Research and Comparison of Time-frequency Techniques for Nonstationary Signals

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***Abstract*—Most of signals in engineering are nonstationary and time-varying. The Fourier transform as a traditional approach can only provide the feature information in frequency domain. The time-frequency techniques may give a comprehensive description of signals in time-frequency planes. Based on some typical nonstationary signals, five time-frequency analysis methods, i.e., the short-time Fourier transform (STFT), wavelet transform (WT), Wigner-Ville distribution (WVD), pseudo-WVD (PWVD) and the Hilbert-Huang transform (HHT), were performed and compared in this paper. The characteristics of each method were obtained and discussed. Compared with the other methods, the HHT with a high time-frequency resolution can clearly describe the rules of the frequency compositions changing with time, is a good approach for feature extraction in nonstationary signal processing.**

***Index Terms*—Nonstationary signal, Short-time Fourier transform, Wavelet transform, Wigner-Ville distribution, Hilbert-Huang transform**

I. Introduction

Many signals in engineering are time-varying. And the frequency features of the signals are very important and can usually be used to distinguish one signal from the others. The Fourier transform and its inversion play important roles in establishment of the relationship between time and frequency domains. They are defined as follows:

$$X(f)=\int x(t)e^{-j2\pi ft}dt \quad (1)$$

$$x(t)=\int X(f)e^{j2\pi ft}df \quad (2)$$

Based on the Fourier transform, the description and energy distribution of a signal in frequency domain can only reflect its frequency features. As the Fourier transform and its inversion are global transform, the signals can only be described entirely in time domain or frequency domain. In practical applications, the Fourier transform is not the best tool, due to most of signals encountered in engineering are nonstationary and time varying, such as the signals of engine noises and vibrations. In order to study the nonstationary signals, the time-frequency techniques are introduced. Fig. 1 shows three descriptions of a chirp signal: (a) is the description in time domain which loses the frequency information, (b) is the description in frequency domain which loses the time information, and (c) is the time-frequency representation which shows the signal energy flowing in a time and frequency plane.

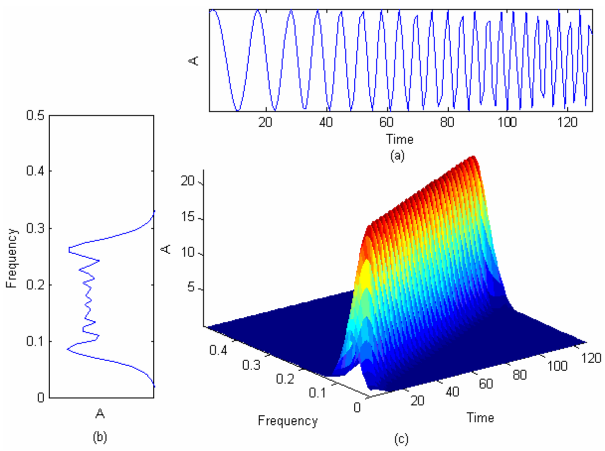

Figure 1. Three description methods of a chirp signal in (a) time, (b) frequency and (c) time-frequency domains.

The basic idea of time-frequency analysis is to design a joint function, which can describe the characteristics of signals on a time-frequency plane [1]. Studies on the time-frequency analysis have become an important research field; and many time-frequency representations were presented. In this paper, five time-frequency techniques, i.e., the short-time Fourier transform (STFT), wavelet transform (WT), Wigner-Ville distribution (WVD), pseudo-WVD (PWVD) and Hilbert-Huang transform (HHT) are investigated and compared.

II. Theory Background

A. Short-time Fourier Transform

The STFT presented by Gabor in 1946 is to intercept the signals by using a window function. The section in the window, which can be regarded as a stationary signal, is treated by the Fourier transform to find its frequency components. The frequency information over time can be obtained by moving the window function along the time axis [2]. The STFT of a signal *x(t)* and its inversion can be described as:

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$$STFT_x(t,f)=\int_{-\infty}^{+\infty}x(\tau)g^*(\tau-t)e^{-j2\pi f\tau}d\tau \quad (3)$$

$$x(t)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}STFT_x(\tau,f)g(t-\tau)e^{j2\pi ft}d\tau df \quad (4)$$

B. Wavelet Transform

In 1980s, the wavelet transform (WT) was developed by Morlet and Grossmam, et al. In recent years, the WT has become a very popular method and is introduced into the analysis of sound and vibration signals in engineering. Due to the high efficiency and superiority in multiresolution analysis of the WT, the analyzed signals can be observed from coarse to fine [3]. The WT of the signal *x(t)* can be described as:

$$WT_x(a,b)=\int_{-\infty}^{+\infty}x(t)\varphi_{a,b}^*(t)dt=\frac{1}{\sqrt{a}}\int x(t)\varphi^*(\frac{t-b}{a})dt \quad (5)$$

If the (6) established, the inversion of WT can be defined as (7).

$$C_\varphi=\int_0^{+\infty}\frac{|\psi(\omega)|^2}{\omega}d\omega<\infty \quad (6)$$

$$x(t)=\frac{1}{C_\varphi}\int_0^{+\infty}\int_{-\infty}^{\infty}WT_x(a,b)\varphi_{a,b}(t)\frac{dadb}{a^2} \quad (7)$$

C. Wigner-Ville Distribution

The Wigner-Ville distribution (WVD) was presented by Wigner in the research of quantum mechanics in 1932 and applied to signal processing by Ville later. The WVD of the signal *x(t)* and its inversion can be described as:

$$WVD_x(t,f)=\int_{-\infty}^{+\infty}x(t+\tau/2)x^*(t-\tau/2)e^{-j2\pi f\tau}d\tau \quad (8)$$

$$x(t)=1/x^*(0)\int WVD_x(t/2,f)e^{j2\pi ft}df \quad (9)$$

D. Pseudo Wigner-Ville Distribution

If the signal has two components $x_1(t)$ and $x_2(t)$, the WVD can be expressed as:

$$WD_x(t,f)=WD_{11}(t,f)+WD_{22}(t,f)+WD_{21}(t,f)+WD_{12}(t,f) \quad (10)$$

Here $WVD_{11}(t, f)$ and $WVD_{22}(t, f)$ are self terms of the WVD, while $WVD_{12}(t, f)$ and $WVD_{21}(t, f)$ are cross terms. The cross terms are involved due to that the WVD belongs to quadratic analysis. To suppress influences of the cross terms, the pseudo-WVD (PWVD) as an equivalent smoothed WVD was developed. The PWVD of *x(t)* can be expressed as:

$$PWVD_x(t,f)=\int_{-\infty}^{+\infty}h(\tau)x(t+\tau/2)x^*(t+\tau/2)e^{-j2\pi f\tau}d\tau \quad (11)$$

E. Hilbert-Huang Transform

A new time-frequency analysis method called Hilbert-Huang transform (HHT) was presented by Norden E. Huang et al in 1998. The HHT contains two important parts: empirical mode decomposition (EMD) and Hilbert transform [4]. First, the EMD is used to obtain the intrinsic mode function (IMF). Then Hilbert transform may be applied to obtain a time-frequency-amplitude distribution, i.e., the Hilbert spectrum [5]. A great contribution of HHT is that it can obtain the instantaneous frequency feature of the signals from the Hilbert spectra [6]. Essentially, the EMD is used for smoothing the nonstationary signals and decomposing them into a set of data sequences with different scale characteristics, i.e., IMFs [7]. The target signal *x(t)* can be expressed as the sum of IMF c_i (i=1,2,3,…,n) and the residue component r_n after the EMD, i.e.,

$$x(t)=\sum_{i=1}^{n}c_i+r_n \quad (12)$$

Imposing Hilbert transform on each IMF component, the Hilbert spectrum of *x(t)* can be obtained by taking the real part of the sum of the Hilbert transform results [8]. Thus, the instantaneous frequency of signals is,

$$x(t,f)=\mathrm{Re}\sum_{i=1}^{n}a_i(t)e^{j\theta_i(t)}=\mathrm{Re}\sum_{i=1}^{n}a_i(t)e^{j\int f_i(t)dt} \quad (13)$$

III. Simulation And Comparison Of Time-Frequency Analysis

There still have no unified criteria for comparison of the time-frequency analysis methods. But, based on the analysis results of a nonstationary signal, characteristics of the time-frequency methods may be compared in three aspects: (a) the time-frequency resolution for local information representation; (b) the rule reflection of the frequency components changing with time axis; and (c) the cross terms and the false information distribution shown in the time-frequency plane.

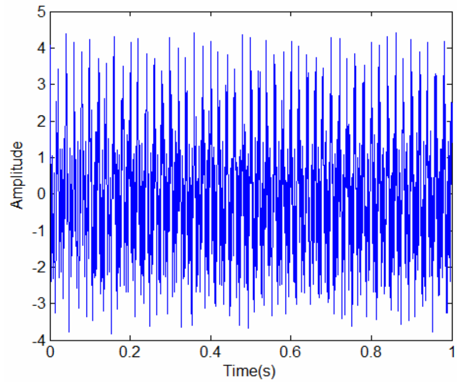


Figure 2. The designed signal *x(t)*

To compare the characteristics of the above mentioned time-frequency methods, a typical nonstationary signal *x(t)* is designed as that in (14). Note that the *x(t)* is a multi-component signal constituted with a cosine component (frequency: 50Hz), a linear frequency modulation component (fundamental frequency: 200Hz) and a sine frequency modulation component (fundamental frequency: 100Hz, modulation frequency: 15Hz). The *x(t)* in time domain and its frequency spectrum are showed in Fig. 2 and Fig. 3, respectively.

$$x(t)=1.5\cos 100\pi t+2\sin(400\pi t+100\pi t^2)+\cos(200\pi t+\sin 30\pi t) \quad .(14)$$

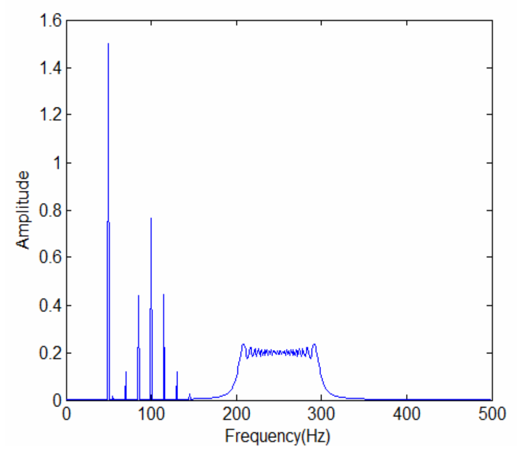


Figure 3. Frequency spectrum of *x(t)*

The STFT, WT, WVD and PWVD are performed to analyze the nonstationary signal *x(t)*, and the time-frequency maps are shown in Fig. 4. It can be seen that, the STFT can recognize the linear frequency component and the cosine component, but the time and frequency resolutions are low. The WT with multiresolution characteristic can improve the resolutions; it has a good frequency resolution in the low frequency region and a good time resolution in the high frequency region. However, the STFT and WT can't exactly recognize the sine frequency modulation component. In contrast, the WVD has higher resolution and can describe the sine frequency modulation, but there are a couple of cross terms appeared in the distribution. The useful informations in the distribution are disturbed seriously by the cross terms. Although the PWVD can suppress the cross terms to a certain extent, it can not eliminate them completely. Meanwhile, the PWVD reduces the resolution and disrupts many mathematical properties in the WVD.

Fig. 5 shows the comparison between the wavelet decomposition and EMD which is the important part of HHT. The (a) is the approach signals decomposed by 6 levels wavelet decomposition with a wavelet base 'db5'. The *a1*, *a2* and *a3* represent the linear frequency modulation, sine frequency modulation and cosine components respectively. Other components can be regard as trend terms. As shown in Fig. 5 (b), the signal *x(t)* was decomposed into five IMF components and one residue component by the EMD. As the EMD decomposes signal from high frequency, the *c1*, *c2* and *c3* represent the linear frequency modulation, sine frequency modulation and cosine components respectively. The *c4* and *c5* are trend terms, and *r5* represents the residue component. Though (a) and (b) look roughly the same, there are signal distortions in (a) in details. A conclusion can be drawn that the EMD can extract the signal components more exactly than wavelet decomposition.

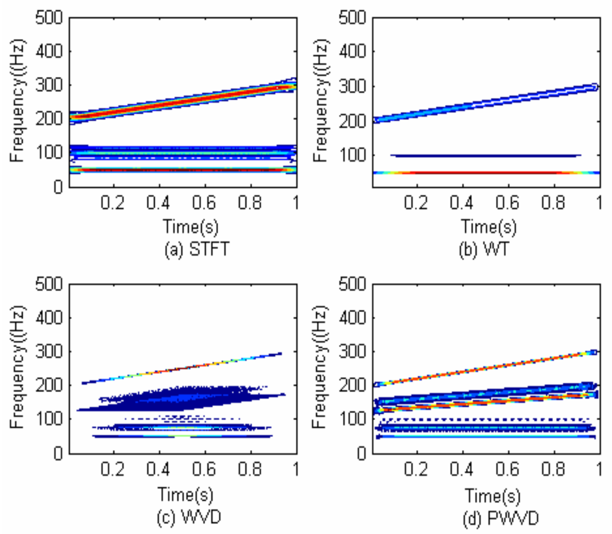
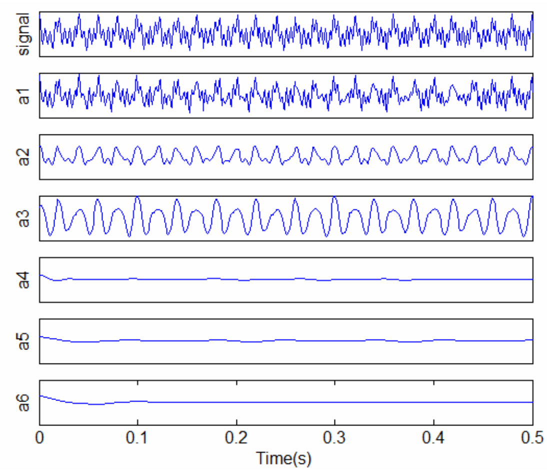
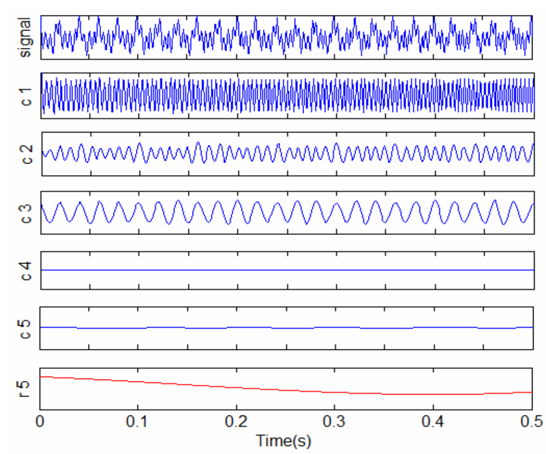


Figure 4. Comparison of time-frequency analysises of the signal x(t).



(a)Wavelet decomposition



(b) EMD

Figure 5. Comparison between wavelet decomposition and EMD of the signal x(t).

The Hilbert time-frequency spectrum of *x(t)* shown in Fig. 6 can be obtained by performing Hilbert transform to the IMF components, following equation (13). Compared with the time-frequency maps in Fig. 4, the Hilbert time-frequency spectrum with higher time-frequency resolution can reflect all frequency components of the signal *x(t)* and give a clear rule of these components changing with time axis. Thus, the local information of *x(t)*, such as instantaneous frequency, can be observed more clearly. For example, the changing rule of sine frequency modulation component, which can not be observed in other methods, can be reflected exactly. The change rule of frequency likes a sine wave.

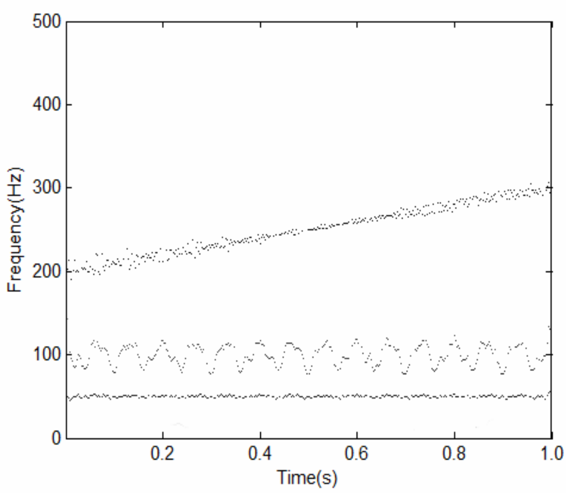


Figure 6. Hilbert time-frequency spectrum

In order to verify their traits in practical signal processing, an engine noise signal, as shown in Fig. 7,was tested by PULSE system made by B&K. The engine signal, which has been disposed by Wavelet packet denoising method, is nonstationary due to more exciting sources and complicated transfer path. As the four cylinders, four strokes engine was running under the speed of 2500 rpm, the noise frequency can be calculated from (15) and (16), which represents the machinery and combustion noise frequency calculation formula respectively. In the formula, k represents harmonic order number, N represents the rotate speed, i represents the cylinder number, τ represents the stroke coefficient. τ is 2 when the stroke number is 4. The calculated frequency (41 Hz and 82 Hz) can also be seen from Fig.8.

$$f = kN/60 . \quad (15)$$

$$f = kNi/60\tau . \quad (16)$$

The five time-frequency analysis methods were performed to the noise signal. The comparison results shown in Fig.9 and Fig.10 are similar to the results of simulation signal *x(t)*. Thus, the same conclusion can be drawn when the time-frequency analysis methods are applied to the practical signals. In contrast, the HHT with higher time-frequency resolution can reflect the local information more clearly. The instantaneous frequency can be observed exactly in Fig.10. Compared with the STFT, WT, WVD and PWVD methods, the HHT which has adaptive characteristic is based on the signal itself. Thus, in view of applications, the HHT may be a better method for time-frequency analysis of nonstationary signals.

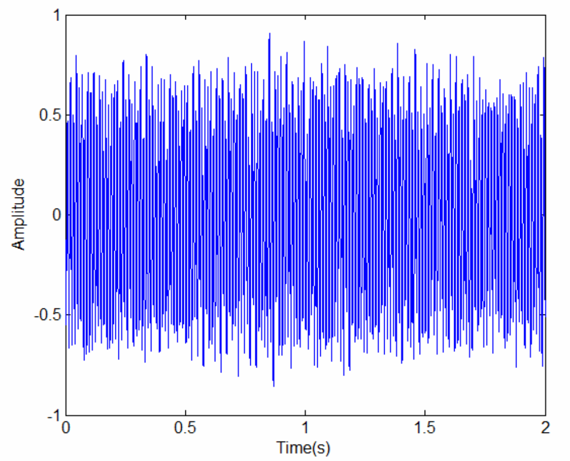


Figure 7. The engine noise signal.

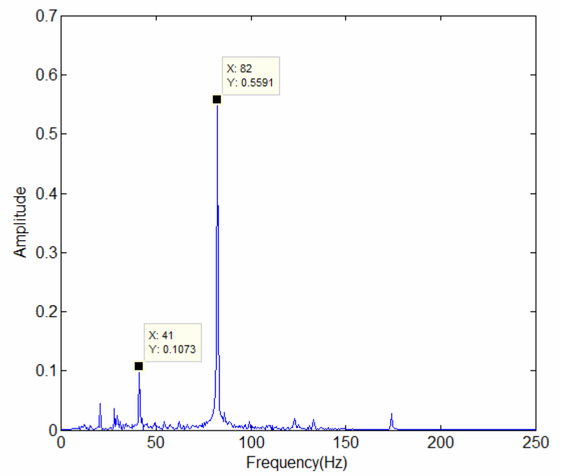


Figure 8. Frequency spectrum of noise signal.

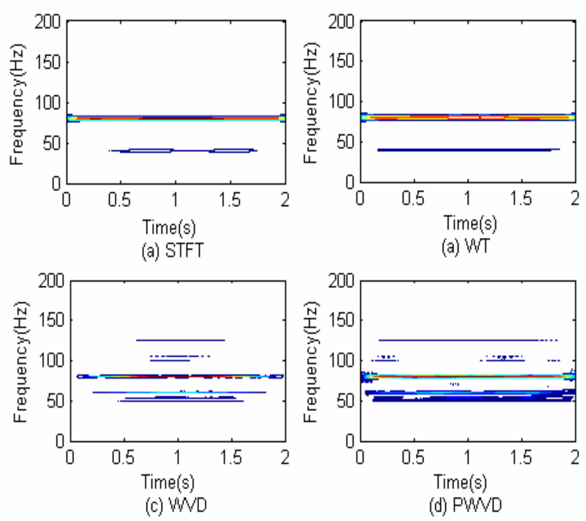


Figure 9. Comparison of time-frequency analysises of the noise signal

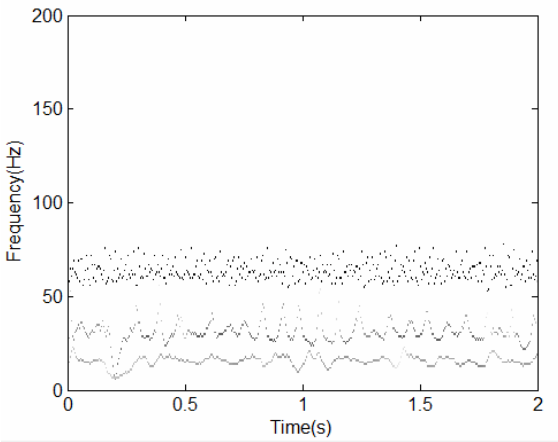


Figure10. Hilbert time-frequency spectrum.

IV. Conclusions

In this paper, five time-frequency analysis methods, i.e., the STFT, WT, WVD, PWVD and HHT, are compared and discussed in nonstationary signal processing. The merits and demerits of each method are investigated. Among the five methods in discussion, the HHT, which has adaptive characteristic, can exactly express the local information of nonstationary signals in a high time-frequency resolution. It overcomes the irreconcilable contradiction between the time-frequency aggregation and cross term. A conclusion are drawn that, since the HHT can exactly reflect instantaneous frequency components, it is more suitable and powerful for feature extraction of the nonstationary signals in engineering.

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