Repeller or Attractor? Selecting the Dynamical Model for the Onset of Turbulence in Pipe Flow

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The collapse of turbulence, observable in shear flows at low Reynolds numbers, raises the question if turbulence is generically of a transient nature or becomes sustained at some critical point. Recent data have led to conflicting views with the majority of studies supporting the model of turbulence turning into an attracting state. Here we present lifetime measurements of turbulence in pipe flow spanning 8 orders of magnitude in time, drastically extending all previous investigations. We show that no critical point exists in this regime and that in contrast to the prevailing view the turbulent state remains transient. To our knowledge this is the first observation of superexponential transients in turbulence, confirming a conjecture derived from low-dimensional systems.

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Finding appropriate models and concepts describing fluid turbulence is one of the outstanding challenges in the physical sciences. Shear flows with a linearly stable laminar state, such as pipe, channel, duct, or Couette flow have proven to be particularly intricate in this regard [1]. Here the laminar and the turbulent state coexist [2,3] without a clear transition point, yet at large flow rates the laminar state becomes increasingly susceptible to perturbations. Once a disturbance is large enough the transition to turbulence occurs suddenly without any intermediate states [4–6]. Surprisingly, at relatively low Reynolds numbers ($\mathrm{Re} \lesssim 2000$) the turbulent state is not stable and after long times suddenly collapses [7–12]. This behavior is reminiscent of memoryless processes in nonlinear systems. In phase space the dynamics can be described by a complex structure giving rise to the disordered dynamics, a so-called chaotic repeller [13]. Underlying such a structure are unstable states and for pipe flow unstable solutions to the governing equations have been identified in the form of traveling waves [14,15]. Surprisingly clear transients of such traveling waves were observed in experiments [16,17] confirming their relevance to the turbulent dynamics. More recently traveling wave transients were also reported in numerical studies [18,19].

A way to probe the validity of this model is to measure the lifetime of turbulence in the transient regime. Previous experimental and numerical lifetime measurements have shown approximately exponential probability distributions [8,10,11,20,21] which suggests that the probability for a turbulent structure to decay is independent of its age and hence that this process is memoryless as would be expected for the escape from a chaotic saddle. Here the probability for a flow to still be turbulent after a time t at a fixed Reynolds number (Re) is then given by

$$P(t - t_0, \mathrm{Re}) = \exp[-(t - t_0)/\tau(\mathrm{Re})], \qquad (1)$$

where τ is the characteristic lifetime (τ^{-1} can be also interpreted as the escape rate) and t_0 is the initial time period required for turbulence to form after the disturbance has been applied to the laminar flow at $t = 0$. The fate of the chaotic repeller is then determined by the functional form of the characteristic lifetime $\tau(\mathrm{Re})$, and different suggestions have been made in the past. The majority of studies reported that τ^{-1} decays linearly and reaches zero at a critical Reynolds number. Here the turbulent state undergoes a boundary crisis [1] leading from transient to sustained turbulence. However there is no quantitative agreement for the value of such a critical point and cited values differ by more than 25%. This view has been challenged in an experimental study [12] carried out in an extremely long pipe where τ^{-1} has been observed to decay exponentially. Crucially it only approaches zero and hence (unless a global bifurcation occurs at larger Re [1]) an infinite lifetime is only reached in the asymptotic limit $\mathrm{Re} \to \infty$. Subsequently a number of studies have questioned this finding and again entertained the occurrence of a boundary crisis [11,22,23]. A clear constraint of all previous investigations is the limited range in lifetimes measured. Typically scaling laws were postulated from data covering 2 orders of magnitude. Numerical simulations are particularly problematic because in order to capture the quantitatively correct behavior computations have to be carried out in large domains, which severely restricts the number of realizations N that are manageable ($N < 50$) [11]. Consequently the statistics are often insufficiently resolved resulting in ambiguous probability distributions [24]. A further difficulty in interpreting the existing data arises from the initial formation time t_0. Most numerical measurements have been carried out at relatively low Reynolds numbers where t_0 can be larger than the actual observation time. Consequently the evaluations of lifetimes in this regime have significant uncertainties.

The experiments presented here were carried out in four pipe setups located in three different laboratories. On all four occasions the pipes were made of 1 m long precision bore glass tubes and the working fluid was water. The setups mainly differ in the diameters (D) and their total length (L). For two pipes 4 mm ($\pm$ 0.01) bore tubes were used and their lengths were $L/D = 2000$ and 3600; the other two had a diameter of $D = 10$ mm $\pm$ 0.01 and a length of $L/D = 690$ and 600. As in our previous study [12] the flow was driven by a constant pressure head. To avoid fluctuations during transition caused by the differences in drag between the turbulent and the laminar motion, a large constant resistance to the flow was added to the supply line between the constant head reservoir and the flow conditioning section at the pipe entrance. This ensured that the flow rate remained constant to between 0.1% and 0.01% depending on the setup, even when transition occurred. The main improvement over the earlier study by Hof *et al.* [12] was the implementation of an accurate temperature control allowing measurements to be carried out at constant temperatures ($\pm$0.05 K) for several days and hence avoiding Reynolds number changes caused by the temperature dependence of the viscosity.

In order to achieve laminar flows at Reynolds numbers in excess of 2000 the pipe sections need to be very accurately aligned and special care has to be taken at the pipe inlet to avoid turbulence being induced (see Fig. 1). In three of the pipes laminar flow could be achieved up to Re $\geq$ 3000. Detailed tests have shown that at the natural transition point turbulence is always triggered at the pipe inlet and not inside the pipe itself. For these three pipes the inlet consisted of a straight convergence reducing the diameter from 12.5 to 4 mm. In the $L/D = 690$ pipe a more sophisticated inlet was used employing several meshes and a smooth convergence. This resulted in a much higher natural transition point of Re $= 10^4$.

The experimental procedure then was as follows: First a perturbation was applied at a fixed position upstream. The perturbation amplitude was chosen large enough to trigger the transition to turbulence and the duration of the perturbations was set to between 10 and $20D/U$. The perturbed segment then develops into a so-called turbulent puff, which in this Reynolds number regime has a fixed length and travels downstream at approximately the mean velocity U [25]. To determine if this turbulent puff had survived its journey to the end of the pipe or if the flow had relaminarized, the outflow angle at the pipe exit was monitored. Since for a given Reynolds number the turbulent flow has a lower center line velocity than the laminar one, it exits the pipe at a steeper angle (with respect to the pipe axis) [12,26]. In the 10 mm pipes velocities were measured with laser Doppler anenometry (LDA) in addition to monitoring the outflow angle. These velocity measurements made it possible to determine the formation period t_0 more accurately. In the case of the single jet perturbation the value of t_0 was $t_0 = 70 \pm 5$. In order to establish if the type of perturbation used had an influence on the lifetime of the resulting turbulent flow, measurements were carried out at various amplitudes and different perturbation types. For the majority of measurements shown here a single jet was injected for a duration of $10D/U$ through a small (0.5 mm) hole in the wall. In additional studies [27] different types of perturbations were tested including a simultaneous injection and withdrawal of fluid through two small holes and triggering of turbulence at larger flow rates followed by a sudden reduction in the Reynolds number (this perturbation is identical to the one used by [8,11,20]). Outside the formation period t_0 no differences, neither in the observed turbulent structures nor in their statistics were observed. Indeed, this behavior is typical for chaotic systems where the exponential divergence of neighboring trajectories quickly erases the memory of the initial conditions.

The improved temperature control allowed us to base each measurement point on observations of typically $N = 500$ and occasionally even up to $N = 100\,000$ puffs reducing statistical errors by an order of magnitude compared to all previous studies and increasing the range of measurable lifetimes by more than 5 orders of magnitude. The probability distributions obtained in the $D = 4$ mm pipes are shown in Fig. 2 for five different distances between the perturbation and the measurement point ($x = 140$, 270, 930, 1900, and 3500) corresponding to fixed dimensionless times $t = (x/U)/(D/U)$. Our data confirm that probability distributions are S shaped and not simple exponentials as would be expected if $\tau(\text{Re})$ was a linear function as pro-

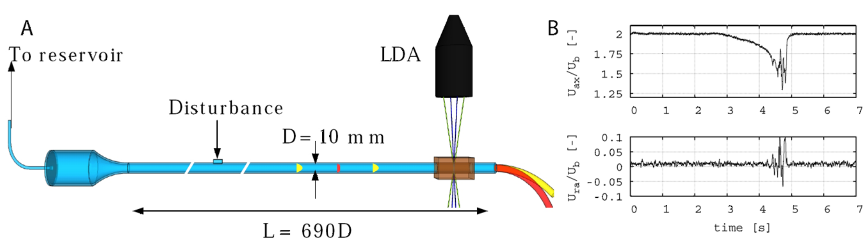

FIG. 1 (color online). (a) Sketch of the general pipe setup. Four different pipes were used, two of them with a 10 mm diameter ($L/D = 690$ and 600) and two with a 4 mm diameter ($L/D = 2000$ and 3600). Pipes were gravity driven and turbulence could be induced by injection and withdrawal of fluid through small holes in the pipe wall at various downstream positions. Turbulence was detected by monitoring the outflow angle and by LDA velocity measurements. (b) LDA velocity trace obtained at the center line of the $D = 10$ mm ($L/D = 690$) pipe during the passage of a turbulent event. The trace shows the well-known [25] signature of a turbulent puff for the axial velocity (top) and the radial velocity (bottom).

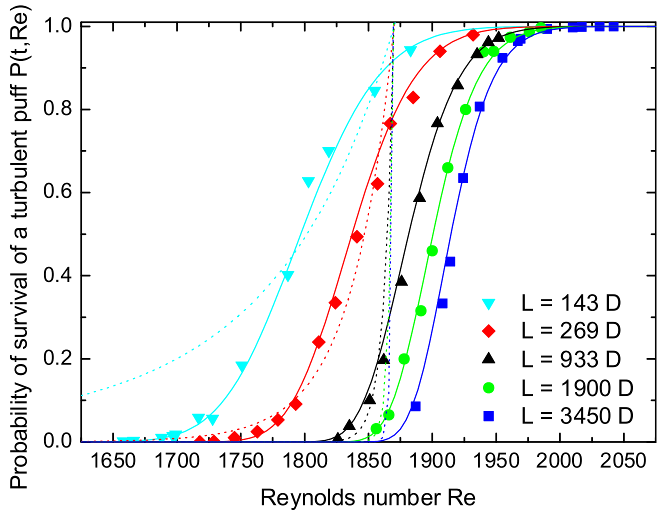

FIG. 2 (color online). Probabilities for the flow to still be turbulent after traveling a fixed distance x. Viewed from left to right the five data sets shown correspond to the following distances: $x = 140$ (down triangles), 270 (diamonds), 930 (up triangles), 1900 (circles), 3500 (squares). The fitted curves follow directly from the superexponetial scaling shown in Fig. 3. The dotted curves show the scaling that would be expected for the critical behavior suggested by Willis and Kerswell [11].

posed in Refs. [8,10,11,20]. In particular the dotted lines show the exponential distributions that follow from the proposed boundary crisis in [11]. Both scalings (exponential and S-shaped curves) agree well for $\mathrm{Re} < 1870$. Here differences only occur for very short pipes, where errors due to uncertainties in t_0 are very large and make a distinction of the decay rates very difficult. For $\mathrm{Re} > 1870$ our data clearly disagree with the proposed exponentially divergent curves and instead fall on the S curves resulting from the fit shown in Fig. 3.

The observed distributions however also differ from the S shape suggested by Hof *et al.* [12]: they are not self-similar but instead their maximum slope (at $P(t) = 0.5$) increases with L/D. For each of the measured probabilities $P(t)$ inverse characteristic lifetimes $\tau^{-1}(\mathrm{Re})$ can be determined using Eq. (1), and the values are plotted in Fig. 3.

In addition to the data obtained in the 4mm pipes, the data of the 10 mm pipes is also included in the graph. All the data collapses onto a single curve which shows that Eq. (1) is the appropriate description for the observed decay of turbulence and hence confirms the model of a chaotic repeller. By resolving values of $P(t)$ up to 0.9999 we were able to determine escape rates down to $\tau^{-1} = 10^{-8}$ which is 4 orders of magnitude smaller than had been measured before. By resolving very small probabilities in a $L/D = 140$ pipe it was possible to determine decay rates down to $\mathrm{Re} = 1670$ while keeping errors due to t_0 at a minimum. In principle lifetimes at even lower Re can be obtained in even shorter pipes, yet as discussed above, the uncertainty in the initial formation time t_0 is considerable when compared to the total observation time, severely restricting measurements in this regime. In addition the numerical data by Willis and Kerswell [11] (open squares) are plotted together with the linear fit proposed in that study. Note that the data point at $\mathrm{Re} = 1580$ of [11] has been refitted as suggested in [24]. The numerical data is in excellent agreement with our measurements (taking the relatively large uncertainties due to t_0 at small Re into account). However the data of our experiments clearly does not follow the linear fit [dashed curve in Fig. 3(a)] proposed in their study. Turbulent puffs are still found to decay well beyond the critical point of $\mathrm{Re}_c = 1870$ postulated by Willis and Kerswell [11]. The exponential scaling suggested by Hof *et al.* [12], shown by the solid black line, gives a reasonable fit only over 2 orders of magnitude in τ^{-1} , but fails over the far larger range measured in the present study. Over these 2 orders of magnitude also the shape of the probability distributions of the present study are indistinguishable to the ones by [12]. Outside this overlap region the S-curves in the present study are observed to become steeper with Re. Such a Re dependence had not been seen in the earlier study [12]. Note that the solid black line in Fig. 3(a) was shifted by $\Delta\mathrm{Re} = -48$

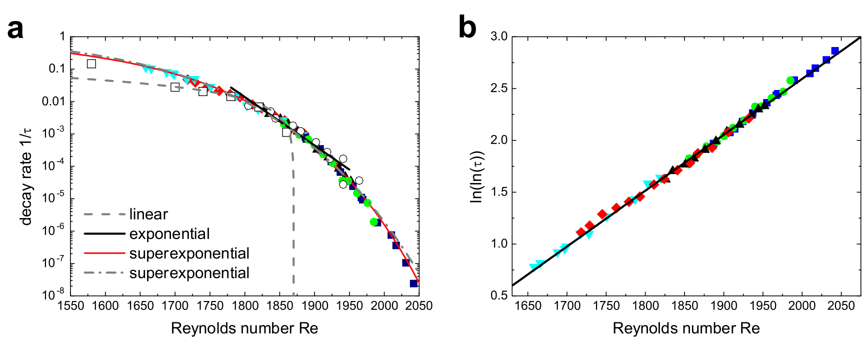

FIG. 3 (color online). (a) Decay rates plotted on a log linear scale. Circles represent data obtained in the $D = 10$ mm pipes while the full symbols were measured in the $D = 4$ mm pipe. The dashed line and the open squares are a reproduction of the data points and the linear fit given by Willis and Kerswell [11]. The black line has the same slope as the exponential scaling observed by Hof *et al.* [12]. The light solid curve assumes a superexponential dependence of the decay rate on Re. The dash-dotted line shows an alternative superexponential fit (see text for details). (b) Data on a log-log linear scale. The data could be fitted by a straight line over the entire regime. This two parameter fit was then used to plot the five curves in Fig. 2 as well as the light solid curve in Fig. 3(a).

with respect to the one shown in [1]. This shift of the data corresponds to a 2.5% difference in the absolute value of Re. In particular the uncertainty of the pipe diameter in [12] with ($\pm 1.5\%$) was comparatively large; furthermore in the present study greater care was taken to measure the absolute value of the temperature allowing to determine the viscosity values more accurately.

The robustness of the scaling behavior was tested by applying a periodic modulation to the flow rate. At a frequency of up to 2 Hz and an amplitude of $\Delta \mathrm{Re} = \pm 10$ the shape of the S curves remained unchanged within experimental errors. Equally small intentional misalignments of the pipe segments did not show any noticeable influence on the distribution shape. S-shaped probability distributions have also been observed in plane Couette [12,28] flow suggesting that this scaling behavior applies to a variety of shear flows.

In Fig. 3(b) the present data are shown on a double log linear scale. On this scale a straight line can be fitted to the data suggesting lifetimes scale superexponentially with Re: $\tau^{-1} = \exp[-\exp(c_1 \mathrm{Re} + c_2)]$, with $c_1 = 0.0057$ and $c_2 = -8.7$. As shown in Fig. 3 this two parameter fit captures the observed escape rate dependence over 8 orders of magnitude. Equally the S-shaped curves plotted in Fig. 2 directly follow from this straight line fit without any additional fitting parameters. While the data allows to rule out functional forms which are subexponential, it should be noted that adequate fits can also be obtained by other superexponential functions.

For instance, $\tau^{-1} = \exp[-(\mathrm{Re}/c)^n]$, with $c = 1549$ and $n = 9.95$ [dash-dotted line in Fig. 3(a)]. Here the magnitude of the exponent n is related to the rate at which the basin of attraction of the laminar state shrinks as Re increases [29]. Discriminating between the different superexponential scalings would require measurements over a substantially larger Reynolds number range. However, due to the rapid increase in lifetimes the parameter space observable in experiments is rapidly approaching its limit. In order to measure the escape rate at $\mathrm{Re} = 2100$ would require an estimated time of 46 yr in our setup, and at $\mathrm{Re} = 2200$ with 10^{12} yr the experimentation time would have to surpass the age of the Universe. Previously long lived transients whose lifetime scales superexponentially with system size, so-called Type-II supertransient [29], had only been observed in low-dimensional dynamical systems.

In conclusion, by increasing the range of measured lifetimes by 6 orders of magnitude and significantly reducing statistical errors the decay rate of turbulence has been measured far more accurately than previously possible. The observation of a critical point reported in many recent studies is not supported. The superexponential behavior found here identifies turbulence in pipe flow as a type-II supertransient [29,30], which had been conjectured as a potential description of turbulence two decades ago [30]. This scaling shows that at least in the intermittent regime, the correct dynamical model of turbulence in linearly stable shear flows is that of a strange repeller.

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