Residual Circulation in the North Sea due to the M_2-tide and Mean Annual Wind Stress

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UDC 551.465.52:551.465.11:551.466.72; North Sea; German Bight

Summary

In a high resolving (mean grid size 11 km) barotropic numerical model of the North Sea tides the M_2-tide is computed. Nonlinearities of the Navier-Stokes-equation yield, in connection with an annual mean wind stress, a residual circulation fitting well to observational data. Some effects of this circulation are discussed in connection with pollution problems.

Restzirkulation zur M_2-Tide in der Nordsee und der Windschub im Jahresmittel (Zusammenfassung)

In einem hochauflösenden (mittlerer Gitterabstand 11 km) barotropen Modell für die Gezeiten der Nordsee wird die M_2-Gezeit berechnet. Verschiedene Nichtlinearitäten der Bewegungsgleichungen führen in Verbindung mit einem mittleren Windschub zu einer Restzirkulation, die mit Beobachtungen in Einklang steht. Einige Effekte dieser Zirkulation in Zusammenhang mit Verschmutzungsproblemen werden diskutiert.

Circulation résiduelle en mer du Nord due à l'onde de marée M_2 et à la force moyenne annuelle du vent (Résumé)

L'onde marée M_2 est calculée dans un modèle numérique barotropique de la mer du Nord de haute résolution (l'espacement moyen entre les lignes de quadrillage étant de 11 km). Des non-linéarités de l'équation Navier-Stokes produisent, en connection avec une force moyenne annuelle du vent, une circulation résiduelle s'accordant bien avec les données observées. Certains effects de cette circulation sont discutés, en relation avec les problèmes de pollution.

1. Introduction

During the past 15 years, hydrodynamic-numerical ("HN") models became an established tool to study tidal phenomena. The tides in the North Sea are dominated by the M_2-Kelvin wave entering from the North Atlantic and reflected at the coasts. This general pattern is well reproduced even within the framework of a global tidal model of 1^0 spatial resolution which only leaves 40 grid points to resolve the North Sea (Zahel [1976]). A good approximation of tidal records from selected coastal stations is obtained with the classical resolution of 37 km (Hansen [1956]).

It must be emphasized, however, that the finite difference method modifies phase velocities more severely than amplitudes. This grid dispersion decreases with increasing resolution but is still a nuisance in Hansen's grid. The coarseness of the grid becomes even more crucial in models of the residual circulation of the North Sea. Maps of this circulation show a complicated fine structure which can only be resolved with a mesh size essentially smaller than 37 km (Böhnecke [1922], Laevastu [1963]).

In this paper, I present a model using a resolution of about 11 km, depending on the geographical position.

The model consists of:

a set of differential equations,

a numerical discretisation scheme,

boundary conditions and discretisized bottom topography.

The differential equations are the Navier-Stokes equations in a vertically integrated form. The usual procedure to incorporate the dissipation of energy by turbulence into these equations introduces two parameters: one for bottom friction and the other for horizontal eddy viscosity. These parameters describe physical processes, but they are also indispensable for the sake of numerical stability. To assign specific values to these parameters is the subject of the non-trivial procedure of parametrization.

The discretisation scheme is the method introduced to oceanography by Hansen. The model uses a staggered grid: the water elevation and the two velocity components are computed at three different points with the advantage of an implicit second order accuracy although explicitly the finite difference equations are written down only up to the first order.

The bottom topography was read out from pilot sea charts. The complex structure of the Dutch and German islands was represented by a constant depth of 2 m.

The boundary conditions turned out to be the crucial problem. Along the coast, the natural boundary condition of zero velocity normal to the coast is sufficient. Along the connection line with adjacent seas, a tidal signal must be prescribed explicitly, either as water elevation or as velocity. At the opening towards the Channel a linear interpolation between the harmonic constants for the tides at Dover and at Dunkerque seems to give good results. The tidal constants at the northern boundary, between Scotland and Norway, are rather poorly known. At this boundary the present model installs harmonic values for the M_2-tide taken out from Heaps' model (Heaps [1974]) of the ambient shelf region. The restriction to only one tidal constituent should be regarded as a shortcoming of the model. In view of the nonlinear character of energy dissipation, the numerical reproduction of the corange lines must suffer from this restriction. On the other hand, the M_2-tide is highly dominant in the North Sea and the restriction provides, in an economic way, a convenient time interval for the computation of the residual circulation.

2. The model

The model is based upon the equations

$$\frac{\mathrm{d}\boldsymbol{U}}{\mathrm{d}t} + \boldsymbol{f} \times \boldsymbol{U} + \frac{r\boldsymbol{U}|\boldsymbol{U}|}{H^2} + gH\nabla\zeta - \nabla^*(A\nabla)\boldsymbol{U}^* = \boldsymbol{F}\cdot H$$

$$\frac{\partial\zeta}{\partial t} + \nabla\boldsymbol{U} = 0$$

where $\boldsymbol{U}$ = horizontal transport, $\boldsymbol{f}$ = Coriolis vector, H = water depth, g = gravity acceleration, ζ = water elevation above the equipotential surface, A = coefficient of horizontal viscosity, $\boldsymbol{F}$ = external acceleration and r = coefficient of turbulent bottom friction. r was chosen to be $r = 0.0025$, A was chosen to be dependent on depth:

$$A = H \times 20\ \mathrm{m/s}.$$

This condition implies free slip along the coastline and due to its special choice, no explicit additional boundary condition at the coasts had to be incorporated. In all other details, the discretisation scheme is identical to the scheme used by Zahel (Zahel [1970]).

The grid is embedded in geographical coordinates. The grid size is 6′ in N-S-direction and 10′ in E-W-direction. The depth values were compiled from the Seekarte No. 101 edited by the Deutsches Hydrographisches Institut. Fig. 1 shows depth contours constructed from the discrete grid point depth values entering the model.

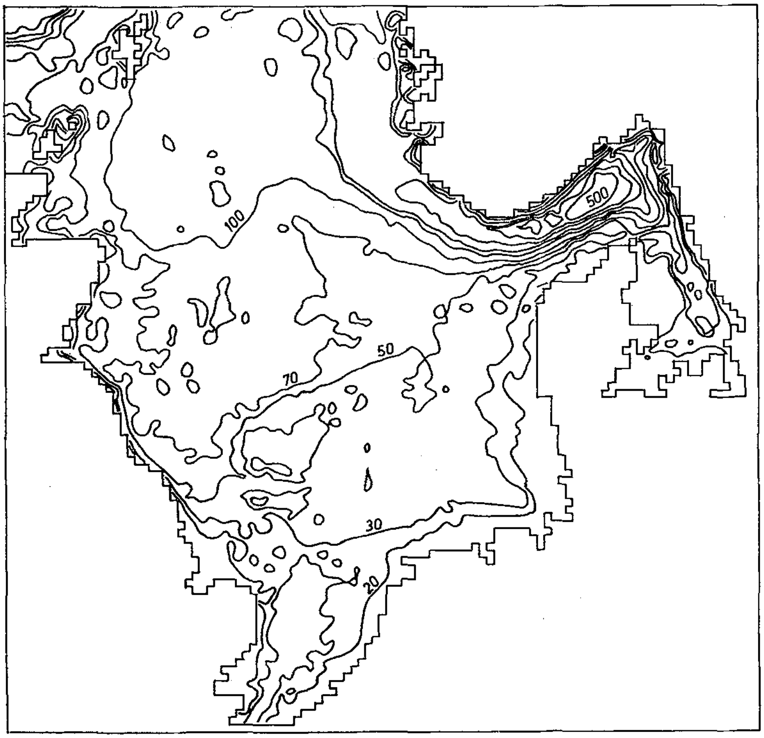

Fig. 1. Depth (m) contours of the discretisized bottom topography

The tide generating force of the M_2 partial tide was taken into account as an external force. An annual mean wind stress field, acting as an additional external force, was constructed in the following manner: Kuhlbrodt's (Kuhlbrodt, Bullig, Bintig et al. [1954]) monthly mean wind vectors for $2^0 \times 2^0$ areas were averaged vectorially over the full year, each component weighted by the monthly wind speed. Linear interpolation of the annual mean wind to the grid points resulted in the rather smooth quadratic wind stress field shown in Fig. 2.

The computed cotidal and corange lines are shown in Fig. 3 together with the harmonic constants at some tide gauge stations for comparison. Such a comparison seems to be reasonable only in the high amplitude regions of the southern and western part of the North Sea. In the Skagerrak and in the Kattegat the amplitudes are too small to give reliable information on the phases. There is some slight evidence, however, for an amphidromic point off the south coast of Norway. The comparison gives an excellent agreement in the phases, whereas the amplitudes seem to be somewhat overestimated by the model. This could be the consequence of too small turbulence parameters, but on the other hand, as said before, the model is driven by the M_2-tide only, and the nonlinear interaction of the M_2-signal with other partial tides could also reduce the amplitudes.

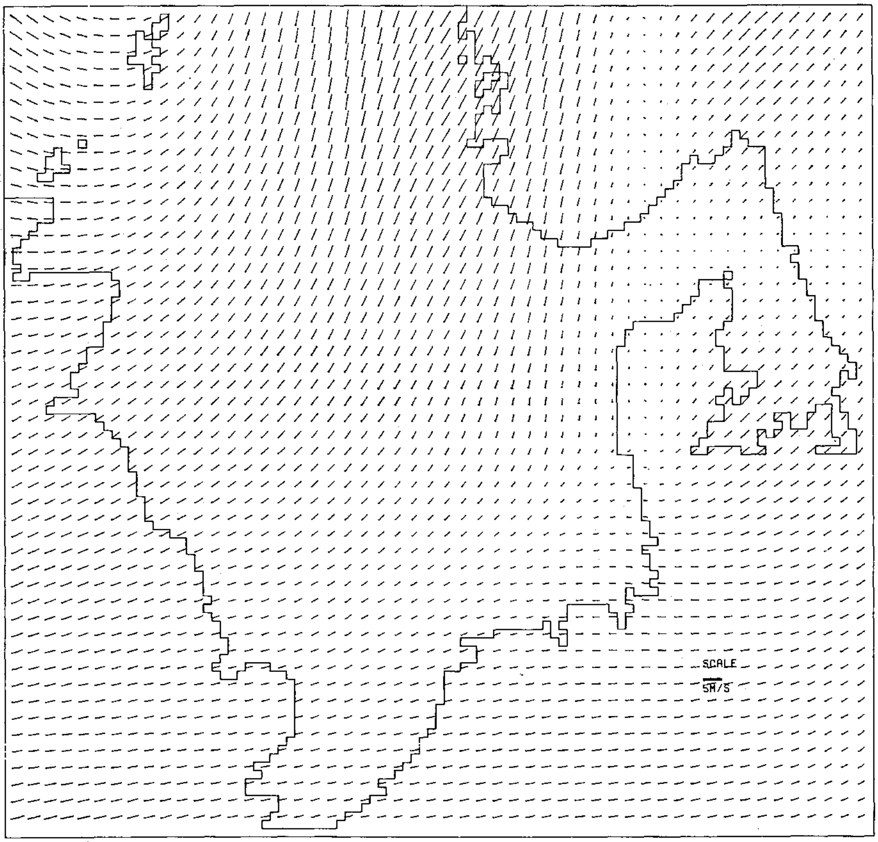

Fig. 2. Annual mean wind stress, weighted average out of monthly mean values in 2×2 degree areas

3. The residual circulation

The mean displacement velocity must be distinguished from the mean velocity as obtained by an integration in time of the data recorded by a fixed current meter. Consider a harmonic wave

$$u = u_0 \cos(\sigma t - kx). \tag{1}$$

The mean velocity at a fixed point is zero. Taking u as the real displacement velocity of water particles, (1) is written as

$$\frac{\mathrm{d}x}{\mathrm{d}t} = u_0 \cos(\sigma t - kx).$$

For $x = 0$ at $t = 0$, the solution is

$$x = \frac{1}{k}\left(\sigma t + 2 \arctan\left(\frac{1 - \frac{u_0 k}{\sigma}}{\sqrt{1 - \left(\frac{u_0 k}{\sigma}\right)^2}} \tan\left(\frac{\sigma t}{2}\sqrt{1 - \left(\frac{u_0 k}{\sigma}\right)^2}\right)\right)\right).$$

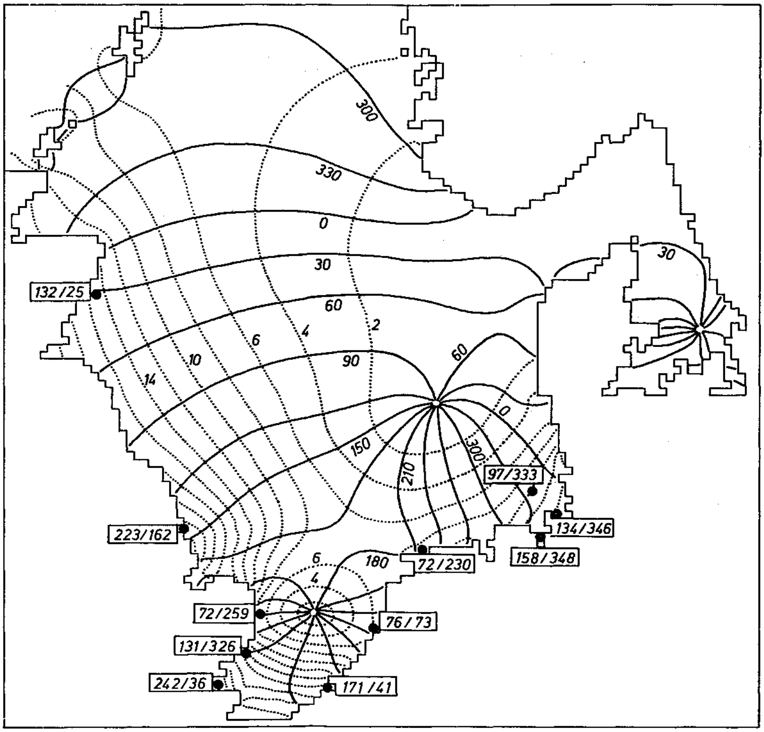

Fig. 3. Computed cotidal (straight lines, degrees) and corange (dotted lines, decimeters) lines and harmonic constants of several tide gauge stations

By expansion into a Taylor series up to the first order, the mean displacement (Lagrangian) velocity turns out to be

$$\bar{u} \approx \frac{u_0^2 k}{2\sigma}.$$

The same result is obtained when weighting the velocity at time t with the water depth at time t in an Eulerian integration: the water elevation corresponding to (1) is

$$\zeta = u_0 \frac{H_0 k}{\sigma} \cos(\sigma t - kx). \tag{2}$$

The mean transport velocity is

$$\bar{u} = \int_0^{\frac{2\pi}{\sigma}} (H_0 + \zeta)\, u \, \mathrm{d}t \Big/ \int_0^{\frac{2\pi}{\sigma}} (H_0 + \zeta)\, \mathrm{d}t = \frac{u_0^2 k}{2\sigma} = u_0 \cdot \frac{u_0}{2\sqrt{gH_0}}. \tag{3}$$

Fig. 4 shows the mean velocity field computed according to this formula. For the sake of a better appearance, the information is slightly reduced: each arrow represents an average of 9 vectors defined at contiguous grid points. A rather good agreement with the Böhnecke (Böhnecke [1922]) chart (Fig. 5) is obvious. There is a number of vortices which in a coarser grid could not be distinguished from numerical instabilities. Fig. 6 shows the full information in terms of streamlines obtained by integration of the V-component with respect to x. The

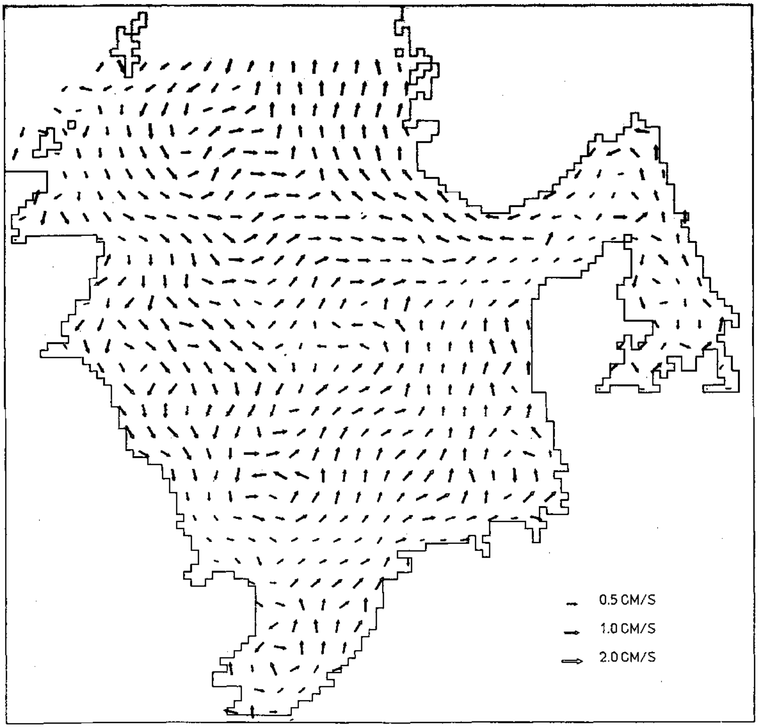

Fig. 4. Mean velocities, M_2-tide and annual mean wind stress

figure shows also an enlargened section of the stream lines in the German Bight which fits well to the compilation of salinity distribution carried out by Goedecke (Goedecke [1941]) (Fig. 7). In both figures, the northernmost part is omitted, because there the doubtful boundary condition obviously falsifies the results. Fig. 8 shows the mean water level as obtained by integration in time of the computed water level. This mean water level is very close to a geostrophic balance with the mean circulation. At the northern boundary, however, the mean water level is permanently kept at zero, preventing a geostrophic adjustment. Fortunately, this inconsistency does not affect but the first 4 or 6 grid lines.

Some authors have argued (Nihoul [1975]) that the mean velocities are smaller than the errors in the computed tidal velocities, and that, consequently, the error in the residual circulation of Figures 4 and 6 should be at least 100 %. This author feels that such an argument is not valid, since (2) implies that the relative error in the residual circulation is just twice the relative error in the tidal currents. Other nonlinearities as bottom friction and advection terms may introduce, of course, additional errors, but they are related too to the dominant phenomenon of the propagating Kelvin wave which is reproduced well in the model.

The computed circulation was applied to simulate the dispersion of matter released continuously at a point source in the Channel. The computation scheme was the stochastic procedure described in a previous paper (Maier-Reimer [1975]). A diffusion coefficient of 25 m^2/s was used in this application. Fig. 9 shows the computed distribution after two years. It compares rather well with the measured distribution of Cs^{137} (Kautsky [1973]) (Fig. 10). In particular, the dispersion of the radionuclide into the inner German Bight was not reproduced in the previous model, and it illustrates an important effect of the mesh refinement.

Knowing the residual circulation, it is possible to evaluate trajectories and to determine for each particle the time spent while leaving the North Sea starting from a particular location.

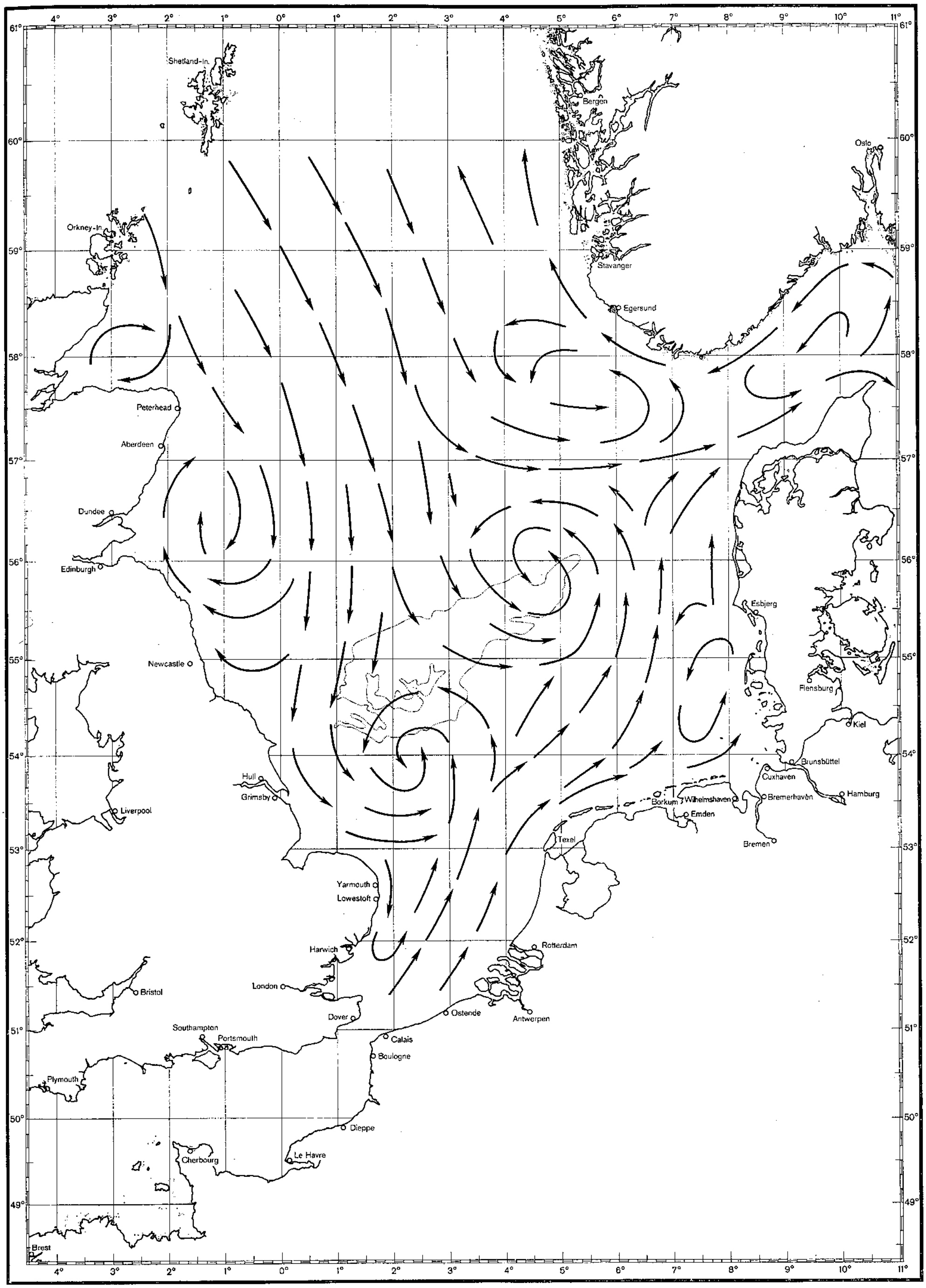

Fig. 5. Average "residual current" in the North Sea (after G. Böhnecke [1922])

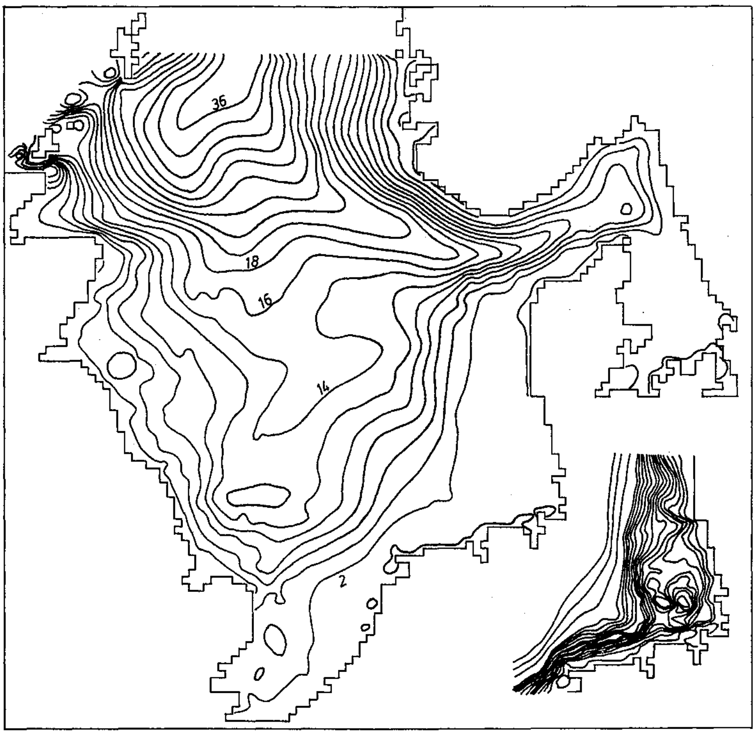

Fig. 6. Mean transport ($10^4 m^3/s$) streamfunction defined as $\int V dx$

Fig. 11 shows the computed lines of equal residence time. The shortest residence times are in the northeastern regions of the Norwegian current. In the western and southern part of the North Sea the residence times are surprisingly constant. The nearshore trajectories are longer than the offshore ones, but they are associated with higher residual velocities. The integration was carried out using a time step of 12 hours; as a consequence of this large time step, particles could leave regions embedded in closed stream lines. These results must be taken to be of hypothetical value only since, at present, there is no chance for a direct verification.

Acknowledgements

The author would like to thank Dr. H. Friedrich and Prof. Dr. W. Hansen for helpful discussions and Miss Krautwald for preparing the bottom topography and assistance in figure caption.

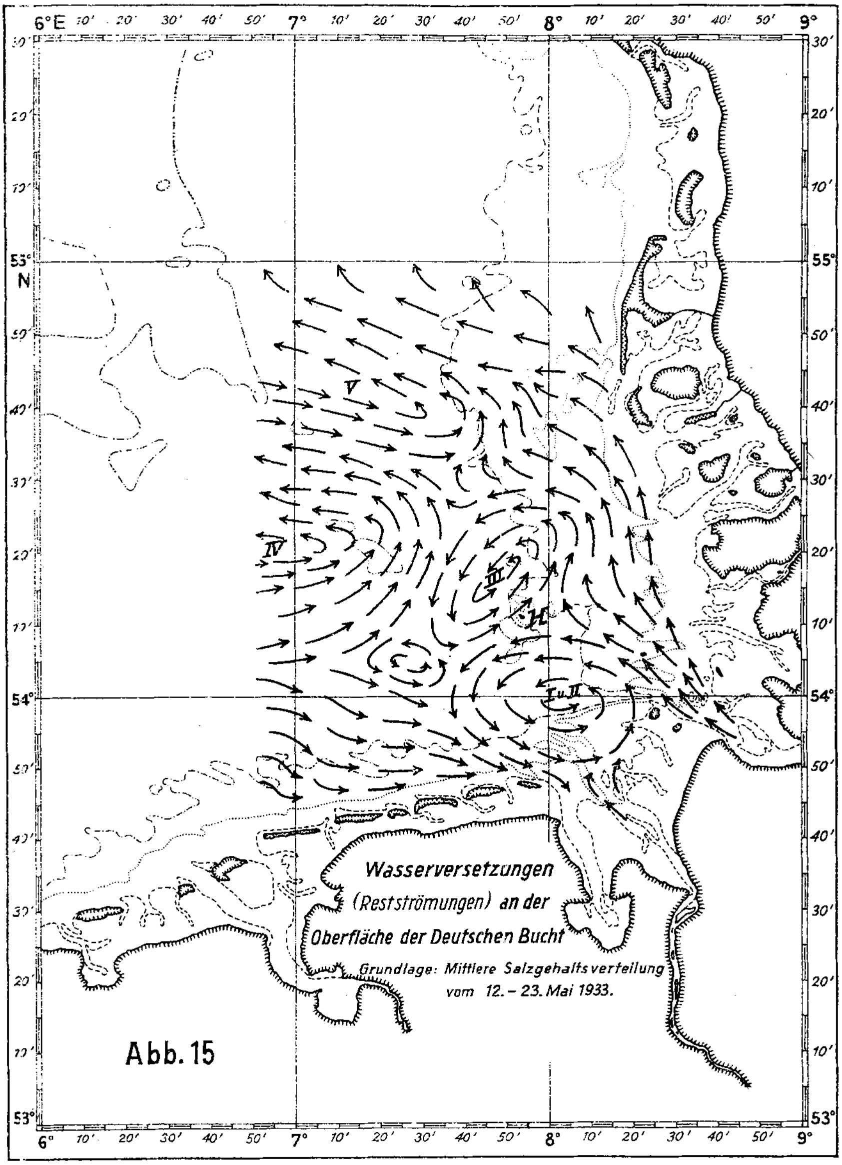

Fig.7. Residual current at the surface of the German Bight (after E. Goedecke [1941])

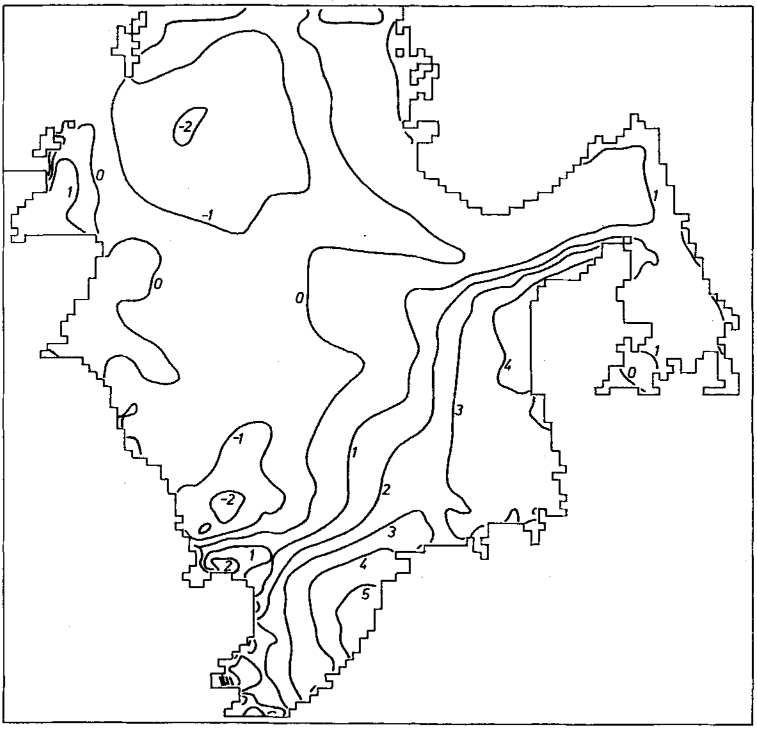

Fig. 8. Mean water level (cm)

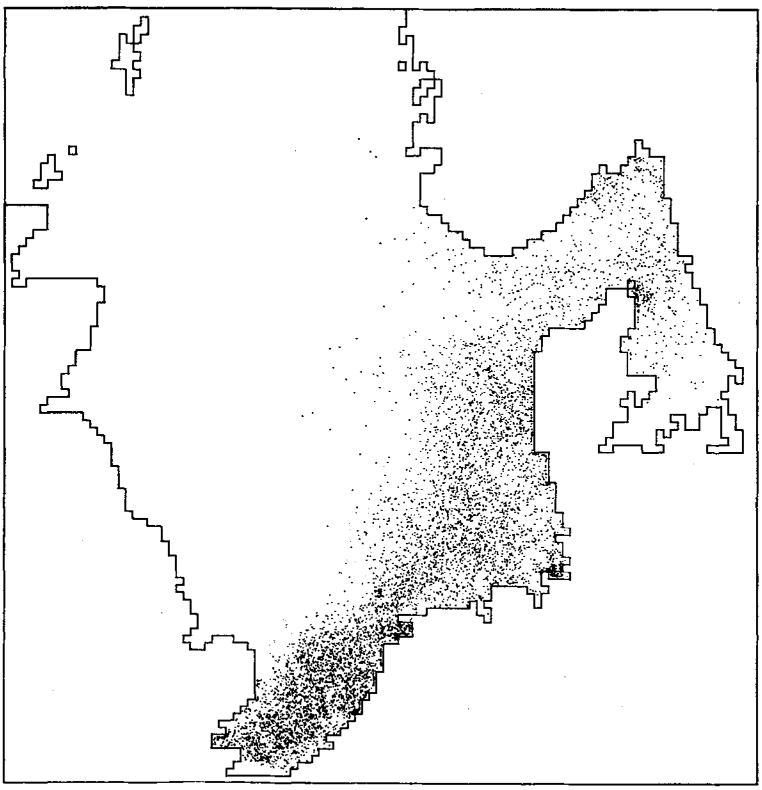

Fig. 9. Dispersion of matter released by a continuous point source in Channel in two years

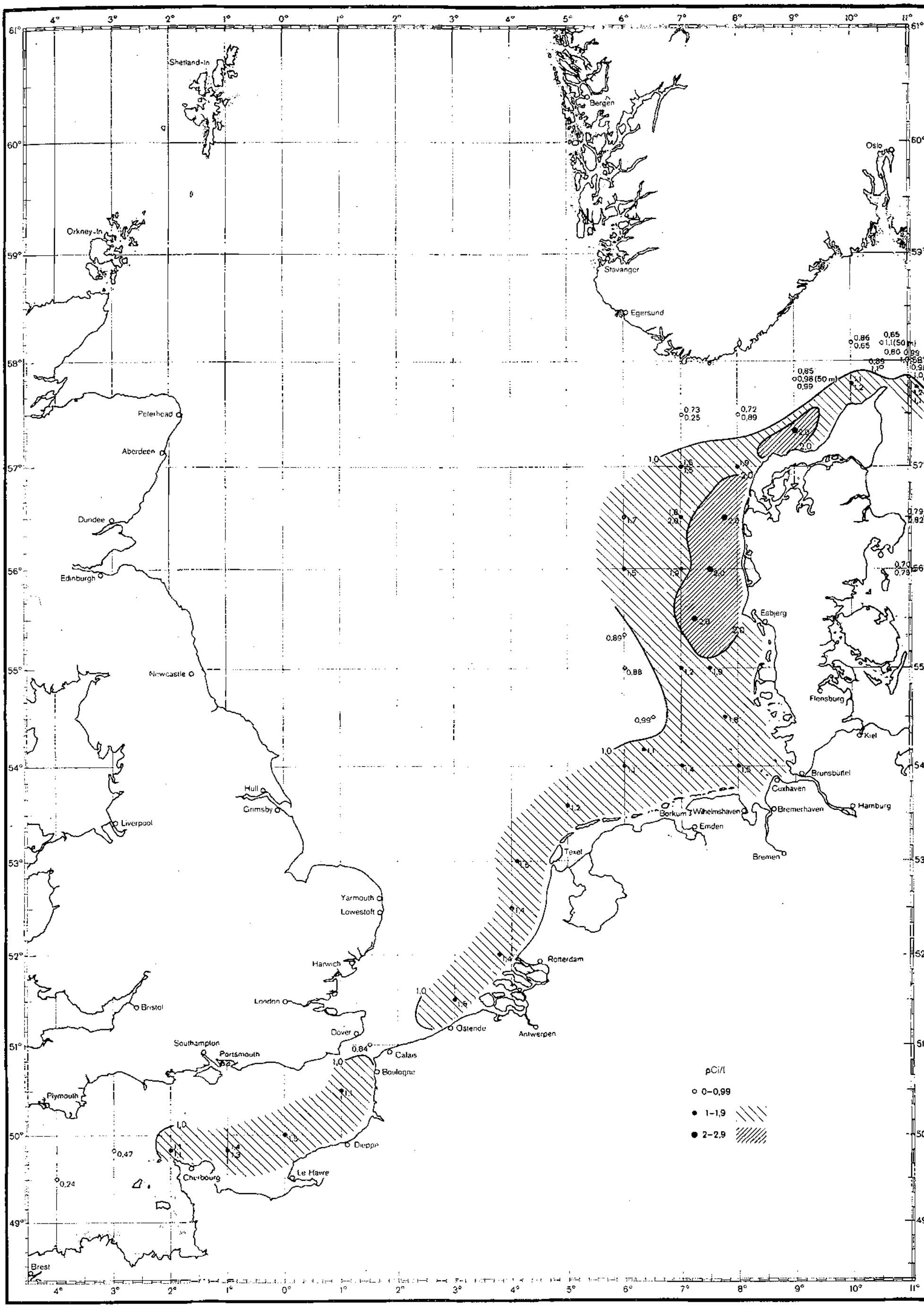

Fig. 10. Cs 137 values, December, 1972. Upper value: in surface water. Lower value: in 100 m depth or in the vicinity of the sea bottom (after H. Kautsky [1973])

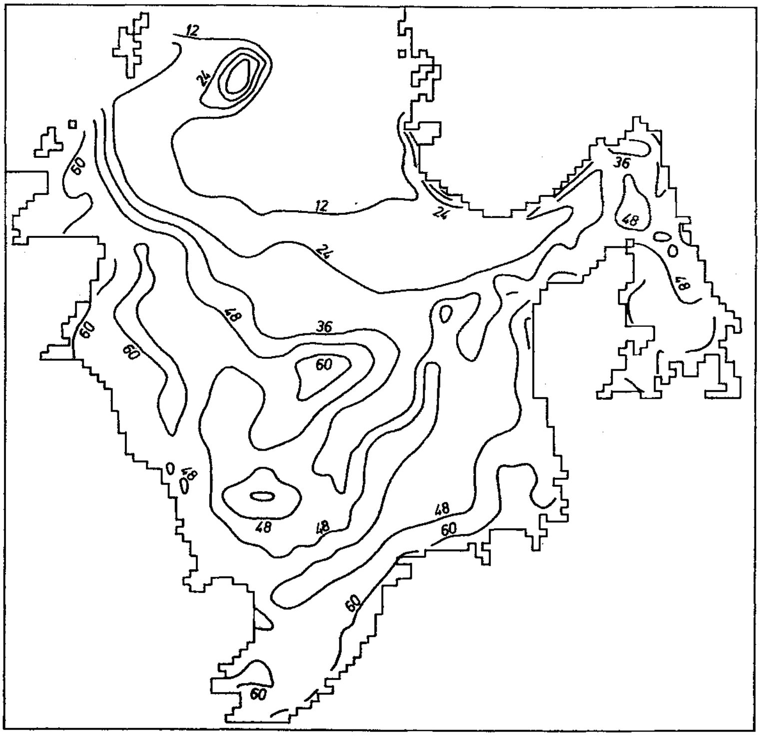

Fig. 11. Residence time due to the residual circulation (months)

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Eingegangen im Januar 1977

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