IC, and then to a PC for display. Codes were reliably read for image distances up to 120 mm, and work is continuing on enhancing the scan length and working distance.

Acknowledgment: The support of EPSRC, decoder construction by C.-L. Chew, and precision machining by P. Jones are gratefully acknowledged.

19 July 1999
Electronics Letters Online No: 19991139
DOI: 10.1049/el:19991139

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Representation of second-order polarisation mode dispersion

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A new expansion for the Jones matrix of a transmission medium is used to describe high-order polarisation dispersion. Each term in the expansion is characterised by a pair of principal states and the corresponding dispersion parameters. With these descriptors, a new expression for pulse deformation is derived and confirmed by simulation.

Introduction: Polarisation mode dispersion (PMD) in singlemode fibres has been extensively studied in recent years. The first-order effects of PMD are conveniently described in terms of the principal states of polarisation (PSPs) and their differential group delay (DGD) [1]. Second-order effects of PMD arise from the variation of these descriptors with optical frequency [2, 3]. Like chromatic dispersion, the effect of second-order PMD becomes increasingly important as the transmission bandwidth increases. It is thus important to have a simple method for representing and characterising the effects of second-order PMD. Until now, the analysis of second-order PMD was based on expanding the Jones matrix of the transmission medium near the carrier frequency using a Taylor series or by considering the variations of the PMD-vector on the Poincaré sphere. In this Letter we propose a new description of PMD which is based on an exponential expansion of the Jones matrix of the transmission medium. According to this description, different orders of PMD are treated separately as different subsystems and the overall behaviour of the transmission medium is obtained from serially concatenating these systems. Generalising Poole and Wagner's phenomenological approach [1], each subsystem is described in terms of a pair of principal states and the corresponding dispersion parameters. With the use of this simple model a new expression for pulse deformation due to second-order PMD is derived. The validity of the model is confirmed using an 'exact' numerical simulation.

PMD representation: Let the field at the input of a linear transmission medium be described by: $\vec{E}_{in} = Ae^{i\omega t}\hat{\varepsilon}_{in}$ where $\hat{\varepsilon}_{in}$ is a unit Jones vector representing the input state of polarisation, ω is the optical frequency and A is the complex amplitude of the field. Using Jones' formulation, the output field is given by: $\vec{E}_{out} = \mathbf{T}(\omega)\vec{E}_{in}$ where $\mathbf{T}(\omega)$ is a frequency-dependent 2×2 complex matrix [4]. In realistic optical transmission media $\mathbf{T}(\omega)$ often possesses a complex frequency dependence. Therefore, to investigate the dispersive properties of the medium, it is useful to expand $\mathbf{T}(w)$ near the carrier frequency ω_0 in powers of $\omega - \omega_0$. In conventional analysis of pulse propagation in dispersive, isotropic media it is customary to represent the frequency dependent phase acquired by the propagating wave by its Taylor expansion near ω_0, rather then expanding the field itself. To generalise this to the non-isotropic case, we propose the following expansion of the medium Jones matrix:

$$\mathbf{T}(\omega_0 + \Omega) = \mathbf{T}_0 e^{\Omega \mathbf{N}_1} e^{\frac{1}{2}\Omega^2 \mathbf{N}_2} e^{\frac{1}{6}\Omega^3 \mathbf{N}_3} \ldots \quad (1)$$

where $\Omega \ll \omega_0$, $\mathbf{T}_0 \equiv \mathbf{T}(\omega_0)$ and the parameters denoted by $\mathbf{N}_k$ (k = 1, 2, 3, ...) are 2×2 complex matrices which can be found by successively differentiating eqn. 1 with respect to Ω and substituting $\Omega = 0$. The first two terms in the expansion are given by

$$\begin{aligned} \mathbf{N}_1 &= \mathbf{T}_0^{-1}\mathbf{T}'\big|_{\omega_0} \\ \mathbf{N}_2 &= \mathbf{T}_0^{-1}\mathbf{T}''\big|_{\omega_0} - (\mathbf{N}_1)^2 \end{aligned} \quad (2)$$

Here the primes denote differentiation with respect to frequency. It can be easily seen that as long as $\mathbf{T}(\omega)$ is non-singular (i.e. the system does not contain an ideal polariser) and differentiable M times, then $\mathbf{N}_k$ will be well defined for k = 1, 2, 3, ..., M. We denote the eigenvectors and eigenvalues of $\mathbf{N}_k$ as $\hat{\varepsilon}_{k\pm}$ and $\alpha_{k\pm}$, respectively. The eigenvectors of $\mathbf{N}_1$, $\hat{\varepsilon}_{1\pm}$, were shown to be the input PSPs of the medium and the imaginary part of $(\alpha_{1+} - \alpha_{1-})$ is the DGD associated with them [5]. We now utilise the well-known theorem that if $\mathbf{A}$ is a non-singular matrix with eigenvalues α_k, then $e^{\mathbf{A}}$ and $\mathbf{A}$ have the same eigenvectors, and the eigenvalues of $e^{\mathbf{A}}$ are e^{α_k}. Therefore

$$\begin{aligned} &\mathbf{T}(\omega_0 + \Omega) = \\ &\mathbf{T}_0 \lfloor \mathbf{P}_1 \mathbf{Q}_1(\Omega) \mathbf{P}_1^{-1} \rfloor \lfloor \mathbf{P}_2 \mathbf{Q}_2(\Omega) \mathbf{P}_2^{-1} \rfloor \lfloor \mathbf{P}_3 \mathbf{Q}_3(\Omega) \mathbf{P}_3^{-1} \rfloor \cdots \end{aligned} \quad (3)$$

where

$$\mathbf{P}_k \equiv [\hat{\varepsilon}_{k+}\ \hat{\varepsilon}_{k-}] \quad \text{and} \quad \mathbf{Q}_k \equiv \begin{bmatrix} e^{\frac{1}{k!}\Omega^k \alpha_{k+}} & 0 \\ 0 & e^{\frac{1}{k!}\Omega^k \alpha_{k-}} \end{bmatrix} \quad k = 1, 2, 3, \ldots \quad (4)$$

The main advantages of this representation is that it enables us to consider the effects of the different orders of PMD separately and that high-order terms have the same form as the first-order term. Each order of PMD is represented as a separate optical system with eigenstates $\hat{\varepsilon}_{k\pm}$ and eigenvalues $\alpha_{k\pm}$. We refer to the parameter denoted by $\hat{\varepsilon}_{k\pm}$ as the input principal states of polarisation of the kth order and to the difference between the imaginary parts of α_{k+} and α_{k-} as the differential group-delay dispersion (DGDD) of the kth order.

First-order PMD: eqn. 3 can be used to study the propagation of short pulses in a polarisation dispersive transmission medium. Let $A(t)$ and $A_f(\Omega)$ describe the amplitude of an optical field at the input of the transmission medium and the corresponding Fourier transform, respectively. We consider first the case where only the zero and first-order terms in eqn. 3 are significant. The output field in this case is given by

$$\vec{E}_{out}(t) \cong \frac{1}{2\pi} \int_{-\infty}^{\infty} A_f(\Omega) e^{i(\omega_0 + \Omega)t} \mathbf{T}_0 \mathbf{P}_1 \mathbf{Q}_1(\Omega) \mathbf{P}_1^{-1} \hat{\varepsilon}_{in} d\Omega \quad (5)$$

In the absence of polarisation dependent loss or gain (PDL/G) $\mathbf{T}(\omega)$ is proportional to a unitary matrix. In this case $\alpha_{1\pm}$ are purely imaginary and integration of eqn. 5 yields

$$\tilde{E}_{out}(t) = \begin{bmatrix} c_{1+} A(t - \alpha_{1+}^i) \\ c_{1-} A(t - \alpha_{1-}^i) \end{bmatrix} e^{i\omega_0 t} \quad (6)$$

where $c_{1\pm}$ are defined by $[c_{1+}\ c_{1-}]^T \equiv P_1^{-1}\hat{\varepsilon}_{in}$, $\alpha_{1\pm}{}^i \equiv \mathrm{Im}\{\alpha_{1\pm}\}$ and $\tilde{E}_{out}(t)$ is given by:

$$\tilde{E}_{out}(t) = (\mathbf{T}_0\mathbf{P}_1)^{-1}\vec{E}_{out}(t) \qquad (7)$$

Identifying $\mathbf{T}_0\mathbf{P}_1$ as a matrix in which the columns are the output PSPs we obtain the well-known result that the output field due to first-order PMD in the basis of the output PSPs is composed of two differently delayed replicas of the input field with weights given by the projections of the input SOP on the input PSPs. In the presence of PDL/G the PSPs are no longer orthogonal and experience differential loss/gain-dispersion. While this necessitates minor modifications to eqn. 6, they will not be considered here for brevity.

Second-order PMD – pulse deformation: The output field in the presence of second-order PMD is given by

$$\vec{E}_{out}(t) \cong \frac{1}{2\pi}\int_{-\infty}^{\infty}\Big\{A_f(\Omega)e^{i(\omega_0+\Omega)t}\mathbf{T}_0[\mathbf{P}_1\mathbf{Q}_1(\Omega)\mathbf{P}_1^{-1}] \times [\mathbf{P}_2\mathbf{Q}_2(\Omega)\mathbf{P}_2^{-1}]\hat{\varepsilon}_{in}\Big\}d\Omega \qquad (8)$$

Introducing the Fourier transform pairs

$$\hat{A}_\pm(t) \equiv \frac{1}{2\pi}\int A_f(\Omega)e^{\frac{1}{2}\Omega^2\alpha_{2\pm}}e^{i\Omega t}d\Omega$$
$$\Leftrightarrow\ \hat{A}_{f\pm}(\Omega) \equiv \int \hat{A}_\pm(t)e^{-i\Omega t}dt \qquad (9)$$

we obtain

$$\vec{E}_{out}(t) \cong \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i(\omega_0+\Omega)t}\mathbf{T}_0\mathbf{P}_1\mathbf{Q}_1(\Omega)\mathbf{P}_1^{-1} \times \left[c_{2+}\hat{A}_{f+}(\Omega)\hat{\varepsilon}_{2+} + c_{2-}\hat{A}_{f-}(\Omega)\hat{\varepsilon}_{2-}\right]d\Omega \qquad (10)$$

where $c_{2\pm}$ are defined by $[c_{2+}\ c_{2-}]^T \equiv \mathbf{P}_2^{-1}\hat{\varepsilon}_{in}$. Finally we use eqn. 6 to obtain

$$\tilde{E}_{out}(t) = c_{2+}\begin{bmatrix} k_{11}\hat{A}_+(t-\alpha_{1+}^i) \\ k_{21}\hat{A}_+(t-\alpha_{1-}^i)\end{bmatrix}e^{i\omega_0 t} + c_{2-}\begin{bmatrix} k_{12}\hat{A}_-(t-\alpha_{1+}^i) \\ k_{22}\hat{A}_-(t-\alpha_{1-}^i)\end{bmatrix}e^{i\omega_0 t} \qquad (11)$$

where the k_{ij} are the elements of the matrix $\mathbf{P}_1^{-1}\mathbf{P}_2$ and the linear loss/gain polarisation dispersion is neglected. This result shows that the output field in the basis of the (first-order) output PSPs is composed of four filtered versions of the input pulse. Two of them correspond to the slow PSP and have the same group delay α_{1+}^i but different envelopes, $\hat{A}_+(\cdot)$ and $\hat{A}_-(\cdot)$. The other two have the same two envelopes $\hat{A}_+(\cdot)$ and $\hat{A}_-(\cdot)$ but are delayed by α_{1-}^i. The weights of the four different components are determined by the projections of the input SOP on the second-order PSPs and the projections of the second-order PSPs on the first order PSPs.

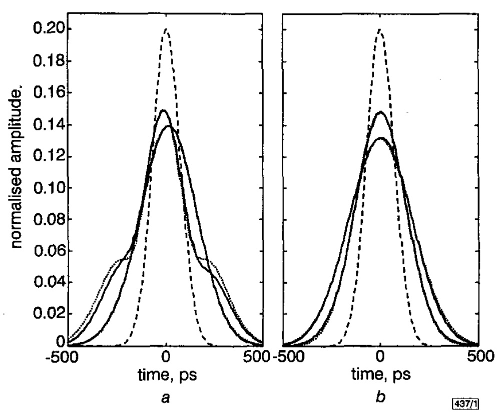

Fig. 1 *Comparison between direct calculation of system response and model presented*

a Input pulse and output pulses in basis of first-order PSPs
b Input pulse and pulses at output of compensated medium on basis of second-order PSPs
– – – – input pulse
——— simulation output pulse
·········· modelled output pulse

Simulation: To check the validity of the model a computer simulation was used. The simulation was composed of the following stages: first the Jones matrix, $\mathbf{T}(\omega_i)$, of a cascade of 1000 randomly oriented linearly-birefringent fibre segments was calculated at 200 consecutive frequencies. Next eqns. 2 and 4 were used to evaluate the matrices $\mathbf{N}_1$, $\mathbf{N}_2$ and $\mathbf{N}_3$ as well as the corresponding principal polarisation states, the DGD and the DGDD at the centre frequency ω_0. To be able to consider the second-order effect of PMD separately, we compensated for the first-order effects by left-multiplying $\mathbf{T}(\omega_i)$ with $e^{-(\omega_i-\omega_0)\mathbf{N}_1}\mathbf{T}_0^{-1}$. We then calculated the time response of the compensated system to a Gaussian input. The pulsewidth, 118ps FWHM, was chosen such that third-order effects would be negligible compared to those of the second-order.

Results: Fig. 1 shows a comparison between the result of the direct calculation of the system response and that according to our model using eqns. 9 – 11. The response of the medium in the basis of the first-order PSPs is plotted on the left side. The two broadened pulses correspond to the fast and slow PSPs and are accordingly delayed differently with a DGD of 26ps. In addition the pulses are non-Gaussian. The response of the compensated medium in the basis of the second-order PSPs is plotted on the right side of Fig. 1. It can be seen that the pulses are now equally delayed. In accordance with our model they retain their Gaussian shape but they are broadened differently with a DGDD of $2900 ps^2$.

Conclusion: We have developed a new representation for high-order PMD. According to this representation the effect of each order of PMD is described by a pair of principal states and the corresponding dispersion parameters. The representation has been used to analyse the deformation of short pulses due to second-order PMD.

26 July 1999
Electronics Letters Online No: 19991136
DOI: 10.1049/el:19991136

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Surface plasmon resonance sensing in capillaries

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A capillary-based surface plasmon resonance sensor is presented. Glass capillaries internally coated with Au are radially illuminated with focused laser light that strikes the interior capillary surface and exits the capillary at angles that vary with the interior incident angle. Measurement of the exit light provides reflectivity against angle spectra useful for chemical sensing.