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Representation and Analysis of Transfer Lines with Machines That Have Different Processing Rates

by

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Abstract

A transfer line is a tandem production system, i.e., a series of machines separated by buffers. Material flows from outside the system to the first machine, then to the first buffer, then to the second machine, the second buffer, and so forth. In some earlier models, buffers are finite, machines are unreliable and the times that parts spend being processed at machines are equal at all machines. In this paper, a method is provided to extend a decomposition method to large systems in which machines are allowed to take different lengths of time performing operations on parts. Numerical and simulation results are provided.

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1 Introduction

Purpose of paper

Consider the tandem production system of Figure 1, a series of machines separated by buffers. Material flows from outside the system to the first machine, then to the first buffer, then to the second machine, the second buffer, and so forth. Finally it reaches the last machine and exits the system. It is important to know the production rate of such a system as well as the average amount of material in each of the buffers.

In Buzacott [1967a and b] and Gershwin and Schick [1983] a model is described in which buffers are finite, machines are unreliable (in that they fail and are repaired at random times), and the times that parts spend being processed at machines are equal at all machines. An approximate analysis method for long lines is provided in Gershwin [1983] which is based on a decomposition technique. In this paper, a method is provided to extend this method to systems in which machines are allowed to take different lengths of time performing operations on parts.

This method is not a new algorithm; rather it is a way of representing machines of different speeds. This new representation allows the use of the earlier decomposition algorithm for this larger class of systems.

Review of previous work

The purpose of this paper is to extend the transfer line model introduced by Buzacott [1967a and b] and analyzed by Gershwin and Schick [1983] and Gershwin [1983] to transfer lines with machines that have different speeds. Other models have been developed, such as those of Buzacott [1972], Gershwin and Berman [1981], Gershwin and Schick [1980], and Wijngaard [1971]. However, they have only been successfully applied to two-machine lines.

In this paper, a new representation of a machine is described. A single machine is represented as two of Buzacott's machines, separated by a buffer of capacity 0. One of the machines captures the unreliability behavior of the original machine; the other represents the processing time. The advantage of this representation is that it can make use of the efficient decomposition algorithm of Gershwin [1983].

The fundamental idea behind this work is the recognition that there are two time scales operating in this system: the part production process, and the failure/repair process. (The emptying and filling of buffers operates at the same time scale as the failure/repair process.) This permits convenient approximations. Because many parts are produced between failure and repair events, the details of the part production process are not important. Consequently, this approach may be applicable to a wide variety of production (service) processes.

Other approximation techniques for machines with different speeds are due to Altiok (1982), Suri and Diehl (1983), and Takahashi et al. (1980). These methods do not explicitly recognize this time scale decomposition.

Outline

Section 2 describes the two-machine representation of a single machine with arbitrary processing time. In Section 3, numerical results are presented. Exact analyses of two-machine lines with exponentially distributed processing times are compared with the approximate results provided by the present representation and the decomposition algorithm. The close agreement indicates that for the class of systems studied, certain details of the models are not important, and that the approximations used here are adequate for many purposes.

Simulation results and comparisons for longer systems are presented in Section 4. Again the close agreement is encouraging. Section 5 concludes.

This paper does not provide analytic proof that the method works. It does not give guaranteed bounds on the performance measures of interest. Instead it provides intuitive justification, suggestive evidence, and hope that such results can be found.

2 Method

The Buzacott Model

The most basic model of a transfer line is that of Buzacott (1967a, 1967b). This model captures the disruptions of otherwise orderly flow due to random machine failures and repairs, and it demonstrates the effects of finite buffers.

In Buzacott's model of a transfer line, workpieces move from station to station at fixed time intervals. The machines at the stations are assumed to have fixed, equal processing times, and it is convenient to call that processing time the time unit. Each machine can be in two states: operational and under repair (or failed). In addition, a machine can be blocked or starved, i.e. the buffer immediately downstream can be full or the buffer upstream can be empty, respectively. When a machine is blocked or starved, it cannot operate, even if it is operational. Therefore, it cannot fail.

In the present version, operational machines have a fixed probability of failing during every time unit the are operating, and a fixed probability of repair during every time unit they are in the failed state.

Let α_i indicate the repair state of machine i. If $\alpha_i = 1$, the machine is operational; if $\alpha_i = 0$, it is under repair. When machine i is under repair, it has probability r_i of becoming operational during each time unit. That is,

$$\text{prob}\left[\ \alpha_i(t+1)=1 \ \middle|\ \alpha_i(t)=0\ \right] = r_i.$$

The repair process is geometric with mean $1/r_i$.

When machine i is operational and neither starved nor blocked, it has probability p_i of failing. That is,

$$\text{prob}\left[\ \alpha_i(t+1)=0 \ \middle|\ n_{i-1}(t)>0,\ \alpha_i(t)=1,\ n_i(t)<N_i\ \right] = p_i.$$

Measured in working time (i.e., during which the machine is neither starved nor blocked), the failure process is geometric with mean $1/p_i$.

The amount of material in a buffer at any time is n, $0 \leq n \leq N$. A buffer gains or loses at most one piece during a time unit. One piece is inserted into the buffer if the upstream machine is operational and neither starved nor blocked. One piece is removed if the downstream machine is operational and neither starved nor blocked. A buffer may fill up only after an operation is complete. Work on the next piece may not begin until the buffer is no longer full.

By convention, repairs and failures are assumed to occur at the beginnings of time units, and changes in buffer levels take place at the end of the time units. (Buzacott followed the opposite convention.) When a failure takes place, the part is considered not to have been affected by the partial operation, and the next operation after the repair has the same probability of failure as any other operation. When Machines i and i+1 are neither starved nor blocked,

$$n_i(t+1) = n_i(t) + \alpha_i(t+1) - \alpha_{i+1}(t+1).$$

The state of the system is

$$s = (n_1, ..., n_{k-1}, \alpha_1, ..., \alpha_k).$$

Performance Measures

The production rate (throughput, flow rate, or line efficiency) of machine M_i, in parts per time unit, is

$$E_i = \text{prob } [\ \alpha_i = 1, n_{i-1} > 0, n_i < N_i\ ].$$

Flow is conserved, so all E_i are equal. The average level of buffer i is

$$\bar{n}_i = \sum_s n_i \text{ prob } (s)$$

Two-Machine Representation

Figure 2 displays a machine with repair rate r (repairs/ time unit), failure rate p (failures/time unit) and processing rate μ (parts/time unit). In this paper we assume that r and p are very small compared with μ. That is, during the time between failures and repairs, there is enough time for many part operations to take place. This assumption is required because we represent the operation process by a different random process. If many operations take place between repair and failure events, the details of the operation process are less important than those of the repair and failure processes.

Figure 2 also shows a two-machine line segment. In this segment, both machines correspond to Buzacott's two-parameter model, so they have unit processing rate. That is, the system operates on discrete parts, and each machine requires the same amount of time for its operation. We use this time as the time unit.

The machines have repair rates r_1 and r_2, respectively. More precisely, the probability of repair of one of these machines during a unit time interval in which it is down is r_1 or r_2. They have failure rates p_1 and p_2. The objective is to choose r_1, p_1, r_2, p_2 so that the pair of two-parameter machines emulates the single three-parameter machine.

The order is not important, but we assume that the first machine represents the processing behavior of the original machine and that the second represents its repair-failure behavior. The first machine fails and gets repaired very quickly, and the second fails and gets repaired just as frequently as the original machine.

To determine the parameters of the first machine, let T be a number on the order of 1/p or 1/r (whichever is smaller). During a period of length T, the expected number of parts the first machine produces is

$$\frac{r_1}{r_1+p_1}T$$

and we choose r_1 and p_1 so that this is equal to μT. If both r_1 and p_1 are large compared to 1/T (i.e., if repairs and failures of the first machine are frequent), then the actual number of these events will be close to the expected value. This is desirable because in the original machine, the expected number of parts produced is equal to μT. In fact the actual number of parts produced by the original machine is close to μT because many parts are produced in an interval of length T.

The parameters of the second machine are obtained by observing that it is responsible for long-term disruptions of flow. Its parameters are selected so that the mean time between long-term failures of the two-machine system and mean time to repair the long-term failures are equal to the corresponding quantities of the single original machine.

Note that since there are only three parameters in that machine, there can be no more than three independent equations for the four parameters of the pair of machines.

The relationship between the original three-parameter machine and the pair of two-parameter machines is determined by the following considerations:

1. When either of the two-parameter machines is stopped, the line consisting of the pair of them is stopped.

2. The average times between repair and failure and between failure and repair in the two systems should be the same.

3. The production rates of the two systems should be the same.

Machine 1 parameters

Assume that the time unit is such that μ is a dimensionless quantity that satisfies

$$0 < \mu < 1.$$

While the three-parameter machine is operational, it produces material at rate μ. While the second machine of the two-parameter pair is operational, the first machine--and thus the two-machine system--produces material at rate

$$\frac{r_1}{r_1+p_1}.$$

Note that this result is based on the assumption that the first machine undergoes many failures and repairs while the second machine is up. Then, since Machine 1 represents the processing time μ, we can choose

$$\frac{r_1}{r_1+p_1} = \mu. \tag{1}$$

Repair rate r_2

Machine 2 represents the repair-failure behavior of the original machine. The parameters of Machine 2 are chosen so that it fails as frequently as the original machine, and takes just as long, on the average, to repair. To determine the repair rate, we have

$$r_2 = r. \tag{2}$$

The corresponding statement about the failure rate is not valid. This is because p_2 is the rate that Machine 2 fails while it is not starved or blocked. However, it is frequently starved by the first machine, so $p_2 \neq p$. To calculate p_2, we must adjust it for the time that Machine 1 is down. Equation (4) provides this adjustment.

Flow rate equality

The rate of flow through the original machine is

$$\frac{\mu r}{r+p}.$$

The rate of flow through the pair of two-parameter machines is (Buzacott, 1967b)

$$\frac{1}{1 + \frac{p_1}{r_1} + \frac{p_2}{r_2}}.$$

Consequently,

$$\frac{\mu r}{r + p} = \frac{1}{1 + \frac{p_1}{r_1} + \frac{p_2}{r_2}}. \qquad (3)$$

Failure rate p_2

Equations (1), (2), (3) together imply that

$$p_2 = \frac{p}{\mu} \qquad (4)$$

This is satisfying because the only time that Machine 2 may fail is when Machine 1 is operational, which is a fraction μ of the time. Thus, the probability of a failure of the second machine is $p_2\mu$, which should be p, the failure rate of the original three-parameter machine.

Magnitude condition: r_1 and p_1 large

There are now three equations ((1), (2), and (4)) for four parameters. A fourth condition comes from the assumption that Machine 1 fails and is repaired much more often than Machine 2. Thus

$$r_1, p_1 >> r_2, p_2. \qquad (5)$$

Numerical experience (reported in Section 3) seems to indicate that a more precise condition is not necessary. That is, as long as the three equations and condition (5) are satisfied, the precise values of r_1 and p_1 are not important.

The justification described below implies that if r_1 and p_1 are too small, the representation loses accuracy. However, numerical experience does not bear this out. The relationship between these and other parameters and the accuracy of this technique is a topic for future investigation.

Justification

Figure 3 shows a typical buffer level trajectory for a buffer following the single three-parameter machine (solid line), and two corresponding trajectories for a buffer following the two-machine system of Figure 2. The solid line represents a scenario in which the buffer is empty at t=0, the machine is operational at t=0, and later the machine fails. The dashed lines correspond to cases in which the buffer starts empty, Machines 1 and 2 start operational, and Machine 2 later fails. While Machine 2 is working, Machine 1 fails and is repaired many times.

The two-machine trajectories have been drawn to stay close to the one-machine trajectory. This is consistent with the choice of the r_1, p_1, r_2, p_2 parameters, which satisfy (1)-(3).

The two two-machine scenarios differ in the values of r_1 and p_1. The dashed line that stays closer to the solid line, in which events occur more frequently, corresponds to larger values of r_1 and p_1.

Figure 3 demonstrates that it is possible to choose parameters of the two-machine system so that trajectories of two-machine systems are close to those of single machines.

THis figure was hand-drawn to illustrate this method; it was not generated by a simulation. To simplify the picture, Figure 3 has been drawn as though the machines are producing material continuously (such as in Wijngaard's (1971) or Gershwin and Schick's (1980) models). A more accurate picture for discrete material produced in fixed time would have replaced the sloped lines by regular staircases. Irregular staircases would have been more appropriate for the exponential processing time model of Gershwin and Berman (1981). As long as repairs and failures are much less frequent the rate at which material is produced, ie,

$$r, p << \mu,$$

this distinction is not important.

There are two time scales operating in this class of systems. The shorter time scale is that of part production, in which events occur at frequency μ. The longer is that of repair and failure of machines and emptying and filling of buffers. Events take place at much lower frequency: r, p, or μ/N.

As long as the difference between the time scales is great, the details of the production process are not important. This is because many short-time-scale events take place between long-time-scale events. The law of large numbers implies that the distribution of the number of short-time-scale events (ie, the amount of material produced) between long-time-scale events is approximately independent of the distribution of the time between short-time-scale events.

A reviewer has pointed out that this cannot be taken to extremes: if there are no failures, the distribution of production times is clearly important. A careful multiple-time-scale analysis is required to resolve this issue.

Procedure

To use this representation on a transfer line consisting of three-parameter machines and finite buffers, use the following procedure:

1. Change the time unit so that the largest μ is exactly 1. That is, divide all r's, p's, and μ's by the largest μ.

2. Replace each machine with μ less than 1 by two machines whose parameters satisfy (1), (2), (4).

3. Analyze the resulting system by an algorithm such as that of Gershwin (1983).

4. Convert production rate back to the original time scale.

3 Numerical Results

This section compares numerical results generated from the exponential two-machine model of Gershwin and Berman (1981), the continuous two-machine model of Gershwin and Schick (1980), and the deterministic k-machine technique of Gershwin (1983). The exponential and continuous results were produced by solving the appropriate equations analytically; the deterministic model results came from an approximation technique. The parameters of the exponential and continuous models were the same; the parameters of the deterministic model were derived from them by the procedure of the previous section.

By deterministic model, we mean the discrete-time, discrete-material model described in Section 2.

Exponential Model

This is a continuous-time, discrete-material model in which all three random processes of a machine are exponentially distributed. The rates of failure, repair, and production are p_i, r_i, and μ_i, respectively. An analytic solution for two-machine lines is presented in Gershwin and Berman (1981), and a decomposition approximation for longer lines appears in Choong and Gershwin (1985).

Continuous Model

This is a continuous-time, continuous-material model in which the production process is deterministic and the failure and repair processes are exponentially distributed. During an interval of length δt in which machine i is operational and neither starved nor blocked, the amount of material that is produced is $\mu_i \delta t$. The rates of failure and repair are p_i and r_i, respectively. An analytic solution for two-machine lines is presented in Gershwin and Schick (1980).

Time Scales and Differences Among Models

While the first machine of these two-machine examples is represented by a pair of machines in accordance with the procedure of the previous section, the second is replaced by a Buzacott-type machine. This is justified by the difference in magnitude between processing times and failure and repair times.

First, it should be noted that the times between successive failure/repair events is either exponentially or geometrically distributed, and that these distributions are close approximations of one another. Second, in discrete-material models, many operations take place between successive failure/repair events. Consequently, the variance of the processing time is not important, and the amount of material produced between these events is nearly the same in all the models.

Note that this implies that the exponential and continuous models are good approximations of one another when $r_i << \mu_i$ and $p_i << \mu_i$ for all i and the buffers are large. These models can only be used as approximations for the deterministic model when $\mu_i = 1$. A comparison among all three models of two-machine lines is shown in the Appendix.

Table 1. Case 1-1

Type	r_1	p_1	μ_1	N	r_2	p_2	μ_2	Production Rate
exponential	.01	.005	.75	2	.006	.005	1	.3071
continuous	.01	.005	.75	2	.006	.005	1	.3551

Type	r_1	p_1	N_1	r_2	p_2	N_2	r_3	p_3	Production Rate
deterministic	.3	.1	0	.01	.00667	2	.006	.005	.3529
deterministic	.6	.2	0	.01	.00667	2	.006	.005	.3529

Table 2. Case 1-2

Type	r_1	p_1	μ_1	N	r_2	p_2	μ_2	Production Rate
exponential	.01	.005	.75	3	.006	.005	1	.3261
continuous	.01	.005	.75	3	.006	.005	1	.3561

Type	r_1	p_1	N_1	r_2	p_2	N_2	r_3	p_3	Production Rate
deterministic	.3	.1	0	.01	.00667	3	.006	.005	.3547
deterministic	.6	.2	0	.01	.00667	3	.006	.005	.3547

Table 3. Case 1-3

Type	r_1	p_1	μ_1	N	r_2	p_2	μ_2	Production Rate
exponential	.01	.005	.75	5	.006	.005	1	.3428
continuous	.01	.005	.75	5	.006	.005	1	.3581

Type	r_1	p_1	N_1	r_2	p_2	N_2	r_3	p_3	Production Rate
deterministic	.3	.1	0	.01	.00667	5	.006	.005	.3570
deterministic	.6	.2	0	.01	.00667	5	.006	.005	.3571

Table 4. Case 1-4

Type	r_1	p_1	μ_1	N	r_2	p_2	μ_2	Production Rate
exponential	.01	.005	.75	10	.006	.005	1	.3563
continuous	.01	.005	.75	10	.006	.005	1	.3629

Type	r_1	p_1	N_1	r_2	p_2	N_2	r_3	p_3	Production Rate
deterministic	.003	.001	0	.01	.006667	10	.006	.005	.3589
deterministic	.006	.002	0	.01	.006667	10	.006	.005	.3592
deterministic	.03	.01	0	.01	.006667	10	.006	.005	.3606
deterministic	.06	.02	0	.01	.006667	10	.006	.005	.3613
deterministic	.3	.1	0	.01	.006667	10	.006	.005	.3625
deterministic	.6	.2	0	.01	.006667	10	.006	.005	.3628

The two sets of cases are similar except that in the first, μ_1 is .75 (compared to μ_2 = 1.) and the second, μ_1 is .9. The cases with the smaller value of μ_1 are shown in Tables 1 - 6; the cases with the larger values of μ_1 are in Tables 6 - 12. In both sets of cases, the buffer sizes are increased from 2 to 1000. Except for the exponential systems with small buffers (in which μ/N is not small compared with μ), the agreement among all sets of models is striking, and is adequate for all practical engineering purposes. Note that in the deterministic cases, the values of r_1 and p_1 do not seem to be important.

Table 2. Case 1-2

Type	r_1	p_1	μ_1	N	r_2	p_2	μ_2	Production Rate
exponential	.01	.005	.75	100	.006	.005	1	.4110
continuous	.01	.005	.75	100	.006	.005	1	.4136
	r_1	p_1	N_1	r_2	p_2	N_2	r_3	p_3	Production Rate
deterministic	.003	.001	0	.01	.006667	100	.006	.005	.4028
deterministic	.3	.1	0	.01	.006667	100	.006	.005	.4219
deterministic	.6	.2	0	.01	.006667	100	.006	.005	.4229

As the buffer sizes grow, the agreement between the exponential and continuous models improves, although the deterministic model shows no clear trend. The effect of the change in μ_1 on the accuracy is also not evident.

A surprising observation is that the accuracy does not fall off as r_1 and p_1 decrease in the deterministic approximations. The justification in Section 2 requires that these quantities be large, but evidently the results are not sensitive to their magnitudes. A careful analysis is clearly required.

Table 6. Case 1-6

Type	r_1	p_1	μ_1	N	r_2	p_2	μ_2	Production Rate
exponential	.01	.005	.75	1000	.006	.005	1	.4901
continuous	.01	.005	.75	1000	.006	.005	1	.4904
	r_1	p_1	N_1	r_2	p_2	N_2	r_3	p_3	Production Rate
deterministic	.003	.001	0	.01	.006667	1000	.006	.005	.4859
deterministic	.3	.1	0	.01	.006667	1000	.006	.005	.4943
deterministic	.6	.2	0	.01	.006667	1000	.006	.005	.4946

Table 7. Case 2-1

Type	r_1	p_1	μ_1	N	r_2	p_2	μ_2	Production Rate
exponential	.01	.005	.9	2	.006	.005	1	.3368
continuous	.01	.005	.9	2	.006	.005	1	.4022

Type	r_1	p_1	N_1	r_2	p_2	N_2	r_3	p_3	Production Rate
deterministic	.9	.1	0	.01	.005555	2	.006	.005	.3999
deterministic	.45	.05	0	.01	.005555	2	.006	.005	.3999

Table 8. Case 2-2

Type	r_1	p_1	μ_1	N	r_2	p_2	μ_2	Production Rate
exponential	.01	.005	.9	3	.006	.005	1	.3583
continuous	.01	.005	.9	3	.006	.005	1	.4033

Type	r_1	p_1	N_1	r_2	p_2	N_2	r_3	p_3	Production Rate
deterministic	.9	.1	0	.01	.005555	3	.006	.005	.4017
deterministic	.45	.05	0	.01	.005555	3	.006	.005	.4017

Table 9. Case 2-3

Type	r_1	p_1	μ_1	N	r_2	p_2	μ_2	Production Rate
exponential	.01	.005	.9	5	.006	.005	1	.3781
continuous	.01	.005	.9	5	.006	.005	1	.4053

Type	r_1	p_1	N_1	r_2	p_2	N_2	r_3	p_3	Production Rate
deterministic	.9	.1	0	.01	.005555	5	.006	.005	.4042
deterministic	.45	.05	0	.01	.005555	5	.006	.005	.4041

Table 10. Case 2-4

Type	r_1	p_1	μ_1	N	r_2	p_2	μ_2	Production Rate
exponential	.01	.005	.9	10	.006	.005	1	.3959
continuous	.01	.005	.9	10	.006	.005	1	.4099

Type	r_1	p_1	N_1	r_2	p_2	N_2	r_3	p_3	Production Rate
deterministic	.9	.1	0	.01	.005555	10	.006	.005	.4102
deterministic	.45	.05	0	.01	.005555	10	.006	.005	.4098

Table 11. Case 2-5

Type	r_1	p_1	μ_1	N	r_2	p_2	μ_2	Production Rate
exponential	.01	.005	.9	100	.006	.005	1	.4534
continuous	.01	.005	.9	100	.006	.005	1	.4575

Type	r_1	p_1	N_1	r_2	p_2	N_2	r_3	p_3	Production Rate
deterministic	.9	.1	0	.01	.005555	100	.006	.005	.4722
deterministic	.45	.05	0	.01	.005555	100	.006	.005	.4715

Table 12. Case 2-6

Type	r_1	p_1	μ_1	N	r_2	p_2	μ_2	Production Rate
exponential	.01	.005	.9	1000	.006	.005	1	.5354
continuous	.01	.005	.9	1000	.006	.005	1	.5359

Type	r_1	p_1	N_1	r_2	p_2	N_2	r_3	p_3	Production Rate
deterministic	.9	.1	0	.01	.005555	1000	.006	.005	.5417
deterministic	.45	.05	0	.01	.005555	1000	.006	.005	.5413

Table 13 shows how the accuracy of the method is affected by the relationship among r and p and μ. When r_1 and p_1 of the exponential and continuous lines are small (Case 1) the three models yield similar results. As they increase, the differences among the models grow. Note that the continuous and deterministic models remain closer than they are to the exponential.

Table 13. Variation of r and p.

Case	Type	r_1	p_1	μ_1	N	r_2	p_2	μ_2	Production Rate
1	exponential	.01	.005	.9	10	.006	.005	1	.3959
2	exponential	.1	.05	.9	10	.06	.05	1	.4297
3	exponential	.2	.1	.9	10	.12	.1	1	.4511
4	exponential	.4	.2	.9	10	.24	.2	1	.4731
1	continuous	.01	.005	.9	10	.006	.005	1	.4099
2	continuous	.1	.05	.9	10	.06	.05	1	.4575
3	continuous	.2	.1	.9	10	.12	.1	1	.4847
4	continuous	.4	.2	.9	10	.24	.2	1	.5114

Case	Type	r_1	p_1	N_1	r_2	p_2	N_2	r_3	p_3	Production Rate
1	deterministic	.9	.1	0	.01	.005555	10	.006	.005	.4102
2	deterministic	.9	.1	0	.1	.055555	10	.006	.005	.4644
3	deterministic	.9	.1	0	.2	.111111	10	.006	.005	.4878
4	deterministic	.9	.1	0	.4	.222222	10	.006	.005	.5170

4 Simulation results

This section presents a set of simulation results that demonstrate how well this method works for larger systems. Table 14 contains the parameters of two lines whose performances are compared. Both lines are deterministic; for each machine, the repair and failure distributions are distributed geometrically and the processing time is constant. (The distributions of these times in actual systems varies, depending on the whether operations are manual or automated, whether there is one or more operator per machine, whether machines are repaired on-line, or replaced by spares and repaired off-line, and other considerations.) Line 1 has one machine (Machine 3) whose processing rate (μ) differs from those of the others. In Line 2, that machine has been transformed into machines 3 and 4 according to the procedure of Section 2.

Table 14. Parameters for Simulation Runs

Line 1 Five Machines					Line 2 Six Machines				
i	r_i	p_i	μ_i	N_i	i	r_i	p_i	μ_i	N_i
1	.05	.07	1	10	1	.05	.07	1	10
2	.05	.07	1	10	2	.05	.07	1	10
3	.2	.05	.5	10	3	.3	.3	1	0
4	.05	.07	1	10	4	.2	1.	1	10
5	.05	.07	1		5	.05	.07	1	10
					6	.05	.07	1	

Table 15 contains the results of a set of simulation runs and approximate analytic evaluations (using the method of Gershwin, 1983). Production rates and corresponding average buffer levels ($\bar{n}_i$) are indicated. (Buffers 4 and 5 of Line 2 correspond to buffers 3 and 4 of Line 1.) Both lines were simulated, but only Line 2 could be evaluated analytically. (Note that buffer 3 in the simulation of Line 2 had a capacity of 2, not 0, because that was the minimum buffer size allowed by the simulation program.)

All simulations were run for 100,000 time units (ie, the time required for Machine 1, while not starved or blocked, to do 100,000 operations). The two simulations of Line 1 informally indicate the magnitude of the variation of performance measures. This is also demonstrated by a comparison of the simulation of Line 2 with the decomposition approximation results.

The representation was motivated by the availabilty of an approximation technique for analyzing a certain class of systems. However, it only depends on different systems behaving in a closely related way, and not at all on the approximation technique. Thus, we compare a simulation of Line 2 with the simulations of Line 1.

The closeness of the simulations of Line 1 to the simulation and approximate analysis of Line 2 indicate that the representation technique works well.

Table 15. Results

Run	Production Rate	$\bar{n}_1$	$\bar{n}_2$	$\bar{n}_3$ ($\bar{n}_4$)	$\bar{n}_4$ ($\bar{n}_5$)
Line 1 simulation 1	.2308	6.742	5.757	4.270	2.977
Line 1 simulation 2	.2295	6.711	5.644	4.129	3.159
Line 2 simulation	.2286	6.683	5.753	3.960	3.183
Line 2 approximation	.2306	7.175	6.146	4.580	3.065

Similar results are demonstrated for another case in Tables 16 and 17.

Table 16. Parameters for Simulation Runs

Line 3 Six Machines					Line 4 Nine Machines				
i	r_i	p_i	μ_i	N_i	i	r_i	p_i	μ_i	N_i
					1	.05	.07	1	10
1	.05	.07	1	10	2	.3	.3	1	0
2	.2	.05	.5	10	3	.2	.1	1	10
3	.05	.07	1	10	4	.05	.07	1	10
4	.2	.05	.5	10	5	.3	.3	1	0
5	.05	.07	1	10	6	.2	.1	1	10
6	.2	.05	.5		7	.05	.07	1	10
					8	.3	.3	1	0
					9	.2	1.	1	

Table 17. Results

Run	Production Rate	$\bar{n}_1$	$\bar{n}_2$ $(\bar{n}_3)$	$\bar{n}_3$ $(\bar{n}_4)$	$\bar{n}_4$ $(\bar{n}_6)$	$\bar{n}_5$ $(\bar{n}_7)$
Line 3 simulation 1	.2480	6.786	5.130	5.011	3.517	2.877
Line 3 simulation 2	.2444	6.729	5.081	4.785	3.200	2.720
Line 4 simulation 1	.2344	6.993	5.006	4.883	3.261	3.008
Line 4 simulation 2	.2424	6.966	5.025	5.168	3.272	3.311
Line 2 approximation	.2550	6.935	5.326	5.255	3.645	3.340

5 Conclusion

A new representation technique for modeling transfer lines has been discussed. The technique was devised to facilitate the analysis of lines with machines that have different processing rates. Such machines are represented by pairs of machines with equal processing rates, which allows the use of such analysis tools as that of Gershwin (1983). Numerical and simulation experience suggest that the method is effective, although analytic results on bounds of the approximation are not yet available.

References

T. Altiok (1982), "Approximate Analysis of Exponential Tandem Queues with Blocking," European Journal of Operations Research, Vol. 11, 1982.

J. A. Buzacott (1967a), "Markov Chain Analysis of Automatic Transfer Lines with Buffer Stock," Ph.D. Thesis, Department of Engineering Production, University of Birmingham.

J. A. Buzacott (1967b), "Automatic Transfer Lines with Buffer Stocks," International Journal of Production Research, Vol. 6 .

J. A. Buzacott (1972), "The Effect of Station Breakdown and Random Processing Times on the Capacity of Flow Lines with In-Process Storage," AIIE Transactions, Volume 4, No. 4, pp. 308-312.

Y. F. Choong and S. B. Gershwin (1985), "...," MIT Laboratory for Information and Decision Systems Report LIDS-P-1476.

S. B. Gershwin (1983), "An Efficient Decomposition Method for the Approximate Evaluation of Tandem Queues with Finite Storage Space and Blocking," MIT Laboratory for Information and Decision Systems Report LIDS-P-1309.

S. B. Gershwin and O. Berman (1981), "Analysis of Transfer Lines Consisting of Two Unreliable Machines with Random Processing Times and Finite Storage Buffers," AIIE Transactions Vol. 13, No. 1, March 1981.

S. B. Gershwin and I. C. Schick (1980), "Continuous Model of an Unreliable Two-Stage Material Flow System with a Finite Interstage Buffer," MIT Laboratory for Information and Decision Systems Report LIDS-R-1039.

S. B. Gershwin and I. C. Schick (1983), "Modeling and Analysis of Three-Stage Transfer Lines with Unreliable Machines and Finite Buffers," Operations Research, Vol. 31, No. 2, pp. 354-380, March-April 1983.

R. Suri and G. Diehl, "A Variable Buffer-Size Model and its Use in Analyzing Closed Queuing Networks with Blocking," Harvard University, Division of Applied Sciences, September, 1983.

Y. Takahashi, H. Miyahara, and T. Hasegawa (1980), "An Approximation for Open Restricted Queuing Networks," Operations Research, Vol. 28, No. 3, Part I, May-June, 1980.

J. Wijngaard (1971), "The Effect of Interstage Buffer Storage on the Output of Two Unreliable Production Units in Series, with Different Production Rates," AIIE Transactions, Vol. 11, No. 1, pp. 42-47.

Appendix: Comparisons Among Models

In this appendix, we compare the behavior of the deterministic, exponential, and continuous two-machine models. A set of cases was evaluated numerically, in which

$$r_1 = .01$$

$$p_1 = .005$$

$$r_2 = .006$$

$$p_2 = .005$$

and where N ranges from 2 to 1000. Production rate results are in Table A.1. Average buffer levels appear in Table A.2.

Note how the three models are not close when N is small, but that production rates and buffer levels are nearly indistinguishable when N is 1000.

Table A.1 Production Rates

N	Deterministic	Exponential	Continuous
2	.4003	.3534	.4300
3	.4296	.3760	.4308
5	.4311	.3969	.4322
10	.4346	.4159	.4357
100	.4785	.4731	.4788
1000	.5431	.5428	.5431

Table A.2 Average Buffer Levels

N	Deterministic	Exponential	Continuous
2	1.200	1.118	1.144
3	1.715	1.688	1.719
5	2.862	2.831	2.869
10	5.746	5.695	5.761
100	61.04	60.12	61.15
1000	805.9	798.0	805.6

Figure 1. Transfer Line

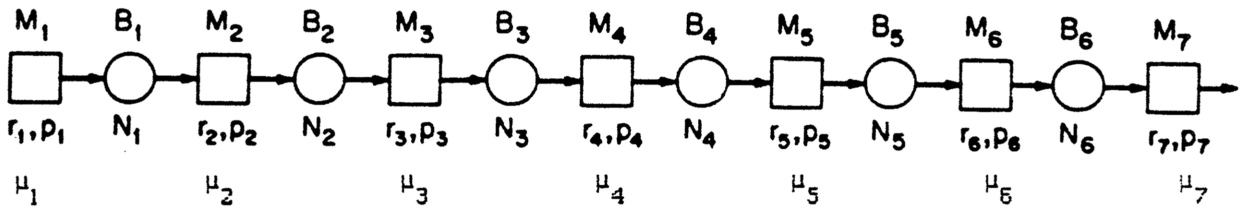
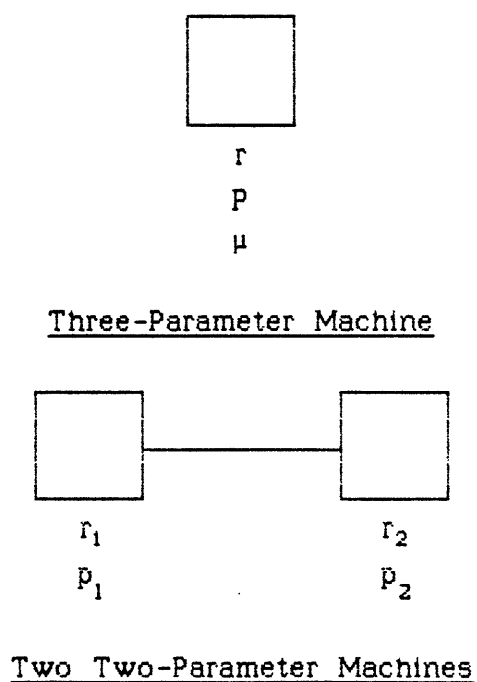


Figure 2. Representation of a Three-Parameter Machine by Two Two-parameter Machines.



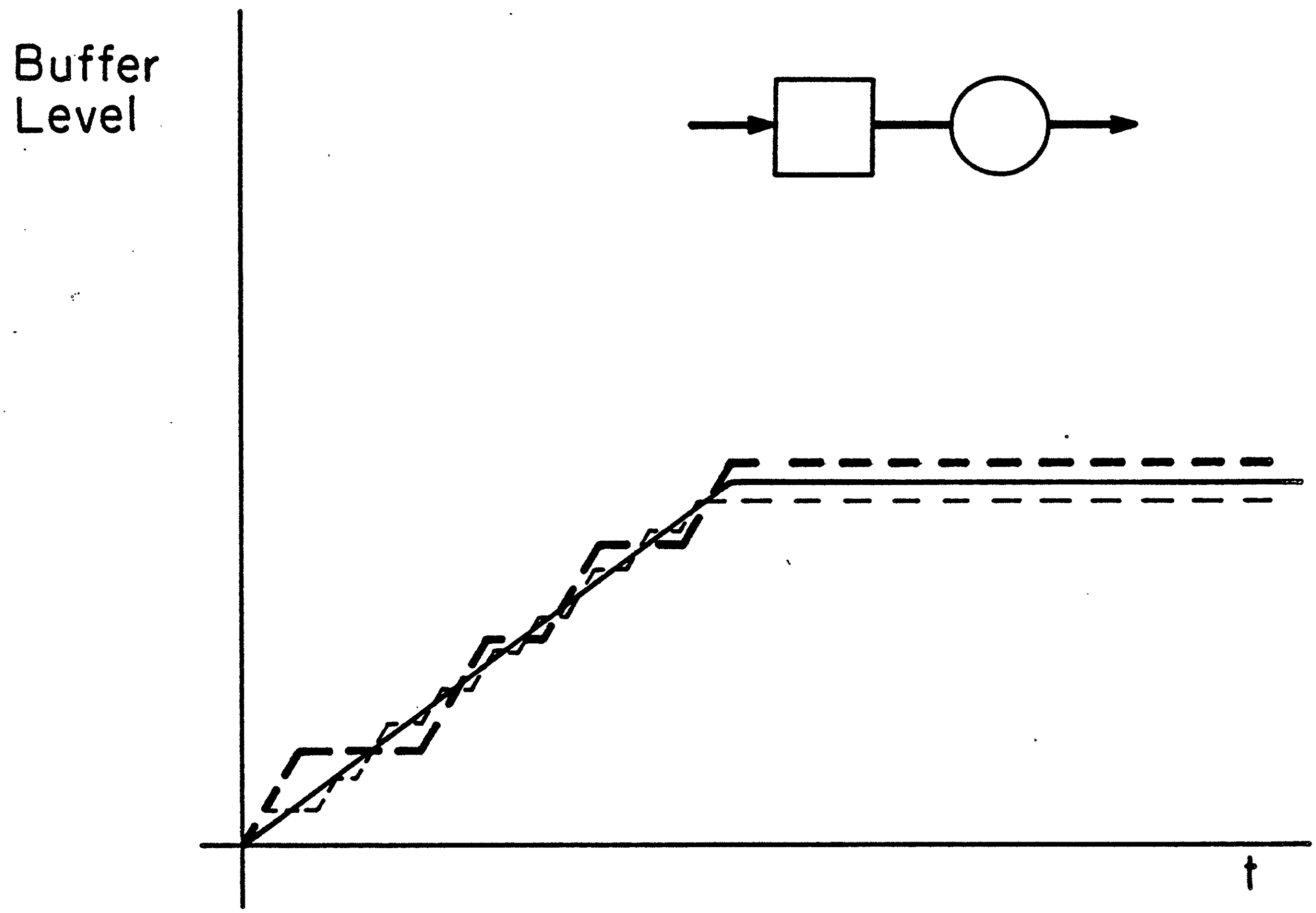


Figure 3. Justification for Representation