Invent. math. 120, 409 – 410 (1995)

Inventiones mathematicae

Addendum

Representations of quantum algebras

H.H. Andersen, P. Polo, K. Wen

Matematisk Institut, Aarhus Universitet, Ny Munkegade, DK-8000 Aarhus C, Denmark

Invent. math. **104**, 1–59 (1991)

It has been pointed out to us (see the Introduction of [1]) that in Section 5 of the above paper we are implicitly using a non-obvious Mackey type result when we apply the results of Section 4. In [1], Lin proves in a more general context that one does indeed have such Mackey properties, at least over a base field (see Remark 2 below).

Here we show that the result we need follows in fact quite easily from arguments in our paper. Let the algebras $U^\flat$ and $U^\flat(i)$, U_i, for $i = 1,\dots,n$, be defined as in our paper, see p. 4 and 2.5. (We take this opportunity to point out that, on p. 4, $U(I)$ should read $U^\flat(I)$.) Let $U_i^\flat$ denote the subalgebra of U_i generated by $K_i^{\pm 1}$ and $F_i^{(r)}$, for $r \geqq 0$. Then, with notation as in our paper, one has:

Proposition 5.0. *Let $M \in \mathscr{C}^\flat$. For any $i = 1,\dots,n$ and $r \geqq 0$, one has natural isomorphisms of U_i-modules*

$$H^r(U^\flat(i)/U^\flat, M)\big|_{U_i} \simeq H^r(U_i/U_i^\flat, M\big|_{U_i^\flat}) .$$

Proof. For $r = 0$ the statement follows directly from the definition of the induction functors in Section 2. To obtain it for all r it is enough (by the usual arguments based on the standard resolutions of 2.17) to check that $H^r(U_i/U_i^\flat, I(\mu)) = 0$, for $r > 0$, where $I(\mu)$ denotes $H^0(U^\flat/U^0, \mathscr{A}_\mu)$, for $\mu \in X$. It follows from the definition that $H^0(U_i/U_i^\flat, -)$ commutes with direct limits. Moreover, by 2.13 the category $\mathscr{C}_i^\flat$ has enough injectives and, using the fact that any object in $\mathscr{C}_i^\flat$ is a union of $\mathscr{A}$-finite $U_i^\flat$-submodules, one obtains that a direct limit of injective objects in $\mathscr{C}_i^\flat$ is injective. By standard arguments it then follows that $H^r(U_i/U_i^\flat, -)$ commutes with direct limits, for $r \geqq 0$. But $I(\mu)$ is the direct limit of $U^\flat$-submodules $V_m \simeq H^0(m\rho) \otimes \mathscr{A}_{m\rho+\mu}$, for $m \geqq 0$, by Lemma 5.3 (note that this lemma does not use the results of 5.1–5.2) and hence, for $r \geqq 0$, $H^r(U_i/U_i^\flat, I(\mu))$ is the direct limit of $H^r(U_i/U_i^\flat, V_m)$, for $m \to \infty$. Finally, using Corollary 3.3 (i) and Proposition 2.16, one obtains

$H^r(U_i/U_i^\flat, V_m) \simeq H^0(m\rho) \otimes H^r(U_i/U_i^\flat, \mathscr{A}_{m\rho+\mu})$ and, by Proposition 4.2, this vanishes for $r > 0$ and $m \gg 0$. The proposition follows.

Remarks. 1. The proposition is used implicitly in 5.1, 5.4–5.8.

2. Let $I \subseteq \{1, \ldots, n\}$. Using Corollary 5.7, applied to the algebra U_I, one then obtains that the proposition generalizes to the case of induction from $U^\flat$ to $U^\flat(I)$. Over a base field this is proved by different arguments in [1].

[1] Lin, Z.: A Mackey Decomposition Theorem and Cohomology for Quantum Groups at Roots of 1. J. Algebra **166**, 100–129 (1994)