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Rescaled Range Analysis in Higher Dimensions

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Abstract: In this study, we give a rescaled range analysis method employed to demonstrate long-range correlations and the presence of periodical features for a time series. An extension of the rescaled range method to estimate the Hurst exponent of high-dimensional fractals is proposed in this study. The two-dimensional rescaled range analysis is used to analyze traffic data, reveal interesting scaling behavior with physical grounds. The relation between the one-dimensional rescaled range method and two-dimensional rescaled range method is interpreted carefully.

Keywords: Binomial multifractal model, fourier transform method, hurst exponent, rescaled range analysis, traffic time series

INTRODUCTION

Recent research has suggested that many physical systems displaying long-range power-law correlation can be characterized with concepts and methods from fractals theory (Alvarez-Ramirez *et al*., 2002; Bird *et al*., 2006; Wang *et al*., 2009). Some observations of nature consist of records in time of observations and their fractal properties are commonly studied by means of a scaling analysis of the underlying fluctuations (Masugi *et al*., 2007; Billat *et al*., 2009). As a robust analysis method, the rescaled range analysis (R/S) was introduced to estimate Hurst exponent H for modeling the fluctuations of Nile River by Hurst (1951). The rescaled range analysis originated in hydrology where it was used by Hurst to determine the design of an optimal reservoir based on the given record of observed discharges from the lake. Mandelbrot and Wallis further developed the rescaled range statistic and introduced a graphical technique for estimating the so-called “Hurst exponent ", a measure of persistence or long-range correlations, in a time series. Hitherto, there are lots of methods for measuring accurate estimates of H, such as the aggregate variance method (Taqqu *et al*., 1995). The detrended fluctuation analysis method (Peng *et al*., 1994; Hu *et al*., 2001) and the wavelet packet method (Chamoli *et al*., 2007; Shen *et al*., 2007). Although, the R/S analysis were originally conceived as hydrological phenomena, where the Hurst exponent H was used to characterize the scaling properties of observed discharges, the theory have successfully been used in diverse disciplines and problems including climatology (Rehman *et al*., 2009, Koutsoyiannis *et al*., 2006) economics time series (Granero *et al*., 2008: Los *et al*., 2008; Kyaw *et al*., 2006) and geology (Hayakawa *et al*., 2004) as well as other fields.

For almost half a century, the interest in and need for investigating and studying on the persistence of traffic conditions have increased with the growing implementation of both traffic management and traveler information systems. Due to the complexity of the traffic problem, there have been growing efforts to understand the fundamental principles governing the flow of vehicular traffic in the theoretical framework of traffic science (Lajunen *et al*., 1999; Vashitz *et al*., 2008; Pandian *et al*., 2009; Logghe *et al*., 2008). Therefore, tracking traffic conditions, reporting real-time travel information and forecasting traffic information can help commuters make educated mode, route and travel time choices. The use of advanced technologies and intelligence in vehicles and infrastructure could make the current highway transportation system much more efficient.

In this study, we focus our attention to generalize the Rescaled range analysis method to high-dimensional versions. Then, the two-dimensional R/S method is used to analyze both synthetic two-dimensional time series and traffic data, revealing interesting scaling behavior that is interpreted from physical meaning.

In this study, we give a rescaled range analysis method employed to demonstrate long-range correlations and the presence of periodical features for a time series. An extension of the rescaled range method to estimate the Hurst exponent of high-dimensional fractals is proposed in this study. The two-dimensional rescaled range analysis is used to analyze traffic data, reveal interesting scaling behavior with physical grounds. The relation between the one-dimensional rescaled range method and two-dimensional rescaled range method is interpreted carefully.

METHOD

One-dimensional R/S Method: For convenience, For a record $\{x(k)\}$,a brief description of the one-dimensional R/S analysis is given. Let $x(k)$, $k = 1, 2, 3, \ldots, n$ be a set which have an expectation value $E[x(k)]$; the scaled, adjusted range is given by:

$$\frac{\mathrm{R(n)}}{\mathrm{S(n)}} = \frac{\max(0, w_1, w_2, \cdots, w_n) - \min(0, w_1, w_2, \cdots, w_n)}{S(n)} \quad (1)$$

where, $S(n)$ = The standard deviation and for each k = 1, 2, 3, … , n, w_k, is given by:

$w_k = (x(1)+x(2)+ \ldots + x(k))-kE[x(n)]$, $k = 1, 2, \ldots n$

If the record $\{x(k)\}$ is scaling over a certain domain, $n \in (n_{min}, n_{max})$ the R/S statistics follow a power-law:

$$\frac{R(n)}{S(n)} \sim cn^H \quad (2)$$

The parameter H, called the Hurst exponent, represents the dependence properties of the data. The values of the Hurst exponent range between 0 and 1. If H = 0.5, there is no correlation and the data is an uncorrelated signal (white noise); if H<0.5, the data is anti-correlated; if H>0.5, the data is long-range correlated.

Two-dimensional R/S Method: The R/S analysis method can be easily extended for two dimensions along similar steps. However, as we are interested in applying the R/S analysis for traffic, the two-dimensional case is detailed a follows.

For two-dimensional time series $\{(x_k, y_k)\}$, the two-dimensional R/S analysis method is given by:

$$\frac{R_{xy}(n)}{S_{xy}(n)} = \frac{\{\Delta[0, \omega_1(x), \cdots, \omega_n(x)] \times \Delta[0, \omega_1(y), \cdots, \omega_n(y)]\}^{\frac{1}{2}}}{S_{xy}(n)} \quad (3)$$

where,

$\Delta[0, \omega_1(x), \ldots, \omega_n(x)]$ =

$[\max(0, \omega_1(x), \ldots, \omega_n(x))-\min(0, \omega_1(x), \ldots, \omega_n(x))]$

$[\max(0, \omega_1(y), \ldots, \omega_n(y))-\min(0, \omega_1(y), \ldots, \omega_n(y))]$

$\Delta[0, \omega_1(y), \ldots, \omega_n(y)]$ =

$S_{xy}(n) = (\frac{1}{n}\sum_{k=1}^{n}|x_k - E(x_k)||y_k - E(y_k)|)^{1/2}$ is the covariance of x_k, y_k, $\omega_k(x)$ is given by $\omega_k(x) = (x_1 + x_2 + \ldots x_k - k E(x_k))$, $\omega_k(y)$, is given by $\omega_k(x) = (y_1 + y_2 + \ldots y_k - k E(y_k))$, $k = 1, 2, \ldots, n$.

As in the one-dimensional case, the above computation is repeated for different box sizes n, to provide the relationship between $R_{xy}(n)/S_{xy}(n)$ and n. For signals with power-law correlations, the R/S statistics follow the power-law behavior:

$$\frac{R_{xy}(n)}{S_{xy}(n)} \sim cn^{H_{xy}} \quad (4)$$

The parameter H_{xy}, is the two-dimensional scaling exponent or two-dimensional Hurst exponent and represents the correlation properties of the two-dimensional signal.

Higher-dimensional R/S Method: Being a direct generalization, the higher-dimensional R/S statistics have quite similar procedures as the two-dimensional R/S analysis method.

For d-dimensional time series $\{x^1_k, x^2_k, \ldots, x^d_k\}$, the d-dimensional R/S analysis method is detailed as follows:

$$\frac{R(n)}{S(n)} = \frac{\{\Delta[0, \omega_1(x^1), \cdots, \omega_n(x^1)] \times \cdots \times \Delta[0, \omega_1(x^d), \cdots, \omega_n(x^d)]\}^{\frac{1}{2}}}{S(n)} \quad (5)$$

where, $\Delta[0, \omega_1(x^1), \ldots, \omega_n(x^1)]$ =

$[\max(0, \omega_1(x^1), \ldots, \omega_n(x^1))-\min(0, \omega_1(x^1), \ldots, \omega_n(x^1))]$

$\Delta[0, \omega_1(x^d), \ldots, \omega_n(x^d)]$ =

$[\max(0, \omega_1(x^d), \ldots, \omega_n(x^d))-\min(0, \omega_1(x^d), \ldots, (x^d))]$

$S(n) = (\frac{1}{n}\sum_{k=1}^{n}|x^1_k - E(x^1_k)| \times \cdots \times |x^d_k - E(x^d_k)|)^{1/2}$ is the inner product of $x^1_k, x^2_k, \ldots, x^d_k$ and for each k = 1, 2, 3, …, n, $\omega_k(x^l)$, is given by $\omega_k(x^l) = (x^l_1 + x^l_2 + \ldots x^l_k - kE(x^l))$, k = 1, 2, … , n.

As in the two-dimensional case, we plot $R(n)/S(n)$ versus n on a log-log plot and compute the slope for obtaining the generalized Hurst exponent. For signals with power-law correlations, the R/S statistics follow the power-law behavior:

$$\frac{R(n)}{S(n)} \sim cn^H \quad (6)$$

The parameter H, is the d-dimensional scaling exponent or d-dimensional Hurst exponent and represents the correlation properties of the d-dimensional signal.

APPLICATION TO TRAFFIC TIME SERIES

The real traffic data are used to illustrate the performance of the two-dimensional R/S analysis

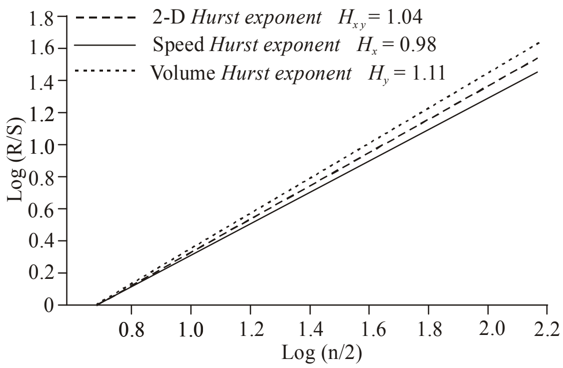

Fig. 1: The two-dimensional Hurst exponent H_{xy} and one-dimensional Hurst exponent H_x, H_y for traffic time series

method. These traffic data were observed on The Renmin Road of Handan over a period of about 4 weeks, from December 6, 2011 to January 2, 2012.

Figure 1 illustrates the relation between the two-dimensional scaling exponent H_{xy} and one-dimensional scaling exponent H_x, H_y for traffic time series, where H_x, H_y is the Hurst exponent of traffic speed and traffic flow. The estimated Hurst scaling exponent is H_{xy} = 1.04, $H_x = 0.98$ and $H_y = 1.11$, respectively. This shows that the average of the scaling exponents H_x and H_y is approximately equal to the scaling exponents of the synthetic data including traffic volume and traffic speed.

The above results have illustrated The the ability of the higher-dimensional R/S algorithm to provide quantitative insights into the intrinsic correlations of higher-dimensional time series. Given its ease of implementation, the higher-dimensional R/S method can be used to investigate and characterize traffic data, meteorological data and many other higher-dimensional time series possessing self-similar properties.

CONCLUSION

Rescaled range analysis is a scaling analysis method estimated long-range power-law correlation exponents in one-dimensional signals. In this study, we propose a higher-dimensional rescaled range analysis method to investigate the correlation of higher-dimensional time series. Then, the two-dimensional rescaled range analysis method is applied to the analysis of the traffic time series to reveal the long-range correlation of the two-dimensional data. Specifically, we find that the two-dimensional scaling exponent H_{xy} is the mean of two one-dimensional Hurst exponent H_x, H_y.

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