Econometrica, Vol. 57, No. 3 (May, 1989), 511–531

REPUTATION IN BARGAINING AND DURABLE GOODS MONOPOLY

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This paper analyzes durable goods monopoly in an infinite-horizon, discrete-time game. We prove that, as the time interval between successive offers approaches zero, all seller payoffs between zero and static monopoly profits are supported by subgame perfect equilibria. This reverses a well-known conjecture of Coase. Alternatively, one can interpret the model as a sequential bargaining game with one-sided incomplete information in which an uninformed seller makes all the offers. Our folk theorem for seller payoffs equally applies to the set of sequential equilibria of this bargaining game.

KEYWORDS: Durable goods monopoly, bargaining, Coase conjecture, reputation, folk theorem.

1. INTRODUCTION

ASSUME THAT A SINGLE FIRM CONTROLS the supply of an infinitely durable good. In a classic paper, Ronald Coase (1972) asked what sales plan this monopolist would adopt to maximize her profits. Coase provided a partial answer by observing that the naive policy of forever offering the good at a static monopoly price is not credible. To paraphrase Martin Hellwig (1975), the monopolist who announces such a policy cannot "keep a straight face"—she has an irresistable temptation to cut the price at future dates, to generate additional sales and profits. Coase supplemented his answer by conjecturing that, with rational consumer expectations, "the competitive outcome may be achieved even if there is but a single supplier." Several subsequent authors have produced models possessing subgame perfect equilibria which support Coase's conjecture.

Nevertheless, Coase's original puzzle concerning the optimal monopoly pricing rule remains essentially unsolved. In this paper, we propose an answer: the firm introduces the durable good at approximately the static monopoly price. She then follows the slowest rate of price descent that enables her to maintain her credibility. As the time interval between successive periods of the game approaches zero, the rate of descent can be made arbitrarily slow while preserving subgame perfection. This enables the supplier to earn nearly static monopoly

[1] The authors' work was supported in part by National Science Foundation Grants SES-85-69701 and IST-86-09129, by the Kellogg School's Paget Research Fund and Esmark Research Chair, and by National Science Foundation Grant SES-83-20464 at the Institute for Mathematical Studies in the Social Sciences, Stanford University, Stanford, California. The appendices make substantial use of ideas and analysis contained in the papers of Fudenberg, Levine, and Tirole (1985) and Gul, Sonnenschein, and Wilson (1986). We learned a great deal from these authors, and we are glad to be able to acknowledge our intellectual debt here. We would also like to thank Varadarajan Chari, Kenneth Judd, David Kreps, and Roger Myerson for helpful suggestions and stimulating discussions, and two anonymous referees for comments which greatly improved the exposition of this paper. The usual disclaimer applies.

profits. Thus it is possible, even in a durable goods market, that a monopoly *is* a monopoly.[2]

The identical reasoning carries over to bargaining games with incomplete information, since essentially the same mathematical model may depict either a continuum of actual consumers with different valuations or a single buyer with a continuum of possible valuations. Thus, in an infinite-horizon bargaining game where the seller makes repeated offers to a buyer whose valuation she does not know, we prove that there exist sequential equilibria where the seller extracts essentially monopoly surplus.

In contrast, the Coase conjecture predicts that the monopolist's initial offer inexorably descends toward marginal cost (and her profits approach zero) as the time interval between periods shrinks to zero. The intuition behind the conjecture is that, once any initial quantity of the good has been sold, the monopolist is always tempted to sell additional output, until the competitive level is reached. But if consumers expect the monopolist to flood the market "in the twinkling of an eye" (Coase (1972)), they will decline to purchase at prices much above marginal cost.

Bulow (1982) analyzed Coase's reasoning in a finite-horizon model. By backward induction, Bulow calculated the monopolist's best action in each period before the last and showed that it is always to charge unambiguously less than the static monopoly price. Stokey (1981) studied the monopolist who lacks commitment powers in an infinite-horizon model. She constructed an equilibrium which is the limit of the unique equilibria of finite-horizon versions of the same model, and demonstrated that it satisfies the Coase conjecture. Gul, Sonnenschein, and Wilson (1986) discovered a continuum of additional subgame perfect equilibria in the infinite-horizon game. However, they proved that these weak-Markov equilibria (subgame perfect equilibria in which buyers use "stationary" strategies) behave qualitatively like the backward induction equilibrium —the initial price converges to marginal cost.[3]

The noncooperative bargaining literature developed in parallel to the study of durable goods monopoly.[4] To escape from Rubinstein's (1982) complete information result that all bargaining is concluded in the first round, this literature introduced incomplete information into the bargaining process. Incomplete information often added a continuum of sequential equilibria (e.g., Fudenberg and Tirole (1983)). One modeling technique, however, offered apparent promise for

[2] This provides an answer to Coase's puzzle by identifying the equilibrium which (subject to credibility) is most favorable to the firm. Given that the firm is a monopolist and has the sole ability to make offers, this equilibrium certainly seems quite sensible. However, we show in Theorem 6.4 that there exists a continuum of other equilibria, in which the monopolist may earn substantially lower profits.

[3] Bond and Samuelson (1984) examined a durable good subject to depreciation. The prospect of replacement sales reduces the monopolist's tendency to cut prices, when the time interval between periods is positive; however, Coase's limiting result continues to hold. Kahn (1986) considered the case of increasing marginal cost and showed that this provides the durable goods monopolist with some incentive not to flood the market instantaneously. Consequently, initial price does not converge to marginal cost.

[4] For a thorough review of this literature, see Rubinstein (1987).

yielding results with greater predictive value: restricting the game to one-sided offers with one-sided uncertainty. When the uninformed party makes all the offers (the informed party only responding with "yes" or "no"), the complications of strategic communication largely disappear. One may have (unjustifiably and, we will show, incorrectly) conjectured from this literature that the multiplicity of equilibrium outcomes vanished as well.

Sobel and Takahashi (1983) wrote the first paper to explore this approach. Their results mirror those obtained by Bulow (1982) and Stokey (1981) in the durable goods monopoly context. Fudenberg, Levine, and Tirole (1985) analyzed two distinct cases in the infinite-horizon game. In the case where the buyer's valuation is known discretely to exceed the seller's, they proved that the model generically has a unique sequential equilibrium and that there is a finite time by which all negotiations conclude. In the second (and, we think, more reasonable) case where there is no gap between the lowest buyer valuation and the seller's valuation, they demonstrated the existence of a backward induction equilibrium. All of the equilibria they constructed are weak-Markov and can be shown to satisfy the Coase conjecture.

The main result of our paper is a folk theorem for seller payoffs, for the "no gap" case. As the time interval between successive periods approaches zero in durable goods monopoly (bargaining), the set of monopolist (seller) payoffs associated with subgame perfect (sequential) equilibria expands to the entire interval from zero to static monopoly profits.

We prove our Folk Theorem by constructing "reputational equilibria" consisting of a main price path and a punishment path. The main path starts with an arbitrary initial price and follows with an arbitrarily-slow (but positive) real-time rate of sales. The punishment path is taken from a weak-Markov equilibrium. As the time interval between periods approaches zero, adherence to the main path becomes subgame perfect, because (by the Coase conjecture) the punishment becomes increasingly severe. Continuously varying the initial price and the subsequent rate of sales yields all levels of profit.

Let us provide an interpretation of these equilibria. Initially, consumers believe they are facing a strong monopolist who will continue to adhere to the main price path specified in the equilibrium. However, the moment a deviation from the main price path occurs, consumers decide they are dealing with a weak monopolist who has read the Coase (1972) paper (and believes its message). The prospect of ruining her reputation thus deters the monopolist from ever deviating.

Observe that the vast multiplicity of equilibria in the current game is not due to the presence of incomplete information. "Reputation," in our equilibria, does not involve the seller's type[5]—buyers have no beliefs to be updated when off-equilibrium behavior is observed. Indeed, the durable goods monopoly model is a game of complete information. In the bargaining interpretation of the model, the only (observable) off-equilibrium buyer behavior which continues the game is

[5] One can also introduce reputation effects by adding buyer uncertainty about the monopolist's marginal cost.

rejection of a nonpositive offer, and so there is little or no scope for the seller to make alternative inferences about the buyer's type. Hence, an equilibrium refinement which acts only to constrain off-equilibrium beliefs about type would have no effect on the set of equilibrium payoffs. Alternatively, one can limit the set of outcomes by restricting attention to weak-Markov equilibria. We do not find this restriction completely natural and, in any case, it is interesting to see what equilibria arise when the Markovian assumption is relaxed.

We extend and use two types of results (which seemingly endorse the Coase conjecture) to prove our folk theorem (which reverses the Coase conjecture). After describing the model (Section 2) and presenting a linear example (Section 3), we first demonstrate a general existence result on weak-Markov equilibria (Section 4). We then show that price paths associated with weak-Markov equilibria are uniformly low compared to the demand curve (Section 5). In Section 6, we proceed to establish the folk theorem for seller payoffs, under very general conditions. We conclude with Section 7.

2. THE MODEL

We consider a market for a good which is infinitely durable, and which is demanded only in quantity zero or one. There is a continuum of infinitely-lived consumers, indexed by $q \in I = [0,1]$. The preferences of these consumers are completely specified by a monotone nonincreasing function f: $[0,1] \rightarrow \mathbb{R}_+$ satisfying $f(q) > 0$ for $q \in [0,1)$, where $f(q)$ denotes the reservation value of customer q, and by a common discount rate r. More precisely, if individual q purchases the good at time t for the price p_t, he derives a net surplus of $e^{-rt}[f(q) - p_t]$. Consumers seek to maximize their net surplus. The monopolist, meanwhile, faces a constant marginal cost of production, which we assume (without loss of generality) to equal zero. Her objective is to maximize the net present value of profits, using the same discount rate as consumers.

The monopolist offers the durable good for sale at discrete moments in time, spaced equally apart. The symbol z will denote the time interval between successive offers, and so sales occur at times $t = 0, z, 2z, \ldots, nz, \ldots$. We will sometimes refer to the "period" n rather than to the "time" t $(= nz)$. Within each period, the timing of moves is as follows: first, the monopolist names a price; then, consumers who have not previously purchased decide whether or not to buy. After a time interval z elapses, play repeats.

A strategy for the monopolist specifies the price she will charge in each period, as a function of the history of prices charged in previous periods and the history of purchases by consumers. A strategy for a consumer specifies, in each period, whether or not to buy in that period, given the current price charged and the history of past prices and purchases. Formally, let $G(z, r)$ denote the above game when the time interval between successive sales is z and payoffs are discounted at the rate r. Let σ be a *pure* strategy for the monopolist. Then σ is a sequence of functions $\{\sigma^n\}_{n=0}^{\infty}$. The function σ^n at date nz determines the monopolist's price in period n as a function of the prices she charged in previous periods and the actions chosen by consumers in the past. We impose measurability restrictions on joint consumer strategies below which imply that the set of consumer acceptances in period n, Q_n, will be a measurable set, i.e., $Q_n \in \Omega$, where Ω is the Borel σ-algebra on I. Then σ^n: $Y^n \times \Omega^n \to Y$, with $Y = [0, f(0)]$ and Y^n and Ω^n the n-fold Cartesian products of Y and Ω. A strategy combination for consumers is a sequence of functions $\{\tau^n\}_{n=0}^{\infty}$ where τ^n: $Y^{n+1} \times \Omega^n \times I \to \{0,1\}$ is such that for each $y^{n+1} \in Y^{n+1}$ and each $B^n \in \Omega^n$, $\tau^n(y^{n+1}, B^n, \cdot)$ is measurable. Decision "0" is to be interpreted as a decision not to buy in the current period; decision "1" indicates that a sale takes place in the current period. Obviously, we require that $\tau^n(y^{n+1}, B^n, q) = 0$ for all $q \in \bigcup_{j=0}^{n-1} Q_j$.[6]

Let Σ be the pure strategy space for the monopolist, and Υ be the set of pure strategy combinations for consumers. The strategy profile $\{\sigma, \tau\}$, with $\tau = \{\tau^n\}_{n=0}^{\infty}$, generates a path of prices and sales which can be computed recursively. The pattern of prices and sales over time in turn determines the payoffs to the players. Let $\pi(\sigma, \tau)$ be the net present value of profits generated by the strategy profile $\{\sigma, \tau\}$, and let $u_q(\sigma, \tau)$ be the discounted net surplus derived by consumer q. The profile $\{\sigma, \tau\}$ is a Nash equilibrium if and only if

$$\pi(\sigma, \tau) \geqslant \pi(\sigma', \tau), \qquad \forall \sigma' \in \Sigma, \qquad \text{and}$$
$$u_q(\sigma, \tau_q, \tau_{-q}) \geqslant u_q(\sigma, \tau_q', \tau_{-q}), \qquad \forall \tau_q' \in \Upsilon_q, \ q\text{-a.e.},$$

where τ_q is the projection of τ onto the qth component (and similarly for Υ_q). An n-period history of the game is a sequence of prices in periods $0, \ldots, (n-1)$ and a specification of the set of consumers who bought in each period prior to n. We denote such a history by the symbol H_n. Thus, $H_n \in Y^n \times \Omega^n$. The symbol H_n' refers to H_n followed by a price announced by the monopolist in period n. Thus, $H_n' \in Y^{n+1} \times \Omega^n$. The strategy profile (σ, τ) induces strategy profiles $(\sigma|_{H_n}, \tau|_{H_n})$ and $(\sigma|_{H_n'}, \tau|_{H_n'})$, after the histories H_n and H_n', respectively. The strategy pair (σ, τ) is a *subgame perfect* equilibrium if and only if $(\sigma|_{H_n}, \tau|_{H_n})$ is a Nash equilibrium in the game remaining after the history H_n, for all n and all H_n, and similarly after any history H_n'. In order to ensure the existence of an equilibrium, we will have to allow the monopolist to mix at any stage of the game. $\hat{\Sigma}$ will denote her set of behavioral strategies. It should be clear to the reader how to extend the above definitions when behavioral strategies are allowed. We will henceforth restrict attention to equilibria in which deviations by sets of measure zero of consumers change neither the actions of the remaining consumers nor those of the monopolist. This requirement reflects our quest for equilibria in which consumers act as price takers.[7] Since $f(\cdot)$ is monotone, and given the measure-zero restriction, there is no further loss of generality in assuming that $f(\cdot)$ is left-continuous.

[6] For notational simplicity, we chose to indicate the domain of definition of σ^n to be $Y^n \times \Omega^n$ and the domain of definition of τ^n to be $Y^{n+1} \times \Omega^n \times I$. However, only elements of Ω^n consistent with the restriction that a consumer can accept at most one offer by the monopolist can occur. One should restrict the domain of definition of σ^n and τ^n accordingly.

[7] This restriction may affect the equilibrium set, as demonstrated in Gul, Sonnenschein, and Wilson (1986, p. 170).

Let $q_n = m(\bigcup_{j=0}^{n-1} Q_j)$ be the (Lebesgue) measure of customers who have purchased. The next lemma, whose proof is given in Fudenberg, Levine, and Tirole (1985, Lemma 1), will imply that, along any equilibrium path, the remaining buyer valuations are a truncated sample of the original ones. Consequently, the single number q_n (incompletely) summarizes prior consumer actions.

LEMMA 2.1: *In any subgame perfect equilibrium, after any history H_n, and for any current price p, there exists a cutoff valuation $\beta(p, H_n)$ such that every consumer with valuation exceeding $\beta(p, H_n)$ accepts the monopolist's offer of p and every consumer with valuation less than $\beta(p, H_n)$ rejects.*

In general, a buyer's accept/reject decision may depend not only on the current price, p, but also on the history, H_n. We define a *weak-Markov equilibrium* to be a subgame perfect equilibrium in which (after histories that contain no simultaneous buyer deviations of positive measure; see footnote 9, below) the accept/reject decisions of all (remaining) buyers depend only on the current price. The set of all weak-Markov equilibria is denoted by the symbol $E^{wm}(f, z)$.

The buyer's strategy in a weak-Markov equilibrium can be described by an acceptance function $P(\cdot)$, where consumer q accepts a price p if and only if $p \leqslant P(q)$. When $f(\cdot)$ is strictly monotone, Lemma 2.1 implies that $P(\cdot)$ is nonincreasing. When $f(\cdot)$ has flat sections, $P(\cdot)$ may be nonmonotone. However, any consumers who violate monotonicity for $P(\cdot)$ have identical valuations, so by permuting them, we may (without loss of generality) assume that $P(\cdot)$ is monotone. Since deviations by sets of measure zero of consumers do not affect the equilibrium, we further assume (still without loss of generality) that $P(\cdot)$ is a left-continuous, nonincreasing function. Thus (after histories that contain no buyer deviations), the set of remaining buyers is an interval $(q, 1]$, where $0 \leqslant q \leqslant 1$.

For a given weak-Markov equilibrium, consider the net present value of profits to the monopolist after any history for which the set of remaining buyers (except for sets of measure zero) equals $(q, 1]$. Since buyer acceptances depend only on the prices which the monopolist will henceforth charge, this value is a function $R(\cdot)$ of q only, and must satisfy the dynamic programming equation:

$$(2.1) \qquad R(q) = \max_{y \in [q,1]} \{P(y)(y-q) + \delta R(y)\},$$

where $\delta \equiv e^{-rz}$. Observe that $P(y) > 0$, for $y \in [0,1)$, and hence that $R(y) > 0$ on the same domain.[8] Consequently, (2.1) implies that there are sales in every period in any weak-Markov equilibrium, until consumers are exhausted. Moreover, in the case of "no gap" (i.e., $f(1) = 0$), sales necessarily occur over infinite

[8] In any subgame perfect equilibrium, the monopolist only charges nonnegative prices (Fudenberg, Levine, and Tirole (1985, Lemma 2)). Thus, a rational consumer y will always accept a price of $(1-\delta)f(y)$, as $f(y) - (1-\delta)f(y) \geqslant \delta^k\{f(y) - p\}$, for all $p \geqslant 0$ and $k \geqslant 1$. Since $f(y) > 0$ for all $y \in [0,1)$, this establishes $P(y) \geqslant (1-\delta)f(y) > 0$.

time. This is because only nonpositive prices can clear the market entirely and because it is suboptimal for the monopolist to ever charge a nonpositive price (since $P(y) > 0$ for $y \in [0,1)$).

Let $T(q)$ be the argmax correspondence in (2.1) and let $t(q) = \inf\{T(q)\}$. Then the monopolist's equilibrium action when customers $(q,1]$ remain is always to charge a price of $P(y)$, for some $y \in T(q)$. Since $T(\cdot)$ is monotone, it is single-valued except at possibly a countable set of q. Excluding this set, the monopolist's action depends only on the summary statistic q, and in fact is to charge the price $S(q) \equiv P(t(q))$. Meanwhile, suppose that the set of remaining customers was brought to $(q,1]$ by an offer of $P(q)$, where q has the property that $T(q)$ is single-valued. Buyer optimization requires that consumer q was indifferent between the price $P(q)$ and the deferred offer $S(q)$. Consequently:

$$f(q) - P(q) = \delta[f(q) - S(q)]. \tag{2.2}$$

When $T(q)$ is multiple-valued, the monopolist may now mix among prices in the set $P(T(q)) \equiv \{P(y)\colon\ y \in T(q)\}$. A variant of (2.2) still holds, where $S(q)$ is replaced by an element of the convex hull of $P(T(q))$ which has the interpretation of expected price. If p_{-1} was the price charged in the previous period, the monopolist should now play a (possibly) mixed strategy such that the expected price, $\bar{p}$, satisfies:

$$\begin{aligned} &f(q) - p_{-1} \geqslant \delta[f(q) - \bar{p}], && \text{but} \\ &f(q') - p_{-1} \leqslant \delta[f(q') - \bar{p}], && \text{for all } q' \in (q,1]. \end{aligned} \tag{2.3}$$

Such a mixed strategy justifies the decision of q to purchase in the previous period and of all $q' \in (q,1]$ to reject. Proposition 4.3 will demonstrate that randomization cannot occur along the equilibrium path except, possibly, in the initial period.

Equation (2.2) and inequality (2.3) establish that it is sufficient for a monopolist, in optimizing against consumers who use an acceptance function $P(q)$, to utilize a strategy which only depends on q and the previous price p_{-1}. It is convenient to restrict attention to equilibria which have this property. We will henceforth consider this restriction part of the definition of weak-Markov equilibrium. Note, via Proposition 4.3, that requiring the monopolist to condition only on q and p_{-1} does not affect the players' equilibrium payoffs attainable in weak-Markov equilibria.

Perhaps a more natural Markovian restriction would be to limit the monopolist to condition her strategy on the payoff-relevant part of the history, namely q, only. Unfortunately, such equilibria (termed strong-Markov equilibria) do not, in general, exist (see Fudenberg, Levine, and Tirole (1985)). In fact, if (P, R) is associated with a weak-Markov equilibrium and $P(\cdot)$ is discontinuous, it is possible to show that randomization is (generically) necessary whenever p_{-1} lies in a discontinuity of the range of $P(\cdot)$.

One final remark: for expositional ease, all of our subsequent definitions, theorems and proofs will be phrased in the language of durable goods monopoly.

However, all of our results also hold for the bargaining game, provided one substitutes "sequential equilibrium" whenever the phrase "subgame perfect equilibrium" appears. It should then be understood that if $F(v)$ denotes the (commonly known) distribution function of buyer valuations, $F(v) = 1 - y$, where $y = \inf\{q\colon f(q) = v\}$. Furthermore, q_n then corresponds to the seller's point of truncation, after history H_n, of her prior distribution on the buyer's valuation.

3. A LINEAR EXAMPLE

Consider a linear demand example with unit slope and unit intercept, i.e., $f(q) = 1 - q$. Let z be the time interval between periods. For this case, Stokey (1981) and Sobel and Takahashi (1983) proved the existence of a strong-Markov equilibrium in which the monopolist charges a price equalling $\alpha_z(1-q)$ after any history in which all consumers $(q, 1]$ remain, and earns a corresponding profit of $R(q) = (\alpha_z/2)(1-q)^2$. These authors also showed that $\lim_{z\downarrow 0}\alpha_z = 0$, thereby confirming the Coase conjecture.

We will now indicate how to construct reputational equilibria which yield the monopolist, for sufficiently small z, essentially any payoff between zero and static monopoly profits. Consider a strategy in which the monopolist follows an exponentially descending price path $p(t) = p_0 e^{-\eta t}$ (confined to the grid of times $\{0, z, 2z, \dots\}$), as long as no deviation from this rule has occurred in the past, and reverts to the strong-Markov equilibrium described above, otherwise. Consumers adopt strategies which are optimal given this behavior.

Fix the real-time rate of descent $\eta > 0$ to be sufficiently slow that, independently of z, the sales in the initial period are bounded away from zero. For any time interval $z > 0$, let the (equilibrium path) price in period n be $p_n \equiv p(nz)$. Then, by consumer indifference, the set of consumers remaining after period n equals $(q_{n+1}, 1]$, where q_{n+1} satisfies: $f(q_{n+1}) - p_n = \delta[f(q_{n+1}) - p_{n+1}]$. Hence, $f(q_{n+1}) = p_n[(1-\delta e^{-\eta z})/(1-\delta)]$. Since demand is linear, this establishes that sales exponentially descend at the same rate η, and that the price and sales in every period are constant multiples of $(1-q_n)$. Consequently, along the equilibrium path, the continuation profits evaluated in any period $n \geqslant 1$ are a constant multiple, λ_z, of $(1-q_n)^2$. Observe that as the interval z approaches zero, consumers purchase at arbitrarily close to the times that they would against a continuous-time price path $p_0 e^{-\eta t}$. Thus, $\lambda_z \to \lambda > 0$, where λ is the constant derived from a profit calculation along a continuous time path.

In every period, the monopolist must weigh continuation profits against the payoff from optimally deviating. Any deviation causes the consumers to instantly adopt the acceptance function from the strong-Markov equilibrium. Hence, the optimal derivation when customers $(q_n, 1]$ remain yields profits of exactly $R(q_n) = (\alpha_z/2)(1-q_n)^2$. Observe that there exists $z_1 > 0$ such that whenever the time interval satisfies $0 < z < z_1$, we have $\alpha_z/2 < \lambda_z$, deterring deviations from the continuation path in all periods $n \geqslant 1$. Meanwhile, let $\pi_0(\eta, z)$ denote the seller's equilibrium profits evaluated in period zero. Since $\lim_{z\downarrow 0}\pi_0(\eta, z) > 0$, there exists z_2 $(0 < z_2 < z_1)$ such that whenever the time interval satisfies $0 < z < z_2$, we have $\alpha_z/2 < \pi_0(\eta, z)$, deterring deviations in period zero as well.

Finally, note that $\lim_{z \downarrow 0} \pi_0(\eta, z) = p_0(1 - p_0)$ and that static monopoly profits equal $1/4$. We conclude that by continuously varying the initial price p_0 and the rate of descent η, and by making z sufficiently small, *every* level of profits in $(0, 1/4)$ can be sustained.

4. EXISTENCE OF WEAK-MARKOV EQUILIBRIA

In order to extend the reasoning of the previous section to general demand curves, we need to demonstrate two facts which were demonstrated by formula for the linear case. We lay this groundwork here and in the next section. First, we show the existence of weak-Markov equilibria for general demand curves (see also Appendix A). This gives us well-defined secondary paths, reverted to in case of deviation from the proposed equilibrium path. Then, in Section 5, we demonstrate that these secondary paths become uniformly low (compared to the highest valuation remaining) as z approaches zero, enabling them to be effective deterrents.

We begin by defining general demand curves.

DEFINITION 4.1: An (inverse) *demand curve* f is a nonnegative-valued, left-continuous, nonincreasing function on $[0,1]$ which, without loss of generality, we normalize so that $f(0) = 1$ and $f(q) > 0$ whenever $0 \leqslant q < 1$.

Using this terminology, we prove the following theorem in Appendix A:[9]

THEOREM 4.2 (Existence of Weak-Markov Equilibria): *Let f be any (inverse) demand curve. Then for every $r > 0$ and every $z > 0$, there exists a weak-Markov equilibrium.*

This theorem strengthens results by Fudenberg-Levine-Tirole (1985), who prove existence for differentiable demand curves with derivative bounded below and above, and Gul-Sonnenschein-Wilson (1986), who prove existence for demand curves with $f(1) > 0$ that satisfy a Lipschitz condition at 1. For example, Theorem 4.2 extends existence to nondifferentiable and possibly discontinuous curves with $f(1) = 0$. It also contains the case where $f(1) > 0$ but $f'(1)$ is infinite. The proof of Theorem 4.2 may be of some general interest, since it uses a version

[9] We follow the tradition of Gul, Sonnenschein, and Wilson (1986, pp. 159–160) of not specifying equilibrium behavior following *simultaneous* deviations by consumers. This is formally correct in the case of a durable goods monopoly since any such deviation will lead to a rescaled (see Definition 5.2) demand curve satisfying the conditions of Theorem 4.2. We can thus specify a subgame perfect equilibrium which is played from that node onward. Note, however, that neither of the prior existence theorems would guarantee existence of equilibria following (off-equilibrium) nonmonotone purchase behavior by consumers. Notice also that in the bargaining interpretation of the model, simultaneous deviations are unobservable and, hence, are not an issue.

of the maximum theorem which does not require objective functions to be continuous. In Appendix A, we also establish the following theorem:

PROPOSITION 4.3: *Along any weak-Markov equilibrium path, the monopolist does not randomize, except (possibly) in the initial period.*

5. THE UNIFORM COASE CONJECTURE

In this section, we strengthen the "Coase conjecture" by presenting a theorem that guarantees *uniformly* low prices for all weak-Markov equilibria of *families* of demand curves.

While the uniform Coase conjecture is of independent interest, we require it here as an intermediate step for use in the main result of the paper: the folk theorem of Section 6. It should be observed that there is a straightforward reason why we did not need to examine families of demand curves to treat the linear case in Section 3: given linear demand, every derived residual demand curve is linear as well.[10] For generic demand curves, however, the residual demand curves are no longer rescaled versions of the original one. Thus, considerations of subgame perfection lead us naturally to study families of demand curves. We will demonstrate, for all residual demand curves arising from a demand curve f, that all price paths derived from weak-Markov equilibria are uniformly low compared to the highest remaining consumer valuation. This establishes that weak-Markov price paths may be used to deter deviation from the main price paths of reputational equilibria.

Define $\mathscr{F}_{L,M,\alpha}$ to be the family of demand curves which are enveloped by scalar multiples of demand curves $(1-q)^{\alpha}$, for some positive α. To be precise, we have the following definition:

DEFINITION 5.1: For $0 < M \leqslant 1 \leqslant L < \infty$ and $\alpha > 0$, $\mathscr{F}_{L,M,\alpha}$ is the set of all (inverse) demand curves $f(\cdot)$ such that $M(1-x)^{\alpha} \leqslant f(x) \leqslant L(1-x)^{\alpha}$, for all $x \in [0,1]$.

The only significant restriction implicit in the definition of $\mathscr{F}_{L,M,\alpha}$ is that $f(1) = 0$. Otherwise, the family is very general. It allows, for example, differentiable demand curves with derivatives bounded above and bounded away from zero, demand curves which are not Lipschitz-continuous at 1, and demand curves which are severely discontinuous.

Let us also define a rescaled residual demand curve as a normalized version of the demand that remains after any proportion of customers have purchased:

[10] Identical reasoning applies to the case where $f(q) = (1-q)^{\alpha}$, considered by Sobel and Takahashi (1983). Furthermore, this is the only family that is closed under the joint operation of truncation and rescaling (see Definition 5.2).

DEFINITION 5.2: Let f be any demand curve. We define f_q to be the *rescaled residual (inverse) demand curve of f at q* $(0 \leqslant q < 1)$ by:

$$f_q(x) = \frac{f[q+(1-q)x]}{f(q)}, \qquad \text{for all } x \in [0,1].$$

LEMMA 5.3: *If $f \in \mathscr{F}_{L,M,\alpha}$, then for every q $(0 \leqslant q < 1)$, $f_q \in \mathscr{F}_{L/M,M/L,\alpha}$.*

PROOF: Observe that

$$\begin{aligned} f(q+(1-q)x) &= f(1-(1-q)(1-x)) \\ &\leqslant L(1-q)^{\alpha}(1-x)^{\alpha} \end{aligned}$$

and

$$f(q+(1-q)x) \geqslant M(1-q)^{\alpha}(1-x)^{\alpha}.$$

Also,

$$M(1-q)^{\alpha} \leqslant f(q) \leqslant L(1-q)^{\alpha},$$

so

$$\frac{M}{L}(1-x)^{\alpha} \leqslant \frac{f[q+(1-q)x]}{f(q)} \leqslant \frac{L}{M}(1-x)^{\alpha},$$

proving the desired result. *Q.E.D.*

Let $L = L'/M'$ and $M = M'/L'$. Lemma 5.3 demonstrates that if $f \in \mathscr{F}_{L',M',\alpha}$ then all residual demand curves arising from f are elements of $\mathscr{F}_{L,M,\alpha}$. Hence, if we can show that the *initial price* is uniformly low for all demand curves in the family $\mathscr{F}_{L,M,\alpha}$, then we will also have established that all price paths arising from weak-Markov equilibria are uniformly low (compared to residual demand). We prove this fact in the following theorem:

THEOREM 5.4 (The Uniform Coase Conjecture): *For every $L \geqslant 1$, $0 < M \leqslant 1$, $0 < \alpha < \infty$ and $\varepsilon > 0$, there exists $\bar{z}(L, M, \alpha, \varepsilon)$ such that for every $f \in \mathscr{F}_{L,M,\alpha}$, for every z satisfying $0 < z < \bar{z}(L, M, \alpha, \varepsilon)$, and for every weak-Markov equilibrium $(P, R) \in E^{wm}(f, z)$, the monopolist charges an initial price less than or equal to ε (and earns profits less than ε).*

PROOF: See Appendix B.

A uniform Coase conjecture also holds when $f(1) > 0$. Consider the family of demand curves satisfying $f(1) \leqslant c$ and $M(1-x)^{\alpha} \leqslant f(x) - f(1) \leqslant L(1-x)^{\alpha}$, for all $x \in [0,1]$. Then an analogous result to Theorem 5.4 holds for this family, provided one substitutes "$f(1) + \varepsilon$" for the bound on the initial price and profits.

6. THE FOLK THEOREM FOR SELLER PAYOFFS

In this section, we prove Theorem 6.4, the main result of the paper. First, let $SE(f, r, z)$ denote, for the durable goods monopoly model, the set of all monopolist payoffs arising from subgame perfect equilibria when the demand curve is f, the interest rate is r, and the time interval between periods is z. For the bargaining game with one-sided incomplete information, the same expression denotes the set of all seller payoffs arising from sequential equilibria. Theorem 6.4 will establish that $SE(f, r, z)$ expands to the entire interval from zero to static monopoly profits, as the time interval z approaches zero. Its proof utilizes the fact that $f \in \mathscr{F}_{L,M,\alpha}$ has the uniform Coase property, which we now define:

DEFINITION 6.1: We will say that f has the *uniform Coase property* if, for some $z_1 > 0$:

(6.1) there exists a subgame perfect equilibrium (σ_z, τ_z) for all games with time interval z between periods, where $0 < z < z_1$, and,

(6.2) for every $\varepsilon > 0$, there exists $\bar{z}(\varepsilon)$ $(0 < \bar{z}(\varepsilon) < z_1)$ such that $S_z(q)/f(q) \leqslant \varepsilon$, for all z $(0 < z < \bar{z}(\varepsilon))$ and all q $(0 \leqslant q < 1)$,

where $S_z(q)$ denotes the supremum of all prices that the monopolist charges using strategy σ_z when the current state equals q (the supremum is taken over all possible price histories).

LEMMA 6.2: *If $f \in \mathscr{F}_{L,M,\alpha}$, then f has the uniform Coase property.*

PROOF: Suppose $f \in \mathscr{F}_{L,M,\alpha}$. Then by Theorem 4.2 there exists $(\sigma_z, \tau_z) \in E^{wm}(f, z)$ for all $z > 0$. We wish to show that $\{\sigma_z, \tau_z\}_{z>0}$ satisfies (6.2). Observe, for any z, that (σ_z, τ_z) induces a weak-Markov equilibrium for any residual demand curve arising from f. Define $(\sigma_{z,q}, \tau_{z,q})$ by multiplying all prices in (σ_z, τ_z) by $f(q)$; observe that $(\sigma_{z,q}, \tau_{z,q})$ is a weak-Markov equilibrium for the rescaled residual demand curve f_q, for all $0 \leqslant q < 1$. (See Definition 5.2.)

Using the notation in Definition 6.1, observe that $S_z(q) = f(q)S_{z,q}(0)$. Further observe, by Lemma 5.3, that $f_q \in \mathscr{F}_{L',M',\alpha}$, where $L' \equiv L/M$ and $M' \equiv M/L$. By the uniform Coase conjecture (Theorem 5.4), for every $\varepsilon > 0$, there exists $\bar{z}(\varepsilon)$ such that the initial price in any weak-Markov equilibrium is less than ε, for any z $(0 < z < \bar{z})$ and any demand curve in $\mathscr{F}_{L',M',\alpha}$. We conclude that (6.2) holds. *Q.E.D.*

Fudenberg, Levine, and Tirole (1985) have shown that $S_z(q) \geqslant f(1)$, for all $q \in [0,1)$, in any subgame perfect equilibrium. Hence, in the case of a "gap" (i.e., $f(1) > 0$), the uniform Coase property cannot hold. Indeed, these authors and Gul, Sonnenschein, and Wilson (1986) demonstrated that when $f(1) > 0$ and $f(q) - f(1) \leqslant L(1-q)$, for some $L < \infty$, there exists a (generically) unique subgame perfect equilibrium. Obviously, if $f \in \mathscr{F}_{L,M,\alpha}$, there is no gap, and hence there is scope for a folk theorem.

Let p^i denote the price actually charged in period i $(0 \leqslant i \leqslant n-1)$.

DEFINITION 6.3: For any $\vec{p} = \{p_n\}_{n=0}^{\infty}$, any $\vec{q} = \{q_n\}_{n=0}^{\infty}$, and any monopolist strategy σ, define the *reputational price strategy* $(\vec{p}, \vec{q}, \sigma)$ by:

$$p^n = \begin{cases} p_n, & \text{if } p^i = p_i \text{ and } Q_i = [q_i, q_{i+1}] \text{ for all } i \ (0 \leqslant i \leqslant n-1), \\ \sigma^n, & \text{otherwise,} \end{cases}$$

where the set equality is up to sets of measure zero.

We will further call $(\vec{p}, \vec{q}, \sigma)$ a *reputational equilibrium* if this reputational price strategy, in conjunction with optimal consumer behavior, forms a subgame perfect equilibrium.

Observe that the definition of reputational equilibrium requires that strategy σ, by itself, be associated with a subgame perfect equilibrium. A reputational equilibrium is defined analogously for the bargaining game; note that "$\vec{q}$" is then omitted. We can now state and prove the main result.

THEOREM 6.4 (The Folk Theorem for Seller Payoffs): *Let f belong to $\mathscr{F}_{L,M,\alpha}$ and let π^* denote static monopoly profits. Then for every real interest rate $r > 0$ and for every $\varepsilon > 0$, there exists a $\bar{z} > 0$ such that whenever the time interval between successive offers satisfies $0 < z < \bar{z}$:*

$$(6.3) \qquad [\varepsilon, \pi^* - \varepsilon] \subset SE(f, r, z).$$

PROOF: By Lemma 6.2, f has the uniform Coase property. Let $\{\sigma_z, \tau_z\}_{0<z<z_1}$ be the family of subgame perfect equilibria guaranteed by (6.1), and let $\{S_z\}_{0<z<z_1}$ be defined as in Definition 6.1. Define the function $g(z) = \sup\{S_x(q)/f(q)\colon 0 < x \leqslant z \text{ and } 0 \leqslant q < 1\}$. Observe that $g(z)$ is well defined for $0 < z < z_1$, since S_z is defined and $S_z(q)/f(q)$ is uniformly bounded above by 1. Further observe, by (6.2), that $\lim_{z\to 0} g(z) = 0$. By definition, $q_0 = 0$. Choose arbitrary sales q_1 in period zero ($0 \leqslant q_1 < 1$). Let us define an exponential rate of subsequent sales by:

$$(6.4) \qquad 1 - q_{n+1} = e^{-naz(rz+g(z))}(1 - q_1), \qquad \text{for any } a > 0 \text{ and all } n \geqslant 0.$$

Our first step is to construct a price sequence $\{p_n\}_{n=0}^{\infty}$ which yields sales of $(q_{n+1} - q_n)$ in period n (for all $n \geqslant 0$). Observe that if $0 < q_1 < 1$, then (6.4) implies sales in all periods. To make consumer q_{n+1} indifferent between purchasing at price p_n in period n and at price p_{n+1} one period later, we must have:

$$(6.5) \qquad f(q_{n+1}) - p_n = \delta[f(q_{n+1}) - p_{n+1}], \qquad \text{for all } n \geqslant 0,$$

where $\delta = e^{-rz}$. Solving for p_n and telescoping the resulting summation yields:

$$(6.6) \qquad p_n = (1 - \delta) \sum_{k=0}^{\infty} \delta^k f(q_{n+1+k}), \qquad n \geqslant 0.$$

Furthermore, the price sequence $\vec{p} \equiv \{p_n\}_{n=0}^{\infty}$ implied by (6.4) and (6.6) satisfies $f(q_{n+1}) \geqslant p_n$ (for all $n \geqslant 0$) and equation (6.5), proving that consumers optimize along the sales path $\vec{q} \equiv \{q_n\}_{n=1}^{\infty}$.

CLAIM 1: For any q_1 ($0 \leqslant q_1 < 1$), there exists $a > 0$ and $\bar{z} > 0$ such that $(\vec{p}, \vec{q}, \sigma_z)$ defined by (6.4) and (6.6) is a reputational equilibrium for all z satisfying $0 < z < \bar{z} < z_1$.

PROOF OF CLAIM 1: Let π_n denote profits starting from period n if the price path $\vec{p}$ is followed in all periods. Define m to be the least integer greater than

$1/z$. Certainly $\pi_n \geqslant \delta^{m-1}[q_{n+m} - q_n]p_{n+m}$. Observe that $\delta^{m-1} \equiv e^{-(m-1)rz} \geqslant e^{-r}$ and, by (6.4), $q_{n+m} - q_n = (1-q_n) - (1-q_{n+m}) \geqslant (1-q_n)(1-e^{-a(rz+g(z))})$, for all $n \geqslant 1$. Meanwhile, by (6.6), $p_{n+m} \geqslant (1-\delta)\Sigma_{k=0}^{m-1}\delta^k f(q_{n+m+1+k}) \geqslant f(q_{n+2m})(1-\delta)\Sigma_{k=0}^{m-1}\delta^k \geqslant (1-e^{-r})f(q_{n+2m})$. Hence:

$$\pi_n \geqslant e^{-r}(1-q_n)(1-e^{-a(rz+g(z))})(1-e^{-r})f(q_{n+2m}), \qquad n \geqslant 1, \tag{6.7}$$

and by similar reasoning,

$$\pi_0 \geqslant q_1 p_0 \geqslant q_1(1-e^{-r})f(q_m). \tag{6.8}$$

Now, let π_n^z denote profits starting from period n if (σ_z, τ_z) is followed. Let q $(0 \leqslant q < 1)$ denote a customer and let p_q denote the price at which customer q purchases, according to (σ_z, τ_z). Let p_q' denote the next (expected) price charged after p_q, following σ_z. Observe by the definition of $g(z)$ that $p_q' \leqslant g(z)f(q)$. By consumer optimization, $f(q) - p_q \geqslant \delta[f(q) - p_q']$. Together these inequalities imply $p_q \leqslant [1-\delta+\delta g(z)]f(q)$, for all q $(0 \leqslant q < 1)$, and so:

$$\pi_n^z \leqslant [1-\delta+\delta g(z)]\int_{q_n}^1 f(q)\,dq, \qquad \text{for all } n \geqslant 0. \tag{6.9}$$

Observe that the bound of (6.9) is a consequence of the uniform Coase property, but does not follow from the ordinary Coase conjecture.

Let $a = (8L/M)[e^{-r}(1-e^{-r})]^{-1}$. To prove subgame perfection, we must show that $\pi_n \geqslant \pi_n^z$, for all $n \geqslant 0$. Observe that there exists z_2 such that for all z $(0 < z < z_2)$: $1 - e^{-a[g(z)+rz]} > (a/2)[g(z)+rz]$. Hence, (6.7) implies that $\pi_n > 4(L/M)(g(z)+rz)(1-q_n)f(q_{n+2m})$ for all $n \geqslant 1$ and $0 < z < z_2$. Since $q_n < q_{n+2m} < 1$ and f is monotone nonincreasing:

$$\int_{q_n}^1 f(q)\,dq \leqslant (q_{n+2m} - q_n)f(q_n) + (1-q_{n+2m})f(q_{n+2m}).$$

Observe that $f \in \mathscr{F}_{L,M,\alpha}$ implies $f(q) \leqslant (L/M)\beta^{-\alpha}f(q')$ whenever $(1-q')/(1-q) = \beta$. By (6.4), $(1-q_{n+2m})/(1-q_n) = \beta(z)$, for all n, where $\lim_{z\to 0}\beta(z) = 1$. Consequently, there exists $z_3 > 0$ such that $f(q_n) \leqslant (2L/M)f(q_{n+2m})$, and hence $\int_{q_n}^1 f(q)\,dq \leqslant (2L/M)(1-q_n)f(q_{n+2m})$, for all $n \geqslant 1$ and for all z $(0 < z < z_3)$. Finally, there exists $z_4 > 0$ such that $[1-\delta+\delta g(z)] \leqslant 2[g(z)+rz]$ for all z $(0 < z < z_4)$. Hence, for all z satisfying $0 < z < \min\{z_1, z_2, z_3, z_4\}$ and for all $n \geqslant 1$, we have by (6.9) that $\pi_n \geqslant \pi_n^z$. It can easily be shown that we may set $\bar{z}$ so that $\pi_0 \geqslant \pi_0^z$ for all z $(0 < z < \bar{z})$ as well.

Claim 2: For any q $(0 < q < 1)$ and any λ $(0 < \lambda < 1)$, there exists $\bar{z} > 0$ such that for every z $(0 < z < \bar{z})$, there is a reputational equilibrium with profits at least $\lambda q f(q)$.

Proof of Claim 2: Set $q_1 = \sqrt{\lambda}\,q$. Define m to be the least integer greater than $-\log(1-\sqrt{\lambda})/rz$. (Observe that $e^{-rmz} \approx 1-\sqrt{\lambda}$.) Now define $\{q_n\}_{n=2}^{\infty}$ by (6.4). Then for arbitrary $a > 0$, there exists z_5 such that for every z $(0 < z < z_5)$, $q_m < q$. By (6.6), $p_0 > (1-\delta)\Sigma_{k=0}^{m-1}\delta^k f(q_{k+1}) \geqslant (1-\delta^m)f(q_m) \geqslant [1-(1-\sqrt{\lambda})]f(q) = \sqrt{\lambda}f(q)$, whenever $0 < z < z_5$. Hence, $\pi_0 \geqslant p_0 q_1 \geqslant \lambda q f(q)$.

Using Claim 1, there exists $\bar{z} > 0$ such that $(\vec{p}, \vec{q}, \sigma_z)$ defined using $q_1 = \sqrt{\lambda}\, q$, (6.4) and (6.6) is a reputational equilibrium, for all z $(0 < z < \bar{z})$, proving Claim 2.

REMAINDER OF PROOF OF THEOREM 6.4: Given any q_1, let the quantity path $\vec{q}$ be defined by (6.4), let the price path $\vec{p}$ be defined by (6.6), and let $\pi(q_1, z)$ denote the profits associated with $\vec{q}$ and $\vec{p}$. Then:

$$\pi(q_1, z) = \sum_{k=0}^{\infty} e^{-krz}(q_{k+1} - q_k)p_k = q_1 p_0 + \sum_{k=1}^{\infty} \delta^k (q_{k+1} - q_k)p_k. \tag{6.10}$$

Suppose $q_1' > q_1$, and define $\vec{q}'$ and $\vec{p}'$ analogously. Observe, by (6.4), that $q_{k+1}' - q_k' < q_{k+1} - q_k$ for all $k \geqslant 1$, and by (6.6), that $p_k' \leqslant p_k$ for all $k \geqslant 0$. Hence, using (6.10), $\pi(q_1', z) \leqslant \pi(q_1, z) + |q_1' - q_1|$. Define $\tilde{\pi}(q_1, z) = \sup\{\pi(q, z)\colon 0 \leqslant q \leqslant q_1\}$. Observe that $\tilde{\pi}$ is monotone nondecreasing in q_1 and also satisfies $\tilde{\pi}(q_1', z) \leqslant \tilde{\pi}(q_1, z) + |q_1' - q_1|$. Thus, $\tilde{\pi}$ is continuous in q_1, for any $z > 0$.

Let $\pi^* = \sup_{0 \leqslant q \leqslant 1} qf(q)$ and choose q^* so that $\pi^* = q^* f(q^*)$. Given ε $(0 < \varepsilon < \pi^*)$, define $\lambda = [\pi^* - \varepsilon]/\pi^*$. By Claim 2, there exists $z_6 > 0$ such that there exists a reputational equilibrium with profits at least $\lambda\pi^* = \pi^* - \varepsilon$ whenever $0 < z < z_6$. Also, using (6.10), observe that $\lim_{z \to 0} \pi(0, z) = 0$, and so there exists $z_7 > 0$ such that $\pi(0, z) < \varepsilon$ whenever $0 < z < z_7$. Finally, by Claim 1, there exists $z_8 > 0$ such that $(\vec{p}, \vec{q}, \sigma_z)$ defined from $q_1 = 0$ is a reputational equilibrium whenever $0 < z < z_8$.

Define $\bar{z} = \min\{z_6, z_7, z_8\}$. Then for any z satisfying $0 < z < \bar{z}$, $\pi(0, z) < \varepsilon$ and $\pi(\sqrt{\lambda}\, q^*, z) > \pi^* - \varepsilon$. Furthermore, we have already shown that $\pi_n \geqslant \pi_n^z$ for $0 < z < \bar{z}$ and $n \geqslant 1$, so $(\vec{p}, \vec{q}, \sigma_z)$ is a reputational equilibrium for all q_1 that yield $\pi_0 \geqslant \pi(0, z)$. Finally, since $\tilde{\pi}(q_1, z)$ is continuous in q_1, the set $\{\pi(q_1, z)\colon 0 \leqslant q_1 \leqslant \sqrt{\lambda}\, q^*$ and $\pi(q_1, z) \geqslant \pi(0, z)\}$ is an interval. Since ε and $\pi^* - \varepsilon$ are both contained in that interval, we have established (6.3). *Q.E.D.*

7. CONCLUSION

Consider the outcome of durable goods monopoly (or bargaining) when the time interval between successive periods approaches *infinity*. In this situation, the monopolist (seller) has close to unlimited commitment power, and thus her maximum equilibrium payoff approaches static monopoly profits. Meanwhile, as we demonstrated in the Folk Theorem, the same outcome is attainable in the limit as the time interval between periods approaches zero. We conclude that the "maximum possible seller surplus" is minimized at some intermediate time interval—let us call this the *time interval of least commitment*.

We explain this phenomenon as the result of two countervailing forces. When the time interval between periods is short, reputational effects may be devastatingly effective in preserving monopoly power. When the time interval between periods is long, reputational effects are superfluous. The most adverse circumstance for the monopolist may be when the time interval is just long enough to preclude reputational equilibria (but still sufficiently short that the inability to commit is a problem).

Let us also explain the somewhat unexpected discontinuity in the equilibrium set, based on whether or not there is separation between seller and buyer valuations. Fudenberg, Levine, and Tirole (1985) demonstrated that in the case of a "gap" between seller and buyer valuations (and subject to some regularity conditions) there is a uniform finite bound to the number of periods in which sales can occur in any subgame perfect equilibrium. Backward induction then forces subgame perfect equilibria to be Markovian, and the Coase conjecture drives the initial price near the lowest buyer valuation. However, in the case of "no gap" treated here, sales necessarily occur over infinite time. There is no last period from which to apply backward induction, and reputation supports equilibria which approximate static monopoly pricing.

We can also draw an interesting comparison between the present monopoly model and the analogous oligopoly model (Ausubel and Deneckere (1987) and Gul (1987)). Folk theorems for joint profits hold in the durable goods oligopoly, even when there is a "gap," because oligopolists can extend sales over infinite time. This defeats monopoly results driven by backward induction. Moreover, the oligopolists' joint profits may exceed the monopolist's theoretical maximum (when the time interval between periods is short), since Bertrand competition is a more severe "punishment" than Coase pricing.

A limitation of the present analysis is that our Folk Theorem is only stated in terms of *seller* payoffs, and that we examine a model where only the *uninformed* party makes offers. We extend our results to buyer payoffs and to other extensive forms in a sequel (Ausubel and Deneckere (1989)). In particular, there is a folk theorem for seller payoffs in the alternating-offer game if and only if the (lowest) monopoly price does not exceed one-half the highest buyer valuation.

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Manuscript received July, 1986; final revision received August, 1988.

APPENDIX A

EXISTENCE OF WEAK-MARKOV EQUILIBRIA

DEFINITION A.1: Let $P(\cdot)$ and $R(\cdot)$ be left-continuous functions on $[\bar{q}, 1]$, where $0 \leqslant \bar{q} < 1$, and let $P(\cdot)$ be nonincreasing and nonnegative. We will say that (P, R) *supports* a weak-Markov equilibrium on $[\bar{q}, 1]$ for (inverse) demand curve $f(\cdot)$ if equations (2.1) and (2.2) are satisfied for every $q \in [\bar{q}, 1)$.

We have already argued in Section 2 that if (P, R) is associated with a weak-Markov equilibrium, then (2.1) holds for all $q \in [0,1]$ and (2.2) is satisfied at all continuity points of $t(\cdot)$ in $[0,1]$. The left-continuity of $f(\cdot)$, $P(\cdot)$, and $t(\cdot)$, and the monotonicity of $T(\cdot)$, imply that (2.2) must hold everywhere. Conversely, suppose (2.1) and (2.2) are satisfied for all elements of $[\bar{q}, 1)$. Then if the state $q \in [\bar{q}, 1)$ and if the previous price, p_{-1}, was in the range of $P(\cdot)$, we specify a weak-Markov equilibrium by requiring that all consumers y accept if and only if $P(y) \geqslant p_{-1}$ and that the monopolist charge $S(q')$ in the next period, where $q' = \sup\{y \in [q,1): P(y) \geqslant p_{-1}\}$. If p_{-1} was not in the range of $P(\cdot)$, consumers accept using the same rule, but the monopolist randomizes in the next period between $\sup P(T(q))$ and $\inf P(T(q))$ in such a way that the expected price, $\bar{p}$, satisfies (2.3). That such a randomization is possible can be demonstrated using (2.2).

LEMMA A.2: *Suppose that (P, R) supports a weak-Markov equilibrium on $[\bar{q},1]$. Then $R(\cdot)$ is decreasing and Lipschitz continuous, satisfying: $0 < R(q_1) - R(q_2) \leqslant q_2 - q_1$, whenever $\bar{q} \leqslant q_1 < q_2 \leqslant 1$.*

PROOF: Observe that $R(q_1) \geqslant [t(q_2) - q_1]P(t(q_2)) + \delta R(t(q_2)) > [t(q_2) - q_2]P(t(q_2)) + \delta R(t(q_2)) = R(q_2)$, since $P(x) \geqslant (1-\delta)f(x) > 0$ for all $x \in [\bar{q},1)$. Meanwhile, define $t^1(q_1) = t(q_1)$, $t^2(q_1) = t(t(q_1))$, etc. Then the monopolist, starting from q_2, has the option of selecting a sales path equal to $\max\{q_2, t^k(q_1)\}$, for $k = 1,2,3,\ldots$. This assures $R(q_2) \geqslant R(q_1) - (q_2 - q_1)$. *Q.E.D.*

LEMMA A.3:[11] *Suppose that $(P_{\bar{q}}, R_{\bar{q}})$ supports a weak-Markov equilibrium on $[\bar{q},1]$, where $0 < \bar{q} < 1$. Then there exists (P, R) which supports a weak-Markov equilibrium on $[0,1]$, with the property that $P(q) = P_{\bar{q}}(q)$ and $R(q) = R_{\bar{q}}(q)$ for all $q \in [\bar{q},1]$.*

PROOF: We proceed constructively. Let $\bar{q}' = \max\{0, \bar{q} - (1-\delta)R_{\bar{q}}(\bar{q})/2\}$. Observe that $\bar{q} < 1$ implies $R(\bar{q}) > 0$ and so $\bar{q}' < \bar{q}$. Let us extend $R_{\bar{q}}(\cdot)$ to $R_{\bar{q}'}(\cdot)$ defined on $[\bar{q}',1]$ by:

$$R_{\bar{q}'}(q) = \max_{y \in [q,1] \cap [\bar{q},1]} \left\{[y-q]P_{\bar{q}}(y) + \delta R_{\bar{q}}(y)\right\}, \tag{A.1}$$

and define $t_{\bar{q}'}(q)$ to be the infimum of the argmax correspondence of (A.1). Also extend $P_{\bar{q}}(\cdot)$ to $P_{\bar{q}'}(\cdot)$ defined on $[\bar{q}',1]$ by:

$$P_{\bar{q}'}(q) = (1-\delta)f(q) + \delta P_{\bar{q}}\big(t_{\bar{q}'}(q)\big). \tag{A.2}$$

It should now be observed that $(P_{\bar{q}'}, R_{\bar{q}'})$ satisfy:

$$R_{\bar{q}'}(q) = \max_{y \in [q,1]} \left\{[y-q]P_{\bar{q}'}(y) + \delta R_{\bar{q}'}(y)\right\}, \qquad \text{for all} \quad q \in [\bar{q}',1], \tag{A.3}$$

using (A.1) and the fact that, for $v \in [q,\bar{q}]$, $[v-q]P_{\bar{q}'}(v) + \delta R_{\bar{q}'}(v) \leqslant (1/2)(1-\delta)R_{\bar{q}}(\bar{q}) + \delta R_{\bar{q}'}(v) \leqslant (1/2)(1-\delta)R_{\bar{q}'}(q) + \delta R_{\bar{q}'}(q) < R_{\bar{q}'}(q)$. Thus, $(P_{\bar{q}'}, R_{\bar{q}'})$ supports a weak-Markov equilibrium on $[\bar{q}',1]$. A finite number of repetitions of the above argument extends $(P_{\bar{q}}, R_{\bar{q}})$ to the entire unit interval. *Q.E.D.*

We will now introduce some notation and state a result which we use in proving Theorem 4.2. Let X and Y be compact, nonempty subsets of $\mathbb{R}$. Let J_n $(n = 1,2,\ldots)$ and J: $X \to Y$ be upper semicontinuous functions. Define $R(J_n) = \max\{J_n(x): x \in X\}$, and $T(J_n) = \{x \in X: J_n(x) = R(J_n)\}$, and similarly for $R(J)$ and $T(J)$. Also define: $\bar{J}_n(x) = \mathrm{conv}\{y: y = \lim_{i\to\infty} J_n(x_i)$, for some $\langle x_i\rangle_{i=1}^{\infty} \subset X$ such that $x_i \to x\}$; $G(J_n) =$ graph of $\bar{J}_n$; and $B_\varepsilon(J_n) = \{(x', y') \in X \times Y: \|(x', y') - (x, y)\| < \varepsilon$ for some $(x, y) \in G(J_n)\}$. Finally, define:

$$\rho(J, J_n) = \inf\left\{\varepsilon > 0: G(J) \subset B_\varepsilon(J_n) \text{ and } G(J_n) \subset B_\varepsilon(J)\right\}. \tag{A.4}$$

We may now state a generalization of the theorem of the maximum which does not require continuity of the objective. A proof may be found in Ausubel and Deneckere (1988):

THEOREM OF THE MAXIMUM: *Suppose $J_n(\cdot)$ and $J(\cdot)$ are upper semicontinuous functions from X into Y, and suppose $\lim_{n\to\infty} \rho(J, J_n) = 0$. Then $\lim_{n\to\infty} R(J_n) = R(J)$ and any cluster point from $\{T(J_n)\}_{n=1}^{\infty}$ is an element of $T(J)$.*

We are now ready to prove the following theorem:

[11] This lemma builds on Fudenberg, Levine, and Tirole (1985, Lemma 3) and Gul, Sonnenschein, and Wilson (1986, Lemma 5).

THEOREM 4.2 (Existence of Weak-Markov Equilibria): *Let f be any (inverse) demand curve satisfying Definition* 4.1. *Then for every $r > 0$ and every $z > 0$, there exists a weak-Markov equilibrium.*

PROOF: Consider the sequence of demand curves:

$$f_n(q) = \begin{cases} f(q), & \text{if } 0 \leqslant q \leqslant (n-1)/n, \\ (n-nq)f((n-1)/n) + (1-n+nq)f(1), & \text{if } (n-1)/n < q \leqslant 1. \end{cases}$$

Observe that f_n and f differ only on $((n-1)/n, 1]$ and that, for every n, f_n is linear on the latter interval. Hence, one can explicitly calculate a linear-quadratic pair $(\tilde{P}_n, \tilde{R}_n)$ which supports a strong-Markov equilibrium on $[(n-1)/n, 1]$ for f_n. (See Section 3.) By Lemma A.3, this pair can be extended to (P_n, R_n) which supports an equilibrium on the entire unit interval.

Without loss of generality, we may assume that $\{P_n\}_{n=1}^{\infty}$ converges pointwise for all rationals in $[0,1]$. (This can be assured by taking successive subsequences and applying a diagonal argument.) For every rational $r \in [0,1]$, let $\Phi(r) = \lim_{n\to\infty} P_n(r)$. Define $P(0) = \Phi(0)$ and, for every $x \in (0,1]$, define $P(x) = \lim_{k\to\infty} \Phi(r_k)$, where each r_k is rational and $r_k \uparrow x$. Observe that $P(\cdot)$ is well-defined, nonincreasing, and left continuous. Without loss of generality, we may also assume that $\{R_n\}_{n=1}^{\infty}$ converges uniformly to a continuous function, which we denote by $R(\cdot)$. (This is made possible by Lemma A.2, which implies that $\{R_n\}_{n=1}^{\infty}$ is an equicontinuous family which thus has a uniformly convergent subsequence.) The remainder of the proof will establish that the constructed (P, R) supports a weak-Markov equilibrium on $[0,1]$ for the (limit) demand curve f.

Define $J_n(q, y) = [y-q]P_n(y) + \delta R_n(y)$ and $J(q, y) = [y-q]P(y) + \delta R(y)$. Also define $T_n(q) = \operatorname{argmax}\{J_n(q, y)\colon\ y \in [q,1]\}$ and $T(q)$ analogously. Finally, let $t_n(q) = \inf T_n(q)$ and $t(q) = \inf T(q)$. We will now argue that $\rho(J(q,\cdot), J_n(q,\cdot)) \to 0$. The theorem of the maximum is then applicable, establishing that (2.1) is satisfied, for all $q \in [0,1]$.

Select arbitrary $\varepsilon > 0$. Cover the closure of $G(P)$ with $\varepsilon/5$-balls. Take a finite subcover, denoting the centers $(x_i, P(x_i))_{i\in I}$, where $x_i < x_{i+1}$ for all $i \in I$. By the definition of $P(\cdot)$, there exist rationals $\{y_i\}_{i\in I}$ such that $|y_i - x_i| < \varepsilon/5$ and $|\Phi(y_i) - P(x_i)| < \varepsilon/5$. Furthermore, there exists N_1 such that for all $n \geqslant N_1$, and all $i \in I$, $|P_n(y_i) - \Phi(y_i)| < \varepsilon/5$. Hence, the distance from $(y_i, P_n(y_i))$ to $(x_i, P(x_i))$ is less than $3\varepsilon/5$, and so the ε-ball centered at $(y_i, P_n(y_i))$ contains the $\varepsilon/5$-ball centered at $(x_i, P(x_i))$, for all $i \in I$. Consequently, $B_\varepsilon(P_n) \supset G(P)$, for all $n \geqslant N_1$.

Consider any consecutive x_i, x_{i+1}. Note that $0 \leqslant x_{i+1} - x_i < 2\varepsilon/5$, and:

$$\text{(A.5)} \qquad \begin{aligned} &P_n(y_i) < P(x_i) + 2\varepsilon/5, \qquad \text{and} \\ &P_n(y_{i+1}) > P(x_{i+1}) - 2\varepsilon/5. \end{aligned}$$

Let us observe that for every $v \in [P(x_{i+1}), P(x_i)]$, there exists $w(v) \in [x_i, x_{i+1}]$ such that $v \in \underline{\bar{P}}(w(v))$. Consequently, the union of all ε-balls around the points $\{(w(v), v)\colon\ v \in [P(x_{i+1}), P(x_i)]\}$ covers the rectangle $D \equiv \{(y, v)\colon\ x_i - \varepsilon/5 \leqslant y \leqslant x_{i+1} + \varepsilon/5$ and $P(x_{i+1}) - 2\varepsilon/5 \leqslant v \leqslant P(x_i) + 2\varepsilon/5\}$. Using (A.5), note that $(y_i, P_n(y_i)) \in D$ and $(y_{i+1}, P_n(y_{i+1})) \in D$. By the monotonicity of $P_n(\cdot)$, it follows that $G(P_n) \subset B_\varepsilon(P)$, demonstrating that $\rho(P, P_n) < \varepsilon$ for all $n \geqslant N_1$.

Since $R_n \to R$ uniformly, there also exists N_2 such that for all $n \geqslant N_2$, $\rho(R, R_n) < \varepsilon$. Using the fact that $|y - q| \leqslant 1$, we conclude that $\rho(J(q,\cdot), J_n(q,\cdot)) < 2\varepsilon$, for $n \geqslant \max\{N_1, N_2\}$. Consequently, the hypothesis of the theorem of the maximum is satisfied, so (2.1) holds for all $q \in [0,1]$.

It remains to be argued that (2.2) is also satisfied. Consider any $q \in [0,1]$ where $t(\cdot)$, $P(\cdot)$, and $P(t(\cdot))$ are continuous. Observe that each of these functions is monotone: hence this restriction excludes at most countably many points. First, the theorem of the maximum implies that every cluster point of $\{t_n(q)\}_{n=1}^{\infty}$ is an element of $T(q)$. Now $T(\cdot)$ is single-valued at q since $t(\cdot)$ is continuous: hence $\lim_{n\to\infty} t_n(q) = t(q)$. Second, observe from the definition of $P(\cdot)$ that $P(\cdot)$ is continuous at q if and only if $\Phi(\cdot)$ is continuous at q. Let p be any accumulation point of $\{P_n(q)\}_{n=1}^{\infty}$ and let $r_k \uparrow q$ and $s_k \downarrow q$ be sequences of rationals. Then, for all k, $P_n(r_k) \geqslant P_n(q) \geqslant P_n(s_k)$, and hence $\Phi(r_k) \geqslant p \geqslant \Phi(s_k)$. The continuity of $\Phi(\cdot)$ implies p is unique and $p = \lim_{k\to\infty} \Phi(r_k) \equiv P(q)$, demonstrating that $\lim_{n\to\infty} P_n(q) = P(q)$. Third, since $t(\cdot)$ and $P(t(\cdot))$ are continuous at q, $P(\cdot)$ is continuous at $t(q)$ and, hence, $\Phi(\cdot)$ is continuous at $t(q)$. Let p' be any accumulation point of $\{P_n(t_n(q))\}_{n=1}^{\infty}$ and let $r'_k \uparrow t(q)$ and $s'_k \downarrow t(q)$ be sequences of rationals. Observe that for every $k > 0$, there exists $N(k)$ such that $t_n(q) \in (r'_k, s'_k)$ for all $n \geqslant N(k)$. Consequently, $P_n(r'_k) \geqslant P_n(t_n(q)) \geqslant P_n(s'_k)$, for all $n \geqslant N(k)$, and $\Phi(r'_k) \geqslant p' \geqslant \Phi(s'_k)$, for all k. As before, we can conclude $\lim_{n\to\infty} P_n(t_n(q)) = P(t(q))$. Finally, by our construction of $f_n(\cdot)$, $f_n \to f$ uniformly, and so $\lim_{n\to\infty} f_n(q) = f(q)$. Observe now that since (P_n, R_n) supports a weak-Markov equilibrium for f_n, we have for each n:

$$(A.6) \qquad f_n(q) - P_n(q) = \delta[f_n(q) - P_n(t_n(q))].$$

By taking limits as $n \to \infty$, we see that (2.2) is satisfied for all but (possibly) countably many q.

Now consider any of the (at most countably many) excluded $q \in (0,1]$. Select a sequence of nonexcluded q_k such that $q_k \uparrow q$. Since (2.2) is satisfied for all q_k, and since $f_n(\cdot)$, $P_n(\cdot)$, and $P_n(t_n(\cdot))$ are left-continuous, we conclude that (2.2) is satisfied for q as well. This completes the proof of the theorem. *Q.E.D.*

Finally, we obtain the following proposition:

PROPOSITION 4.3: *Along any weak-Markov equilibrium path, the monopolist does not randomize, except (possibly) in the initial period.*

PROOF: Suppose otherwise. Then there exists a history after which the state is q and the monopolist randomizes among elements of $P(T(q))$ to yield an expected price $p^2 < p^1 \equiv \sup P(T(q))$. Define $\hat{p}^i$ by $f(q) - \hat{p}^i = \delta[f(q) - p^i]$, $i = 1,2$. By (2.2) the price in the previous period must have been $\hat{p}^2$. We now claim that $P(q) \geqslant \hat{p}^1$, showing that the monopolist could have profited by setting $\hat{p}^1$ instead. To see this, first observe that $T(\cdot)$ is a monotone increasing correspondence and so $P(T(\cdot))$ is a nonincreasing correspondence. Next, let q_n be such that $q_n \uparrow q$ and $P(T(q_n))$ is single-valued. Then, using (2.2): $P(q) = \lim_{n\to\infty} P(q_n) = \lim_{n\to\infty} [(1-\delta)f(q_n) + \delta S(q_n)] \geqslant (1-\delta)f(q) + \delta p^1 = \hat{p}^1$, establishing the proposition. *Q.E.D.*

APPENDIX B

THE UNIFORM COASE CONJECTURE

PROOF OF THEOREM 5.4: Suppose not. Then there exist $\varepsilon > 0$, a sequence $\{f_n\}_{n=1}^{\infty} \subset \mathscr{F}_{L,M,\alpha}$, a sequence of positive numbers $\{z_n\}_{n=1}^{\infty} \to 0$, and a sequence of weak-Markov equilibria $\{P_n, R_n\}_{n=1}^{\infty}$ such that the initial price $S_n(0) \geqslant \varepsilon$ for all $n \geqslant 1$. Construct (P, R) as in the proof of Theorem 4.2. Let $y = \inf\{r\colon P(r) < \varepsilon\}$. By left continuity, $P(y) \geqslant \varepsilon$, but $P(r) < \varepsilon$ for every $r > y$. Recall that any weak-Markov equilibrium has sales in every period; hence $S_n(0) \geqslant \varepsilon$ implies $P_n(0) \geqslant \varepsilon$ for all n. But since P_n is monotone and $f_n(q) \leqslant L(1-q)^{\alpha}$, we have $P_n(1-(\varepsilon/2L)^{1/\alpha}) \leqslant f_n(1-(\varepsilon/2L)^{1/\alpha}) \leqslant \varepsilon/2$, for all $n \geqslant 1$, implying that $0 \leqslant y < 1$.

Case I: Suppose $R(y) > 0$.

Since $R_n \to R$ uniformly, there exists a rational q $(y < q < 1)$ and an integer $\bar{n}_1$ such that $R_n(q) > R(y)/2$, for all $n \geqslant \bar{n}_1$. Since $q > y$, $P(q) < \varepsilon$, so there exists $\omega > 0$ and integer $\bar{n}_2$ such that $P_n(q) < \varepsilon - \omega$ for all $n \geqslant \bar{n}_2$. We will establish a lower bound on the real time t before which the price can drop by ω, in equilibrium, and hence before which consumer q purchases. Consumer 0 prefers to purchase at the initial equilibrium price, which is at least ε, to buying at a price below $\varepsilon - \omega$ at time t, so $1 - \varepsilon \geqslant e^{-rt}[1-(\varepsilon-\omega)]$, or $e^{-rt} \leqslant 1 - \omega/(1-\varepsilon+\omega)$. This gives an upper bound on the profits attainable by the monopolist:

$$(B.1) \qquad R_n(0) \leqslant \int_0^q P_n(x)\,dx + e^{-rt}R_n(q), \qquad \text{for all} \qquad n \geqslant \bar{n}_2.$$

Choose any integer m. Then for any consumer reservation price function P_n, the monopolist may charge prices $(m-1)/m, (m-2)/m, \ldots, 1/m$, respectively, in the first $(m-1)$ periods. This earns the monopolist within $1/m$ of all "available surplus," within a factor $e^{-(m-2)z}$ of discounting. Hence $R_n(0) \geqslant e^{-r(m-2)z}\{\int_0^1 P_n(x)\,dx - 1/m\}$. Since $z_n \to 0$, there exists $\bar{n}_3(m)$ such that $rz_n \leqslant 1/m^2$ for all $n \geqslant \bar{n}_3(m)$, and so:

$$(B.2) \qquad R_n(0) \geqslant e^{-1/m}\left\{\int_0^1 P_n(x)\,dx - 1/m\right\}, \qquad \text{for all} \qquad n \geqslant \bar{n}_3(m).$$

Since $0 < R(y)/2 < R_n(q)$ for all $n \geqslant \bar{n}_1$, there exists an integer m such that (B.1) and (B.2) are contradictory for $n \geqslant \max\{\bar{n}_1, \bar{n}_2, \bar{n}_3(m)\}$.

Case II: Suppose $R(y) = 0$.

By hypothesis, (P_n, R_n) is a subgame perfect equilibrium for all n. Suppose that, in the initial period, the monopolist chooses to deviate by charging a price of $\varepsilon/2$. This defines a subgame. We will show that, for sufficiently large n, the posited behavior under (P_n, R_n) in this subgame cannot be optimal for both the monopolist and consumers.

Observe that any weak-Markov equilibrium has sales in every period; hence $P_n(0) \geqslant \varepsilon$ for all n. Customer 0 is optimizing when he purchases at price $\varepsilon/2$, so he must believe that the price will not drop rapidly thereafter. In particular, let t_n be the first (real) time in which the price will drop below $\varepsilon/4$. Then $1 - \varepsilon/2 \geqslant e^{-rt_n}[1 - \varepsilon/4]$. Letting $e^{-rt} = (1 - \varepsilon/2)/(1 - \varepsilon/4)$, we have $t_n \leqslant t$ for all n. Thus a price less than or equal to $\varepsilon/4$ is not charged until at least time t.

Recall that y has been defined so that $P(y) \geqslant \varepsilon$. Therefore, there exists a sequence of rationals $y_n \uparrow y$ such that $P_n(y_n) \geqslant \varepsilon/2$ for all n. For arbitrarily chosen $z > 0$, there exists $\bar{n}_1$ such that $z_n < z$ for all $n \geqslant \bar{n}_1$. Since $R(y) = 0$, $R_n \to R$ uniformly, and $y_n \to y$, there also exists $\bar{n}_2$ such that $R_n(y_n) < (1/4)\varepsilon z e^{-rt}$ for all $n \geqslant \bar{n}_2$. Write n for $\max\{\bar{n}_1, \bar{n}_2\}$. Meanwhile, let m be the greatest even integer less than t/z. Let $p_1, \ldots, p_m$ denote the first m prices charged by the monopolist along a subgame arising after the monopolist charges an initial price $p_0 = \varepsilon/2$. (When a mixed strategy is called for in period 1, let p_1 be the largest price which the monopolist randomizes over.) Observe that $p_m > \varepsilon/4$. Following Gul-Sonnenschein-Wilson (1986), let $a_i = \varepsilon/2 - (2i/m)[\varepsilon/2 - p_m]$ for $0 \leqslant i \leqslant m/2$. Define an alternative sequence $p'_0, \ldots, p'_{m/2}$ by $p'_i = \min\{a_i, p_i\}$ (for $0 \leqslant i \leqslant m/2$). Observe that, by following $p'_0, \ldots, p'_{m/2}$, the monopolist "does not lose time" on any sale and loses at most $2(p_0 - p_m)/m$ on each sale. Furthermore, since $R_n(y_n) < (1/4)\varepsilon z e^{-rt}$ and since each sale before time t is at a price greater than $\varepsilon/4$, the total number of customers sold to at $p_1, \ldots, p_m$ is less than z.

Let V_n denote the net present value of profits from following the equilibrium price path $p_1, p_2, p_3, \ldots$ after a price $p_0 = \varepsilon/2$ was charged. Let V'_n denote the value from following $p'_1, \ldots, p'_{m/2}$ in the first $m/2$ periods and then continuing optimally. Let V''_n denote the value to the monopolist of playing optimally, beginning in the period after a price p_m is charged. Then:

$$V'_n - V_n \geqslant [e^{-rt/2} - e^{-rt}]V''_n - (2/m)(p_0 - p_m)z.$$

We now place a lower bound on V''_n. Observe that, in the period after p_m is charged, customer $1 - (\varepsilon/4L)^{1/\alpha}$ remains in the market, since by the upper bound on f_n: $P_n(1 - (\varepsilon/4L)^{1/\alpha}) \leqslant f_n(1 - (\varepsilon/4L)^{1/\alpha}) \leqslant \varepsilon/4$. Meanwhile, customer $1 - (\varepsilon/4L)^{1/\alpha}/2$ prefers to purchase at a price of $[1 - e^{-rz}]f_n(1 - (\varepsilon/4L)^{1/\alpha}/2)$ this period to purchasing at a price of zero next period. By Definition 5.1, $f_n(1 - (\varepsilon/4L)^{1/\alpha}/2) \geqslant M[1 - (1 - (\varepsilon/4L)^{1/\alpha}/2)]^{\alpha} = M(\varepsilon/4L)2^{-\alpha}$. Hence, a price of $(M/2^{\alpha})(\varepsilon/4L)(1 - e^{-rz})$ induces all customers in the interval $[1 - (\varepsilon/4L)^{1/\alpha}, 1 - (\varepsilon/4L)^{1/\alpha}/2]$ to purchase, so $V''_n \geqslant [1 - e^{-rz}](M/2^{1+\alpha})(\varepsilon/4L)^{1+(1/\alpha)}$.

Recall that $(p_0 - p_m) \leqslant \varepsilon/4$ and $m \approx t/z$. Hence, for sufficiently small z (and the implied n):

$$\begin{aligned} V'_n - V_n &\geqslant (e^{-rt/2} - e^{-rt})(1 - e^{-rz})\big(M/2^{1+\alpha}\big)(\varepsilon/4L)^{1+(1/\alpha)} - (\varepsilon/3t)z^2 \\ &\geqslant (1 - e^{-rz})\Big\{(e^{-rt/2} - e^{-rt})\big(M/2^{1+\alpha}\big)(\varepsilon/4L)^{1+(1/\alpha)} \\ &\qquad - (\varepsilon/3t)\big(z^2/(1 - e^{-rz})\big)\Big\}. \end{aligned}$$

Since $\lim_{z \to 0}(z^2/(1 - e^{-rz})) = 0$, $V'_n - V_n > 0$ for sufficiently small choice of z. This contradicts our hypothesis that, for all n, (S_n, P_n) is a subgame perfect equilibrium. *Q.E.D.*

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