Resilience and entropy as indices of robustness of water distribution networks

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ABSTRACT

The use of entropy and resilience indices for measuring robustness of water distribution networks has been investigated. The effects on network performance, caused by the failure of one or two links, have been evaluated by means of several indices for two existing medium sized water distribution networks serving two towns in southern Italy. All the possible network configurations obtained by suppressing one or two links have been studied, excluding only the cases in which disconnection of some nodes from the remaining part of the network occurred. The hydraulic simulations, carried out with a demand-driven approach by means of the EPANET2 software, have shown that, unlike entropy, resilience may represent a useful index of network robustness with regard to link failures.

Key words | entropy, performance indices, resilience, robustness, water supply network

INTRODUCTION

The design of water distribution networks has been carried out for decades by *a priori* assigning network topology, under the assumption that a densely looped layout allows overcoming of local pipe failures and peaks in the water demand spatial and temporal pattern. Pipe diameters are then assigned aimed at the minimization of the overall network cost (capital cost plus operating cost), usually assumed to depend on pipe diameter by means of a simplified relationship (Shamir 1974; Alperovits & Shamir 1977). Such optimal design is usually carried out for the conventional case of peak hour demand, considered to be the most frequent among the possible critical operating conditions. Due to computational difficulties, even for medium sized networks, such a task is usually accomplished for a simplified scheme of network layout in which only some of the pipes, arbitrarily considered as 'main pipes', are retained. The remaining pipes are then added in a second phase, assigning them the minimum pipe size (usually 100 mm), without any further consideration of their cost (Todini 2000). The hydraulic performance of the obtained network is then tested for several conventional loading scenarios (i.e., peak hour, fire extinguishment, local pipe failures, and so on).

The advent of low-cost computing has more recently made it possible to define more sophisticated network design procedures, explicitly taking into account water distribution system performance requirements under various network operating conditions.

In this respect, the concept of system reliability, usually defined as its ability of performing its required functions under stated conditions for a specified period of time, has been introduced in water distribution networks design. However, such definition is hardly applicable to the case of water distribution systems, due to their inherent complexity and to the uncertainty about possible operating conditions. Indeed, early studies, which considered among the possible operating scenarios only the case of failure of some of network mechanical components (i.e., pipes, pumps, tanks, valves, etc.), mistook network mechanical reliability for network connectivity, thereby neglecting the effects of failures upon hydraulic head losses (Goulter & Coals 1986; Wagner *et al.* 1988). More recently, general definitions of network hydraulic reliability have been proposed, directly related to the probability that the network is able to deliver the required demand at nodes under the required

pressure heads (Su *et al*. 1987; Bao & Mays 1990; Fujiwara & De Silva 1990; Cullinane *et al*. 1992; Tanyimboh & Templeman 2000; Setiadi *et al*. 2005; Martínez-Rodríguez 2010, 2011; Martínez-Rodríguez *et al*. 2011). Only in a few cases has the uncertainty about water demand also been taken into account (Gargano & Pianese 2000; Babayan *et al*. 2005; Surendran *et al*. 2005). An overview about reliability applied to water distribution systems can be found in Ostfeld (2004).

A common shortcoming of most of the proposed reliability-based hydraulic network design procedures, besides the computational burden, is that they rely upon the knowledge of the probability of the considered unfavourable events, such as pipe failures or critical demand distributions. Indeed, few data sets exist allowing either to calculate the probability of mechanical components unavailability (Walski & Pelliccia 1982; Su *et al*. 1987; Cullinane *et al*. 1992; Tabesh *et al*. 2009) or to calibrate the probability distribution of water demand (Khomsi *et al*. 1996; Babayan *et al*. 2005; Surendran *et al*. 2005). The extrapolation of the obtained probabilities to networks others than those to which the available data originally referred seems questionable.

Therefore, in this paper the deterministic concept of robustness, e.g., the capability of a system of maintaining given performance levels in the presence of unfavourable variations of operating conditions, is investigated and applied to real water distribution systems. To such an aim, network behaviour will be modelled only under a critical condition, namely after the failure of one or more pipes, irrespective of its probability of occurrence. The scope of the study is to define indices that, evaluated for the network under normal operating conditions, allow measuring of to what extent it will be capable of retaining its performance level after the failure of some pipes. The definition of such indices would help prioritizing rehabilitation interventions on water distribution systems at a regional decision-making level.

In particular, the potential use of network entropy (Tanyimboh & Templeman 1993) and resilience index (Todini 2000) as synthetic indices of network robustness is investigated.

Network entropy, which has been proposed by several authors as a surrogate for network reliability (Tanyimboh & Templeman 2000; Tanyimboh & Sheahan 2002; Setiadi *et al*. 2005), is related to looped network redundancy, thus inherently prone to measure network robustness (Ang &

Jowitt 2003). Several examples can be found in the literature in which entropy is included within the objectives of optimization procedures for water distribution network design (i.e., Sousa *et al*. 2007; Geem 2009; Tanyimboh & Kalungi 2009).

Resilience indexes have been introduced in order to evaluate the usefulness of pressure head surplus in normal operating conditions to allow the network to overcome critical operating conditions (Todini 2000; Prasad & Park 2004; Jayaram & Srinivasan 2008), and have been recently tested by several authors in cost-based multi-objective optimal network design procedures (i.e., Farmani *et al*. 2006; Saldarriaga *et al*. 2010; Vasan & Simonovic 2010; Baños *et al*. 2011).

To test the usefulness for evaluating network robustness of the above-mentioned indices, two existing medium sized networks serving two towns close to the city of Naples, southern Italy, namely Villaricca (NA), with 30,000 inhabitants, and Parete (CE), with 11,000 inhabitants, have been studied.

The performance of the two networks has been evaluated by means of a demand-driven hydraulic model, with reference to all the possible combinations of simultaneous failures of some pipes, simulated as closures of links in the model. For all the obtained network configurations, the hydraulic simulation has been carried out and the proposed robustness indices have been calculated. The relationships between the two indices, as well as with other commonly used indices of network hydraulic performance, are discussed.

MATERIALS AND METHODS

Demand-driven hydraulic simulations of the network under peak conditions, by considering all the possible combinations of simultaneous failure of one or more links, have been carried out by running EPANET2 software (Rossman 2000) in the MATLAB® environment. In particular, the followed procedure consists of four steps:

1. Hydraulic simulation of the network;
2. Evaluation of entropy and resilience indices;
3. Hydraulic simulation of all the possible network configurations obtained by suppressing one link;
4. For each of the obtained configurations, evaluation of hydraulic performance indices.

Steps 2 to 4 can be repeated by suppressing more links until the number of considered network configurations becomes so large to make the calculations infeasible. Such a limit depends on the initial number of links in the considered network: as will be shown in this paper, for medium sized networks (between 100 and 200 links), more than two simultaneous failures can be hardly considered.

The relationship between the values of resilience and entropy indices for all the considered network configurations has been studied, in order to get more insight about the information carried by each of the proposed robustness indices. Furthermore, the suitability of such indices to quantify network robustness has been investigated by evaluating the relationship between each of the proposed robustness indices, calculated for a given network configuration, and the values of hydraulic performance indices for all the possible configurations obtainable by depriving such a network of one link. In the following sections all of the adopted indices are briefly defined.

Network entropy

The concept of entropy of a water supply network has been derived from Shannon's information entropy (Shannon 1948), by considering all the possible N_p flow paths of water through the network, from source nodes to delivery nodes, and assuming that the probability P_k of water flowing through a pipe belongs to k-th path might be expressed as the ratio between path flow Q_k and total flow delivered by the network through demand nodes, Q. In such a way, the N_p flow paths constitute a set of mutually exclusive and completely exhaustive events, for which the entropy function may be written as:

$$S = -\sum_{k=1}^{N_p} P_k \ln P_k \qquad (1)$$

It is worth noting that, for a given network topology, the set of possible paths connecting all the source nodes to all the delivery nodes is unequivocally defined, as well as the flow belonging to each of them, for a given operating scenario, by means of an hydraulic model of the network. Thus, once network topology, diameters and roughness

parameters of pipes, and a set of demands at delivery nodes have been assigned, a value of entropy S is unequivocally associated to the network.

For the sake of automating the computational procedure, in this study, the calculation of S has been carried out making use of the equivalent recursive expression, based on multiple probability space formulation, proposed by Tanyimboh & Templeman (1993) and applicable to general network layouts.

For any given network topology, the above defined entropy is a measurement of the redundancy of the paths available for water flow in the network, and becomes maximum when all the possible paths carry the same flow $Q_k = Q/N$, say when there are no 'main' paths in the network. Such a feature explains why entropy has been proposed as a surrogate for network reliability, assuming that the more a network is redundant, the more it is reliable (Tanyimboh & Templeman 2000; Tanyimboh & Sheahan 2002; Setiadi *et al.* 2005).

However, such an assumption should be handled with care, since it has been already argued that entropy is more related to connectivity than to hydraulic reliability (Ostfeld 2004), while there is no direct relation between entropy and energy losses: paradoxically, two topologically identical networks, with diameters and roughness coefficients changed in such a way that energy losses along all the links maintain their mutual ratios, would share the same pipe flows, and thus the same entropy, in a demand-driven analysis, even in the case of an increase in energy losses causing pressure deficit at some nodes. In the same case, a pressure-driven analysis would predict a reduction of delivered flows at some nodes, and thus a change of calculated flows through pipes, which would not necessarily produce a reduction of network entropy, because both Q_k and Q could decrease, unpredictably affecting the values of P_k in Equation (1).

Network resilience

The concept of resilience introduced by Todini (2000) immediately resembles the above definition of network robustness. The proposed resilience index represents the fraction of the total available power which is not dissipated in the network for delivering the design demands $Q_{D,j}^*$ at

nodes:

$$I_r = \frac{P^*_{OUT} - P^{MIN}_{OUT}}{P_{IN} - P^{MIN}_{OUT}} = \frac{\sum_{j=1}^{N} Q^*_{D,j} H_j - \sum_{j=1}^{N} Q^*_{D,j} H^*_{D,j}}{\sum_{i=1}^{N_R} Q_{R,i} H_{R,i} - \sum_{j=1}^{N} Q^*_{D,j} H^*_{D,j}} \tag{2}$$

In Equation (2), P^*_{OUT} represents the power associated with the delivery of the design demands at the N nodes of the network under the actual heads, H_j; P^{MIN}_{OUT} is the value assumed by the same quantity whereas the design demands are delivered exactly under the minimum design heads, $H^*_{D,j}$; P_{IN} is the total power carried by the flows $Q_{R,i}$ entering the network through the N_R nodes connected to reservoirs with head $H_{R,i}$.

The resilience index is in the range [0, 1] if the design requirements are fulfilled and represents the residual amount of available power which may allow the network to properly operate under stress conditions, such as the failure of one or more links, and/or unpredicted demand concentrated peaks at some nodes.

In this paper, the resilience index, which is tested as the network robustness index, has also been calculated for situations in which design requirements are not fulfilled. In these cases, the resilience index may assume negative values.

Hydraulic performance indices

The test of the capability of the proposed indices to synthesize network robustness is carried out by means of some indices commonly used to evaluate the hydraulic performance of water supply networks. In particular, the following indices are calculated:

- mean hydraulic head at network nodes,

$$H_{med} = \frac{\sum_{j=1}^{N} H_j}{N} \tag{3}$$

- hydraulic head standard deviation,

$$\sigma_H = \sqrt{\frac{\sum_{j=1}^{N} (H_j - H_{med})^2}{N - 1}} \tag{4}$$

- mean head deficit,

$$H_D = \frac{\sum_{j=1}^{N} H_{D,j} Q^*_{D,j}}{Q} \text{ with}$$

$$\begin{cases} H_{D,j} = 0 & \forall j : H_j \geq H^*_j \\ H_{D,j} = H^*_j - H_j & \forall j : H_j < H^*_j \end{cases} \tag{5}$$

- mean head surplus,

$$H_S = \frac{\sum_{j=1}^{N} H_{S,j} Q^*_{D,j}}{Q} \text{ with}$$

$$\begin{cases} H_{S,j} = H_j - H^*_j & \forall j : H_j > H^*_j \\ H_{S,j} = 0 & \forall j : H_j \leq H^*_j \end{cases} \tag{6}$$

- hydraulic performance index,

$$HPI = \frac{\sum_{j=1}^{N} \alpha_j Q^*_{D,j}}{Q} \text{ with}$$

$$\begin{cases} \alpha_j = 0 & \forall j : H_j < z_j \\ \alpha_j = \left(\frac{H_j - z_j}{H^*_j - z_j}\right)^{1/2} & \forall j : z_j < H_j < H^*_j \\ \alpha_j = 1 & \forall j : H_j > H^*_j \end{cases} \tag{7}$$

In the above equations, z_j represents the ground elevation at network nodes; HPI is a simplified version of the hydraulic performance index proposed by Gargano & Pianese (2000).

Case studies

The suitability of entropy and resilience as robustness indices has been tested for two medium sized hydraulic networks, serving two towns near Naples (Italy): Villaricca, with 30,000 inhabitants (Di Nardo & Di Natale 2011) and Parete, with 11,000 inhabitants (Di Nardo & Di Natale 2010). Figure 1 shows a sketch of the layout of the two networks. The main characteristics of the studied networks are summarized in Table 1.

Amongst all the network layouts obtained by considering all the possible combinations of simultaneous failure
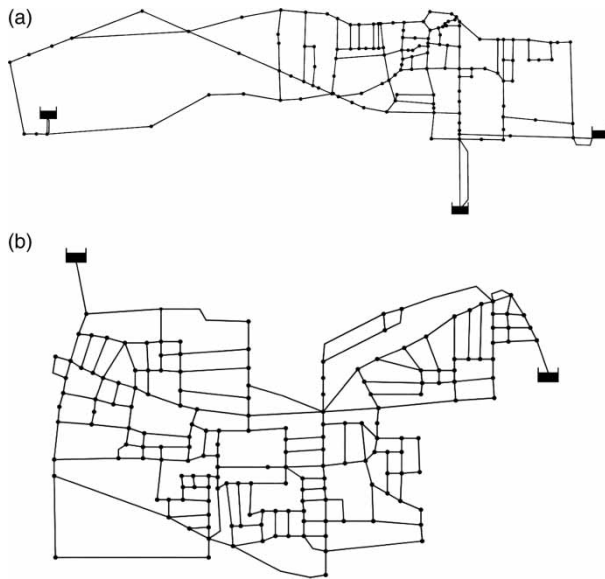
Figure 1 | Sketch of the layouts of the two networks: (a) Villaricca and (b) Parete.

of some pipes, the few configurations giving rise to disconnection of nodes from the rest of the network have been excluded from the set used for the comparison of the indices, because reliable hydraulic head values at such nodes could not be calculated.

RESULTS AND DISCUSSION

Figure 2 shows the scatter plots of the values of S vs. I_r computed for all the considered layouts of the two studied networks. The two diagrams show clearly that no relationship exists between resilience and entropy, suggesting that different information is provided by the two indices. Indeed, while it can be shown that resilience index is strictly related to network average hydraulic head at nodes, one may expect that entropy, being a measure of network

Table 1 | Main characteristics of the two tested hydraulic networks

| Network characteristics	Hydraulic network	
	Parete	Villaricca
Number of nodes, N	184	139
Number of links, N_L	282	183
Number of reservoirs, N_R	2	3
Hydraulic head of reservoirs [m]	110.0	159.0; 160.0; 161.0
Total pipes length, L_{TOT} [km]	32.7	36.0
Minimum ground elevation, z_{MIN} [m]	53.1	103.8
Maximum ground elevation, z_{MAX} [m]	78.6	122.0
Pipes materials	Cast iron	Cast iron and steel
Pipes diameters [mm]	60 (57.9%); 80 (4.6%);	80 (8.3%); 90 (17.9%);
(percentage of total network pipes length)	100 (6.2%); 110 (2.0%);	100 (4.4%); 110 (2.0%);
	125 (1.4%); 150 (9.0%);	150 (12.9%); 200 (36.2%);
	200 (19.0%)	300 (18.4%)
Peak demand, Q [m^3/s]	0.110	0.208
Design pressure head, h^* [m]	25	30
Maximum number of simultaneous pipe failures	2	2
Number of studied network layouts	39,903	16,836
Number of network layouts with disconnected nodes	44	82
Network entropy, S	5.771	5.047
Network resilience, I_r	0.351	0.552
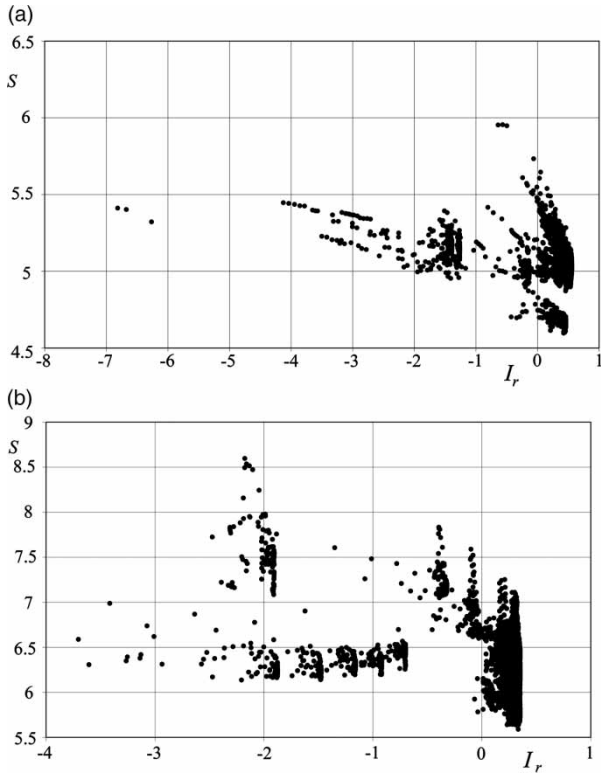
Figure 2 | Relationship between resilience, I_r, and entropy, S, for all the considered network configurations: (a) network of Villaricca and (b) network of Parete.

Figure 3 | Scatter plots of resilience vs. mean hydraulic head at nodes: (a) network of Villaricca; (b) network of Parete.

redundancy, should exhibit negative correlation with standard deviation of hydraulic head at nodes.

For a network delivering the same demand $Q_D^* = Q/N$ at all nodes, it results

$$I_r = \frac{H_{\mathrm{med}} - \left(\bar{h}^* + \bar{z}\right)}{\bar{H}_R - \left(\bar{h}^* + \bar{z}\right)} \tag{8}$$

In Equation (8) $\bar{H}_R$ represents the flow-weighted average head of reservoirs; $\bar{h}^*$ is the mean design pressure head at nodes; $\bar{z}$ is the mean ground elevation at nodes. Although demand at nodes is not evenly distributed, the scatter plots of Figures 3(a) and 3(b) show that the relationship between calculated I_r and H_{med} is not far from Equation (8), for both the considered networks. Conversely, as shown in Figures 4(a) and 4(b), in any case no relationship can be recognized between S and H_{med}.

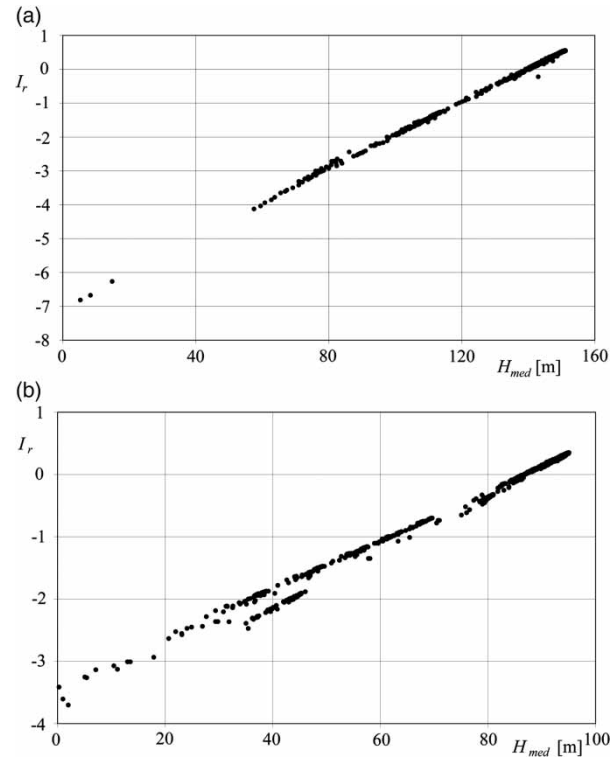Although high entropy, which implies a high degree of interconnection between network nodes, should lead to a smoother distribution of hydraulic head at nodes, surprisingly, for all the studied networks, resilience index appears more related to σ_H than entropy, as indicated by the scatter plots of Figures 5 and 6.

The correlation coefficients reported in Table 2 for both the studied networks confirm that resilience results more correlated than entropy with the considered statistical indices of hydraulic head distribution at nodes. In particular, resilience index, at the same time positively correlated with H_{med} and negatively correlated with σ_H, appears more suitable than entropy to be used as a measurement of network robustness.

In order to test if this expectation was confirmed for the two studied networks, the values assumed by the performance indices H_D, H_S and HPI of all the network layouts, derived from all the configurations with one link failure (indicated in the following as *mother* network layouts) by suppressing one more link (indicated in the following as *son* network layouts), have been calculated. In this way,
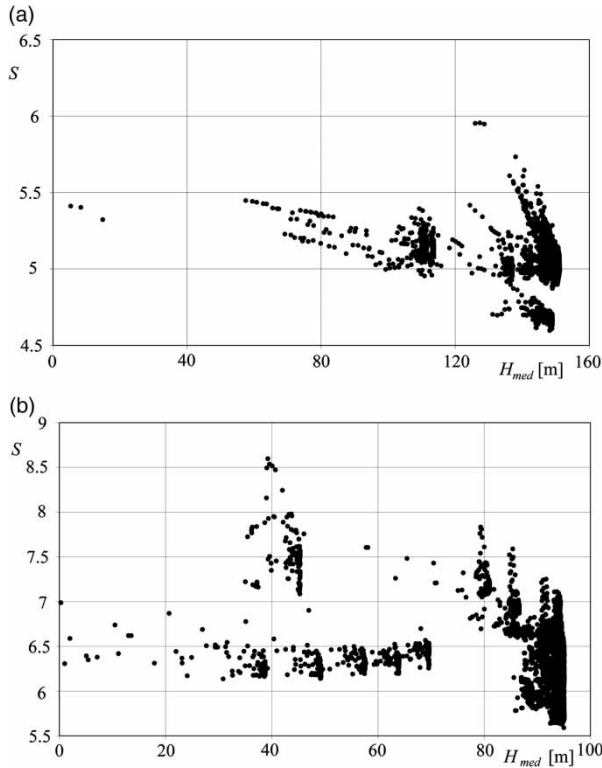
Figure 4 | Scatter plots of entropy vs. mean hydraulic head at nodes: (a) network of Villaricca; (b) network of Parete.

Figure 5 | Scatter plots of resilience vs. standard deviation of hydraulic head at nodes: (a) network of Villaricca; (b) network of Parete.

for each of the studied networks, it was possible to investigate N different mother layouts, and to evaluate at what extent hydraulic performance was retained after the failure of one more link.

For the hydraulic network of Parete, the distribution of the values of the performance indices H_D, H_S and HPI of all the *son* network layouts, are plotted in Figures 7 and 8, in the form of box and whiskers plots (Tukey 1977), against resilience and entropy, respectively, of the *mother* network layout. The subscript of the indices indicates the number of simultaneous link failures of the considered network layouts. The five lines of the box and whiskers plots represent the 10, 25, 50, 75 and 90% quantiles. Figures 9 and 10 show the same diagrams for the network of Villaricca.

In all cases the distributions of the values of the various indices are such that the 50, 75 and 90% quantiles are nearly identical, so that the relevant lines in box plots collapse into each other; in most cases, the same also happens for the 25% quantile, so that the only whisker clearly
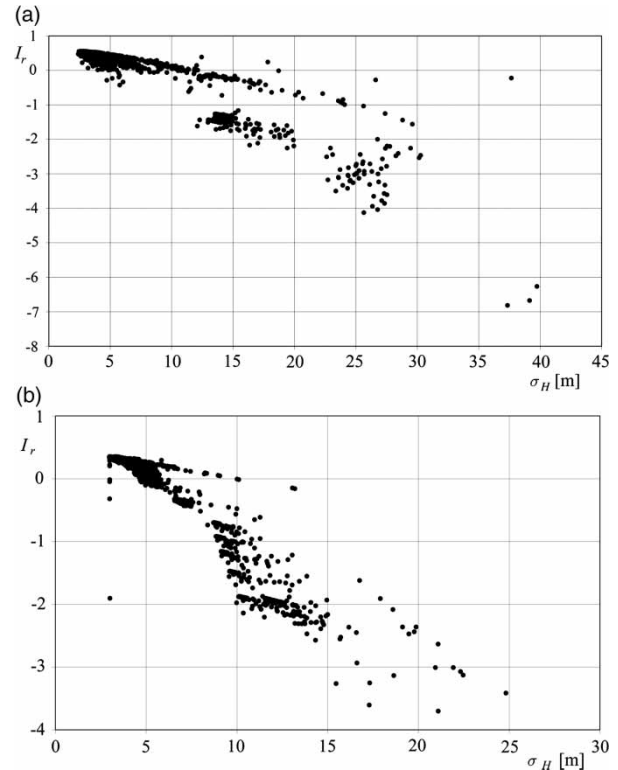
distinguishable from the others is the 10% quantile. These features indicate that most of the network configurations deriving from a common *mother* layout assume the highest values of all the considered performance indices, while the few cases for which the indices indicate lower performances correspond to configurations in which the failure of links that are particularly crucial due to network topology has been considered.

The diagrams show that the entropy of the *mother* network does not allow predicting the performance of the *son* networks, whatever performance index is used. High values of HPI_2 and H_{D2} are achieved regardless the entropy S_1 of the *mother* network, while the values of H_{S2} are chaotically spread.

Conversely, all the considered performance indices of the *son* network layouts show a strong dependence on the resilience of the *mother* layout, indicating bad performances of the *son* networks when the resilience of the *mother* layout is negative.
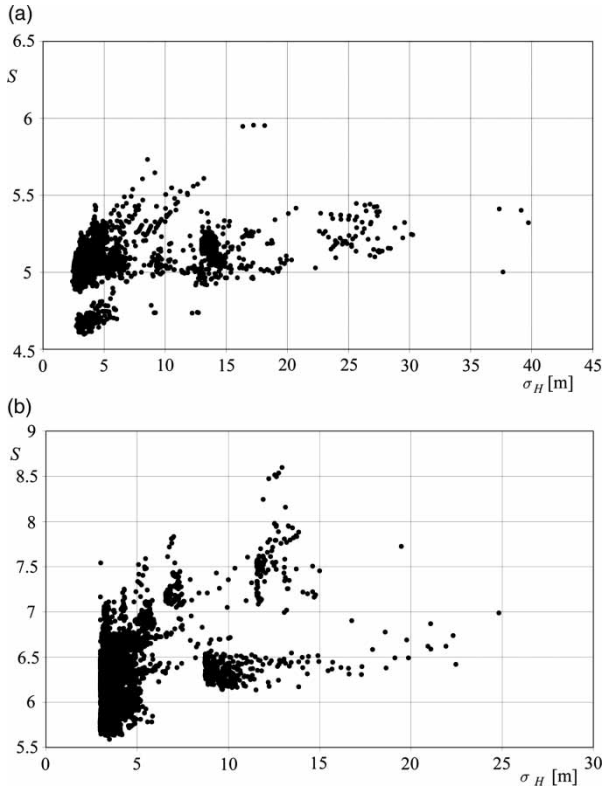
Figure 6 | Scatter plots of entropy vs. standard deviation of hydraulic head at nodes: (a) network of Villaricca; (b) network of Parete.

Table 2 | Correlation coefficients between entropy and resilience and statistical indices calculated for all the considered connected network layouts.

	Parete		Villaricca	
	S	I_r	S	I_r
S	1	−0.398	1	−0.295
I_r	−0.398	1	−0.295	1
H_{med}	−0.376	0.998	−0.292	1.000
σ_H	0.444	−0.971	0.319	−0.919

In particular, for the case study of Parete, the mean head surplus is less than 1.0 m until the resilience of the *mother* layout I_{r1} is negative, and shows a rapid linear growth in the range of positive I_{r1}; similarly, the values of H_{S2} of the Villaricca network slowly grow until the resilience of the *mother* layout is smaller than 0.2, and then show a rapid linear growth for larger I_{r1}.

The trends of the H_{D2} and HPI$_2$ results are similar for both the case studies, showing a constant improvement
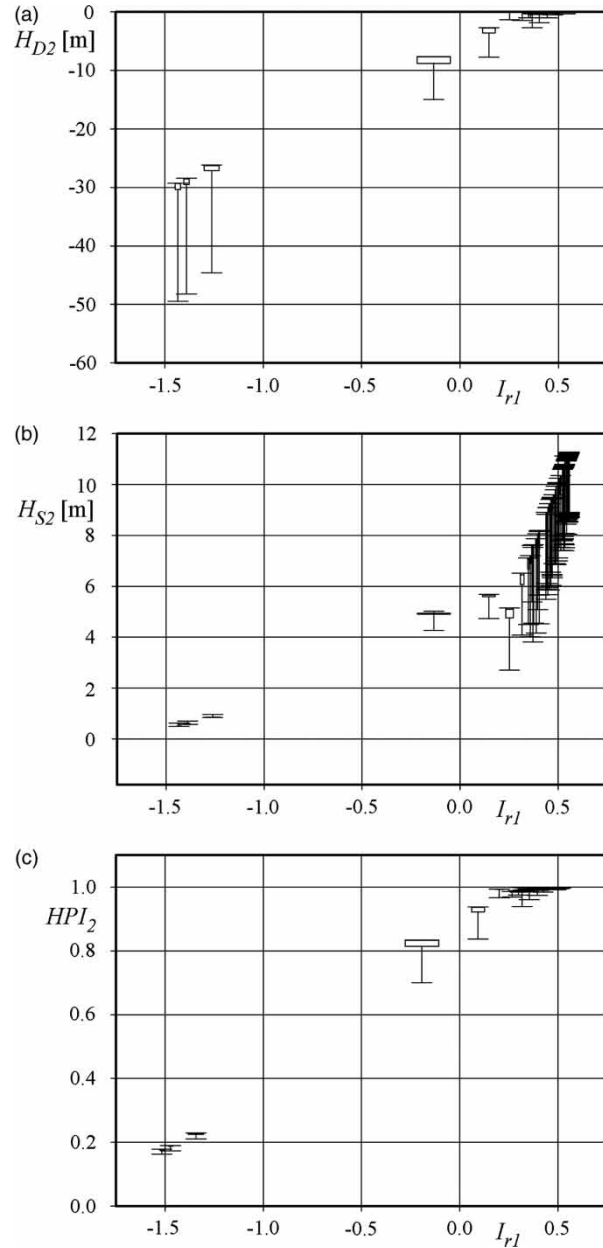

Figure 7 | Network of Villaricca. Box plots of the distribution of hydraulic performance indices of all the network configurations with two link failures deriving from a *mother* network configuration with one link failure vs. resilience of the *mother* network: (a) mean hydraulic head deficit; (b) mean hydraulic head surplus; (c) hydraulic performance index.

in *son* networks performances with *mother* network resilience I_{r1}, until it reaches the value of 0.2, beyond which at least 90% of the *son* networks still provide high performances.
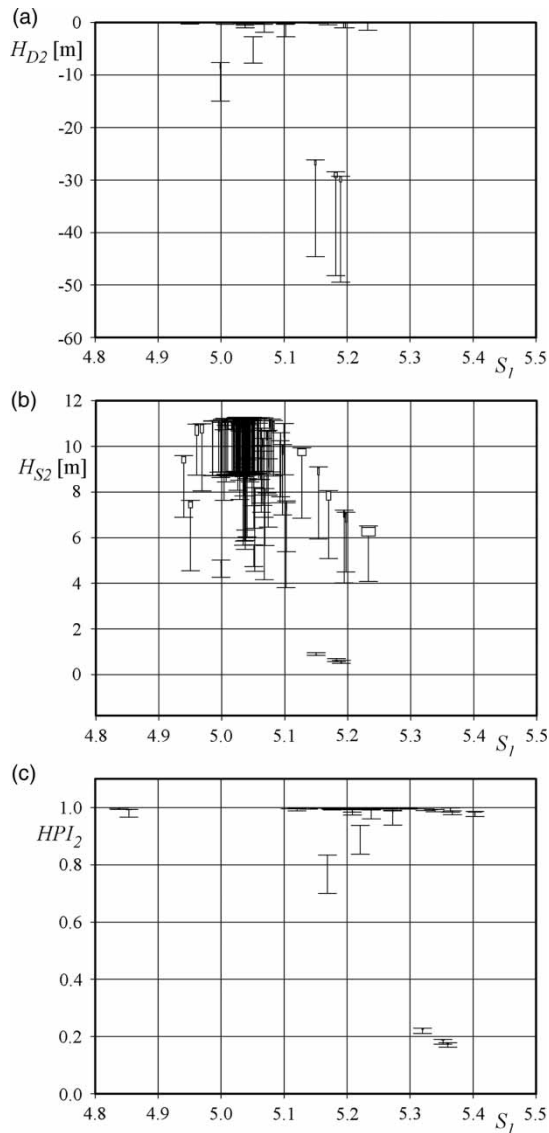
Figure 8 │ Network of Villaricca. Box plots of the distribution of hydraulic performance indices of all the network configurations with two link failures deriving from a *mother* network configuration with one link failure vs. entropy of the *mother* network: (a) mean hydraulic head deficit; (b) mean hydraulic head surplus; (c) hydraulic performance index.

Figure 9 │ Network of Parete. Box plots of the distribution of hydraulic performance indices of all the network configurations with two link failures deriving from a *mother* network configuration with one link failure vs. resilience of the *mother* network: (a) mean hydraulic head deficit; (b) mean hydraulic head surplus; (c) hydraulic performance index.

CONCLUSIONS

Entropy and resilience have been tested as indices for measuring robustness of water distribution networks with regards to link failures. The deterioration of network performance, owing to the failure of one or two links, has been evaluated for two medium sized water distribution networks serving Villaricca and Parete, two small towns in southern Italy, with
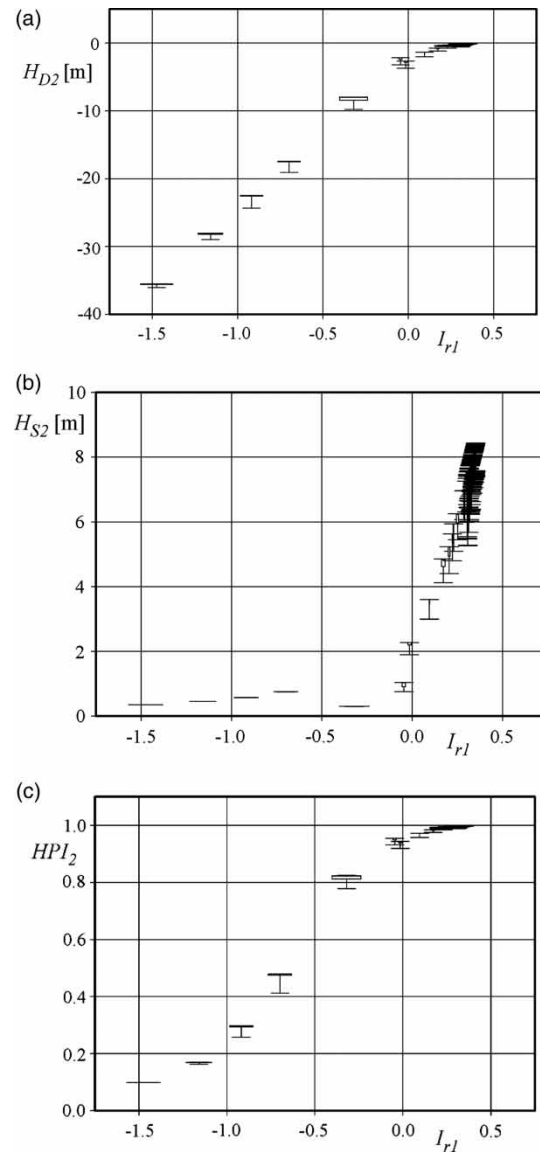
30,000 and 11,000 inhabitants, respectively. All the possible network configurations obtained by suppressing one or two links have been studied, excluding only the cases in which disconnection of some nodes from the remaining part of the network occurred. The hydraulic simulations have been carried out, with a demand-driven approach, by means of the EPANET2 software in a MATLAB® environment. The performance of the various considered network configurations
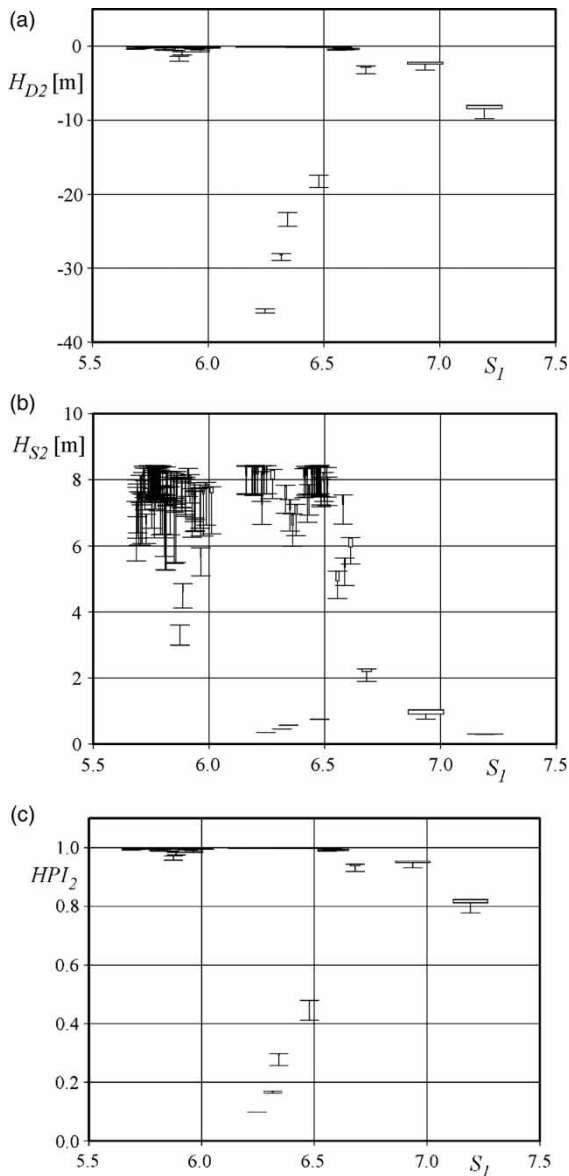
Figure 10 | Network of Parete. Box plots of the distribution of hydraulic performance indices of all the network configurations with two link failures deriving from a *mother* network configuration with one link failure vs. entropy of the *mother* network: (a) mean hydraulic head deficit; (b) mean hydraulic head surplus; (c) hydraulic performance index.

different information about network characteristics: indeed, for both the case studies, no relationship exists between the values assumed by the two indices for all the considered network configurations. Furthermore, resilience shows strong positive correlation with mean hydraulic head at nodes and, to a lesser degree, negative correlation with hydraulic head standard deviation. Conversely, no relationship between such indices of hydraulic head distribution and entropy has been found, although a correspondence between smoothness of hydraulic head distribution and entropy was expected, since the latter is usually interpreted as a measurement of the redundancy of looped networks.

Second, network configurations with high resilience are more robust with respect to link failures: indeed, the distributions of the values assumed by the performance indices, for all the possible network configurations obtained by suppressing one link starting from a network configuration, indicate that the higher the resilience, the more the performance level is maintained in the case of link failures. Conversely, no relationship has been found between the entropy of a network configuration and the values assumed by the performance indices for the configurations obtained by depriving the starting configuration of one link.

Therefore, for the two studied networks, it was shown that high values of resilience indicate high robustness of the network with regards to link failures; conversely, network entropy, although it represents a surrogate of topological reliability useful in network design procedures, does not provide information about the capability of the network to assure good performances after the occurrence of link failures.

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has been synthesized by evaluating the mean hydraulic head at nodes, the standard deviation of hydraulic head, the demand-weighted mean head deficit and surplus, and a hydraulic performance index expressing the degree of fulfilment of the design pressures and demands at nodes.

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