(l_3)+m^2(e_3)+m_1^2+m_6^2]\\&+6hh^*[m^2(q_3)+m^2(n_3)+m_1^2+m_8^2]+6\tilde{h}\tilde{h}^*m_9^2-g'^2S\},\end{aligned} \tag{10}$$

$$\begin{aligned}\mu\frac{dm_2^2}{d\mu}=\frac{1}{(4\pi)^2}\{&-(6g^2+2g'^2)m_4^2-6g^2M_2^2-2g'^2M_1^2+2ff^*m_5^2+6hh^*m_7^2\\&+6\tilde{h}\tilde{h}^*[m^2(q_3)+m^2(p_3)+m_2^2+m_{10}^2]+g'^2S\},\end{aligned} \tag{11}$$

$$\mu\frac{dm_3{}^2}{d\mu}=\frac{1}{(4\pi)^2}\{-(3g^2+g'^2-ff^*-3hh^*-3\tilde{h}\tilde{h}^*)m_3{}^2$$
$$+2ff^*m_5m_6+6hh^*m_7m_8+6\tilde{h}\tilde{h}^*m_9m_{10}-6g^2m_4M_2-2g'^2m_4M_1\}, \tag{12}$$

$$\mu\frac{dm_4}{d\mu}=\frac{1}{(4\pi)^2}(-3g^2-g'^2+ff^*+3hh^*+3\tilde{h}\tilde{h}^*)m_4, \tag{13}$$

$$\mu\frac{dm_5}{d\mu}=\frac{1}{(4\pi)^2}\{-2(3g^2+g'^2)m_4+(3g^2+g'^2+ff^*-3hh^*+3\tilde{h}\tilde{h}^*)m_5+6hh^*m_7\}, \tag{14}$$

$$\mu\frac{dm_6}{d\mu}=\frac{1}{(4\pi)^2}(-6g^2M_2-6g'^2M_1+8ff^*m_6+6hh^*m_8), \tag{15}$$

$$\mu\frac{dm_7}{d\mu}=\frac{1}{(4\pi)^2}\{-2(3g^2+g'^2-2\tilde{h}\tilde{h}^*)m_4+2ff^*m_5+(3g^2+g'^2-ff^*+3hh^*+\tilde{h}\tilde{h}^*)m_7-2\tilde{h}\tilde{h}^*m_9\}, \tag{16}$$

$$\mu\frac{dm_8}{d\mu}=\frac{1}{(4\pi)^2}\left(-\frac{32}{3}g_c{}^2M_3-6g^2M_2-\frac{14}{9}g'^2M_1+2ff^*m_6+12hh^*m_8+2\tilde{h}\tilde{h}^*m_{10}\right), \tag{17}$$

$$\mu\frac{dm_9}{d\mu}=\frac{1}{(4\pi)^2}\{-2(3g^2+g'^2-2hh^*)m_4-2hh^*m_7+(3g^2+g'^2+ff^*+hh^*+3\tilde{h}\tilde{h}^*)m_9\}, \tag{18}$$

$$\mu\frac{dm_{10}}{d\mu}=\frac{1}{(4\pi)^2}\left(-\frac{32}{3}g_c{}^2M_3-6g^2M_2-\frac{26}{9}g'^2M_1+2hh^*m_8+12\tilde{h}\tilde{h}^*m_{10}\right), \tag{19}$$

$$\mu\frac{dm^2(l_r)}{d\mu}=\frac{1}{(4\pi)^2}(-6g^2M_2{}^2-2g'^2M_1{}^2-g'^2S),\quad (r=1,\ 2) \tag{20}$$

$$\mu\frac{dm^2(l_3)}{d\mu}=\frac{1}{(4\pi)^2}\{-6g^2M_2{}^2-2g'^2M_1{}^2+2ff^*[m^2(l_3)+m^2(e_3)+m_1{}^2+m_5{}^2+m_6{}^2-2m_4{}^2]-g'^2S\}, \tag{21}$$

$$\mu\frac{dm^2(e_r)}{d\mu}=\frac{1}{(4\pi)^2}(-8g'^2M_1{}^2+2g'^2S),\qquad (r=1,\ 2) \tag{22}$$

$$\mu\frac{dm^2(e_3)}{d\mu}=\frac{1}{(4\pi)^2}\{-8g'^2M_1{}^2+4ff^*[m^2(l_3)+m^2(e_3)+m_1{}^2+m_5{}^2+m_6{}^2-2m_4{}^2]+2g'^2S\}, \tag{23}$$

$$\mu\frac{dm^2(q_r)}{d\mu}=\frac{1}{(4\pi)^2}\left(-\frac{32}{3}g_c{}^2M_3{}^2-6g^2M_2{}^2-\frac{2}{9}g'^2M_1{}^2+\frac{1}{3}g'^2S\right),\quad (r=1,\ 2) \tag{24}$$

$$\mu\frac{dm^2(q_3)}{d\mu}=\frac{1}{(4\pi)^2}\left\{-\frac{32}{3}g_c{}^2M_3{}^2-6g^2M_2{}^2-\frac{2}{9}g'^2M_1{}^2\right.$$
$$+2hh^*[m^2(q_3)+m^2(n_3)+m_1{}^2+m_7{}^2+m_8{}^2-2m_4{}^2]$$
$$\left.+2\tilde{h}\tilde{h}^*[m^2(q_3)+m^2(p_3)+m_2{}^2+m_9{}^2+m_{10}^2-2m_4{}^2]+\frac{1}{3}g'^2S\right\}, \tag{25}$$

$$\mu\frac{dm^2(p_r)}{d\mu}=\frac{1}{(4\pi)^2}\left(-\frac{32}{3}g_c{}^2M_3{}^2-\frac{32}{9}g'^2M_1{}^2-\frac{4}{3}g'^2S\right),\quad (r=1,\ 2) \tag{26}$$

$$\mu\frac{dm^2(p_3)}{d\mu}=\frac{1}{(4\pi)^2}\left\{-\frac{32}{3}g_c{}^2M_3{}^2-\frac{32}{9}g'^2M_1{}^2\right.$$
$$\left.+4\tilde{h}\tilde{h}^*[m^2(q_3)+m^2(p_3)+m_2{}^2+m_9{}^2+m_{10}^2-2m_4{}^2]-\frac{4}{3}g'^2S\right\}, \tag{27}$$

$$\mu\frac{dm^2(n_r)}{d\mu}=\frac{1}{(4\pi)^2}\left(-\frac{32}{3}g_c{}^2M_3{}^2-\frac{8}{9}g'^2M_1{}^2+\frac{2}{3}g'^2S\right),\quad (r=1,\ 2) \tag{28}$$

$$\mu\frac{dm^2(n_3)}{d\mu}=\frac{1}{(4\pi)^2}\left\{-\frac{32}{3}g_c{}^2M_3{}^2-\frac{8}{9}g'^2M_1{}^2\right.$$
$$\left.+4hh^*[m^2(q_3)+m^2(n_3)+m_1{}^2+m_7{}^2+m_8{}^2-2m_4{}^2]+\frac{2}{3}g'^2S\right\}, \tag{29}$$

where

$$S=m_2^2-m_1^2+\sum_{r=1}^{3}\{m^2(e_r)+m^2(q_r)+m^2(n_r)-m^2(l_r)-2m^2(p_r)\}. \tag{30}$$

Our numerical results given in Ref. 5) are slightly modified for large Yukawa couplings. However the essential feature of the paper is unchanged. In particular, the lower bound on the top quark mass in our scenario ($\sim$60 GeV) is not sensibly affected.

In Ref. 5), we assumed that the supersymmetry was broken only through gauge fermion mass terms at the unification mass scale.[5] On the other hand, in the case of grand unified theories coupled with the $N=1$ supergravity, almost all of breaking parameters systematically take nonzero boundary values of the order of the gravitino mass.[7] Especially the boundary value of m_3^2 at the unification mass scale, which has been taken to be zero in the most of previous papers,[5),9),10)] turns out to be nonzero when a supersymmetric Higgs mass is incorporated. In this case the results of previous phonomenological analyses receive considerable modifications. There exists even a possibility that the top quark mass is as light as the present experimental lower bound.[*] Detailed discussion on the above and related problems will be given in a separate paper.

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*) This possibility is also noticed by Kounnas et al.[11]