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Reset Complexity of Ideal Languages

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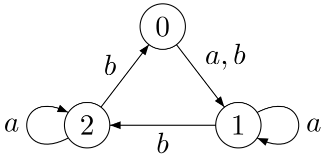
Background and motivation

- A DFA $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ is called *synchronizing*, if there exists a word $w \in \Sigma^*$ which leaves the automaton in one particular state no matter which state in Q it starts at:
  $\delta(q_1, w) = \delta(q_2, w)$ for all $q_1, q_2 \in Q$.
  $|\delta(Q, w)| = 1$.
- Any such word is said to be *synchronizing* (or *reset*) for the DFA $\mathscr{A}$.
- $Syn(\mathscr{A})$ – the language of all words synchronizing $\mathscr{A}$.



A reset word is *abba*: applying it at any state brings the automaton to the state 1.

Černý conjecture

Any synchronizing n-state automaton possesses a synchronizing word of length at most $(n-1)^2$.

Notion of reset complexity

- The *state complexity* of the language L (the number of states in the minimal DFA recognizing L) is denoted by $sc(L)$.
- In what follows we consider only ideal languages: $L = \Sigma^* L \Sigma^*$.

Lemma 1.

Let L be an ideal language and $\mathscr{A}$ the minimal automaton recognizing L. Then $\mathscr{A}$ is synchronizing and $Syn(\mathscr{A}) = L$.

- *Reset complexity* of an ideal language L is the minimal possible number of states in a synchronizing automaton $\mathscr{A}$ such that $Syn(\mathscr{A}) = L$.

Results

- $rc(L) \leq sc(L)$.

Proposition 1.

Let L be an ideal language over a unary alphabet. Then $sc(L) = rc(L) = \ell + 1$, where ℓ is the minimum length of words in L.

- A synchronizing automaton, whose shortest reset words have length close to $(n-1)^2$, is called *"slowly"* synchronizing.

Proposition 2.

For every "slowly" synchronizing automata $\mathscr{A}_n$ with n states, $sc(Syn(\mathscr{A}_n)) = 2^n - n$ and $rc(Syn(\mathscr{A}_n)) = n$.

- For the language $L = (a+b)^*(b^3ab^2a + a^2b^3a + abab^3a + ab^2ab^3a)(a+b)^*$ there exist two different synchronizing automata for which L serves as the language of synchronizing words.