

Research Issues and Challenges for Multiple Digital Signatures

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Abstract

In this paper, we surveyed several well-known batch verification multiple digital signatures. These schemes can batch verify multiple digital signatures which need only one verification instead of t verifications. However, a number of weaknesses of these schemes are pointed out. According to our proposed issues and challenges, we compared and analyzed them. To sum up these schemes, a secure and efficient multiple digital signatures scheme which needs only one verification remains an open problem.

Keywords: Cryptography, digital signature, DSA, RSA

1 Introduction

Digital signatures is a method for the Internet which is similar to traditional signatures. People sign their true names on papers in traditional signatures. No one can forge other signatures as it is difficult to imitate others handwritings. To provide a digital signature, it uses the known public key cryptosystem. Each one has a pair key, private key and a public key. The private key is kept secret and the public key is made public. A sender can sign a electronic document known as a digital signature using his/her private key and a receiver can verify the digital signature by the sender's public key. No one can forge others' digital signatures as the private key is safe guarded. A digital signature scheme has the following properties [2, 15, 17]:

- 1) Only the sender can sign an electronic document.
- 2) The receiver can verify the validity of the digital signature.

- 3) No one can forge the digital signatures of others.
- 4) It can achieve integrity. An attacker should not be able to substitute a false document for a legitimate one.
- 5) It can achieve non-repudiation. A sender should not be able to deny that he/she sent a document.

There are two famous public key cryptosystems to provide digital signatures. One is the RSA digital signature scheme [9, 14]. The security of this scheme is based on the difficulty of solving the factor problem. The scheme can apply to various areas. The other is ElGamal digital signature scheme [3]. The security of this scheme is based on the difficulty of solving the discrete logarithm problem [8]. It also can apply to various other areas. Here, we briefly explain the ElGamal digital signature scheme as follows. Let p is a large prime and g is a generator of Z_p^* . A key pair (private key, public key) is (x, y) , where $y = g^x \bmod p$. (p, g, y) are public. Assume that a sender named Alice, wants to send a message m to a receiver named Bob, where $1 \leq m \leq p - 1$. Alice chooses a random number, k , to satisfy $GCD(k, p - 1) = 1$. Then, computes (r, s) as $r = g^k \bmod p$, and $s = k^{-1}(m - xr) \bmod (p - 1)$. (r, s) is digital signature for message m . Alice sends (m, r, s) to Bob. After receiving (m, r, s) , Bob can verify the digital signature by using the equation $g^m = y^r r^s \bmod p$. It is seen that the ElGamal digital signature scheme can achieve the above properties of digital signature scheme.

If Alice wants to send a message m , where $m \geq p$, to Bob, the m must be divided into t copies m_1, m_2, \dots, m_t . Then Alice signs these messages t times to generate multiple digital signatures and sends these messages and multiple digital signatures to Bob. Upon receiving these mes-

sages and multiple digital signatures, Bob should spend t times to verify the validity of the multiple digital signatures. It is seen that it costs many modular exponentiations. In 1994, Naccache et al. proposed an efficient batch verifying multiple digital signatures [13]. The verifier can verify these multiple digital signatures by the signer's public key which needs only one verification instead of t verifications. However, Lim and Lee pointed out their scheme can be easily forged multiple digital signatures to make a false batch verification valid [12]. And then Harn proposed a secure interactive batch verification protocols [4]. In 1998, Harn proposed two batch types verifying multiple digital signatures [5, 6]. However, Hwang et al. pointed out their schemes are also insecure [7, 10]. An attacker can forge multiple digital signatures to make a false batch verification valid. Therefore, they proposed two improvements [11]. In 2001, Shao also proposed an improvement on Harn's scheme [16]. We can see that if the multiple digital signatures are forged by an attacker, the verifier should verify each digital signature. It is back to the original digital signature scheme which need t verifications. In 2002, Changchien and Hwang proposed an efficient detecting forged multiple digital signatures [1].

From the above related researches, some issues and challenges for multiple digital signatures are discussed and are then addressed in the design of multiple digital signatures. These are explained in the following:

- 1) In multiple digital signatures, only the valid signer can sign multiple electronic documents by his/her private key.
- 2) No one can forge the multiple digital signatures of others to make a false batch verification valid. Many papers cannot go against this challenge. An attacker should not forge multiple digital signatures to make a false batch verification valid.
- 3) Any verifier can batch verify the validity of the multiple digital signatures. Any verifier can verify these multiple digital signatures by the signer's public key which needs only one verification instead of t verifications. It is clear that batch verifying multiple digital signatures is more efficient than original method which verifies each digital signature individually.
- 4) It should achieve integrity. An attacker should not be able to substitute false documents for legitimate ones. In other words, multiple documents should not be modified, deleted, or replaced.
- 5) It should achieve non-repudiation. If a sender can forge multiple digital signatures to make a false batch verification valid. The sender can deny that he/she sent these documents. It cannot achieve non-repudiation. In multiple digital signatures, a sender should be unable to deny that he/she sent these documents.

- 6) It should be able to detect forged multiple digital signatures efficiently when multiple digital signatures are modified or replaced.

This paper is organized as follows. Section 2 describes various multiple digital signatures and their weaknesses. In Section 3, are their comparisons. Finally, the conclusion is explained in Section 4.

2 Related Works

2.1 Naccache et al.'s Scheme and Its Weakness

In this section, we first briefly outline the DSA digital signature [11]. To speed up verification of multiple digital signatures, Naccache et al. proposed a scheme to batch verifying multiple digital signatures [13]. Then, Lim and Lee pointed out Naccache et al.'s scheme has a security flaw [12]. As demonstrated in this section.

2.1.1 DSA

Assume that Alice wants to send an electronic document to Bob. Alice applies DSA algorithm to create a digital signature and sends along a document to Bob. Upon receiving the document and digital signature, Bob can verify the correctness of the document. The signing signature of DSA algorithm is briefed as follows. In Table 1, we list the abbreviations and notations used in DSA algorithm.

Table 1: Notations

x	private key of signer
y	public key of signer; $y = g^x \bmod p$
p	a large prime
q	a large prime divisor of $p - 1$
g	an element in Z_p of order q

(g, p, q, y) are public parameters, and x is kept secure by signer.

- 1) Alice randomly chooses a number $k \in Z_q$. Then she computes r and s as follows:

$$\begin{aligned} r &= (g^k \bmod p) \bmod q, \\ s &= k^{-1}(m + xr) \bmod q. \end{aligned}$$

r and s are DSA digital signature for message m .

- 2) Alice sends (m, r, s) to Bob.

The verifying signature of DSA algorithm is briefed as follows:

- 1) After receiving (m, r, s) from Alice, Bob can verify the correctness of the signature using the following equation:
- 2) Computes $s_b^{-1} \bmod q$ to satisfy $v_b = r_b s_b^{-1} \bmod q$, $b = 1, 2, \dots, t-2$.
- 3) The attacker can derive the s_{t-1} and s_t from the following equations.

$$r = (g^{ms^{-1}} y^{rs^{-1}} \bmod p) \bmod q. \quad (1)$$

- 2) If the above equation holds, Bob can confirm that the digital signature r and s are signed by Alice.

It is seen that If Alice wants to send t multiple digital signatures to Bob, Bob must verify the t multiple digital signatures t times using the verifying signature of DSA algorithm.

2.1.2 Naccache et al.'s Scheme

To speed up the verification of multiple digital signatures, Naccache et al. proposed a scheme to batch verifying multiple digital signatures. The verifier can verify multiple digital signatures by the signer's public key which needs only one verification instead of t verifications. The scheme is as follows:

- 1) Assume that signer, Alice, wants to send t messages (m_1, m_2, \dots, m_t) and multiple digital signatures $(r_1, s_1), (r_2, s_2), \dots, (r_t, s_t)$ to Bob. The multiple digital signatures are created by the signing signature of DSA algorithm. It is identical to the above DSA signature.
- 2) Upon receiving the t messages m_1, m_2, \dots, m_t and multiple digital signatures $(r_1, s_1), (r_2, s_2), \dots, (r_t, s_t)$, Bob can verify the correctness of these multiple digital signatures on messages m_1, m_2, \dots, m_t using Alice's public key y in the following equation:

$$\prod_{i=1}^t r_i = (g^{\sum_{i=1}^t m_i s_i^{-1}} y^{\sum_{i=1}^t r_i s_i^{-1}} \bmod p) \bmod q. \quad (2)$$

- 3) If the above equation holds, Bob can confirm that the multiple digital signatures $(r_1, s_1), (r_2, s_2), \dots, (r_t, s_t)$ are signed by Alice.

It is clear that Bob can batch verify multiple digital signatures which needs only one equation 2. Therefore, Naccache et al.'s scheme is more efficient to batch verifying multiple digital signatures.

2.1.3 The Weakness of Naccache et al.'s Scheme

Lim and Lee shown that Naccache et al.'s scheme is not secure. It has a security flaw in which an attacker can forge the multiple digital signatures and pass the batch verification equation 2. The attack is briefed as follows.

- 1) An attacker randomly chooses random numbers (u_i, v_i) , $i = 1, 2, \dots, t$, and computes $r_i = g^{u_i} y^{v_i} \bmod p$, $i = 1, 2, \dots, t$.

$$\begin{aligned} \sum_{i=1}^t u_i &= \sum_{i=1}^t m_i s_i^{-1} \bmod q, \\ \sum_{i=1}^t v_i &= \sum_{i=1}^t r_i s_i^{-1} \end{aligned}$$

It is seen that the attacker can forge t messages m_1, m_2, \dots, m_t and multiple digital signatures $(r_1, s_1), (r_2, s_2), \dots, (r_t, s_t)$ to pass the batch verification equation 2. However, each of the forged signatures do not pass the DSA verification equation 1 individually.

2.2 Harn's DSA-type Multiple Digital Signatures Scheme and Its Weakness

To go against Lim-Lee's attack, Harn proposed a batch verifying multiple DSA-type digital signatures. In this section, we first brief DSA-type digital signature. Then, we review Harn's batch verifying multiple digital signatures [5]. Next, Hwang et al. pointed out Harn's scheme has a security flaw [10]. We show them in this section.

2.2.1 DSA-type

The DSA-type is similar to DSA algorithm. Assume that Alice wants to send an electronic document to Bob. Alice applies DSA-type algorithm to create a digital signature and sends along with the document to Bob. Upon receiving the document and digital signature, Bob can verify the correctness of the document. The signature of DSA-type algorithm is briefed as follows. The algorithm also uses the same abbreviations and notations in Table 1.

- 1) Alice randomly chooses a number $k \in Z_q$. Then she computes r and s as follows:

$$\begin{aligned} r &= (g^k \bmod p) \bmod q, \\ s &= rk - mx \bmod q. \end{aligned}$$

r and s are DSA-type digital signature for message m .

- 2) Alice sends (m, r, s) to Bob.

The verifying signature of DSA-type algorithm is briefed as follows:

- 1) After receiving (m, r, s) from Alice, Bob can verify the correctness of the signature using the following equation:

$$r = (g^{sr^{-1}} y^{mr^{-1}} \bmod p) \bmod q. \quad (3)$$

- 2) If the above equation holds, Bob can confirm that the digital signature r and s are signed by Alice.

It is also seen that If Alice wants to send t multiple digital signatures to Bob, Bob must verify the t multiple digital signatures t times using the verifying signature of DSA-type algorithm.

2.2.2 Harn's DSA-type Scheme

Harn proposed a batch verifying DSA-type multiple digital signatures to speed up verification of multiple digital signatures. The scheme is as follows:

- 1) Assume that Alice wants to send t messages m_1, m_2, \dots, m_t and multiple digital signatures $(r_1, s_1), (r_2, s_2), \dots, (r_t, s_t)$ to Bob. The multiple digital signatures are created by the signing signature of DSA-type algorithm. It is the same as the above mentioned DSA-type signature.
- 2) Upon receiving the t messages m_1, m_2, \dots, m_t and multiple digital signatures $(r_1, s_1), (r_2, s_2), \dots, (r_t, s_t)$, Bob can verify the correctness of these multiple digital signatures on messages m_1, m_2, \dots, m_t using Alice's public key y in the following equation:

$$\prod_{i=1}^t r_i = (g^{\sum_{i=1}^t s_i r_i^{-1}} y^{\sum_{i=1}^t m_i r_i^{-1}} \bmod p) \bmod q. \quad (4)$$

- 3) If the above equation holds, Bob can confirm that the multiple digital signatures $(r_1, s_1), (r_2, s_2), \dots, (r_t, s_t)$ are signed by Alice.

It is also seen that Bob can batch verify multiple digital signatures which need only one equation 4. Therefore, Harn's scheme is more efficient to batch verifying multiple digital signatures.

2.2.3 The Weakness of Harn's Scheme

Hwang et al. shown that Harn's scheme is not secure. It has a security flaw that a signer can forge the multiple digital signatures and pass the batch verification equation 4. Then, the signer can deny the digital signatures. It cannot achieve the property non-repudiation. The attack is briefed as follows.

- 1) Alice sends the forged pairs $(m_i, r_i, s'_i), i = 1, 2, \dots, t$, to Bob, where $s'_i = s_i + a_i r_i \bmod q$ and a_i is an integer such that $\sum_{i=1}^t a_i = 0$.
- 2) Upon receiving forged pairs (m_i, r_i, s'_i) , Bob can pass the batch verification equation 4. And Bob can confirm that the multiple digital signatures $(r_1, s'_1), (r_2, s'_2), \dots, (r_t, s'_t)$ are signed by Alice.

It is seen that each of the forged signatures does not pass the DSA-type verification equation 3 individually. Therefore, Alice can deny these multiple digital signatures. It cannot achieve the property non-repudiation because of $r_i \neq (g^{s'_i r_i^{-1}} y^{m_i r_i^{-1}} \bmod p) \bmod q, i = 1, 2, \dots, t$.

2.3 Harn's RSA Multiple Digital Signatures Scheme and Its Weakness

In this section, we first briefly explain Harn's RSA multiple digital signatures scheme [6] and then show its weakness [7].

2.3.1 Harn's Scheme

The scheme is based on RSA algorithm. Let p and q are two large primes. Computes $n = p \times q$ and chooses e and d such that $e \times d \bmod (p-1)(q-1) \equiv 1$. (e, n) is a signer's public key and d is signer's private key. Assume that a signer, Alice, wants to send a message to receiver, Bob. Alice generates her digital signature S by using RSA algorithm, where $S = h(m)^d \bmod n$ and $h(\cdot)$ is a public one-way hash function. Alice sends (m, S) to Bob. After receiving them, Bob can verify the correctness of the signature by checking the equation $h(m) = S^e \bmod n$. If it holds, Bob can confirm that the digital signature S are signed by Alice.

Now, assume that Alice wants to send t messages m_1, m_2, \dots, m_t and multiple digital signatures S_1, S_2, \dots, S_t to Bob. The multiple digital signatures are created by the above RSA algorithm. Then, Alice sends $(m_i, S_i), i = 1, 2, \dots, t$, to Bob.

Upon receiving the t messages m_1, m_2, \dots, m_t and multiple digital signatures S_1, S_2, \dots, S_t , Bob can verify the correctness of these multiple digital signatures on messages m_1, m_2, \dots, m_t using Alice's public key e in the following equation:

$$\left(\prod_{i=1}^t S_i \right)^e = \prod_{i=1}^t h(m_i) \bmod n. \quad (5)$$

If the above equation holds, Bob can confirm that the multiple digital signatures S_1, S_2, \dots, S_t are signed by Alice.

It is seen that Bob can batch verify multiple digital signatures which needs only one equation 5. Therefore, Harn's scheme is more efficient to batch verifying multiple digital signatures.

2.3.2 The Weakness of Harn's Scheme

Hwang et al. proposed two attacks on Harn's scheme. They show that a signer can forge the multiple digital signatures and pass the batch verification equation 5. Then, the signer can deny the digital signatures. It cannot achieve the property non-repudiation. The attacks is briefed as follows.

Attack 1:

Alice sends her forged pairs $(m_i, S'_i), i = 1, 2, \dots, t$ to Bob, where $S'_i = h(m_{f(i)})^d \bmod n, i = 1, 2, \dots, t; f(\cdot)$ is a one to one and onto function such that $f(i) = j, i = 1, 2, \dots, t$ and $j = 1, 2, \dots, t$. Upon receiving forged pairs (m_i, S'_i) , Bob can pass the batch verification equation 5. And Bob can confirm that the multiple digital signatures S'_1, S'_2, \dots, S'_t are signed by Alice.

Attack 2:

Alice sends her forged pairs (m_i, S'_i) , $i = 1, 2, \dots, t$ to Bob, where $S'_i = a_i \times S_i \bmod q$, $i = 1, 2, \dots, t$ and $\prod_{i=1}^t a_i = 1$. Upon receiving forged pairs (m_i, S'_i) , Bob can pass the batch verification equation 5. And Bob can confirm that the multiple digital signatures S'_1, S'_2, \dots, S'_t are signed by Alice.

It is seen that each of the forged signatures does not pass the RSA verification equation $h(m_i) = S'_i{}^e \bmod n$ individually. Therefore, Alice can deny these multiple digital signatures. It cannot achieve the property non-repudiation.

2.4 Hwang et al.'s Multiple Digital Signatures Schemes

To remedy the weaknesses of Harn's DSA-type multiple digital signatures scheme and RSA multiple digital signatures scheme, Hwang et al. proposed two improved schemes [11]. One is improved DSA-type multiple digital signatures scheme (abbreviated as BV-DSA). The other is improved RSA multiple digital signatures scheme (abbreviated as BV-RSA). Two improved schemes are shown as follows.

2.4.1 Improved DSA-type Multiple Digital Signatures Scheme

The improved scheme is similar to Harn's scheme. The only difference is in equation 4. We modified it to

$$\prod_{i=1}^t r_i^{v_i} = (g^{\sum_{i=1}^t s_i r_i^{-1} v_i} y^{\sum_{i=1}^t m_i r_i^{-1} v_i} \bmod p) \bmod q, \quad (6)$$

where v_i , $i = 1, 2, \dots, t$, are small random numbers which are selected by a verifier.

2.4.2 Improved RSA Multiple Digital Signatures Scheme

The improved scheme is similar to Harn's scheme. The only difference is in equation 5. We modify it to

$$\left(\prod_{i=1}^t S_i^{v_i}\right)^e = \prod_{i=1}^t h(m_i)^{v_i} \bmod n. \quad (7)$$

where v_i , $i = 1, 2, \dots, t$, are small random numbers which are chosen by a verifier.

2.5 Shao's DSA-type Multiple Digital Signatures Scheme

In 2001, Shao proposed a DSA-type multiple digital signatures scheme [16]. His scheme is similar to Hwang et al.'s scheme. The only difference is in equation 7. The batch verifying criterion is as follows.

$$\prod_{i=1}^t (e_i(s_i))^{u_i} = \prod_{i=1}^t (f_i(s_i))^{u_i} \bmod p, \quad (8)$$

where $u_i \in (1, 2^{32})$, $i = 1, 2, \dots, t$, are random numbers which are chosen by a verifier and s_i are multiple digital signatures individually satisfying the verification equations: $e_i(s_i) = f_i(s_i) \bmod p$, $i = 1, 2, \dots, t$.

2.6 Changchien et al.'s Scheme

In section 4, it is seen that If the batch verification fails, that is $(\prod_{i=1}^t S_i)^e \neq \prod_{i=1}^t h(m_i) \bmod n$, $i = 1, 2, \dots, t$, the receiver, Bob, must check each multiple digital signatures using the verification equation $h(m_i) = S_i^e \bmod n$. It needs t exponential computations to detect the forged digital signature. In 2002, Changchien and Hwang proposed a scheme to detect forged multiple digital signature [1], which only needs one exponential and t modulus computations.

Changchien and Hwang redefine $h()$ as a prime one-way hash function and $\prod_{i=1}^t h(m_i) \leq n$. This can make the length of $h()$ is $\lfloor |n|/t \rfloor$ bits, where $\lfloor \cdot \rfloor$ is floor function and $|n|$ is length of n . To detect the forged multiple digital signature, we can use the following steps:

- 1) Bob calculates $L = (\prod_{i=1}^t S_i)^e \bmod n$.
- 2) Bob can detect the forged digital signature S_i by checking whether $L \bmod h(m_i) = 0$, for $i = 1, 2, \dots, t$.

3 Comparisons

A lot of multiple digital signatures have been proposed in recent years. The security of these schemes are based on the difficulty of solving the factoring problem [14] and discrete logarithm problem [3]. In this section, we compare them in terms of properties and efficiency.

3.1 Properties

In the Introduction, some issues and challenges for multiple digital signatures are discussed and then addressed in the design of multiple digital signatures. These are also the properties of multiple digital signatures. Table 2 shows these schemes in properties. We compare them as follows:

- 1) *Only a valid signer can sign multiple electronic documents.*
All schemes can meet this property. If the signer has his/her private key, he/she can do that.
- 2) *No one can forge the multiple digital signatures.*
In these schemes, it is seen that only Hwang's BV-DSA, Hwang's BV-RSA, and Shao's scheme can meet this property. The multiple digital signatures of their scheme cannot be forged to make a false batch verification valid.
- 3) *Any verifier can batch verify the validity of the multiple digital signatures.*

All schemes can meet this property. If the verifier has the signer's public key, he/she can batch verify the correctness of the multiple digital signatures which needs only one verification.

4) *It should achieve integrity.*

All schemes can meet this property. An attacker should not be able to substitute false documents for legitimate ones because he/she is not aware of the private key of the signer. Only the signer can generate digital signatures for particular documents.

5) *It should achieve non-repudiation.*

If the multiple digital signatures can be forged by a sender, it cannot meet this property because the sender can deny that he/she signed these multiple digital signatures. Therefore, only Hwang's BV-DNA, Hwang's BV-RSA, and Shao's scheme can meet this property.

6) *It should be able to detect forged multiple digital signatures efficiently.*

Most of these schemes cannot meet this property. Only Changchien et al.'s scheme can meet this property. When the multiple digital signatures are forged, the verifier can detect these forged multiple digital signatures efficiently.

3.2 Efficiency

Compared with the original verification of digital signature, these multiple digital signatures can speed up the verification of multiple digital signatures. These schemes batch verify multiple digital signatures which needs only one verification instead of t verifications. It can save many modular exponentiations.

4 Conclusions

In this paper, we have reviewed some multiple digital signatures and proposed some issues and challenges for them. Compared with these multiple digital signatures, we can see that not all these schemes can go against these issues and challenges. However, any verifier in these schemes can batch verify the validity of the multiple digital signatures. They can save many modular exponentiations. In the future, a secure and efficient multiple digital signatures scheme which meets all issues and challenges is still an open problem.

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Table 2: Comparisons among the multiple digital signatures schemes

	IC1	IC2	IC3	IC4	IC5	IC6	SP
Naccache et al.[13]	Y	N	Y	Y	N	N	DLP
Harn's DSA[5]	Y	N	Y	Y	N	N	DLP
Harn's RSA[6]	Y	N	Y	Y	N	N	FP
Hwang's BV-DSA[11]	Y	Y	Y	Y	Y	N	DLP
Hwang's BV-RSA[11]	Y	Y	Y	Y	Y	N	FP
Shao[16]	Y	Y	Y	Y	Y	N	DLP
Changchien et al.[1]	Y	N	Y	Y	N	Y	FP

ICi: Proposed issues and challenges in Section 1, Y: Supported, N: Not supported, SP: Based on the difficulty of solving problem, DLP: Discrete logarithm problem, FP: Factoring problem.



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