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Reset Observers for Linear Time-Delay Systems. A Delay-Independent Approach

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Abstract— **A Reset observer (ReO) is a novel sort of observer consisting of an integrator, and a reset law that resets the output of the integrator depending on a predefined condition over its input and/or output. The introduction of the reset element in the adaptive laws can decrease the overshooting and settling time of the estimation process without sacrificing the rising time. Motivated by the interest in the design of state observers for systems with time-delay, which is an issue that often appears in process control, this paper contributes with the extension of the ReO to the time-delay system framework. The time-independent stability analysis of our proposal is addressed by means of linear matrix inequalities (LMIs). Simulation results show the potential benefit of the proposed reset observer compared with traditional linear observers.**

I. INTRODUCTION

State observers are recursive algorithms that play a key role in many applications such as failure detection and recovery, monitoring and maintenance, or fault tolerant control. Initially, the research on state observers was focused on linear time invariant (LTI) systems [1], and afterwards on nonlinear systems [2], and on time-delay systems as well [3]. All those works are characterized by having only a proportional feedback term of the output observation error, and are known as proportional observers (POs). This proportional approach ensures a bounded estimation of the state and the unknown parameter, assuming a persistent excitation condition as well as the lack of disturbances. The performance of proportional observers can be improved by adding an integral term to the adaptive laws, and the resultant observers are known as proportional integral observers (PIOs) [4]. This additional integral term can increase the steady state accuracy and improve the robustness against modeling errors and disturbances [5]. Although PIOs were initially introduced in LTI systems for robustness improvement and loop transfer recovery, their effectiveness have been also checked with nonlinear system [6] and time-delay systems [7].

However, since the adaptive laws are still linear, they have the inherent limitations of linear feedback control. Namely, they cannot decrease the settling time and the overshoot of the estimation process simultaneously. Therefore, a trade-off between both requirements is needed. Nevertheless, this fundamental limitation can be overcome by adding a reset element. A simple reset element consists of an integrator and a reset law that resets the output of the integrator as long as the reset condition holds. This reset element is commonly referred to as the Clegg integrator after the work of Clegg in 1958 [8], who proposed an integrator which was reset to zero when its input is zero (zero crossing reset law). In 1974, Horowitz generalized that initial work substituting the Clegg integrator by a more general structure called the first order reset element (FORE) [9]. During the last years, the research on the stability analysis and stabilization for reset systems is attracting the attention of many academics and engineers. A main difference between the state-of-art reset control works is the definition of the reset law: in [10] reset instants are fixed and thus the stability analysis is much simpler; the work [11] develops stability conditions for the zero crossing reset law; in [12] stability conditions are obtained when the reset is performed at those instants in which the input and output of the reset element have different signs (sector reset condition); finally in [13] a dwell-time stability condition over reset instants is given that is applicable to any reset law. It is important to note that all these definitions of reset laws results in different reset systems dynamics, and all of them have advantages and disadvantages in relation to obtain stability and performance specifications in control practice. It should be noted that although there are a relevant number of stability results for the different types of reset law, synthesising reset elements for optimal performance is still an open issue.

Recently, stability analysis of reset control systems has been also extended to time-delay systems for the case of zero crossing reset law. There are two main approaches to study the stability of time-delay systems, which depend on whether the time-delay is included in the stability analysis or not [14]. Regarding the stability of reset control systems, the delay-dependent approach was addressed in [15], whereas the delay-independent stability analysis was given in [16]. In both cases, stability conditions were given using a set of LMI.

Although the research on reset elements is still an open and challenging topic, this research has been mainly focused on control issues. The first application of the reset elements to the state observer framework is [17]. There, the authors proposed a new sort of observer called reset observer (ReO). A ReO is an state observer whose integral term has been substituted for a reset element. The introduction of the reset element in the adaptive laws can improve the performance of the observer, as it is possible to decrease the overshoot and settling time of the estimation process simultaneously.

This paper extends our previous work about ReO [17], [18] to the time-delay system framework, using the ideas

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developed in [15]. The design of state observers for time-delay systems has attracted the attention of many researchers and practitioners, since time-delays often appear in many control applications [19]. In this paper, we adopt a delay-independent approach for the stability analysis. In addition, we consider the two most popular reset conditions, that is, the zero crossing reset condition (see for instance [11], [13], [15], [16]), and the sector condition (see for instance [12]).

This paper is organized as follows. In Section II, the ReO formulation for time-delay linear systems is presented. In Section III, LMI-based stability conditions that guarantee the convergence and stability of the estimation process for both sort of ReOs are developed. Note that the delay-independent stability result for the sector reset condition has value by itself and can be also applied for stability analysis of reset control systems. A simulation example is presented in Section IV in order to test the performance of our proposed ReO compared with traditional POs and PIOs. Finally, concluding remarks are given in Section V.

Notation: In the following, we use the notation $(x, y) = \left[x^T\ y^T\right]^T$. Given a state variable $x(t)$ of a hybrid time-delay system with resets, we will denote its time derivative with respect to the time by $\dot{x}(t)$, and the value of the state variable after the resets, that is, the value of $x(t+\delta)$ with $\delta \to 0^+$ by $x(t^+)$.

II. Reset Observer Formulation

In this paper, we address the problem of the state estimation of linear time-delay systems [3], which are described by:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-h) + Bu(t) + B_w w(t),\ \ t \geq 0 \\ x(t) &= \phi(t),\ \ t \in [-h, 0) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $\phi(t)$ is the continuous initial function, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}$ is the input vector, $w(t) \in \mathbb{R}^n$ is the disturbance vector, $y(t) \in \mathbb{R}$ is the output vector, h is the constant known time delay , $A \in \mathbb{R}^{n\times n}$, $A_d \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times 1}$, $B_w \in \mathbb{R}^{n\times 1}$, and $C \in \mathbb{R}^{1\times n}$ are known constant matrices. We consider single-input single-output (SISO) systems only, since a suitable formulation of reset elements for multiple-input multiple-output (MIMO) systems is still an open research topic. Moreover, $u(t)$ is assumed persistently exciting, and the pair (A, C) is assumed observable.

Now two different ReO formulations for time-delay systems such as (1) are presented. The difference lies in the reset condition chosen. Firstly, we present a ReO based on the zero crossing reset condition, that is, it is reset when the output estimation error $\tilde{y}(t) = 0$. Secondly, a ReO based on the sector reset condition is presented, and in this case, it is reset when the output estimation error and the integral estimation error have different signs $\tilde{y}(t)\zeta(t) \leq 0$. It should be noted that both reset laws are not equivalent in general, and that in principle they may give a difference performance over the estimation error. The goal of this work is to give formal stability conditions for both reset law, a formal analysis of performance is still an open issue. However, it will be illustrated how both definitions of reset laws may overcome the performance of a linear observer. Therefore, it should be noted that the sector reset condition allows a significant relaxation of the stability conditions, and that for some class of reset systems (like low-pass filters) the two reset conditions are equivalent, and the resultant systems behave in the same way except under some special initial conditions.

A. ReO Based on Zero Crossing Reset Condition

In this case, ReO dynamics are given by:

$$\left.\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + A_d\hat{x}(t-h) + Bu(t) \\ &+ K_P\tilde{y}(t) + K_I\zeta(t) \\ \dot{\zeta}(t) &= A_\zeta\zeta(t) + B_\zeta\tilde{y}(t) \\ \hat{y}(t) &= C\hat{x}(t) \\ \dot{\tau}(t) &= 1 \end{aligned}\right\} \quad \text{if } \eta(t) \notin \mathcal{M} \vee \tau \leq \rho, \tag{2}$$

$$\left.\begin{aligned} \hat{x}(t^+) &= \hat{x}(t) \\ \hat{\zeta}(t^+) &= A_r\zeta(t) \\ \hat{y}(t^+) &= \hat{y}(t) \\ \dot{\tau}(t^+) &= 0 \end{aligned}\right\} \text{if } \eta(t) \in \mathcal{M} \wedge \tau \leq \rho, \tag{3}$$

where $\eta(t) = [\tilde{x}(t)\ \zeta(t)]^T$, $\hat{x}(t)$ is the estimated state, $\tilde{x}(t) = x(t) - \hat{x}(t)$ is the state error, K_I and K_P represent the integral and proportional gain respectively and $\tilde{y}(t) = y(t) - \hat{y}(t)$ is the output estimation error, $\zeta(t)$ is the reset integral term, $A_\zeta \in \mathbb{R}$ and $B_\zeta \in \mathbb{R}$ are two tuning scalars which regulate the transient response of ζ, and A_r is the reset matrix. Specifically, we define $A_r = 0$, since the reset integral term ζ is reset to zero when $\zeta(t) \in \mathcal{M}$.

Therefore, to complete closed-loop system equations the set $\mathcal{M}$, that will be referred to as the reset surface, needs to be defined. Another set, the after-reset surface $\mathcal{M}_{\mathcal{R}}$, also plays an important role in the definition of closed-loop system solutions. Note that reset actions occur when the augmented state error $\eta(t)$ contacts the reset surface $\mathcal{M}$ at some instant t, that is $\eta(t) \in \mathcal{M}$, and then the reset term jumps to $A_r\eta(t) \in \mathcal{M}_{\mathcal{R}}$. In general, the set $\mathcal{M}_{\mathcal{R}}$ will be defined as $\mathcal{M}_{\mathcal{R}} = \mathcal{R}(A_r) \cap \mathcal{N}(C)$ where $\mathcal{R}(X)$ and $\mathcal{N}(X)$ stands for the image and null subspace of the linear operator given by the matrix X, respectively. Thus, $\mathcal{M}_{\mathcal{R}}$ is the set of states $\eta(t)$ that belong both to the null space of C (and then the output $C\eta(t) = 0$), and to the image space of A_r (they are after reset states). In addition, the set $\mathcal{M}$ is defined as $\mathcal{M} = \mathcal{N}(C)\backslash\mathcal{M}_{\mathcal{R}}$.

To avoid Zeno solutions, the reset term dynamics (2)-(3) includes temporal regularization. We use the notation proposed by [12], based on including an auxiliary variable τ which guarantees that the time interval between any two consecutive resets is not higher than $\rho \in \mathbb{R}^+$.

B. ReO Based on Sector Reset Condition

In this case, ReO dynamics are described as follows:

$$\left.\begin{array}{rcl}\dot{\hat{x}}(t) & = & A\hat{x}(t) + A_d\hat{x}(t-h) \\ & + & Bu(t) + K_P\tilde{y}(t) + K_I\zeta(t) \\ \dot{\zeta}(t) & = & A_\zeta\zeta(t) + B_\zeta\tilde{y}(t) \\ \hat{y}(t) & = & C\hat{x}(t) \\ \dot{\tau}(t) & = & 1\end{array}\right\} \quad \text{if } \eta(t) \in \mathcal{F} \vee \tau \leq \rho, \tag{4}$$

$$\left.\begin{array}{rcl}\hat{x}(t^+) & = & \hat{x}(t) \\ \hat{\zeta}(t^+) & = & A_r\zeta(t) \\ \hat{y}(t^+) & = & \hat{y}(t) \\ \dot{\tau}(t^+) & = & 0\end{array}\right\} \text{if } \eta(t) \in \mathcal{J} \wedge \tau \leq \rho, \tag{5}$$

As it was shown in [17], as long as the sector condition is preferred the ReO can be regarded as a hybrid system with a *flow set* $\mathcal{F}$ and a *jump or reset set* $\mathcal{J}$. On one hand, when $\eta(t) \in \mathcal{F}$, that is, if $\tilde{y}(t)$ and $\zeta(t)$ have the same sign, the ReO behaves as a proportional integral observer. On the other hand, if $\eta(t) \in \mathcal{J}$, that is, if $\tilde{y}(t)$ and $\zeta(t)$ have different sign, the integral term is reset according to the reset map A_r. According to these statements and since $\tilde{y}(t) = C\tilde{x}(t)$, the definition of both sets can be formalized by using the following augmented representation:

$$\mathcal{F} := \left\{\eta(t) : \eta^T(t)M\eta(t) \geq 0\right\}, \tag{6}$$

$$\mathcal{J} := \left\{\eta(t) : \eta^T(t)M\eta(t) \leq 0\right\}, \tag{7}$$

where $M = M^T$ is defined as

$$M = \begin{bmatrix} 0 & C^T \\ C & 0 \end{bmatrix}. \tag{8}$$

Since the ReO flows not only when $\eta^T(t)M\eta(t) \geq 0$ but also when $\tau \leq \rho$, the flow set $\mathcal{F}$ becomes slightly inflated. That inflated flow set is formally defined as $\mathcal{F}_\epsilon := \left\{\eta(t) : \eta^T(t)M\eta(t) + \epsilon\eta^T(t)\eta(t)\right\}$ where $\epsilon(\rho) \geq 0$ represents how the set is inflated [12]. Since $\epsilon \to 0$ as $\rho \to 0$, an arbitrarily small ρ can be chosen so that the effect of ϵ is small enough to be neglected [21]. Given that $\mathcal{F}_\epsilon$ slightly overflows into the jump set $\mathcal{J}$, the following assumption is needed to guarantee that the solution will be mapped to the flow set $\mathcal{F}$ after each reset and, consequently, there are no trajectories that keep flowing and jumping within $\mathcal{J}$.

Assumption 1. *The reset observer described by (4)-(5) is such that* $\eta(t) \in \mathcal{J} \;\Rightarrow\; A_R\,\eta(t) \in \mathcal{F}$, *where* $A_R = \begin{bmatrix} I & 0 \\ 0 & A_r \end{bmatrix}$ *is the reset map.*

It is important to note that this assumption is commonly used in many of the reset system formulations based on the sector reset condition, that are available in literature [12], [21].

III. Stability and Convergence Analysis

In this section we state computable sufficient conditions for quadratic stability of both ReOs presented in the previous section applied to time-delay systems described by (1).

To this end, let us begin analyzing the corresponding error system dynamics of both observers. Defining the state estimation error as $\tilde{x}(t) = x(t) - \hat{x}(t)$, and by using the previously defined augmented state error $\eta(t) = [\tilde{x}(t) \;\; \zeta(t)]^T$, $\eta(t-h) = [\tilde{x}(t-h) \;\; \zeta(t-h)]^T$, the error dynamics of the ReO based on the reset condition are given by

$$\left.\begin{array}{rcl}\dot{\eta}(t) & = & A_\eta\,\eta(t) + A_{\eta_d}\,\eta(t-h) + B_\eta\,w(t) \\ \tilde{y}(t) & = & C_\eta\,\eta(t)\end{array}\right\} \quad \text{if } \eta(t) \notin \mathcal{M} \vee \tau \leq \rho, \tag{9}$$

$$\left.\begin{array}{rcl}\eta(t^+) & = & A_R\,\eta(t) \\ \tilde{y}(t^+) & = & \tilde{y}(t)\end{array}\right\} \text{if } \eta(t) \in \mathcal{M} \wedge \tau \leq \rho, \tag{10}$$

and the error dynamics of the ReO based on the sector condition are as follows

$$\left.\begin{array}{rcl}\dot{\eta}(t) & = & A_\eta\,\eta(t) + A_{\eta_d}\,\eta(t-h) + B_\eta\,w(t) \\ \tilde{y}(t) & = & C_\eta\,\eta(t)\end{array}\right\} \quad \text{if } \eta(t) \in \mathcal{F} \vee \tau \leq \rho, \tag{11}$$

$$\left.\begin{array}{rcl}\eta(t^+) & = & A_R\,\eta(t) \\ \tilde{y}(t^+) & = & \tilde{y}(t)\end{array}\right\} \text{if } \eta(t) \in \mathcal{J} \wedge \tau \leq \rho, \tag{12}$$

where in both cases

$$A_\eta = \begin{bmatrix} A - K_PC & -K_I \\ B_\zeta C & A_\zeta \end{bmatrix}, \tag{13}$$

$$A_{\eta_d} = \begin{bmatrix} A_d & 0 \\ 0 & 0 \end{bmatrix}, \tag{14}$$

$$B_\eta = \begin{bmatrix} B_w \\ 0 \end{bmatrix}, \tag{15}$$

$$C_\eta = \begin{bmatrix} C & 0 \end{bmatrix}. \tag{16}$$

A. Asymptotic stability of ReOs Based on Zero Crossing Reset Condition

In this case the stability analysis follows directly from Propositions 1-2 in [16] where quadratic stability of time-delay reset systems subject to the zero crossing reset condition is addressed. As long as the same reset condition is used, those results can be particularized for the stability of ReOs as follows.

Proposition 1. *For given* A_η, A_{η_d}, B_η *and* A_R *the augmented error dynamics shown in (9)-(10) with* $B_w = 0$ *is quadratically stable, if for any matrix* Θ *with* $Im\,\Theta = Ker\,C$ *there exist some matrix* $P = P^T > 0$, $Q = Q^T > 0$ *subject to*

$$\begin{bmatrix} A_\eta^T P + PA_\eta + Q & PA_{\eta_d} \\ A_{\eta_d}^T P & -Q \end{bmatrix} < 0,$$
$$\Theta^T(A_R^T P A_R - P)\Theta \leq 0, \tag{17}$$

which is a linear matrix inequality problem in the variables P, Q.

Proof. It is quite similar to the proof shown in Proposition 3 in [16]. Firstly, let us introduce the Lyapunov-Krasovskii functional [14], that is defined as:

$$V(\eta_t) = \eta^T(t)P\eta(t) + \int_{-h}^{0} \eta^T(t+\theta)Q\eta(t+\theta)d\theta, \quad (18)$$

for some symmetric and positive definite matrices P, Q with size $n \times n$. Notice that $V(\eta_t) \geq 0$, and that $V(\eta_t) = 0$ only if $\eta_t(\theta) = 0 \in \mathbb{R}^n$, for each $\theta \in [-h, 0]$.

Then, to prove the quadratic stability of our proposed ReOs based on the zero crossing reset condition, we have to check that:

$$\begin{array}{ll} \frac{d}{dt}V(\eta_t) < 0 & \eta(t) \notin \mathcal{M} \\ V(\eta_{t^+}) - V(\eta_t) \leq 0 & \eta(t) \in \mathcal{M} \end{array} \quad (19)$$

Note that these inequalities are the standard stability requirements for reset systems, and they guarantee that there exists a common Lyapunov function that decreases when the reset system is flowing, and does not grow when the reset system is within the reset region.

Then, let us take derivative of (18) to obtain

$$\begin{aligned} \frac{d}{dt}V(\eta_t) &= \dot{\eta}(t)^T P\eta(t) + \eta(t)^T P\dot{\eta}(t) \\ &+ \eta(t)^T Q\eta(t) - \eta(t-h)^T Q\eta(t-h) \\ &= \left(\eta(t)^T A_\eta^T + \eta(t-h)^T A_{\eta_d}^T\right) P\eta(t) \\ &+ \eta(t)^T P \left(A_\eta \eta(t) + A_{\eta_d}\eta(t-h)\right) \\ &+ \eta(t)^T Q\eta(t) - \eta(t-h)^T Q\eta(t-h), (20) \end{aligned}$$

and using the first term of (19), (20) can be rearranged as an equivalent LMI problem in the variables $P, Q > 0$

$$\left[\begin{array}{cc} A_\eta^T P + PA_\eta + Q & PA_{\eta_d} \\ A_{\eta_d}^T P & -Q \end{array} \right] < 0, \quad (21)$$

which is the first inequality of (17).

It remains to prove the second term of (19). Since the reset action is only active when $\eta(t) \in \mathcal{M}$, and thus does not affect the delay buffer $\eta(t+\theta)$ for any $\theta \in [-h, 0)$ then the integral part of the Lyapunov functional (20) does not contribute to the jump, thus the reset jump in second term of (19) results in that

$$V(\eta_{t^+}) - V(\eta_t) = \Theta^T(A_R^T P A_R - P)\Theta \leq 0 \quad (22)$$

for every $\eta(t) \in \mathcal{M} = \text{Ker}C$, which is the second inequality of (17) and completes the proof. □

B. Asymptotic stability of ReOs Based on Sector Reset Condition

In this case the quadratic stability is addressed in similar way than [17], although in this time the standard quadratic Lyapunov function candidate must be substituted for the previously presented Lyapunov-Krasovskii functional.

Proposition 2. *For given A_η, A_{η_d}, B_η and A_R the augmented error dynamics shown in (11)-(12) with $B_w = 0$ is quadratically stable, if there exist some matrix $P = P^T > 0$, $Q = Q^T > 0$ and scalars $\tau_F \geq 0$ and $\tau_J \geq 0$ subject to*

$$\begin{gathered} \left[\begin{array}{cc} A_\eta^T P + PA_\eta + Q + \tau_F(M + \epsilon I) & PA_{\eta_d} \\ A_{\eta_d}^T P & -Q \end{array} \right] < 0, \\ A_R^T P A_R - P - \tau_J M \leq 0, \quad (23) \end{gathered}$$

which is a linear matrix inequality problem in the variables P, Q, τ_F and τ_J.

Proof. In this case, to prove the quadratic stability of the ReO based on the sector reset condition, we have to check that:

$$\begin{array}{ll} \frac{d}{dt}V(\eta_t) < 0 & \eta(t) \in \mathcal{F}_\epsilon \\ V(\eta_{t^+}) - V(\eta_t) \leq 0 & \eta(t) \in \mathcal{J} \end{array} \quad (24)$$

where V is the Lyapunov-Krasovskii functional (18) and $\frac{d}{dt}V(\eta_t)$ is as it is defined in (20).

Since $\mathcal{F}_\epsilon := \left\{\eta(t) : \eta^T(t)\, M\, \eta(t) + \epsilon\eta^T(t)\eta(t) \geq 0\right\}$ and employing the S-procedure [22], the first term of (24) is equivalent to the existence of $\tau_F \geq 0$ such that

$$\frac{d}{dt}V(\eta_t) < -\tau_F \eta^T(t)(M + \epsilon I)\eta(t) \quad (25)$$

Rearranging terms of equations (25) and (20), the first term of (24) holds if the following inequality is satisfied

$$\begin{aligned} &\eta(t)^T \left(A_\eta^T P + PA_\eta + Q + (M + \epsilon I)\right)\eta(t) \\ +\ &\eta(t-h)^T A_{\eta_d}^T P\eta(t) + \eta(t)^T PA_{\eta_d}\eta(t-h) \\ -\ &\eta(t-h)^T Q\eta(t-h) < 0, \quad (26) \end{aligned}$$

which can be rearranged as an equivalent LMI problem in the variables $P, Q > 0$ and $\tau_F \geq 0$

$$\left[\begin{array}{cc} A_\eta^T P + PA_\eta + Q + (M + \epsilon I) & PA_{\eta_d} \\ A_{\eta_d}^T P & -Q \end{array} \right] < 0, (27)$$

which is the first inequality of (23) and consequently, proves the first equation of (24).

Similarly, employing again the S-procedure, the second term of (24) holds if there exists $\tau_J \geq 0$ such that

$$V(\eta_{t^+}) - V(\eta_t) \leq \eta^T(t)\tau_J M\eta(t), \quad (28)$$

which is equivalent to

$$\eta(t)^T A_R^T P A_R \eta(t) - \eta^T(t)P\eta(t) - \eta(t)^T \tau_J M\eta(t) \leq 0. \quad (29)$$

Rearranging terms, (29) can be also rewritten as an equivalent LMI problem in the variables $P > 0$ and $\tau_J \geq 0$ as follows

$$A_R^T P A_R - P - \tau_J M \leq 0, \quad (30)$$

which is analogous to the second inequality of (23) and proves the second equation of (24) and, as a consequence, completes the proof of the proposition. □

Remark 1. For stability purposes, we have to prove that $\frac{d}{dt}V(\eta_t)$ is negative in any region wherein the state $\eta(t)$ can flow, and that $\Delta V(\eta_t) \leq 0$ for any region wherein the state $\eta(t)$ is reset. In particular, since $\mathcal{F} \subseteq \mathcal{F}_\epsilon$, $\frac{d}{dt}V(\eta_t)$ must be proven for all $\eta(t) \in \mathcal{F}_\epsilon$. If we only check $\frac{d}{dt}V(\eta_t)$ when

$\eta(t) \in \mathcal{F}$, we cannot guarantee that the Lyapunov function is decreasing when the state $\eta(t)$ overflows into the adjacent reset region because of the temporal regularization. For this reason, we have to check $\frac{d}{dt}V(\eta_t)$ for all $\eta(t) \in \mathcal{F}_\epsilon$, and $\Delta V(\eta_t) \le 0$ for all $\eta(t) \in \mathcal{J}$.

IV. Simulation Results

In this section, an example is presented in order to show the effectiveness of our proposed ReOs. To this end, we compare the simulation results obtained by a ReO based on the zero crossing reset condition, and a ReO based on the sector reset condition with a PO and with a PIO. On the one hand, the PO will be tuned to minimize the overshooting and, as a consequence, it provides a smooth response. On the other hand, the PIO will be designed to minimize the rising time, and hence, it gives an oscillating and faster response. The next simulation example will show that our proposed ReOs can achieve both requirements (i.e. a smooth and quick response) simultaneously. These simulation results have been obtained by using Simulink with the ode3 solver.

Let us consider the following time-delay system:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} -2 & 0.1 \\ 0.1 & -0.9 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \\ &+ \begin{bmatrix} -0.75 & 0 \\ -0.75 & -0.75 \end{bmatrix} \begin{bmatrix} x_1(t-h) \\ x_2(t-h) \end{bmatrix} \\ &+ \begin{bmatrix} 0.5 \\ 2.5 \end{bmatrix} u(t) + \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} w(t) \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{aligned} \tag{31}$$

with $x(t=0) = x(t=h) = [1, -1]^T$, $u(t) = sin(5t)$, and $h = 0.2\ sec$. The aim is to develop an state observer for the system described by (31) which satisfies that the state estimation error tends to zero without overshooting.

Additionally, let us outline the tuning parameter for each observer (i.e. PO, PIO, ReO). Notice that when it is possible, the tuning parameters are equal for each observer in order to make the results more comparable. Since the tuning process of each observer involves several parameters, let us outline how all these tuning parameters have been determined. Firstly, we have designed the PO in such a manner that its rising time is around 1.5 seconds without overshooting. After that, to design the oscillating PIO we have increased the K_I gain until its rising time is roughly equal to 0.5 seconds, that implies an oscillating estimation process. Finally, to make the results more comparable, the ReOs have the same K_I and K_P than the oscillating PIO.

Specifically, the parameters of the PO are $\hat{x}(t=0) = \hat{x}(t=h) = [0,0]^T$, $K_P = [0.05, -1]^T$, whereas the tuning parameters of the oscillating PIO are $\hat{x}(t=0) = \hat{x}(t=h) = [0,0]^T$, $z(t=0) = z(t=h) = 0$, $A_z = -0.5$, $B_z = 1$, $K_P = [0.05, -1]^T$, and $K_I = [16, -20]^T$. For a further discussion on the structure of PIOs and PIOs the reader is referred to [6] and [17]. On the other hand, both ReOs for the system (31) have the same tuning parameters. Namely, $\hat{x}(t=0) = \hat{x}(t=h) = [0,0]^T$, $\zeta(t=0) = \zeta(t=h) = 0$, $A_\zeta = -0.5$, $B_\zeta = 1$, $K_P = [0.05, -1]^T$, $K_I = [16, -20]^T$, $A_r = 0$, $\epsilon = 0$. Notice that the K_P and K_I gains of the ReOs are equal to the gains of the oscillating PIO. Finally, it is worth mentioning that the stability of both ReOs can be easily checked with the propositions given in Section III.

To analyze the effect of the disturbance $w(t)$ on the performance of our proposal, let us consider that the system (31) is noise-free, and thus, $w(t) = 0$. In this case, the state estimation error $\tilde{x}(t) = [\tilde{x}_1(t), \tilde{x}_2(t)]^T$ of all the observers is shown in Fig. 1. It is evident that our proposed ReOs have a better performance compared with traditional PIOs, since it has a response as quick as the oscillating PIO but without overshooting. Notice that if we decrease the integral gain K_I of the oscillating PIO to avoid overshooting it will behave as the conservative PIO and, thus, its rising time will be higher than the obtained by ReOs. On the other hand, if we increase the integral gain K_I of the conservative PIO to reduce its rising time, it will behave as the oscillating PIO and, as a consequence, its response will be oscillating.

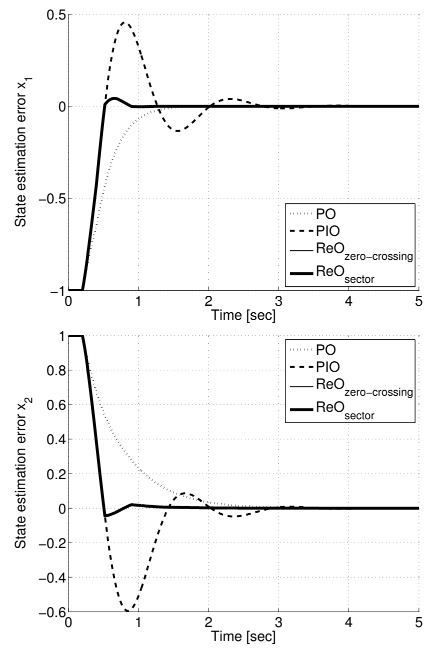

Fig. 1. State estimation error $\tilde{x}(t)$ obtained for each observer when the system is noise-free. Dotted lines have been obtained by using the proportional observer. Dashed lines have been obtained by using the oscillating proportional integral observer. Thin solid lines have been obtained by using the ReO based on the zero crossing reset condition. Thick solid lines have been obtained by using the ReO based on the sector reset condition. Note that the behavior of both ReOs is exactly the same.

In this case both reset conditions are equivalent, however the differences appear when the initial state of the integral term ζ is not zero. To see this, Figs. 2-3 show the behavior of the previously presented ReOs when $\zeta(t=0) = \zeta(t=h) = 2$ and $\zeta(t=0) = \zeta(t=h) = -0.15$ respectively, and $w(t) = sin(15t)$. These results underline the key role played by the reset condition in the behavior of reset systems, and that depending on the initial conditions of the system, some reset conditions can behave better than others. However, it is not clear which one is generally the best. It remains for future research to answer this challenging question.

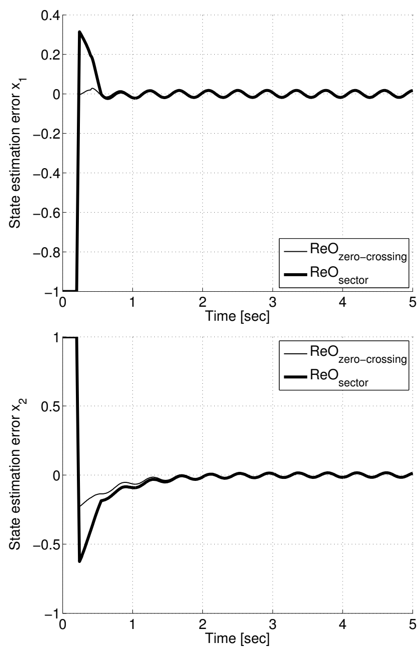

Fig. 2. State estimation error $\tilde{x}(t)$ obtained for each observer when the system is noise-corrupted. Thin solid lines have been obtained by using the ReO based on the zero crossing reset condition. Thick solid lines have been obtained by using the ReO based on the sector reset condition. Note that $\zeta(t=0)=\zeta(t=h)=2$.

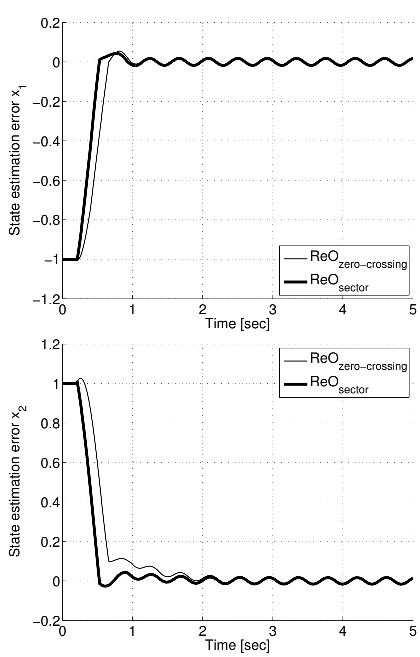

Fig. 3. State estimation error $\tilde{x}(t)$ obtained for each observer when the system is noise-corrupted. Thin solid lines have been obtained by using the ReO based on the zero crossing reset condition. Thick solid lines have been obtained by using the ReO based on the sector reset condition. Note that $\zeta(t=0)=\zeta(t=h)=-0.15$.

V. CONCLUSION

This paper has extended our previously developed ReO [17] to the time-delay system framework. The two most popular reset conditions have been considered. Stability analysis independent of the time delay has been given for both cases.

Reset elements can decrease the overshoot and settling time of the estimation process without sacrificing the rising time for some kind of systems. Simulation results have been given to underline their potential benefits. Independently of the reset condition chosen, the proposed ReOs introduce improvements even in the presence of disturbances.

REFERENCES

[1] G. Luders and K. S. Narendra. A new canonical form for an adaptive observer. *IEEE Trans. on Automatic Control*, 19(2):117–119, 1974.

[2] Q. Zhang. Adaptive observer for multiple-input multiple-output (MIMO) linear time-varying systems. *IEEE Trans. on Automatic Control*, 47(3):525–529, 2002.

[3] M Trinh, P. S. Teh, and T. L. Fernando. Time-delay systems: Design of delay-free and low-order observers. *IEEE Trans. on Automatic Control*, 55(10):2434–2438, 2010.

[4] B. Shafai, S. Beale, H. H. Niemannv, and J. Stoustrup. LTR design of discrete-time proportional integral observer. *IEEE Trans. on Automatic Control*, 41(7):1056–1062, 1996.

[5] E. Vahedforough and B. Shafai. Design of proportional integral adaptive observers. In *Proc. of the IEEE American Control Conference*, pages 3683–3688, 2008.

[6] J. Jung, J. Hwang, and K. Huh. Optimal proportional-integral adaptive observer design for a class of uncertain nonlinear systems. In *Proc. of the IEEE American Control Conference*, pages 1931–1936, 2007.

[7] H. Sadaka, B. Shafai, and R. Sipahi. Robust PI observer design for linear time-delay systems. In *Proc. of the Multi-conference on Systems and Control*, pages 1209–1213, 2009.

[8] J. Clegg. A nonlinear integrator for servomechanisms. *Trans. of the A.I.E.E.*, 77(Part-II):41–42, 1958.

[9] I. Horowitz and P. Rosenbaum. Nonlinear design for cost of feedback reduction in systems with large parameter uncertainty. *International Journal of Control*, 21(6):977–1001, 1975.

[10] Y. Guo, Y. Wang, L. Xie, and J. Zheng. Stability analysis and design of reset systems: Theory and an application. *Automatica*, 45(2):492–497, 2009.

[11] O. Beker, C. Hollot, Y. Chait, and H. Han. Fundamental properties of reset control systems. *Automatica*, 40(6):905–915, 2004.

[12] D. Nesic, L. Zaccarian, and A. R. Teel. Stability properties of reset systems. *Automatica*, 44(8):2019–2026, 2008.

[13] A. Baños, J. Carrasco, and A. Barreiro. Reset times dependent stability of reset systems. *IEEE Trans. on Automatic Control*, 56(1):217–223, 2011.

[14] K. Gu, V. L. Kharitonov, and J. Chen. *Stability of Time-Delay Systems*. Birkhauser, 2003.

[15] A. Barreiro and A. Baños. Delay-dependent stability of reset systems. *Automatica*, 46(1):216–221, 2010.

[16] A. Baños and A. Barreiro. Delay-independent stability of reset systems. *IEEE Trans. on Automatic Control*, 52(2):341–346, 2009.

[17] D. Paesa, C. Franco, S. Llorente, G. Lopez-Nicolas, and C. Sagues. Reset adaptive observers and stability properties. In *Proc. of the 18th IEEE Mediterranean Control Conference*, pages 1435–1440, 2010.

[18] D. Paesa, A. Baños, and C. Sagues. Optimal reset adaptive observer design. *Systems and Control Letters*, 60(10):877–883, 2011.

[19] A. Vidal and A. Baños. Reset compensation for temperature control: Experimental application on heat exchangers. *Chemical Engineering Journal*, 159:170–181, 2010.

[20] T. Loquen, S. Tarbouriech, and C. Prieur. Stability analysis for reset systems with input saturation. In *Proc. of the IEEE Conference on Decision and Control*, pages 3272–3277, 2007.

[21] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory*. SIAM - Society for Industrial Applied Mathematics, 1994.