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REPRESENTATIONS OF PETRI NET INTERACTIONS

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Paper available on my homepage

KLEENE'S THEOREM

- Classic result in theory of sequential computation with finite state
- **Finite automata**
  - graphical representation
  - semantics given globally
- **Regular expressions**
  - syntactic representation
  - semantics given inductively

- Why do we teach this to undergraduates?

WHY IS KLEENE IMPORTANT?

- Much of Computer Science is about **syntax**
  - how to capture **dynamic** notions of computation by an **efficient** syntax?
    - programming languages
    - process calculi
    - specification logics
- Kleene's theorem is about capturing the **essence** of **sequential computation** with **finite state** (finite automata) with an **efficient syntax** (regular expressions)

WHAT ABOUT CONCURRENCY?

- Kleene's theorem is about capturing the **essence** of **sequential computation** with **finite state** (finite automata) with an **efficient syntax** (regular expressions)
- what is the essence of **concurrent computation** with finite state? (one answer: **finite Petri nets**)
  - intuitive and popular
  - non-compositional
- we have many syntaxes: **process calculi** of various sorts
  - intuitive and popular with process-calculists
  - compositional with SOS semantics

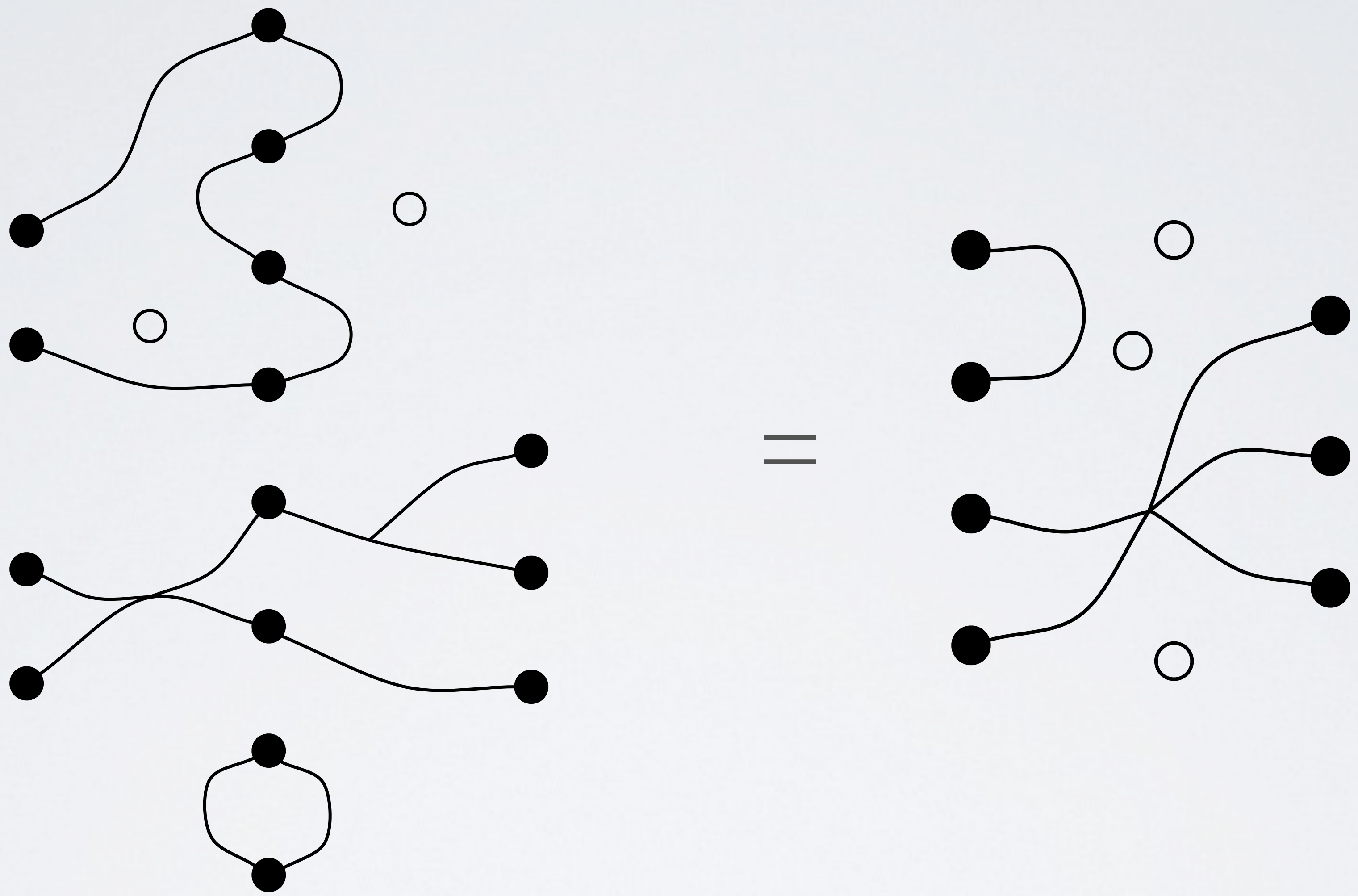
THE CONTRIBUTION

- people have tried to go from nets to calculi and vice-versa but with limited success
  - is the model "wrong" or is the syntax "wrong"?
- we show that the expressive power of an open variant of nets is the same as that of a process calculus
  - most well-known process calculi are based on a binary parallel composition || operator
  - process calculus in this talk has fundamentally different operations

PART 1: GRAPHICAL STRUCTURE

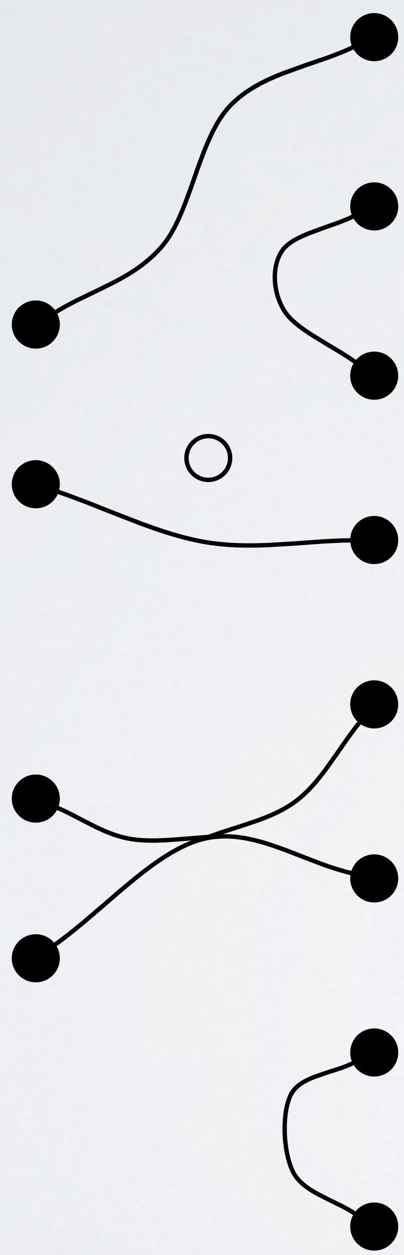
Link graphs and Petri nets

LINK GRAPHS



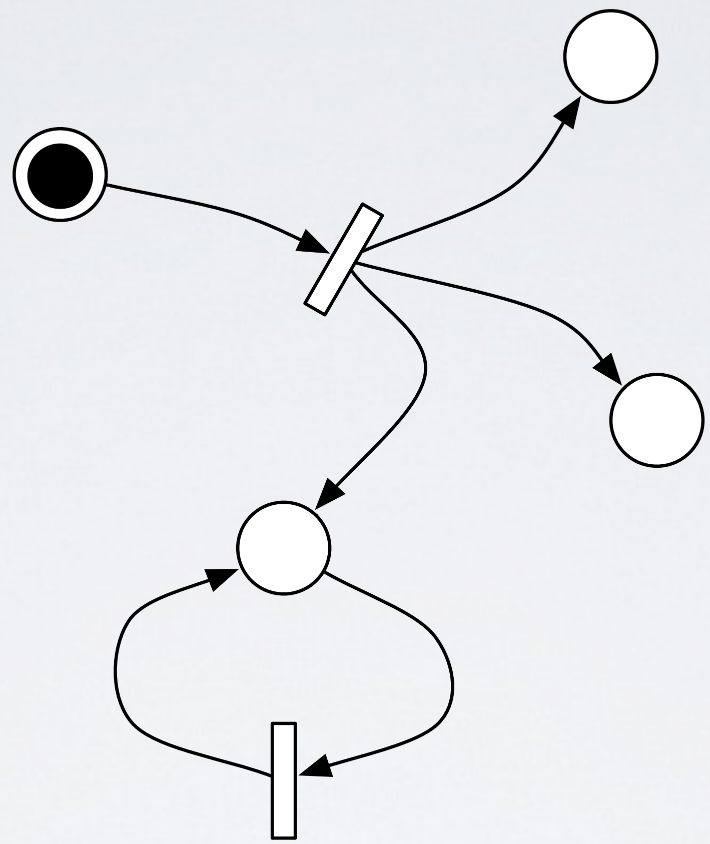
- composition by **synchronisation** of links
- this category is equivalent to *free compact closed category on a self-dual object* (Abramsky, Calco `05)

CATEGORY OF LINK GRAPHS



- Composition explained
  - objects are (finite) ordinals
  - arrows are cospans of functions
  - composition is by pushout

FINITE NETS

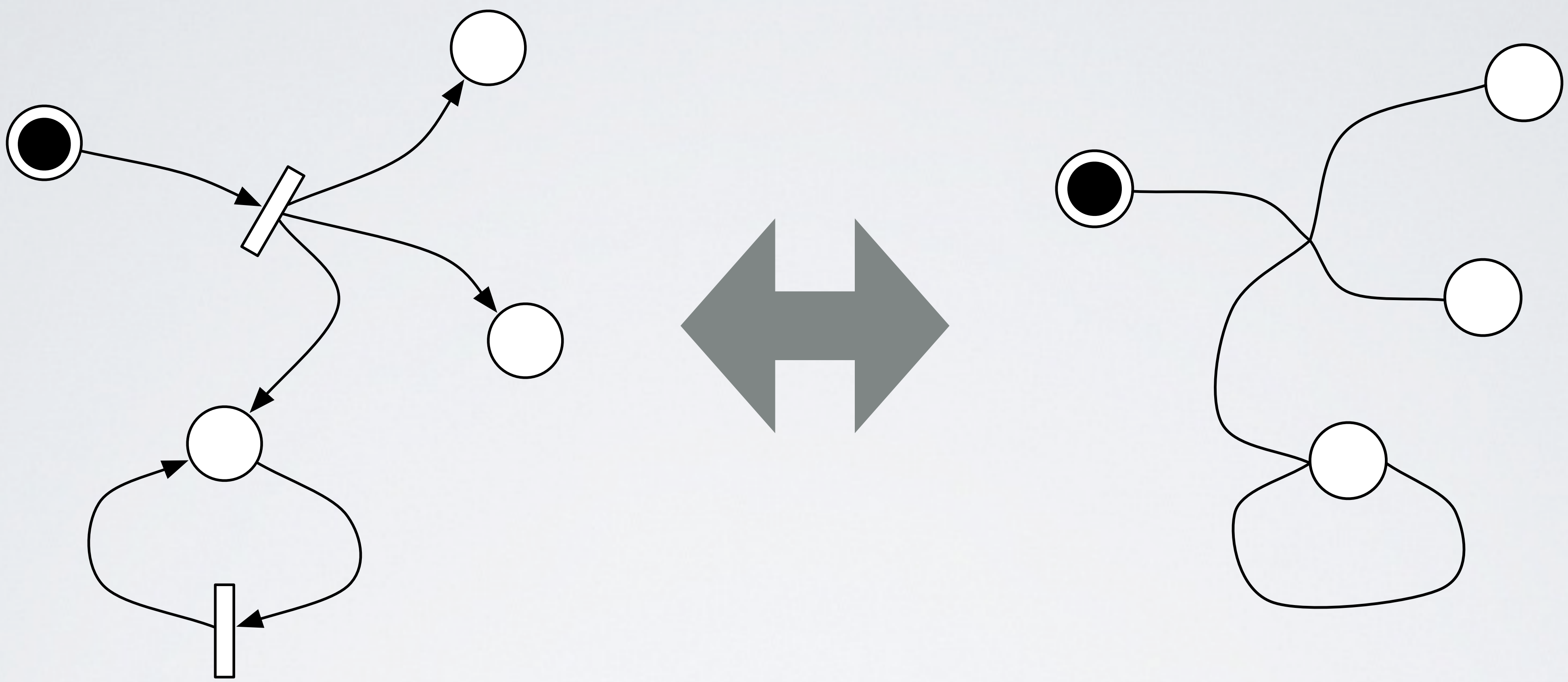


- Definition
  - Finite set of places P
  - Finite set of transitions T
  - Functions °-, -° : T → 2^P

SEMANTICS OF NETS

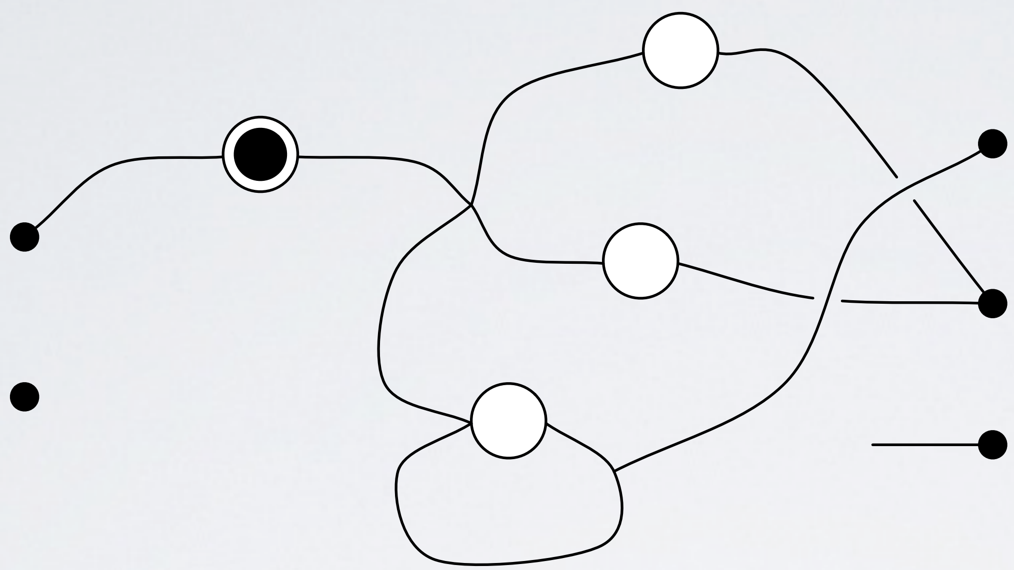
- trans $t, u \in T$ *independent* when ${}^{\circ}t \cap {}^{\circ}u = \emptyset$ and $t^{\circ} \cap u^{\circ} = \emptyset$
- Suppose $X, Y \subseteq P$
  - $X \to Y$ if there exists a set $U \subseteq T$ of mutually independent transitions such that ${}^{\circ}U \subseteq X$, $U^{\circ} \subseteq Y$ and $X \setminus {}^{\circ}U = Y \setminus U^{\circ}$

PETRI NETS



- (multi) link graphs plus
  - two kinds (marked/unmarked) of nodes
  - each with two ports (in/out)

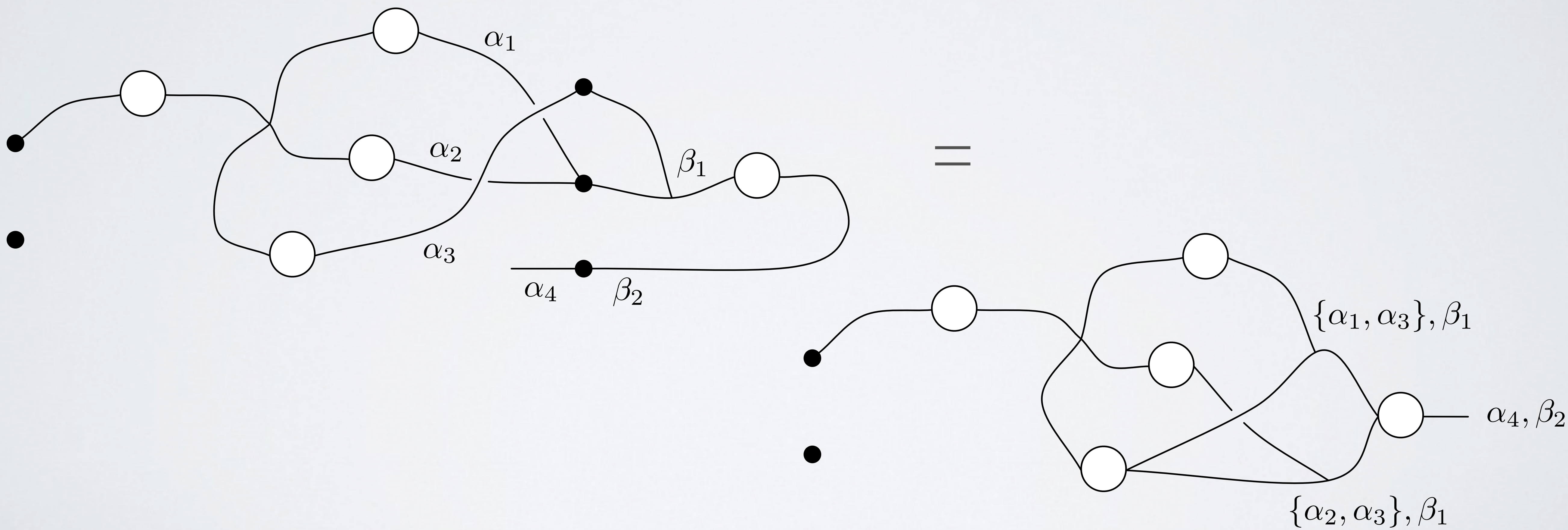
PETRI NETS WITH BOUNDARIES



- Definition, net $N : k \to l$ (k, l finite ordinals)
  - finite set of places P
  - finite set of transitions T
  - functions $^{\circ}-, -^{\circ} : T \to 2^P$
  - functions $^{\bullet}- : T \to 2^k$, $-^{\bullet} : T \to 2^l$

COMPOSITION

- A **synchronisation** is a pair (U,V) where
  - U∪V ≠ ∅
  - U• = •V
- The set of transitions of the composed net is the set of **minimal** synchronisations



is this universal in some sense?

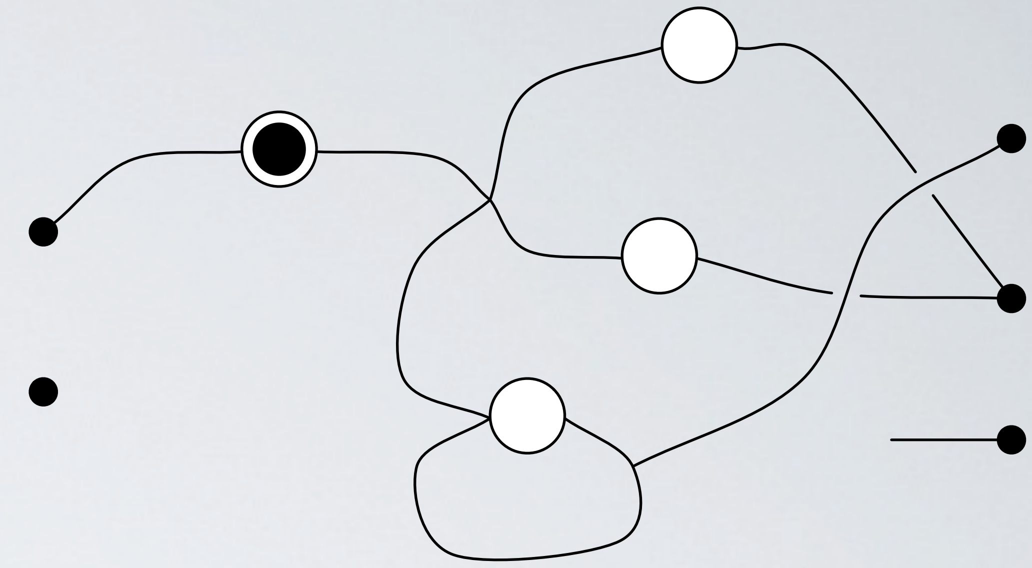
TRANSITION SYSTEMS

- Possible signals are 0 (no signal) and 1 (signal)
- for k, l ∈ **N** a (k, l)-transition is a labelled transition of the form

$$P \xrightarrow[\vec{b}]{\vec{a}} Q, \quad \#(\vec{a}) = k,\ \#(\vec{b}) = l$$

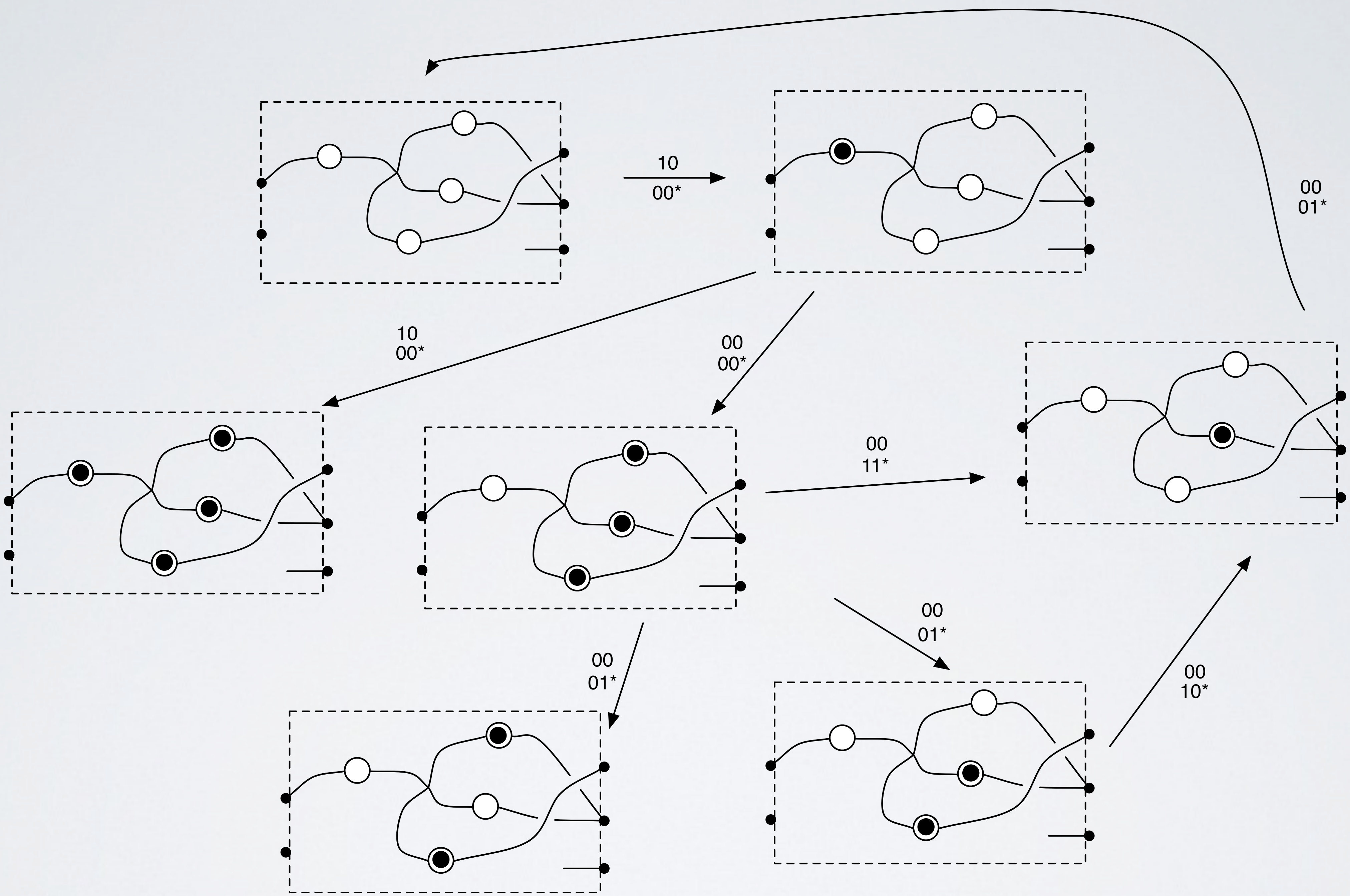
  - where each label is a vector of 0,1s
- a (k, l)-transition system is consists of (k, l)-transitions

SEMANTICS OF BOUNDED NETS



- $t, u \in T$ *independent* when $^{\circ}t \cap {}^{\circ}u = \varnothing$, $t^{\circ} \cap u^{\circ} = \varnothing$, $^{\bullet}t \cap {}^{\bullet}u = \varnothing$, and $t^{\bullet} \cap u^{\bullet} = \varnothing$
- Suppose $X, Y \subseteq P$
  - $X \xrightarrow[\vec{b}]{\vec{a}} Y$ if there exists a set $U \subseteq T$ of mutually independent transitions such that $^{\circ}U \subseteq X$, $U^{\circ} \subseteq Y$, $X \setminus {}^{\circ}U = Y \setminus U^{\circ}$ and
    - $a_i = 1$ iff $i \in {}^{\bullet}u$ for some (necessarily unique) $u \in U$
    - $b_i = 1$ iff $i \in u^{\bullet}$ for some (necessarily unique) $u \in U$

EXAMPLE (PARTIAL STATE SPACE)



expressive power = class of LTSs (up to bisimilarity) that one can define

PART 2: SYNTACTIC STRUCTURE

"PETRI CALCULUS"

$$P \;::=\; P\,;P \mid P\otimes P \mid \bigcirc \mid \odot \mid \mathsf{I} \mid \mathsf{X} \mid \Delta \mid \bot \mid \nabla \mid \top \mid \Lambda \mid {\downarrow} \mid \vee \mid {\uparrow}$$

- any well-formed process is the "Petri calculus" has a sort $(k,\, l)$ and its semantics will be an LTS of $(k,\, l)$-transitions

Calculus of connectors + one-place buffer

$$\frac{}{\bigcirc \xrightarrow[0]{1} \odot}\ (\textsc{TkI}) \qquad \frac{}{\odot \xrightarrow[1]{0} \bigcirc}\ (\textsc{TkO1}) \qquad \frac{}{\odot \xrightarrow[1]{1} \odot}\ (\textsc{TkO2}) \qquad \frac{}{\mathsf{I} \xrightarrow[1]{1} \mathsf{I}}\ (\textsc{Id}) \qquad \frac{a,b\in\{0,1\}}{\mathsf{X} \xrightarrow[ba]{ab} \mathsf{X}}\ (\textsc{Tw})$$

$$\frac{}{\Delta \xrightarrow[11]{1} \Delta}\ (\Delta) \qquad \frac{}{\nabla \xrightarrow[1]{11} \nabla}\ (\nabla) \qquad \frac{}{\perp \xrightarrow{1} \perp}\ (\perp) \qquad \frac{}{\top \xrightarrow[1]{} \top}\ (\top) \qquad \frac{(a\in\{0,1\})}{\wedge \xrightarrow[(1-a)a]{1} \wedge}\ (\wedge a) \qquad \frac{(a\in\{0,1\})}{\vee \xrightarrow[1]{(1-a)a} \vee}\ (\vee a)$$

$$\frac{P \xrightarrow[\mathbf{c}]{\mathbf{a}} Q \qquad R \xrightarrow[\mathbf{b}]{\mathbf{c}} S}{P;R \xrightarrow[\mathbf{b}]{\mathbf{a}} Q;S}\ (\textsc{Cut}) \qquad \frac{P \xrightarrow[\mathbf{b}]{\mathbf{a}} Q \qquad R \xrightarrow[\mathbf{d}]{\mathbf{c}} S}{P\otimes R \xrightarrow[\mathbf{bd}]{\mathbf{ac}} Q\otimes S}\ (\textsc{Ten}) \qquad \frac{P:(k,l)}{P \xrightarrow[0^l]{0^k} P}\ (\textsc{Refl})$$

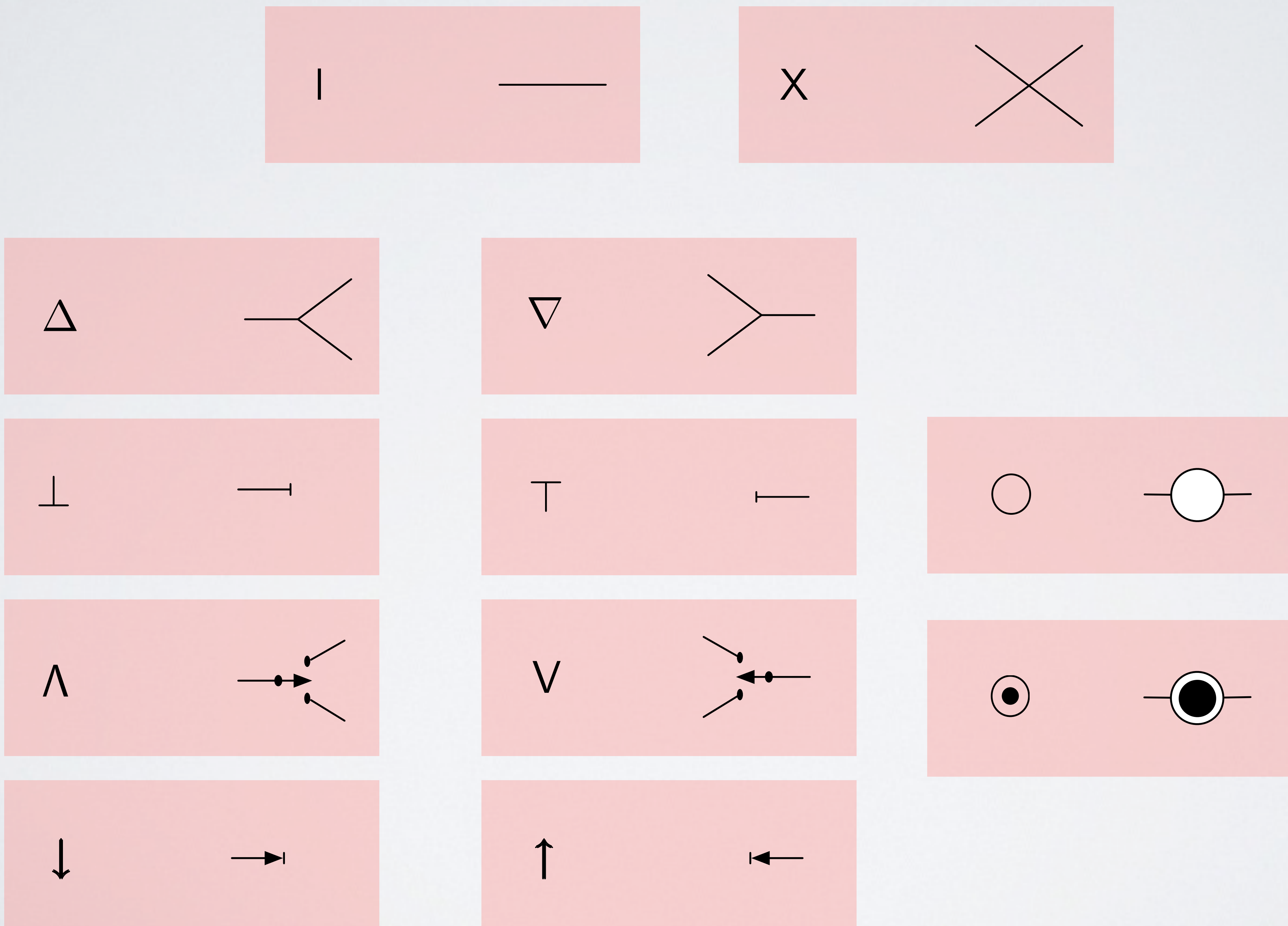
true concurrency!

BASIC PROPERTIES

- strong ($\sim$) and weak bisimulation ($\approx$) are congruences
- sequential composition is associative up to $\sim$ and up to $\approx$
- categories with objects natural number arrows terms up to $\sim$ or $\approx$
- categories have tensor product induced by $\otimes$ in the obvious way

- all follow from "A non-interleaving calculus for multi-party synchronisation" via an encoding into the wire calculus, my controversial IFIP talk from '09

CIRCUIT DIAGRAMS



RELATIONAL FORMS

$$\theta \in \{\mathsf{X}, \Delta, \nabla, \bot, \top, \wedge, \vee, \downarrow, \uparrow\}$$

$$T_\theta \ ::= \ \theta \mid I \mid T_\theta \otimes T_\theta \mid T_\theta \ ; T_\theta .$$

- Right relational form: $t = t_\bot \ ; t_\Delta \ ; t_{\mathsf{X}} \ ; t_\vee \ ; t_\uparrow$
  - dually, left relation form: $t_\downarrow \ ; t_\wedge \ ; t_{\mathsf{X}} \ ; t_\nabla \ ; t_\top$
- **Lemma**: for any function $f : k \to 2^l$ there exists a term in right relational form such that:

$$\frac{}{\rho_f \xrightarrow[V]{U} \rho_f}$$

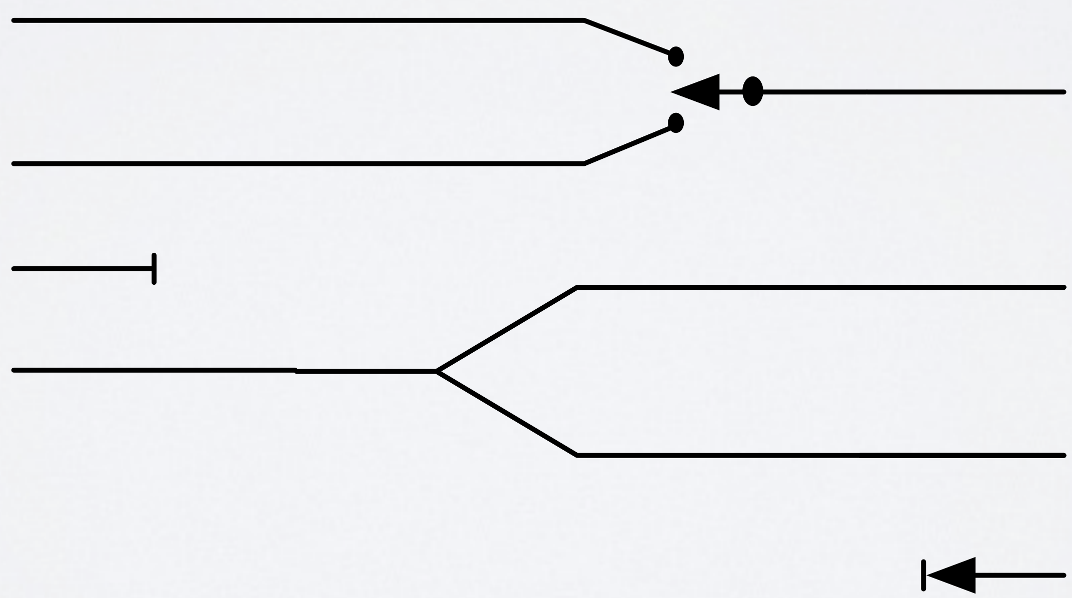
$$\Leftrightarrow$$

$$U \subseteq \underline{k} \text{ s. t. } \forall u, v \in U.\ u \neq v \Rightarrow f(u) \cap f(v) = \varnothing \ \& \ V = f(U)$$

EXAMPLE

$f: 4 \to 2^4$ $\qquad f(0),\ f(1) = \{0\},\ f(2) = \varnothing,\ f(3) = \{1,2\}$

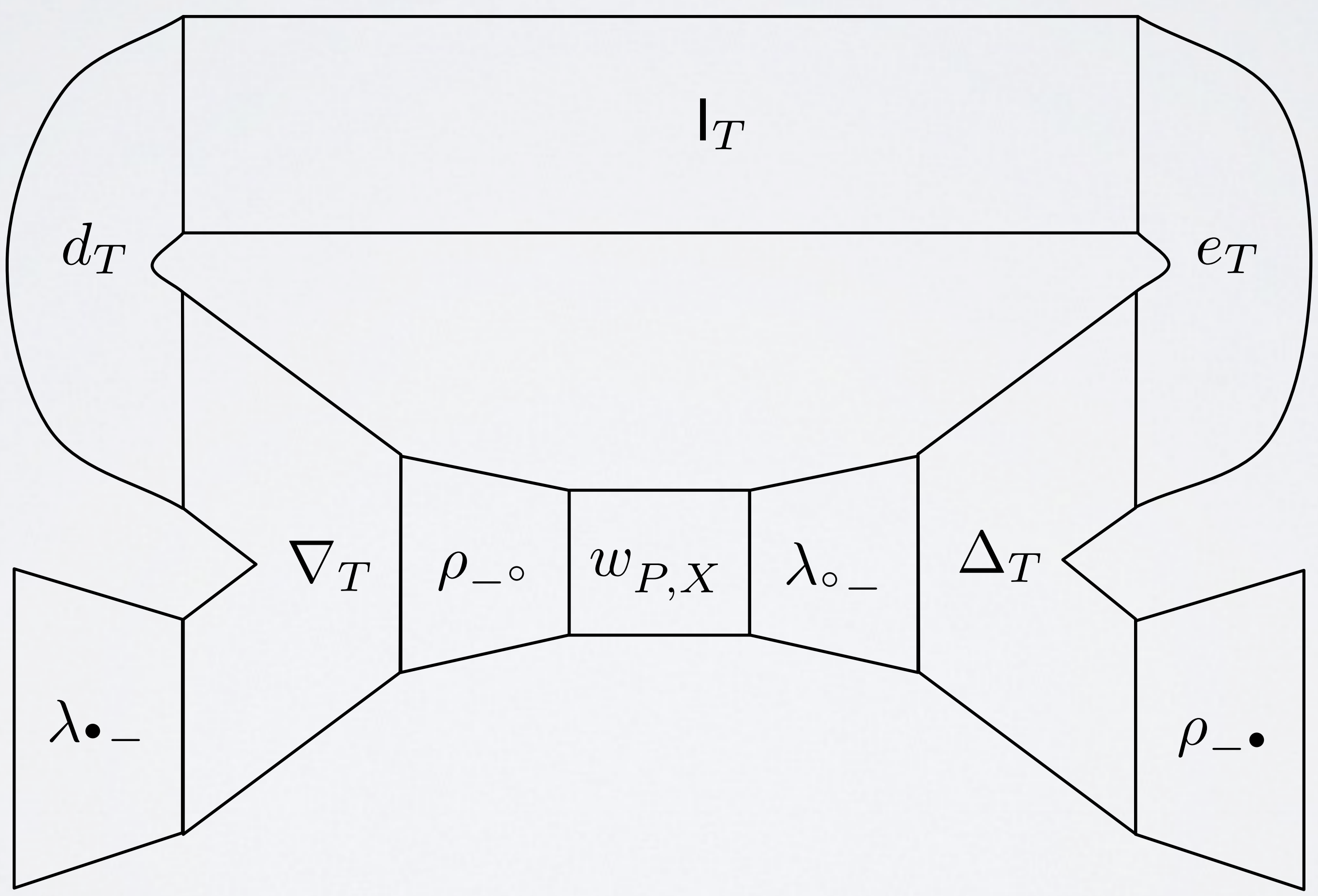
Term in right relational form



PART 3: A “KLEENE THEOREM”

FROM NETS TO SYNTAX

$N \colon m \to n = (P, T, {}^{\circ}-, -^{\circ}, {}^{\bullet}-, -^{\bullet})$ finite net, X marking



THEOREM

$$(N, X) \xrightarrow[\beta]{\alpha} (N, Y) \quad \Rightarrow \quad T_{N,X} \xrightarrow[\beta]{\alpha} T_{N,Y}$$

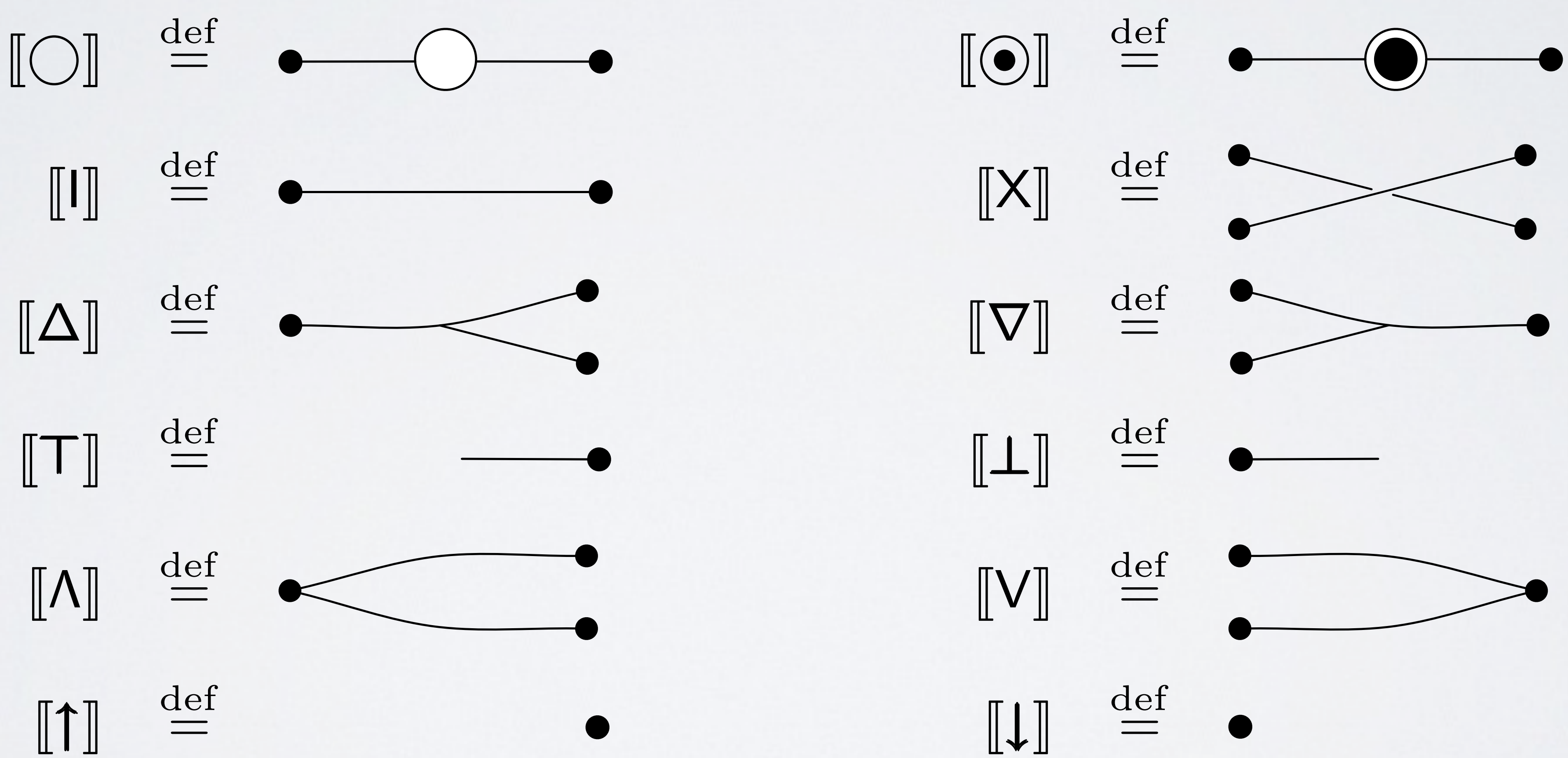
$$T_{N,X} \xrightarrow[\beta]{\alpha} Q$$

$$\Rightarrow$$

$$\exists Q.\; Q = T_{N,Y} \;\wedge\; (N, X) \xrightarrow[\beta]{\alpha} (N, Y)$$

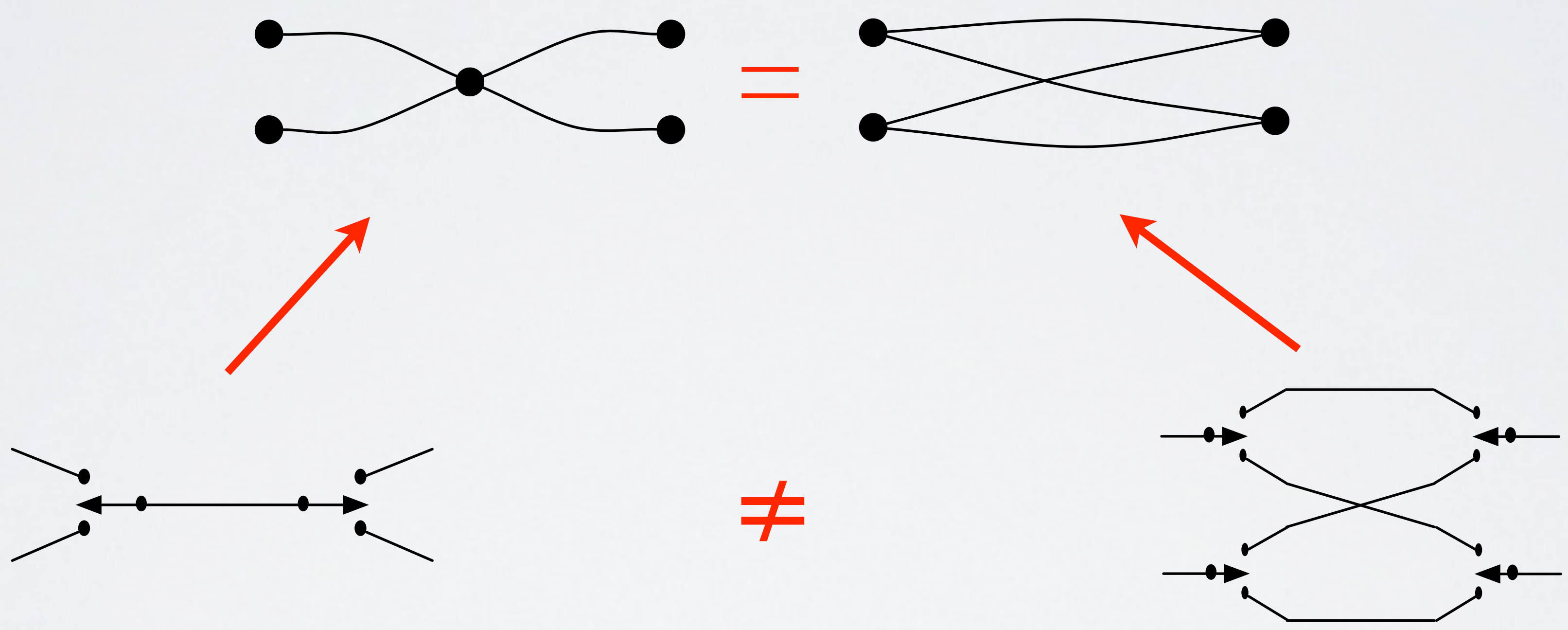
FROM SYNTAX TO NETS

- Naive translation: each syntactic atom corresponds to a simple net, eg.

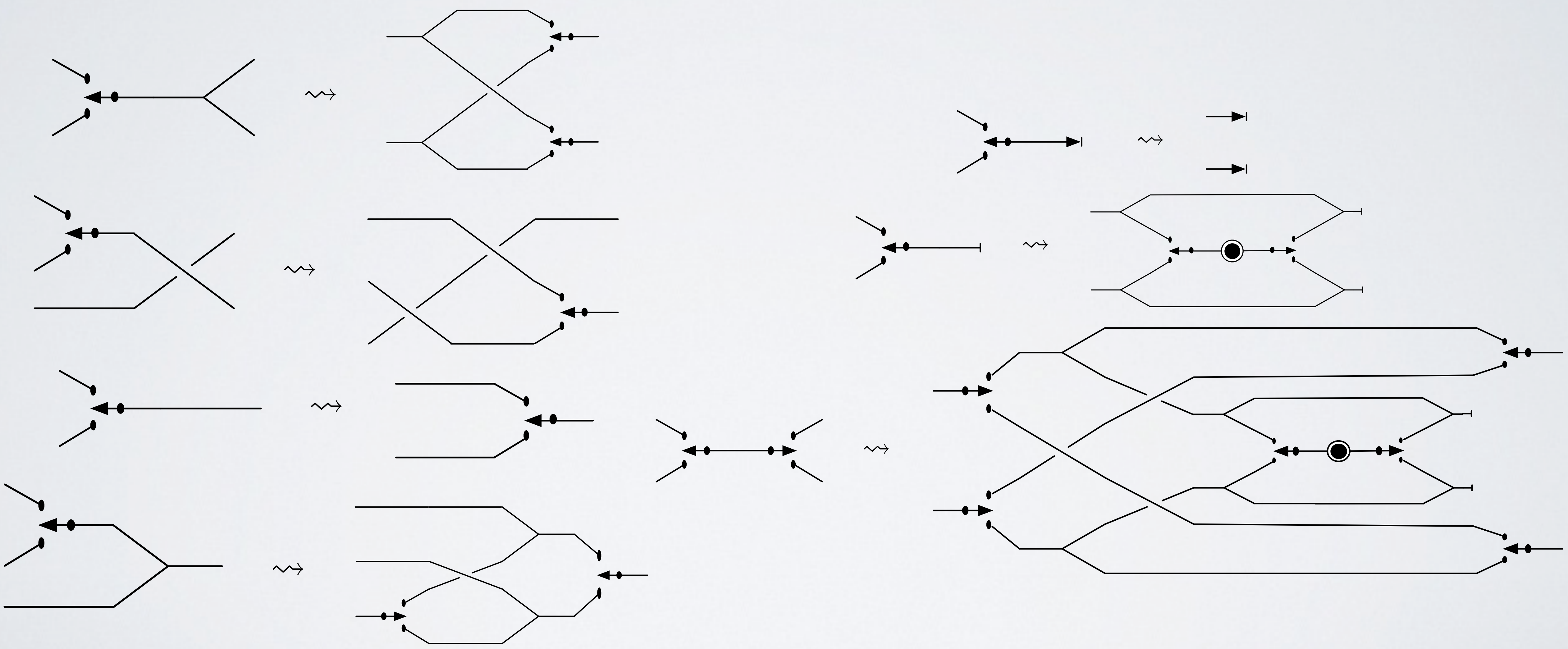


- Then theorem should follow by induction....

PROBLEMATIC COMPOSITIONS



SOLUTION - NORMALISE



THEOREM

- For each term t, there exists a net N_t such that $t \sim N_t$

CONCLUSION

- A “Kleene theorem for concurrency”:
  - Petri nets with boundaries
    - graphical model
    - global semantics
  - “Petri calculus”
    - real syntax (no structural congruence or other cheating)
    - inductive semantics
- Extensions to P/T nets with boundaries, other forms of nets?
  - wires with buffers