Representation and Control of Infinite Dimensional Systems, 2^{nd} edition

by A. Bensoussan, G. Da Prato, M. C. Delfour, S. K. Mitter

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It is perhaps difficult to recall how new the treatment of the first 1992-1993 two-volume edition [2] was in using the modern functional analytically oriented PDE theory of evolution equations (as well as a corresponding semi-group treatment of delay equations) to present a treatment of topics that had only recently obtained a satisfactory formulation. The present work (hereafter, BDDM2) has now streamlined and smoothed this treatment into a coherent one-volume book. It is extremely impressive that this felicitous combination of authors has been able to develop such a coherent structure. As before, the book is neither an exhaustive monograph nor a text, but rather is intended to show how a particular approach can be used effectively to treat a variety of problems.

Having said this, it is important to be clear as to just what is presented. There are two, quite different, choices one might make in constructing a book with such a title, namely, either one attempts broad coverage of the considerable variety of material on distributed parameter systems (systems where the state space is infinite dimensional, governed by partial differential equations or functional differential equations) or one restricts attention thematically to the treatment of a core problem; for example, the similarly titled [4] makes the first choice, whereas BDDM2 adopts the second.

What, then, is the theme of BDDM2?

Control theory is an interdisciplinary branch of engineering and mathematics dealing with the behavior of dynamical systems. If we look back to the inception of control theory as a subject for theoretical analysis with [1, 5] in the mid-19$^{\text{th}}$ century, we see as a focus the stabilization of a system—against unpredictable disturbances φ, including the modeling error of linearization—in the neighborhood of a stationary setpoint. This formulation leads to treatment of the autonomous, deterministic linear system $\dot{x} = Ax$ with inputs given by

$$\dot{x} = Ax + Bu + \varphi(t), \qquad x(0) = x_0, \tag{1}$$

and to the linear-quadratic optimal control problem of minimizing the quadratic cost

$$\mathcal{J} = \int \left[|Cx(t)|^2 + |u|^2 \right] \, dt \tag{2}$$

subject to (1). Besides consideration of controllability/observability, the thematic choice of BDDM2 is that the appropriate core problem is the quadratic cost problem (2)-(1) with an emphasis on the feedback form of the control, where the primary objective is not directly the existence and characterization of the optimal control itself (open loop), but rather the linear operator $x(t) \mapsto u_{opt}(t)$, which can be characterized through a Riccati equation. As noted by the authors:

"The study of the quadratic cost problem over a finite and an infinite time interval is essentially a study of the Riccati differential equation over the finite time interval $[s, T]$*."* (p. 33)

So far, this setting could equally well describe an approach to lumped parameter control theory and, indeed, is initially presented in an elegant discussion of the finite-dimensional theory for ordinary differential equations. The complementary implicit assertion of BDDM2 is that distributed parameter systems, governed by partial differential equations or delay equations, are essentially similar to lumped parameter systems with finite-dimensional state spaces; these systems are still ordinary differential equations, with the same abstract form (1), although now set in Hilbert spaces. Thus, the distinction is primarily a matter of overcoming the technical difficulties attendant on working with infinite-dimensional state spaces rather than in $\mathbb{R}^n$. Now A, B in (1) typically become unbounded operators between infinite-dimensional spaces whose topology becomes a significant structural choice in the treatment.

The thrust of BDDM2is thus to show how to follow this program, namely, how to handle the technical difficulties of replacing linear algebra by functional analysis in treating the Riccati equation for various classes of distributed parameter systems. For the reader already familiar with the finite-dimensional theory, BDDM2 is a superb reference for solving this focal linear-quadratic problem.

We further note the inclusion in this edition of new material, particularly, a 40-page chapter in Part I devoted to dynamic game theory. This problem corresponds, of course, to considering the external input $\varphi(\cdot)$ of (1) as generated by an optimizing opponent, but also serves to exhibit a context for the Riccati equation leading to nonpositive solutions, contrasting with the more usual settings in optimal control.

What is in the book?

BDDM2 is organized with particular approaches in mind along with the task of providing the requisite background development of the tools for this approach. The book is thus divided into five parts, each subdivided into chapters, beginning with the motivating Part I.

The foundation of this structure is Part II, which develops the basic representation theory of the title in terms of operator semigroups and appropriate Sobolev spaces. Note that a partial differential operator, for example, the Laplacian with Neumann boundary conditions, appears here in two forms, namely, as an unbounded operator $\mathcal{X} \supset \mathcal{D} \to \mathcal{X}$ and in variational form as a bounded operator: $\mathcal{V} \to \mathcal{V}'$ with $\mathcal{V} \subset \mathcal{X} = \mathcal{X}' \subset \mathcal{V}'$, for example, with $\mathcal{X} = L^2(\Omega)$, $\mathcal{V} = H_0^1(\Omega)$, $\mathcal{V}' = H^{-1}(\Omega)$. The first form is used for the semigroup representation (and later for spectral expansion with separation of variables), showing how standard examples of DPS problems can be formulated as first-order systems (1) giving a variation of parameters representation as the mild solution

$$x(t) = e^{tA}x_0 + \int_0^t e^{(t-s)A}\,[Bu(s) + \varphi(s)]\,ds. \tag{3}$$

The significant differences between forms of distributed parameter systems now show up in the fact that $t \mapsto e^{tA}$ is an analytic semigroup for the heat equation, a group for the time-reversible wave equation, and a differentiable semigroup for delay equations. In fact, I remember being surprised, years back, to discover that the time-reversible Euler plate equation could give an analytic semigroup.

As examples, it is shown for the heat equation $x_t = \Delta x$ that the representation (3) is straightforward for the case of interior control. For physically reasonable situations, direct access may be available only at the boundary (so that control appears as inhomogeneous boundary conditions such as $\partial_\nu x = u$ at $\partial\Omega$), and the representation continues to work straightforwardly for such Neumann boundary conditions (albeit with an unbounded operator as B), while the less regular Dirichlet control is treated in the variational form through the method of transposition. The treatment of such unbounded operators for a variety of function spaces is necessary for these infinite-dimensional problems and requires delicate care. A principal accomplishment here is an approach that makes the treatment closely resemble the finite-dimensional theory.

A sophisticated self-contained treatment of the semigroup representation for delay equations also appears in Part II, but treatment of second-order systems is deferred to Part III, which uses spectral methods in the formulation for skew-symmetric generators, corresponding to the wave, plate, and Maxwell equations. Part III is further devoted to the use of these methods in showing controllability for a variety of problems of interior and boundary control for the heat equation and for these second-order systems.

Parts IV and V of the book provide the culmination of the original theme. Part IV treats the operator Riccati equation

$$P' = A^*P + PA - PBB^*P + C^*C, \qquad P(0) = P_0 \tag{4}$$

for optimal feedback $u(t) = -B^*P(T-t)x(t)$ in the finite-horizon problem (1) (without φ) on $[0,T]$, minimizing $\mathcal{J}$ with the added terminal cost $\langle P_0 x(T), x(T)\rangle$. Part V then treats the algebraic Riccati equation

$$A^*P + PA - PBB^*P + C^*C = 0 \tag{5}$$

for the infinite-horizon problem and feedback stabilization. Again, this material involves considerable technical detail in the treatment of unbounded operators, which is given a clear exposition.

The thematic choice made here certainly means that I have occasionally found BDDM2 somewhat unsatisfying in its omission of various topics that, to my own taste, are both interesting and important. For example, I would have liked more treatment of exact null controllability, significant as dual to continuous observability. Further, while spatial localization and the effects of spatial geometry, having no counterpart in the finite-dimensional theory, might perhaps be viewed as a distraction from a purely control-theoretic analysis, my personal reaction to the loss of these most interesting considerations for partial differential equations is to feel this omission as a possible disadvantage of an approach based largely on the semigroup representation. In fact, I missed this concern for spatial geometry most especially in reading the controllability material in Part III, whose approach cannot, for example, treat the relation between the minimal control time for the wave equation and the finite speed of propagation.

Thus, this is not the book that I might have written and, quite unfairly, I occasionally found myself wishing to have seen the authors provide an equally masterful treatment of my own selection of topics—perhaps, for example, updating to include an exposition of the use of Carlemann estimates, cf., e.g., [3]. On the other hand, almost every time I looked into the book to work on the review I found myself browsing, distracted by some exposition of one item or another of fascinating material. I concur with the authors' comment in the Preface to this second edition:

> Over the years [2] has been recognized as a key reference in the field ... Even if some good books on the control of infinite dimensional linear systems have appeared since then, we felt that the original material has not aged too much and that the breadth of its presentation is still attractive and very competitive.

What is presented in BDDM2 is presented well, and I will continue to find the book valuable as a reference. Such a good treatment of this material is extremely welcome. That was indeed the case for the well-received original 2-volume version [2] of 1992-1993, and is now again the case for this second edition.

References

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