Available online at www.tjnsa.com
J. Nonlinear Sci. Appl. 9 (2016), 5821–5829
Research Article

Journal of Nonlinear Science and Applications
Print: ISSN 2008-1898 Online: ISSN 2008-1901

Residual power series method for time-fractional Schrödinger equations

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Communicated by A. Atangana

Abstract

In this paper, the residual power series method (RPSM) is effectively applied to find the exact solutions of fractional-order time dependent Schrödinger equations. The competency of the method is examined by applying it to the several numerical examples. Mainly, we find that our solutions obtained by the proposed method are completely compatible with the solutions available in the literature. The obtained results interpret that the proposed method is very effective and simple for handling different types of fractional differential equations (FDEs).

Keywords: Fractional Schrödinger equation, residual power series, fractional power series, exact solution.
2010 MSC: 47H10, 26A33, 34A08, 34A34.

1. Introduction

The fractional differential equation (FDEs) which is generalized form of classical differential equation, has the gained considerable importance during the past decades, mainly due to its applications in diverse fields of different branches of sciences. Various definitions and basic concepts of fractional calculus (FC) are present in many books [3, 12, 16]. Therefore, for the study of numerical solutions of FDEs, there are variety

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Received 2016-09-13

of analytical methods, which were found in literature. Among them, most useful and common methods are presented in [1, 4, 5, 8, 9, 11, 14, 15, 17–19].

Recently, an efficient analytical technique (called the residual power series method (RPSM)) for handling different types of FDEs has been developed. Further, this method was effectively used for finding the solution of various kinds of FDEs [2, 6, 7]. Based on the generalized Taylor series formula method, an approximate analytical solution was given in the form of a convergent series.

The Schrödinger equations were arising in hydrodynamics, optics, chemistry and physics. Some standard Schrödinger equations were solved by Mousa and Ragab [10] and Wazwaz [13] by using the homotopy perturbation method (HPM) and variation iteration method (VIM), respectively. However, the analytical solutions of the linear and nonlinear fractional Schrödinger equations by using the RPSM has not yet been solved by any scientists and researchers. In this paper, we extend the idea of the RPSM for the fractional-order time dependence Schrödinger equations. The structure of the previous paper is as follows. In Section 2, the idea of the RPSM is given. In Section 3, the solutions of the fractional-order time dependent Schrödinger equations are presented. Finally, the conclusion is outlined in Section 4.

2. Method applied

In this section, we introduce the idea of the RPSM.

Definition 2.1 ([6])**.** For $0 \le m-1 < \alpha \le m$, a power series of the form

$$\sum_{k=0}^{\infty}\sum_{l=0}^{m-1} f_{kl}(x)(t-t_0)^{k\alpha+l}, \quad t \ge t_0,$$

is called a multiple fractional power series about $t = t_0$, where t is a variable and $f_{ij}(x)$ are functions of x called the coefficients of the series.

Theorem 2.2 ([2, 6, 7])**.** *Suppose that f has a FPS representation at $t = t_0$ of the form*

$$\sum_{k=0}^{\infty}\sum_{l=0}^{m-1} c_{kl}(x)(t-t_0)^{k\alpha+l}, \quad 0 \le m-1 < \alpha \le m, \quad t_0 \le t < t_0 + R.$$

Further, if $D^{k\alpha+l}f(t)$ are continuous on (t_0, t_0+R), $k = 0, 1, 2, \cdots$, then the coefficients c_{kl} are given by the formula:

$$c_{kl} = \frac{D^{k\alpha+l}f(t_0)}{\Gamma(k\alpha+l+1)}, \quad k = 0, 1, 2, \cdots,$$

where $D^{k\alpha} = D^{\alpha},\ D^{\alpha}, \cdots, D^{\alpha}$ ($k-$times) is a fractional derivative operator (see [3, 12, 16]*) and R is the radius of convergence.*

The above method is called as the RPSM (see [2, 6, 7]).

3. Applications of RPSM to Schrödinger equations

To show potentially, generality and efficiency of the RPSM method, we consider the following time-fractional linear and nonlinear Schrödinger equations.

Example 3.1. We now consider the linear Schrödinger equation [10]

$$D_t^{\alpha}u + iu_{xx} = 0, \tag{3.1}$$

with the initial condition

$$u(x,0) = 1 + \cosh(2x),$$

where $u(x,t)$ is a complex function and $i^2 = -1$.

The exact solution of (3.1) for standard motion, i.e., $\alpha = 1$ is given by [10]

$$u(x,t) = 1 + \cosh(2x)e^{-4it}.$$

According to the RPSM [6, 7], by starting with the initial guess approximation $u_{0,0}(x,t) = 1+\cosh(2x)$, the series solution of (3.1) can be written in the form

$$u(x,t) = 1 + \cosh(2x) + \sum_{k=1}^{\infty} f_{k0}(x)\frac{(t)^{k\alpha}}{\Gamma(k\alpha+1)}.$$

Next, according to the method, the (a,b)-truncated series of $u(x,t)$ is

$$u_{(a,b)}(x,t) = 1 + \cosh(2x) + \sum_{k=1}^{\infty} f_{k0}(x)\frac{(t)^{k\alpha}}{\Gamma(k\alpha+1)}, \quad a = 1,2,3,\cdots, \quad b = 0,$$

and (a,b)-truncated residual function of (3.1) is

$$\mathrm{Res}_{(a,b)}(x,t) = D_t^{\alpha}u_{(a,b)}(x,t) + iu_{xx}(x,t), \quad a = 1,2,3,\cdots, \quad b = 0.$$

In order to find $f_{10}(x)$, by substituting

$$u_{(1,0)}(x,t) = 1 + \cosh(2x) + f_{10}(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)},$$

into

$$\mathrm{Res}_{(1,0)}(x,t) = D_t^{\alpha}u_{(1,0)}(x,t) + i\frac{\partial^2\left(u_{(1,0)}(x,t)\right)}{\partial x^2},$$

it follows that

$$\mathrm{Res}_{(1,0)}(x,t) = f_{10}(x) + 4i\cosh(2x).$$

For $(k,l) = (1,0)$, one can obtain

$$f_{10}(x) = -4i\cosh(2x).$$

As a result, the first residual power series (RPS) solution of (3.1) is given as

$$u_{(1,0)}(x,t) = 1 + \cosh(2x) - 4i\cosh(2x)\frac{t^{\alpha}}{\Gamma(1+\alpha)}.$$

Similarly, by substituting

$$u_{(2,0)}(x,t) = 1 + \cosh(2x) - 4i\cosh(2x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_{20}(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)},$$

into

$$\mathrm{Res}_{(2,0)}(x,t) = D_t^{\alpha}u_{(2,0)}(x,t) + i\frac{\partial^2\left(u_{(2,0)}(x,t)\right)}{\partial x^2},$$

we have

$$\mathrm{Res}_{(2,0)}(x,t) = f_{20}(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + 8\cosh(2x)\frac{t^{\alpha}}{\Gamma(1+\alpha)}. \tag{3.2}$$

By operating D_t^{α} on the both sides of (3.2) and for $(k,l) = (2,0)$, we get the form of equation:

$$f_{20}(x) = -8\cosh(2x).$$

Consequently, the second RPS solution of (3.1) is given as

$$u_{(2,0)}(x,t) = 1 + \cosh(2x) - 4i\cosh(2x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} - 8\cosh(2x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}.$$

As the former, by similar way for (k,l), $k = 3, 4, \cdots$ and $l = 0$, it yields after easy calculations to

$$f_{30}(x) = \frac{32}{3} i\cosh(2x),$$

$$f_{40}(x) = \frac{32}{3}\cosh(2x).$$

Further, if we collect all the last results, then the final solution can be summarized as follows:

$$u(x,t) = 1 + \cosh(2x)\Big(1 - 4i\frac{t^{\alpha}}{\Gamma(1+\alpha)} - 8t^2\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{32}{3}i\frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{32}{3}\frac{t^{4\alpha}}{\Gamma(1+4\alpha)} + \cdots\Big).$$

Remark 3.2. In particular, for the standard case, i.e., for $\alpha = 1$, the RPS solution of (3.1) in term of infinite series is as follows

$$u(x,t) = 1 + \cosh(2x)\Big(1 - 4it - 8t^2 + \frac{32}{3}it^3 + \frac{32}{3}t^4 + ...\Big) = 1 + \cosh(2x)e^{-4it}.$$

The above expression is exactly in accordance with those given by the HPM [10] and VIM [13].

Example 3.3. As the second example, let us consider the linear Schrödinger equation [10]

$$D_t^{\alpha}u + iu_{xx} = 0, \tag{3.3}$$

with the initial condition

$$u(x,0) = e^{3ix},$$

where $u(x,t)$ is a complex function and $i^2 = -1$.

The exact solution of (3.1) for standard motion, i.e., $\alpha = 1$, is given by [10]

$$u(x,t) = e^{3i(x+3t)}.$$

By starting with the initial guess approximation $u_{0,0}(x,t) = e^{3ix}$, the series solution of (3.3) can be written as

$$u(x,t) = e^{3ix} + \sum_{k=1}^{\infty} f_{k0}(x)\frac{(t)^{k\alpha}}{\Gamma(k\alpha+1)}.$$

Similarly, the (a,b)-truncated series of $u(x,t)$ and the (a,b)-truncated residual function of (3.3) are as follows

$$u_{(a,b)}(x,t) = e^{3ix} + \sum_{k=1}^{\infty} f_{k0}(x)\frac{(t)^{k\alpha}}{\Gamma(k\alpha+1)}, \quad a = 1, 2, 3, \cdots, \quad b = 0,$$

$$\mathrm{Res}_{(a,b)}(x,t) = D_t^{\alpha}u_{(a,b)}(x,t) + iu_{xx}(x,t), \quad a = 1, 2, 3, \cdots, \quad b = 0.$$

By substituting

$$u_{(1,0)}(x,t) = e^{3ix} + f_{10}(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)},$$

into

$$\mathrm{Res}_{(1,0)}(x,t) = D_t^{\alpha}u_{(1,0)}(x,t) + i\frac{\partial^2\left(u_{(1,0)}(x,t)\right)}{\partial x^2},$$

we have that

$$\mathrm{Res}_{(1,0)}(x,t) = f_{10}(x) - 9ie^{3ix}.$$

For $(k,l) = (1,0)$, we directly obtain

$$f_{10}(x) = 9ie^{3ix}.$$

Consequently, the first RPS solution of (3.3) is written as

$$u_{(1,0)}(x,t) = e^{3ix} + 9ie^{3ix}\frac{t^\alpha}{\Gamma(1+\alpha)}.$$

By substituting

$$u_{(2,0)}(x,t) = e^{3ix} + 9ie^{3ix}\frac{t^\alpha}{\Gamma(1+\alpha)} + f_{20}(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)},$$

into

$$\mathrm{Res}_{(2,0)}(x,t) = D_t^\alpha u_{(2,0)}(x,t) + i\frac{\partial^2\left(u_{(2,0)}(x,t)\right)}{\partial x^2},$$

we easily obtain

$$\mathrm{Res}_{(2,0)}(x,t) = f_{20}(x)\frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{81}{2}e^{3ix}\frac{t^\alpha}{\Gamma(1+\alpha)}. \tag{3.4}$$

By operating D_t^α on the both sides of (3.4) and for $(k,l) = (2,0)$, we easily get

$$f_{20}(x) = -\frac{81}{2}e^{3ix}.$$

As a result, the second RPS solution of (3.3) reads

$$u_{(2,0)}(x,t) = e^{3ix} + 9ie^{3ix}\frac{t^\alpha}{\Gamma(1+\alpha)} - \frac{81}{2}e^{3ix}\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}.$$

For (k,l), $k = 3,4,\cdots$ and $l = 0$ we have that

$$f_{30}(x) = -\frac{243}{2}ie^{3ix},$$

$$f_{40}(x) = \frac{2187}{8}e^{3ix}.$$

The solution of (3.3) is of the form

$$u(x,t) = e^{3ix}\Big(1 + 9i\frac{t^\alpha}{\Gamma(1+\alpha)} - \frac{81}{2}t^2\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{243}{2}i\frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + ...\Big).$$

Remark 3.4. By taking $\alpha = 1$, the compact form of RPS solution of (3.3) is

$$u(x,t) = e^{3ix}\Big(1 + 9it - \frac{81}{2}t^2 - \frac{243}{2}it^3 + \frac{2187}{8}t^4 + ...\Big) = e^{3i(x+3t)}.$$

The above result is exactly in consistent with those given by the HPM [10] and VIM [13].

Example 3.5. As the third example, we consider the nonlinear fractional Schrödinger equation [10, 13]

$$iD_t^\alpha u + u_{xx} + m\left|u\right|^2 u = 0, \tag{3.5}$$

with the initial condition

$$u(x,0) = e^{nix},$$

where m and n are two constants. For $\alpha = 1$, we get the exact solution [10]

$$u(x,t) = e^{-i(nx+(m-n^2)t)}.$$

Due to the RPSM [6, 7], by starting with an initial guess approximation given by

$$u_{0,0}(x,t) = e^{nix},$$

the series solution of (3.5) can be written as

$$u(x,t) = e^{nix} + \sum_{k=1}^{\infty} f_{k0}(x) \frac{(t)^{k\alpha}}{\Gamma(k\alpha+1)}.$$

Similarly, the (a,b)-truncated series of $u(x,t)$ is written as

$$u_{(a,b)}(x,t) = e^{nix} + \sum_{k=1}^{\infty} f_{k0}(x) \frac{(t)^{k\alpha}}{\Gamma(k\alpha+1)}, \quad a = 1,2,3,\cdots, \quad b = 0,$$

and the (a,b)-truncated residual function of Eq. (3.5) is given as

$$\mathrm{Res}_{(a,b)}(x,t) = D_t^{\alpha} u_{(a,b)}(x,t) + i \frac{\partial^2 \left(u_{(a,b)}(x,t)\right)}{\partial x^2}, \quad a = 1,2,3,\cdots, \quad b = 0.$$

For finding $f_{10}(x)$, by substituting

$$u_{(1,0)}(x,t) = e^{nix} + f_{10}(x) \frac{t^{\alpha}}{\Gamma(1+\alpha)},$$

into

$$\mathrm{Res}_{(1,0)}(x,t) = i D_t^{\alpha} u_{(1,0)}(x,t) + \frac{\partial^2 \left(u_{(1,0)}(x,t)\right)}{\partial x^2} + m \left|u_{(1,0)}\right|^2 u_{(1,0)},$$

we obtain

$$\mathrm{Res}_{(1,0)}(x,t) = f_{10}(x) - i(m-n^2)e^{nix}.$$

For $(k,l) = (1,0)$, we easily get the first unknown coefficient as

$$f_{10}(x) = i(m-n^2)e^{nix}.$$

Therefore, the first RPS solution of (3.5) is expressed as

$$u_{(1,0)}(x,t) = e^{nix} + i(m-n^2)e^{nix} \frac{t^{\alpha}}{\Gamma(1+\alpha)}.$$

In a similar manner, the second RPS solution of (3.5) can be expressed by

$$u_{(2,0)}(x,t) = e^{nix} + i(m-n^2)e^{nix} \frac{t^{\alpha}}{\Gamma(1+\alpha)} - \frac{1}{2}(m-n^2)^2 \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}.$$

For (k,l), $k = 3,4,\cdots$ and $l = 0$, we have that

$$f_{30}(x) = -\frac{i}{6}(m-n^2)^3 e^{nix},$$

$$f_{40}(x) = \frac{1}{24}(m-n^2)^4 e^{nix}.$$

Additionally by collecting all the previous results, we have the following

$$u(x,t) = e^{nix}\Big(1 + i(m-n^2)\frac{t^{\alpha}}{\Gamma(1+\alpha)} - \frac{1}{2}(m-n^2)^2 \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{i}{6}(m-n^2)^3 \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \cdots\Big). \tag{3.6}$$

Remark 3.6. When $\alpha = 1$, (3.6) is written in the following pattern

$$\begin{aligned} u(x,t) &= e^{nix}\Big(1 + i(m-n^2)t - \frac{1}{2}(m-n^2)^2t^2 - \frac{i}{6}(m-n^2)^3t^3 - \frac{1}{24}(m-n^2)^4t^4 + \cdots\Big) \\ &= e^{-i(nx+(m-n^2)t)}. \end{aligned}$$

The above expression is exactly in line with those given by the HPT [10] and VIM [13].

Example 3.7. As the fourth example, let us consider the cubic nonlinear fractional Schrödinger equation given by [10, 13]

$$iD_t^\alpha u + u_{xx} + 2\,|u|^2 = 0, \tag{3.7}$$

with the initial condition

$$u(x,0) = 2\,\mathrm{sech}(2x).$$

When $\alpha = 1$, the exact solution of (3.7) is written as [10]

$$u(x,t) = 2\,\mathrm{sech}(2x)e^{4it}.$$

By starting with initial guess approximation given by

$$u_{0,0}(x,t) = 2\,\mathrm{sech}(2x),$$

the series solution of (3.7) can be written as

$$u(x,t) = 2\,\mathrm{sech}(2x) + \sum_{k=1}^{\infty} f_{k0}(x)\frac{(t)^{k\alpha}}{\Gamma(k\alpha+1)}.$$

Next, the (a,b)-truncated series of $u(x,t)$ and the (a,b)-truncated residual function of (3.7) are given as follows

$$u_{(a,b)}(x,t) = 2\,\mathrm{sech}(2x) + \sum_{k=1}^{\infty} f_{k0}(x)\frac{(t)^{k\alpha}}{\Gamma(k\alpha+1)}, \quad a = 1,2,3,\cdots, \quad b = 0,$$

$$\mathrm{Res}_{(a,b)}(x,t) = D_t^\alpha i u_{(a,b)}(x,t) + \frac{\partial^2\left(u_{(a,b)}(x,t)\right)}{\partial x^2} + 2\left|u_{(a,b)}\right|^2, \quad a = 1,2,3,\cdots, \quad b = 0,$$

respectively.

By substituting

$$u_{(1,0)}(x,t) = 2\,\mathrm{sech}(2x) + f_{10}(x)\frac{t^\alpha}{\Gamma(1+\alpha)},$$

into

$$\mathrm{Res}_{(1,0)}(x,t) = iD_t^\alpha u_{(1,0)}(x,t) + \frac{\partial^2\left(u_{(1,0)}(x,t)\right)}{\partial x^2} + 2\left|u_{(1,0)}\right|^2,$$

it follows that

$$\mathrm{Res}_{(1,0)}(x,t) = f_{10}(x) - 8i\,\mathrm{sech}(2x).$$

For $(k,l) = (1,0)$, we get

$$f_{10}(x) = 8i\,\mathrm{sech}(2x).$$

Subsequently, the first RPS solution of (3.7) can be systematized as

$$u_{(1,0)}(x,t) = 2\,\mathrm{sech}(2x) + 8i\mathrm{sech(2x)}\frac{t^\alpha}{\Gamma(1+\alpha)}.$$

By the similar process, the second RPS solution of (3.7) for $(i,j) = (2,0)$ is expressed by

$$u_{(2,0)}(x,t) = 2\,\mathrm{sech}(2x) + 8i\,\mathrm{sech}(2x)\frac{t^\alpha}{\Gamma(1+\alpha)} - 16\,\mathrm{sech}(2x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}.$$

By similar way for (k,l), $k=3,4,\cdots$ and $l=0$, we have that

$$f_{30}(x) = -\frac{64}{3} i \operatorname{sech}(2x),$$

$$f_{40}(x) = \frac{64}{3} \operatorname{sech}(2x).$$

When we collect all the last results, then the RPS solution of (3.7) can be constructed in the form of a infinite series given by

$$u(x,t) = 2\operatorname{sech}(2x)\Big(1 + 4i\frac{t^{\alpha}}{\Gamma(1+\alpha)} - 8\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{32}{3} i \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \cdots\Big).$$

Remark 3.8. For $\alpha = 1$, the RPS solution of (3.7) is written as

$$u(x,t) = 2\operatorname{sech}(2x)\Big(1 + 4it - 8t^2 - \frac{32}{3} it^3 + \frac{32}{3} t^4 + \cdots\Big) = 2\operatorname{sech}(2x)e^{4it}.$$

The above result is exactly in agreement with those given by the HPT [10] and VIM [13].

4. Conclusion

In this work, we proposed new applications of the RPSM to successfully adopt to determine the solutions of the time-fractional Schrödinger equations. It has been observed that there exists a very good agreement between the approximate solutions obtained by the previous method and those available in the literature. The method for the obtained results is quite effective, convenient and practically well to find the exact solutions of such types of fractional PDEs.

Acknowledgment

This work is supported by the State Key Research Development Program of the People's Republic of China (Grant No. 2016YFC0600705) and the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD2014).

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