Resistance Neutralization

An Application of Thermionic Amplifier Circuits

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Review of the Subject.—*A power amplifier may be so connected to the circuit supplying the power to be amplified that it introduces terms into the equation for the current in the power circuit which subtract from the resistance terms. Upon this fact depends the operation of the device as a generator of sustained oscillations, as a regenerative amplifier, and as a resistance neutralizer. In the treatment of oscillating audion or triode circuits this fact has been pointed out by a number of writers but the more general significance of these subtractive terms has never been discussed. This paper points out that, by making use of the idea of resistance neutralization, circuits and systems having all the properties of low-resistance systems and also a number of other unique properties may be obtained.*

The first part of the paper derives and discusses the current and power relations which obtain in circuits having a resistance neutralizer associated with them. One of the things brought out is that not only does the neutralizer supply power to the circuit but it also causes the generator or source of driving e.m. f. to furnish more power to the circuit than it would were the neutralizer not present. Thus if the source of driving e. m. f. is the impinging of electromagnetic waves upon a wireless antenna the neutralizer not only amplifies the power received but it actually causes the impinging waves of the correspondent station to give up more power to the antenna circuit while it causes the waves from detuned station to give up less power to the receiving circuit.

The power relations cited above while important are not the most important results obtained by resistance neutralization. At resonance, the power delivered to a wireless receiving system by impinging waves is inversely proportional to the net resistance, while the power received from detuned stations is practically independent of the net resistance. The neutralizer, by lowering the net resistance of the receiving system, thus causes the ratio of signal to interferent power to increase. In a simple series antenna circuit the ratio of signal to interferent power, where the interferent source may be either atmospheric strays or interferent station, is a function of $(L/R)^n$ *where n is a positive number. The neutralizer thus increases the selectivity of the receiving station against all types of disturbances.*

A general physical argument is given (paragraph 7) to show that a triode may be made to function as a resistance neutralizer. This physical argument is illustrated by a mathematical treatment of the steady and transient states for a particular method of associating the triode with the circuit in which neutralization is desired. This mathematical treatment shows that under all conditions both in the steady and transient state, the neutralizer circuit functions so as to reduce the net resistance of the circuit of interest to some predetermined value. A numerical example is given to illustrate the mathematical theory. In this particular circuit the ratio of the signal to interferent power in the steady state is increased 200 fold by the insertion of the neutralizer.

Section (II) of the paper treats of the optimum conditions obtaining in receiving circuits containing a resistance neutralizer, and gives relations for the designing of an antenna and its circuits so as to obtain the maximum selectivity and maximum power abstraction from impinging waves. The last topic discussed (Section 12) deals with a circuit which neutralizes resistance for a narrow range of frequencies near a desired frequency and introduces resistance and reactance into the circuit for frequencies removed fom this band of frequencies. A numerical example is given to illustrate the theory. The performance curves of the neutralizer are given by Figs. 12 and 13. Fig. 14 shows the ratio of the power received at resonance to the power received from a detuned station by a given antenna circuit; first, without a neutralizer, second, with a pure resistance neutralizer, and third, with a selective neutralizer. For example, if the interferent source is detuned by 3 per cent the circuit without the neutralizer gives practically 1 for the ratio of signal to interferent power; with pure resistance neutralization the ratio is about 1300; with the selective neutralizer the ratio is increased to 3000.

CONTENTS

General Theory of Resistance Neutralization

1. *Purpose.* A power amplifier is a device having three elements so related that the supply of power to the *control* element of the device from an external system controls the delivery of a much larger amount of power from a *local source* of power to the third element, the *power receiving* element.

A power amplifier always comprises the three elements,

1. The control, or trigger, or power input element.

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2. The local source of power.
3. The power receiving, or power output element.

By suitable connections between the elements, the power available in the output element may be utilized in the following ways:

A. The power output may all be expended in the output element to accomplish some desired purpose: this is the case of *simple amplification.*

B. A part of the power output may be diverted to the control element to supplement the power received from the external or original actuating agency in such a manner as to increase the power amplifying ratio: this is the case of *regenerative amplification.*

C. A portion of the power output may be diverted to the external system which supplies the power to the control element in such a manner as to cause the driving forces in the external system to deliver more power to the external system. The amplifier thus supplies a part only of the increased power which is expended in the external system. In the mathematical treatment of the system the amplifier constants enter the equations of the external circuit in the form of terms which *subtract* from the frictional or resistance terms. The amplifier when thus associated with the external agency is therefore said to have the properties of a *negative resistance*: this is the case of *resistance neutralization.*

D. The power may be so diverted to the external system that any disturbance which is set up in the system may result in sustained oscillations (reciprocating motion) in a system in which the only driving force is the continuous force of the power source of the amplifier: this is the case of the *generation of sustained oscillations.*

The trielectrode thermionic amplifier (the audion or triode) has been used in radio telegraphy essentially as an amplifier and as a generator of sustained oscillations (the uses of the "triode" as a translator by rectification from radio to audio frequencies, and as a controller or modulator, are not under discussion) and its applications and possibilities as a *resistance neutralizer* seem to have been overlooked.[1] By this we do not mean to say that it has not been known that the amplifier constants enter into the dissipative terms as negative quantities. We mean that the possibilities of the use of the amplifier in such a manner as to increase the abstracting factor of a radio antenna for power from the correspondent station, and to increase the selective coefficient of an antenna against interferent sources are not generally known.

The present paper has for its object a detailed and quantitative treatment of resistance neutralization with special reference to its applications in increasing the selective properties of radio receiving circuits.

The treatment is divided into three general parts. In the first part, the general theory of resistance neutralization is developed. In this theory the conditions which a device must fulfill in order to function as a resistance neutralizer toward a circuit with which it is associated are arrived at. The second part shows how triode circuits may be designed to fulfill the conditions developed in the general theory, and thus how these circuits may be made to function as resistance neutralizers. The third part treats of the effects of resistance neutralization upon the abstractive and selective properties of radio receiving circuits.

2. *The Conditions Necessary for Resistance Neutralization.* Consider the circuit shown in Fig. 1. Let B represent a device feeding power into the circuit. B may have any voltage characteristic whatsoever. Let A be a device which introduces into the circuit

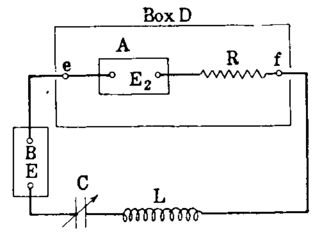

FIG. 1

1 an electromotive force e_2 which at every instant is directly proportional to and in the same direction as the current i in the circuit ($e_2 = N\,i$). The effect of the device A will be called *pure resistance neutralization.* This effect is termed pure resistance neutralization because if we were to inclose the resistance R and the device A in a box D and were to bring out two terminals, e and f, the box would act in all respects like a resistance of magnitude $R - N$.

The proof of this proposition is very simple. It consists in comparing the differential equation of the circuit of Fig. 1 with the differential equation of the same circuit if A were not present. Let the electromotive force of the source B be represented by e and that of the device A be expressed by the equation:

$$e_2 = N\,i \tag{1}$$

Kirchoff's law applied to the circuit of Fig. 1 gives the equation,

$$e - L\frac{d\,i}{d\,t} - R\,i + N\,i - \frac{q}{C} = 0 \tag{2}$$

Differentiating (2) with respect to time gives

$$\frac{d\,e}{d\,t} - L\frac{d^2\,i}{d\,t^2} - (R - N)\,\frac{d\,i}{d\,t} - \frac{i}{C} = 0 \tag{3}$$

If A were not present (3) would be

$$\frac{d\,e}{d\,t} - L\frac{d^2\,i}{d\,t^2} - R\,\frac{d\,i}{d\,t} - \frac{i}{C} = 0 \tag{4}$$

1. Since the above was written, we have found the circuit of Fig. 11 described by Armstrong and Pupin in a French Patent abstracted in the *Revue Generale de L'Ectricite,* Volume 5, page 270. This review of the patent, however, contained no analytic treatment of the circuit.

Equation (**3**) is sufficient to portray the relations in the circuit of Fig. 1 under all conditions and equation (**4**) is sufficient to portray the relations in this same circuit with A not present. The only difference between these equations is that (**3**) contains $R - N$ where (**4**) contains R. Therefore under all conditions and for all types of applied voltages the box D of Fig. 1 acts as a resistance of magnitude $R - N$.

3. *Power Relations.* In the circuit of Fig. 1 let the device B represent an alternator delivering a sine electromotive force whose root-mean-square value is E. Let X represent the net reactance of the circuit.

$$X = \omega L - \frac{1}{\omega C}$$

Then in the steady state, the current flowing in the circuit is,

$$\left.\begin{aligned} I &= \frac{E}{(R - N) + j\left(\omega L - \frac{1}{\omega C}\right)} \\ &= \frac{E}{(R - N) + j X} \\ I &= \frac{E}{\sqrt{(R - N)^2 + X^2}} \end{aligned}\right\} \quad (5)$$

The power P_B delivered by the alternator B is,

$$P_B = \frac{(R - N) E^2}{(R - N)^2 + X^2} \quad (6)$$

The power P_A delivered by the amplifier or resistance neutralizer A is,

$$P_A = \frac{N E^2}{(R - N)^2 + X^2} \quad (7)$$

The total power P_T delivered by both A and B is,

$$P_T = \frac{R E^2}{(R - N)^2 + X^2} \quad (8)$$

The power P_0 delivered by the generator B with the amplifier A removed is,

$$P_0 = \frac{R E^2}{R^2 + X^2} \quad (9)$$

Let the ratio

$$\frac{R - N}{R} \text{ be represented by } \gamma \quad (10)$$

This ratio γ may be called the *reduction factor* of the neutralizer when associated with the particular circuit. It is the factor by which the original circuit resistance must be multiplied in order to obtain the reduced or net resistance.

The ratios of the power delivered by the neutralizer to the power delivered by the alternator, etc. are as follows:

$$P_A/P_B = \frac{N}{R - N} = \frac{1}{\gamma} - 1 \quad (11)$$

$$P_T/P_B = \frac{R}{R - N} = 1/\gamma \quad (12)$$

$$P_B/P_0 = \frac{R - N}{R} \frac{R^2 + X^2}{(R - N)^2 + X^2} \quad (13)$$

$$P_A/P_0 = \frac{N}{R} \frac{R^2 + X^2}{(R - N)^2 + X^2} \quad (14)$$

$$P_T/P_0 = \frac{R^2 + X^2}{(R - N)^2 + X^2} \quad (15)$$

For the case in which the circuit is resonant to the frequency of the alternator B, the power ratios reduce to the following forms. These ratios also apply to the direct current case:

$$P_A/P_B = \frac{N}{R - N} = \frac{1}{\gamma} - 1 \quad (11a)$$

$$P_T/P_B = \frac{R}{R - N} = 1/\gamma \quad (12a)$$

$$P_B/P_0 = \frac{R}{R - N} = 1/\gamma \quad (13a)$$

$$P_A/P_0 = \frac{R}{R - N} \frac{N}{R - N} = (1/\gamma)^2 - 1/\gamma \quad (14a)$$

$$P_T/P_0 = \frac{R^2}{(R - N)^2} = (1/\gamma)^2 \quad (15a)$$

An inspection of equations (**13**) and (**13a**) shows that if the circuit is resonant, the presence of the neutralizer

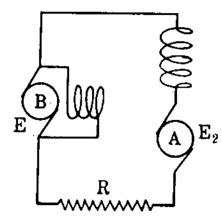

FIG. 2

in the circuit causes the alternator to deliver more power in the ratio of R to $(R - N)$, while if the circuit is so much out of tune that the resistance R is small in comparison with the reactance X, the presence of the neutralizer causes the alternator to deliver less power in the ratio of $(R - N)$ to R.

4. *A Simple Example of Resistance Neutralization.* As a simple example of resistance neutralization, consider the circuit shown in Fig. 2. In this circuit B is a shunt generator and A is a series generator used as a booster. For a limited range of current, the voltage of the booster A is approximately proportional to the current in the circuit. Within this range the following relations hold good:

$$E_2 = N I \qquad I = \frac{E}{R - N}$$

and the series generator acts as a resistance neutralizer.

5. *Departures from Pure Resistance Neutralization.* The conditions necessary for pure resistance neutralization may be summed up as follows:

1. The resistance neutralizer A must introduce into the circuit 1 an electromotive force in series with the electromotive force of the power supplying device.

2. The electromotive force introduced by the resistance neutralizer must at every instant of time be directly proportional to and in the same direction as the current in the circuit. $e_2 = N\,i\,;N > 0$

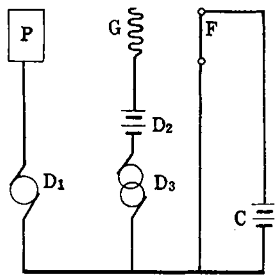

FIG. 3

Condition 2 insures that the following relations are obtained:

a. The voltage of A is in phase with the current i.

b. The voltage of A is of the same wave form as the current i.

c. The voltage of A is of the same frequency as the current i.

Small departures from these relations lead to small departures from the condition of pure resistance neutralization. For instance, a slight departure from the relation (a) leads to the introduction into the circuit equations of terms which add to or subtract from the reactance terms of the impedance, as well as the term which subtracts from the resistance. Thus a slight departure from relation (a) would cause the box D of Fig. 1 to act as a resistance of magnitude $(R - N)$ in series with an inductance or capacity. If the circuit were resonant to the frequency of the driving force and if the voltage of A departs from the relation

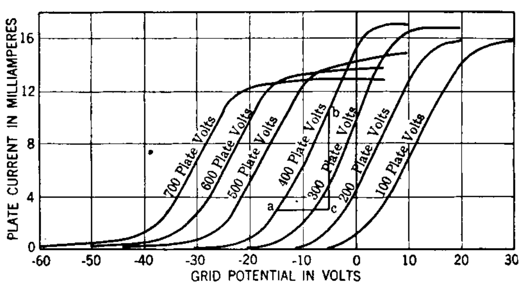

FIG. 4

(a), the circuit would no longer be in tune with the voltage of B. In most cases this can be taken care of by retuning or by tuning while A is included in the circuit.

II. THE THREE-ELEMENT THERMIONIC AMPLIFIER OR TRIODE CIRCUITS AS RESISTANCE NEUTRALIZERS

6. *Definition of Triode Constants, Conventions and Notation.* Before commencing the discussion of the three-element thermionic circuit as a resistance neutralizer, it will be necessary to define the triode constants and to set forth the conventions which will be used in the following discussion.

The elements of the thermionic amplifier or triode are a heated cathode, F, an anode or plate, P, and a third electrode or grid G, interposed between the

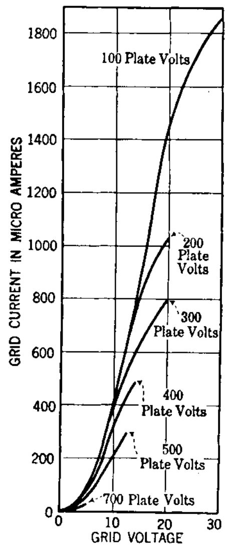

FIG. 5

cathode and anode. (See Fig. 3). In the circuits dealt with in this paper, these three elements are always connected through auxiliary apparatus to a common point or *bus.* Therefore all circuits will be represented in a manner similar to the circuit shown in Fig. 3.

The positive directions in both plate and grid circuits will always be so taken that a + e. m. f. in these circuits is an e. m. f. tending to cause current to flow *through the gas* from plate or grid to filament,—or tending to cause electrons to pass from filament to plate or grid.

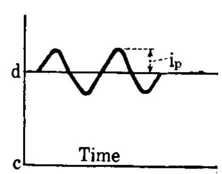

FIG. 6

By the *plate current* is meant the current flowing *from* the plate *through the gas* to filament and grid.

By the grid current is meant the current flowing *from* the grid *through the gas* to filament and plate.

By the *plate and grid potentials* is signified their potentials relative to the negative end of the filament.

Let the characteristics of the amplifier represented in Fig. 3 be given by the curves of Figs. 4 and 5. Let the voltages of the battery D_1 and the generator D_2 be so adjusted that conditions are represented by the point o in Fig. 4. That is, the generator D_2 gives a voltage of 400 volts and D_1 a voltage of 10 volts. There is a continuous current of 6.45 milli-amperes flowing in the plate circuit. Now suppose the alternator D_3 is excited to give a voltage whose peak value is 5 volts. Operation now takes place over the 400 volt characteristic from a to b. The current in the plate circuit is shown in Fig. 6. This current can be broken up into a steady current of magnitude $c\,d$ and an alternating current of amplitude i_p.

Let the ratio of the plate alternating current to the grid alternating potential (the plate potential being kept constant) be called the controlled conductance of the plate *by the grid*, or briefly the *controlled plate conductance*, G_{cp}

$$\frac{\Delta I_p}{\Delta E_g} \text{ is represented by } G_{cp}$$

Let the ratio of the grid alternating current to the grid alternating potential (the plate potential being kept constant) be called the *grid conductance*, G_g.

$$\frac{\Delta I_g}{\Delta E_g} \text{ is represented by } G_g$$

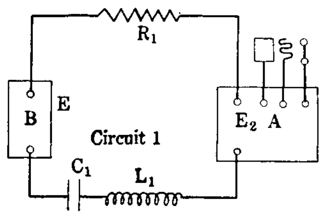

FIG. 7

Now imagine the alternator D_3 shifted to the plate circuit and from similar considerations let the following definitions be adopted:

Let the ratio of the grid alternating current to the plate alternating potential (the grid potential being kept constant) be called the *controlled conductance of the grid* by the plate, or briefly, the *controlled grid conductance*, G_{cg}

$$\frac{\Delta I_g}{\Delta E_p} \text{ is represented by } G_{cg}$$

Let the ratio of the plate alternating current to the plate alternating potential (the grid potential being kept constant) be called the *plate conductance*, G_p.

$$\frac{\Delta I_p}{\Delta E_p} \text{ is represented by } G_p$$

7. *Conditions Leading to Resistance Neutralization by Triode Circuits.* In Fig. 7 let A be a region in which circuit 1 is associated with the three elements of a thermionic amplifier. Let the association represented by Fig. 7 be such that the following conditions are fulfilled:

1. The grid must be excited from the circuit 1.
2. The amplifier must feed power into circuit 1, and the power fed into circuit 1 by the amplifier must be less than the sum of all the losses occurring in circuit 1.

In order to fulfill condition 2 the bulb must operate so as to transform d-c. power into a-c. power. This requires that

(a) The alternating voltages introduced into the plate and grid circuits using the bus as a reference shall be 180 deg. out of phase. Variations from a phase displacement of 180 deg. so long as these displacements do not exceed 90 deg. may obtain without causing the triode to cease to act as a generator. However, the closer these voltages approach to phase opposition, the closer the triode functions as a pure resistance neutralizer.

(b) The ratio of the plate alternating voltage to the grid alternating voltage must be less than the voltage amplification constant of the tube.

(e_p/e_g must be less than G_{cp}/G_p)

Condition 1 above, taken together with requirements (a) and (b), insures that the fundamental of the voltage introduced by the triode into circuit 1 shall be of the same frequency as the current in circuit 1. If operation takes place about a correct point of the characteristic curve of the triode, and if the triode is not pushed too much for output, the harmonics of the voltage wave which it introduces into circuit 1 should be relatively small.

Condition 2, taken together with the considerations just mentioned, insures that the voltage introduced into circuit 1 by the triode will have a component in phase with and proportional to the current in circuit 1. Since the voltage e_2 introduced into circuit 1 by the bulb is of the same frequency as the current in circuit 1, it follows that the most general effect which this voltage can have upon the current in the circuit is to change its magnitude and phase position. Then, that the triode may deliver power to 1, the voltage e_2 must have a component in phase with i_1. Since i_1 and e_2 are practically sine waves, the only manner in which the component of e_2 in phase with i_1 can differ from i_1 is by a constant multiplier N. The amplifier thus reduces the effective resistance of the circuit from R_1 to $R_1 - N$.

The component of e_2 at right angles to i_1 introduces a reactance into circuit 1. By a proper choice of circuits and circuit constants, this reactance can be made large or very small at will. The amplifier thus may be used to neutralize or to add to the reactance of circuit 1. If circuit 1 is a tuned circuit, the reactance introduced by the amplifier can, in most cases, be taken care of by tuning while the amplifier is associated with the circuit.

If condition 2 is reversed, the bulb abstracts power from 1. The bulb may be made to function as a true positive resistance of value N.

8. *Derivation of the Steady State Equations for Fig. 8.* As an illustration of a specific triode circuit satisfying the conditions outlined above, the equations

for the steady state current in the circuit of Fig. 8 will now be derived and discussed. In Fig. 8, B represents an alternator generating a sine electromotive force. The capacities of the condensers C_3 and C_2 are assumed to be so large that the alternating currents which flow through these condensers cause no appreciable variation in the voltage across these condensers. The power circuit containing the alternator (circuit 1) is magnetically coupled in opposite directions with the plate and grid circuits (as illustrated in the figure). That is, if M_p is a positive quantity, M_g is a negative quantity, or vice versa. In the following equations M_p and M_g represent the algebraic values and not the absolute values of the mutual inductances.

Applying Kirchhoff's law to circuit 1, we obtain

$$E - R_1 I_1 - j\,\omega L_1 I_1 + \frac{j I_1}{\omega C_1} - j\,\omega M_p I_p - j\,\omega M_g I_g = 0 \qquad (24)$$

The last term may be made negligibly small and will therefore be dropped from this equation.

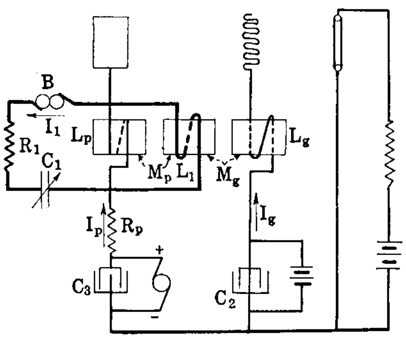

FIG. 8

The plate voltage is,

$$E_p = -j\,(\omega M_p I_1 + \omega L_p I_p) - R_p I_p \qquad (25)$$

The grid voltage is,

$$E_g = -j\,\omega M_g I_1 - j\,\omega L_g I_g - R_g I_g \qquad (26)$$

The last two terms are negligibly small and will be dropped. The plate current is,

$$I_p = E_g G_{cp} + E_p G_p$$

$$I_p = j\,\omega\,[-M_g I_1 G_{cp} - (M_p I_1 + L_p I_p) G_p] - R_p I_p G_p \qquad (27)$$

$$I_p = \frac{j\,\omega\,(-M_g G_{cp} - M_p G_p)\, I_1}{1 + R_p G_p + j\,\omega L_p G_p} \qquad (28)$$

Letting D represent $1 + R_p G_p$ (29)

and X_1 represent $\omega L_1 - \frac{1}{\omega C_1}$

and substituting the value of I_p from (**28**) in (**24**), there results,

$$E - R_1 I_1 - j X_1 I_1 + \frac{\omega^2 M_p (-M_g G_{cp} - M_p G_p)\, I_1}{D + j\,\omega L_p G_p} = 0 \qquad (30)$$

Writing h for $M_p(-M_g G_{cp} - M_p G_p)$ (31)

and solving for I_1, we obtain,

$$I_1 = \frac{E}{\left[R_1 - \frac{\omega^2 h D}{D^2 + \omega^2 L_p^2 G_p^2}\right] + j\left[X_1 + \frac{\omega^3 L_p G_p h}{D^2 + \omega^2 L_p^2 G_p^2}\right]} \qquad (32)$$

Equation (**32**) shows that the triode lowers the effective resistance of the circuit 1 by the amount

$$\frac{\omega^2 h D}{D^2 + \omega^2 L_p^2 G_p^2}\,.$$

The algebraic value of this quantity depends upon and is the same as the algebraic value of h. An inspection of the expression for h, equation (**31**), shows that the only conditions under which h is positive (or under which the triode acts as a resistance neutralizer)are that M_p and M_g shall be of opposite signs and $M_g G_{cp}$ greater than $M_p G_p$. If both of these conditions are not fulfilled the triode circuit increases the effective resistance of the power circuit.

The triode circuit also increases the effective inductive reactance by an amount

$$\frac{\omega^3 L_p G_p h}{D^2 + \omega^2 L_p^2 G_p^2}$$

This latter term can be made large or small at will. Thus the triode circuit may also be used to neutralize reactance or to introduce reactance into a circuit.

In order to see how closely the triode can be made to function as a true resistance neutralizer, it is necessary to obtain some idea of the relative magnitude of the terms involved in equation (**32**). In a particular radio antenna using ordinary laboratory inductances and a 100 watt audion bulb, the circuit constants had the following values:

$R_1 = 80$ ohms
$G_p = 200 \times 10^{-6}$ mhos
$G_{cp} = 3000 \times 10^{-6}$ mhos
$R_p = 5.13$ ohms
$L_p = 3350 \times 10^{-6}$ henrys
$L_g = 172 \times 10^{-6}$ henrys
$L_1 = 20320 \times 10^{-6}$ henrys
$M_p = 3000 \times 10^{-6}$ henrys
$M_g = 432 \times 10^{-6}$ henrys
$C_1 = 1.4 \times 10^{-9}$ farads (antenna)
$\omega_r = 1.875 \times 10^5$ rad/sec.
$R_g = 1.16$ ohms
$\omega_r^2 = 3.515 \times 10^{10}$

For this circuit, $D = 1.00102$, $\omega^2 L_p^2 G_p^2 = 0.0158$. $(D^2 + \omega^2 L_p^2 G_p^2)$ may for most purposes be written equal to unity. This will be done in the following equations:

$$h = 2.1 \times 10^{-9}$$
$$\omega^2 h = 74 \text{ ohms}$$
$$\omega^3 L_p G_p h = 9.23 \text{ ohms}$$

Now $\omega^3 L_p C_p h$ is the net reactance introduced by the neutralizer into the circuit 1. This term can be kept low if desired. If circuit 1 is a tuned circuit, the net reactance can in most cases be made zero by tuning while the neutralizer is associated with the circuit.

Calling $D = 1$, and neglecting $(\omega^2 L_p^2 G_p^2)$ in comparison with unity, equation (**32**) reduces to the simple form,

$$I_1 = \frac{E}{(R_1 - \omega^2 h) + j\,(X_1 + \omega^3 L_p G_p h)} \quad \textbf{(33)}$$

The power delivered by alternator B to the circuit 1 is,

$$P_B = \frac{E^2 (R_1 - \omega^2 h)}{(R_1 - \omega^2 h)^2 + (X_1 + \omega^3 L_p G_p h)} \quad \textbf{(34)}$$

If the net reactance is zero this becomes,

$$P_B = \frac{E^2}{R_1 - \omega^2 h} \quad \textbf{(35)}$$

9. *Complete Equations for Fig. 8.* If the triode circuit is designed to keep the reactive term low, a very close complete solution of the differential equations of the system is given by the following equations:

The current in the circuit 1 is,

$$i_1 = \frac{E}{\sqrt{(R_1 - \omega^2 h)^2 + X_n^2}} \cos(\omega t - \tau - \lambda) + [I_d \cos \beta t + (C_1 \beta E_{cd} + a/\beta I_d) \sin \beta t]\, \epsilon^{-\frac{R_1 - \beta^2 h}{2 L_1} t} \quad \textbf{(36)}$$

The counter electromotive force of the condenser C_1 is

$$e_c = \frac{E}{\omega C_1 \sqrt{(R_1 - \omega^2 h)^2 + X_n^2}} \cos(\omega t + \pi/2 - \tau - \lambda) + \left[E_{cd} \cos \beta t - \left(\frac{E_{cd}\, a}{\beta} + \frac{I_d}{C_1 \beta} \right) \sin \beta t \right] \epsilon^{-\frac{R_1 - \beta^2 h}{2 L_1} t} \quad \textbf{(37)}$$

In these equations the symbols have the following meaning: The alternating voltage impressed in circuit 1 is expressed by the equation $e = E \cos(\omega t - \tau)$. In which time is measured from the instant of switching in the voltage.

τ is the interval in radians from the moment of switching to the first positive peak of the impressed e. m. f.

λ is the angle of lag of the permanent current behind the impressed e. m. f. $= \tan^{-1} \dfrac{X_n}{R_1 - \omega^2 h}$

X_n is the net reactance $= \omega L_1 - \dfrac{1}{\omega c_1} + \omega^3 L_p G_p h$

$$I_d = I_0 - E/Z \cos(\tau + \lambda)$$

$$E_{cd} = E_{c0} - \frac{E X_c}{Z} \cos(\tau + \lambda - \pi/2)$$

Z is the net impedance $= \sqrt{(R_1 - \omega^2 h)^2 + X_n^2}$

$$X_c = \frac{1}{\omega C_1}$$

$$\Omega_r = \frac{1}{\sqrt{L_1 C_1}}$$

$$\beta = \Omega_r \sqrt{\frac{D L_1}{D L_1 + R_1 L_p G_p}}$$

$$a = -\frac{R_1 - \beta^2 h}{2 L_1}$$

These equations are identical in form with the equations for the start of an alternating current in the circuit with the neutralizer omitted. The only difference is that for the circuit without the neutralizer, R must be substituted for $R_1 - \omega^2 h$ and $R_1 - \beta^2 h$, X_1 substituted for X_n, and $\dfrac{1}{\beta L_1}$ written for $C_1 \beta$. This latter substitution is legitimate if β differs little from Ω_r.

III. Some Applications of Resistance Neutralization to Radio Receiving Circuits

10. *Increased Selectivity through Pure Resistance Neutralization.* One of the most important applications of resistance neutralization is in increasing the selective properties of radio receiving circuits. It is this application which will now be taken up.

In discussing the selective properties of radio receiving circuits, it is helpful to define some coefficients, the values of which for any circuit are a measure of the selectivity of the circuit. The steady-state selective coefficient, S_c, of a receiving circuit against a specified detuned frequency has been defined[2] as the ratio of the power delivered to the detector by waves of a frequency such as to make the circuit resonant and the power delivered to the same detector by waves of the same intensity but of the specified detuned frequency, the power being determined after the current builds up to the steady state value.

For the simple series circuit shown in Fig. 8, let E be the voltage induced by the impinging waves. The current in the circuit is

$$I = E/Z$$

and the power delivered to the detector is:

$$P = I^2 R_d = \frac{E^2 R_d}{Z^2}$$

If the impedance of the circuit to the waves of the interferent station is Z_1 and to the correspondent station Z_c, we have

$$S_c = P_c/P_1 = Z_1^2/Z_c^2 \quad \textbf{(40)}$$

If the circuit is resonant to the frequency of the correspondent station $Z_c = R_n$ and,

$$S_c = Z_1^2/R_n^2 \quad \textbf{(41)}$$

Let the interferent source be detuned by a small decimal part, P_d, of the resonant frequency. Then,

$X_n = 4 P_d \pi f_n L$ very closely

$$S_c = \frac{16 \pi^2 P_d^2 f_r^2 L^2 + R_n^2}{R_n^2}$$

2. See "Abstractive and Selective Properties of Radio Antenna Circuits," by Edward Bennett. Journal A. I. E. E. 1920, Vol. 39.

$$= 1 + \left(2\,\pi\,P_d\,f_r\,\frac{2\,L}{R_n}\right)^2$$

$$S_c = 1 + (2\,\pi\,P_d\,f_r\,T_c)^2 \qquad (42)$$

In most cases the second term is large compared to unity, and for these cases we may write

$$S_c = (2\,\pi\,P_d\,f_r\,T_c)^2 \qquad (43)$$

where T_c represents the time constant of the circuit $2\,L/R_n$ and f_r represents the resonant frequency.

Let the selective coefficient (of an antenna circuit) for the interval of excitation, T_e, against a specified detuned frequency be defined to signify the ratio between the energy delivered to the detector by an impressed alternating electromotive force of a frequency such as to make the circuit resonant and the energy delivered to the same detector by an impressed electromotive force of the same value but of the detuned frequency, both electromotive forces being impressed for the same interval of time, T_e.

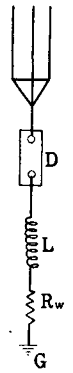

Fig. 9

Let the selective coefficient (of an antenna circuit) for the time interval of excitation T_e (time of Morse dot interval) against a continuous e. m. f. be defined as the ratio of the energy delivered to the detector by the resonant e. m. f. during the time T_e to the energy delivered to the same detector by a continuous e. m. f. both e. m. fs. to have the same peak value.

In a like manner we may define a selective coefficient against a short impulse e. m. f. and a selective coefficient against the first cycle due to a continuous e. m. f.

It is evident that these five coefficients determine to a marked degree the selective properties of an antenna circuit. The first two coefficients determine the selectivity against interferent stations, the last three give indications of the selective properties of the antenna circuit against static.

In the paper referred to above it is shown that for a simple series antenna circuit, similar to the circuit shown in Fig. 9, the values of these coefficients are given by the following expressions

S_c [for steady state against a detuned frequency]

$$= (2\,\pi\,P_d\,f_r\,T_c)^2 \qquad (44)$$

S_c [for interval of excitation against a detuned frequency]

$$= 0.8\,(2\,\pi\,P_d\,f_r\,T_c)^2 \text{ if } T_c = 0.2\,T_e \qquad (45)$$

S_c [against a continuous e. m. f.]

$$= 2\,\pi^2\,b\,T_e\,f_r^2\,T_c \qquad (46)$$

S_c [for first cycle against a continuous e. m. f.]

$$= \pi^2\,b\,f_r^2\,T_c^2 \qquad (47)$$

S_c [against a very short impulse]

$$= \frac{b\,T_e\,T_c}{2\,T_1^2} \qquad (48)$$

In these equations b is a factor expressing the ratio of the energy actually delivered to the circuit (expended and stored) during the interval of excitation to the energy which would be expended in the resistance if the current in the circuit jumped immediately to its final value and remained there during the interval of excitation.[3]

All of these coefficients vary either as the square of the time constant or as the first power of the time constant T_c. By means of a resistance neutralizer T_c can be increased to almost any desired value by making $R_1 - N$ small enough. It has been shown that the triode circuits may be made to function under all conditions, transient and otherwise, as a resistance neutralizer. In the special case considered, $N = \omega^2\,h$, and R_n or $(R_1 - \omega^2\,h)$ can be reduced to a low value, thus making it possible to obtain a large value for T_c.

In the previous paper, it is shown that for continuous wave telegraphy the upper limit of T_c must not be made much longer than 0.01 sec. if there is to be an interval of silence between dots and dashes at a speed of 30 words per minute. Therefore 0.01 second is taken as the upper limit for T_c. While this value for T_c might be obtained by using small aerials and by taking great precautions to reduce wasteful resistances in the antenna circuit, it may be more readily obtained by using a triode as a resistance neutralizer.

As an example of the lengthening of the time constant due to resistance neutralization, consider the data given in Section 8. C_0 is an antenna, and the circuit is designed from ordinary laboratory inductances. Without the triode the time constant is 0.0005 sec. and with the triode it is 0.0068 second. This example merely serves to indicate how the triode lengthens the time constant. By a proper design of circuits, a time constant of 0.01 sec. can readily be obtained.

11. *Effect of Resistance Neutralization upon the Power which can be Abstracted by an Antenna from Impinging Waves.* If resistance neutralization is not resorted to, the value of the detector resistance which will make the delivery of power from sustained waves to the detector a maximum is:

$$R_d = R_r + R_w \qquad (49)$$

where R_d represents the equivalent series detector resistance, R_r the radiation resistance of the antenna with which the detector is associated, and R_w represents the wasteful resistance of the antenna circuit. Let us seek the expressions for the optimum values of R_d with resistance neutralization under the conditions stated in the following problems.

Problem 1. A given antenna has at a given frequency a given radiation resistance R_r and a given wasteful resistance R_w. It is desired to deliver the maximum possible power to a utilization device (detector) having an undetermined resistance R_d.

3. See Curve B, Fig. 6, of previous paper, loc. cit.

A resistance neutralizer is available which will operate reliably (steadily) to reduce the total resistance R_t (or $R_d + R_w + R_r$) to a net resistance R_n, which is γ decimal parts of the total resistance. That is, by the use of the neutralizer

$$R_n = \gamma (R_d + R_r + R_w) \qquad (50)$$

What value should be assigned to the detector resistance R_d to make the power delivered to it a maximum when waves of resonant frequency impinge upon the antenna?

The antenna current caused by an electromotive force of r. m. s. value E of resonant frequency is

$$I = E/R_n = \frac{E}{\gamma (R_d + R_r + R_w)}$$

The power expended in the detector is

$$P_T = I^2 R_d = \frac{E^2 R_d}{\gamma^2 (R_d + R_r + R_w)}$$

The value of R_d which makes the power P a maximum as found by equating the derivative of P with respect to R_d to zero and solving, is

$$R_d = R_r + R_w \qquad (49)$$

If R_d has this value,

$$R_n = 2 \gamma (R_r + R_w) \qquad (51)$$

and

$$P_T = \frac{E^2}{4 \gamma^2 (R_r + R_w)} \qquad (52)$$

Problem 2. In the problem above, no lower limit was placed upon the value of the net resistance, and in satisfying the conditions for maximum power the net resistance was reduced to $2 \gamma (R_r + R_w)$. But suppose this low net resistance results in a circuit time constant which is of prohibitive length. In other words, let it be assumed that it is not permissible to reduce the net resistance R_n below a specified value R_m. Under these limiting conditions the value assigned to R_d must be such as to make R_n or $\gamma (R_d + R_r + R_w)$ not less than R_m. That is, R_d must not be less than (but may be greater than) $(R_m/\gamma) - (R_r + R_w)$. If $[(R_m/\gamma) - (R_r + R_w)] > (R_r + R_w)$, or if $R_m/\gamma > 2 (R_r + R_w)$, the value which must be assigned to R_d in order to limit the net resistance R_m (or the time constant) as specified above is as follows:

$$R_d = (R_m/\gamma) - (R_r + R_w) \qquad (53)$$

This is greater than the value for maximum power delivery.

On the other hand, if $R_m/\gamma \lesseqgtr 2 (R_r + R_w)$, the value to be assigned to R_d is the optimum value specified in equation (**49**).

If $R_m/\gamma = 2 (R_r + R_w)$, this optimum value makes the net resistance R_n just equal to the lower limit R_m for the net resistance, or gives to the time constant the maximum permissible value.

If $R_m/\gamma < 2 (R_r + R_w)$, this optimum value makes the net resistance R_n greater than the lower limit R_m, or has the effect of making the time constant shorter than the maximum permissible value.

Problem 3. Now suppose the problem is not that of making the power delivery to the detector a maximum, but the problem is to make the selective coefficient against a sustained wave detuned station a maximum. What is the optimum value for the detector resistance?

We limit the discussion to the general case in which the interferent station is sufficiently dissonant (2 to 5 per cent) to make the net reactance of the antenna to the dissonant frequency large in comparison with its net resistance. From equation (**42**) it is seen that the selective coefficient of the antenna against an interferent electromotive force which is detuned by a given percentage from the given resonant frequency of the antenna is substantially proportional to the square of the time-constant (T_c) of the antenna. The selective coefficient is independent of the value of the detector resistance, except as the detector resistance may affect the value of the time-constant. If then we are dealing with an antenna of *given* height and capacity, the time-constant of which may not be permitted to exceed a specified value, (such as 0.01 second), two cases arise:

Case I. If the sum of the radiation and the wasteful resistance of the given antenna is so large that $\gamma (R_r + R_w)$ by itself is greater than the resistance R_m which corresponds to the maximum permissible time-constant, then the maximum selective coefficient will be obtained if the detector resistance is allowed to approach zero. However the power delivered to the detector at the resonant frequency is a maximum when $R_d = (R_r + R_w)$, and the power decreases to zero as R_d approaches zero.

Case II. If $\gamma (R_r + R_w)$ is less than R_m, the value of the selective coefficient is fixed by the value assigned to T_c (or to R_m), and is independent of the value assigned to R_d, provided that $\gamma (R_d + R_r + R_w)$ is made equal to R_m.

Before the advent of the resistance neutralizer all antennas fell under Case I. By the proper use of neutralizers all antennas may be made to fall under Case II. The question which now arises is this. If the maximum selective coefficient possible by the use of single tuned circuit has been obtained by satisfying the relation

$$\gamma (R_d + R_r + R_w) = R_m \qquad (53)$$

in which, R_m is the resistance which gives the maximum permissible time-constant, what further conditions should be satisfied to make the power delivered to the detector resistance at the resonant frequency a maximum? Two sub-cases arise under this Case II.

Sub Case A. In this case we have a *given* antenna whose dimensions are not to be changed. The only thing which may be varied is the detector resistance R_d. Since in this case the values of both R_m and of the selective coefficient are fixed by the assignment of a value to the time-constant, and since at resonance, R_m alone determines the flow of current per volt induced in the antenna, and since the power delivered to the detector is

$$P = I^2 R_d = (E/R_m)^2 R_d \qquad (54)$$

we may formulate the following rule:

To obtain from a *given* antenna the maximum power consistent with a specified time-constant (or selective coefficient), the detector resistance R_d should be made as great as possible consistent with the stable reduction of the total resistance to the net value R_m which is fixed by the specified time-constant.

Sub Case B. In this case the problem is to determine the proportions which the antenna itself should have in order to deliver the maximum power (consistent with the specified time-constant) to a detector when the antenna is used with a neutralizer having a fixed resistance reduction factor γ.

Let

C_0 represent the capacity of the antenna.

L_0 represent the inductance of the antenna circuit.

f_r represent the frequency of the correspondent station

T_c represent the desired time-constant.

R_m represent the net resistance for the specified time-constant.

h represent the height of the antenna network in cm.

s represent the velocity of light, 3×10^{10} cm. per sec.

p_0 represent the permittivity of air 8.84×10^{-14} farad-cm.

F_m represent the peak value of the electric intensity at the antenna in volts per cm.

The expression for the power delivered to the detector is,

$$P = \frac{(F_m h)^2 R_d}{2 R_m^2} \qquad (54)$$

To obtain the maximum selective coefficient the value assigned to R_d must satisfy equation (**53**). That is,

$$R_d \text{ must equal } (R_m/\gamma) - (R_r + R_w) \qquad (53)$$

In the subsequent discussion the value of the ratio

$$\frac{R_r + R_w}{R_r} \text{ will be represented by } k \qquad (55)$$

and k will be treated as a constant. It should be recognized that this is not strictly correct but is an approximation only.

Substituting the value of R_d from (**53**) and (**55**) in the equation for the power, it becomes,

$$P = \frac{F_m^2 h^2}{2 R_m^2}\left(\frac{R_m}{\gamma} - k R_r\right) \qquad (54a)$$

An expression for the value of the minimum permissible net resistance R_m in terms of the antenna constants and specified time-constant may be arrived at as follows:

$$f_r = \frac{1}{2\pi\sqrt{L_0 C_0}} \quad \text{or} \quad L_0 = \frac{1}{4\pi^2 f_r^2 C_0}$$

$$T_c = \frac{2 L_0}{R_m} = \frac{1}{2\pi^2 f_r^2 C_0 R_m}$$

From which

$$R_m = \frac{1}{2\pi^2 f_r^2 C_0 T_c} \qquad (56)$$

That is, the value of R_m is fixed by T_c, C_0 and f_r.

In any antenna with an extended capacity area at a height h, the expression for the radiation resistance may be written,

$$R_r = \frac{160\pi^2 h^2}{\lambda^2} = \frac{4\pi h^2 f_r^2}{3 s^3 p_0} \qquad (57)$$

Substituting the values of R_m and R_r as expressed in equations (**56**) and (**57**) in equation (**54a**), we have the following equation for the power delivered to the detector when the antenna capacity, the detector resistance and the reduction factor γ are so related as to give the specified selective coefficient.

$$P = (F_m h)^2 \, 2\pi^4 f_r^4 C_0^2 T_c^2 \left[\frac{1}{2\pi^2 f_r^2 C_0 T_c \gamma} - \frac{4 k \pi h^2 f_r^2}{3 s^3 p_0}\right] \qquad (58)$$

If the radius of the antenna network is so great as compared with the mounting height that the capacity is approximately expressed by the parallel plate formula, namely,

$$C_0 = \frac{p_0 a}{h} \qquad (59)$$

the following equation results from the substitution of the value of C_0 from (**59**) in (**58**)

$$P = F_m^2 \, 2\pi^4 f_r^4 p_0^2 a^2 T_c^2 \left[\frac{h}{2\pi^2 f_r^2 p_0 a \gamma T_c} - \frac{4 k \pi h^2 f_r^2}{3 s^3 p_0}\right]$$

To find the antenna height or the antenna area which will make the power a maximum, we take the derivatives of P with respect to (h) or to (a) respectively, equate the derivatives to zero, and solve the resulting equations. Upon doing so, it is found that the antenna should be so proportioned that,

$$\frac{h}{2\pi^2 f_r^2 p_0 a \gamma T_c} = \frac{8 k \pi h^2 f_r^2}{3 s^3 p_0} \qquad (59)$$

That is, the values assigned to (h) or to (a) must be such that,

$$R_m/\gamma = 2 (k R_r) \qquad (59a)$$

in which case,

$$R_d \text{ will equal } (k R_r) \text{ or } (R_r + R_w) \qquad (49)$$

Equation (**59**) may also be written in the form,

$$a h = \frac{3 s^3}{16 \pi^3 k \gamma T_c f_r^4} \qquad (61)$$

On the other hand if the antenna network is so high that its capacity is approximately expressed by the formula for an elevated circular disk, namely,

$$C_0 = 8\, p_0 \sqrt{a/\pi} \qquad (62)$$

the following equation results from the substitution of this value of C_0 in equation (58)

$$P = (F_m^2 h^2)\, 128\, p_0^2\, \pi^3 f_r^4\, a\, T_c^2 \left[\frac{1}{16\, \pi^{3/2} f_r^2\, p_0\, a^{1/2}\, \gamma\, T_c} - \frac{4\, k\, \pi\, h^2 f_r^2}{3\, s^3\, p_0} \right]$$

Upon taking derivatives of this value of P with respect to (h) and (a), equating to zero and solving, it is found that the antenna should be so proportioned that,

$$\frac{1}{16\, \pi^{3/2} f_r^2\, p_0\, a^{1/2}\, \gamma\, T_c} = \frac{8\, k\, \pi\, h^2 f_r^2}{3\, s^3\, p_0} \qquad (63a)$$

That is, in this case also the values assigned to (h) and to (a) must be such that,

$$R_m/\gamma = 2\,(k\, R_r) \qquad (59a)$$

Equation (**63a**) may also be written in the form

$$a^{1/2}\, h = \frac{3\, s^3}{128\, \pi^{5/2}\, k\, \gamma\, T_c\, f_r^4} \qquad (63)$$

These equations, (**61**) and (**63**), give respectively the dimensions which antennas of the parallel plate type and the elevated disk type must have to permit of the maximum power delivery to the detector, and the maximum selective coefficient against detuned frequencies, which is possible with the given resistance ratio k, reduction factor γ, time constant T_c, and frequency f_r. These equations express, not exactly, but only approximately the optimum relations between the antenna dimensions and the four quantities k, γ, T_c and f_r. They are valid only for antennas of the usual proportions found in high power practice; that is, for antennas whose greatest length is short (one eighth or less) in comparison with the wave length.

The total power delivered to the detector associated with a simple series antenna is,

$$P_T = \frac{h^2 F_m^2}{2} R_d/R_m^2 \qquad (54)$$

If

$$R_d = (R_r + R_w) = k\, R_r \qquad (49)$$

equation (**54**) may be written

$$P_T = \frac{h^2 F_m^2}{8\, \gamma^2\, k\, R_r} \qquad (65)$$

Of the total power P_T, the amount P_B abstracted from the impinging waves is

$$P_B = \frac{h^2 F_m^2}{8\, \gamma\, k\, R_r} \qquad (66)$$

and the amount P_A supplied by the neutralizer is

$$P_A = (1/\gamma^2 - 1/\gamma)\, \frac{h^2 F_m^2}{8\, k\, R_r} \qquad (67)$$

By substituting the value of the radiation resistance from (**57**) in equation (**66**) the following expression is obtained for the power which is abstracted from the impinging waves and delivered to a detector resistance proportioned for maximum power as in equation (**49**).

$$P_B = \frac{3}{16\, \pi\, k\, \gamma} (s\, \lambda^2) \left(\frac{1}{2}\, p_0\, F_m^2\right) \qquad (68)$$

In a previous paper,[4] the factor $(s\, \lambda^2)\,(1/2\, p_0\, F_m^2)$ is shown to represent the power flowing across a *wave length square* at the receiving station. Therefore, the greatest power which can be delivered to a detector by an antenna from impinging sustained waves is

$$\frac{3}{16\, \pi\, k\, \gamma}$$

times the power flowing across a *wave length square* at the receiving station.

In the previous paper the factor $1/k$ was termed the *abstractive factor* of the antenna. With a neutralizer associated directly with an antenna the expression for the abstractive factor A_f of the antenna becomes

$$A_f = \frac{1}{k\, \gamma} \qquad (69)$$

Equation (**69**) shows that the power abstractive factor of any existing antenna can be increased by associating with the antenna a resistance neutralizer, but it should be realized that the increase in the abstractive factor is accompanied by an increase in the time constant of the antenna. If then the antenna circuit without the neutralizer has the longest time constant which is permissible at the sending speed (for example 0.01 second at 30 words per minute), increased power from the waves can be obtained only by increasing the dimensions of the antenna. This may be readily seen by examining the expression giving the proportions which an antenna of the parallel plate type must have for maximum selective coefficient, namely equation (**61**)

$$a\, h = \frac{3\, s^3}{16\, \pi^3\, k\, \gamma\, T_c\, f_r^4} \qquad (61)$$

From this it is seen that if k and f_r are fixed, and if T_c is to remain constant, the volume under the antenna must be proportional to the reciprocal of the reduction factor of the neutralizer.

The time constants of existing antenna circuits are short (from 0.0001 to 0.001 sec.), and the selective coefficients of existing circuits can be greatly increased by the use of properly designed resistance neutralizers with both the antenna circuit and its secondary. Resistance neutralization *in the secondary* is to some extent utilized in present radio sets when the thermionic amplifier associated with the secondary is regeneratively coupled. This effect does not however extend back into the antenna circuit.

From the relations brought out in this and the pre-

4. loc. cit.

ceding section it is evident that *pure resistance neutralization as obtained from the Fig. 8 circuit* does not make it possible to obtain a higher selective coefficient against detuned stations than it is *hypothetically* possible to obtain by proper antenna design. That is to say, the selective coefficient against a frequency detuned by a given per cent is fixed by the value of the operating frequency and of the time constant (equation **43**), and calculations indicate that the maximum permissible time constant at a sending speed of 30 words per minute (0.01 second) can be obtained without resorting to resistance neutralization. (This has not been verified by experiment). In general, however, and particularly in the case of the long wave lengths, it will be far cheaper to obtain the desired time constant and selectivity by associating resistance neutralizers directly with antenna, secondary and tertiary circuits rather than by the construction of circuits which in themselves have time constants of 0.01 second.

Pure resistance neutralization as obtained in Fig. 8 increases the selective coefficient of a circuit at the expense of an increase in the time constant of the circuit. A limit to the selective coefficient which may be obtained by this method is set by the fact that in a

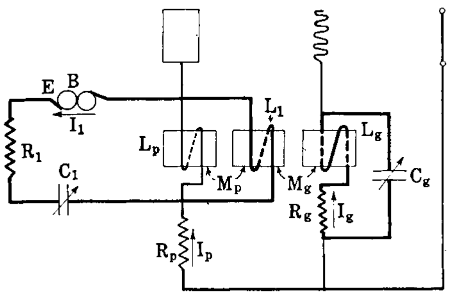

FIG. 10

make and break telegraph system, the time constant cannot be allowed to exceed a definite value. We now proceed to consider types of neutralizer circuits which will increase the selective coefficient of a circuit by methods other than the straightforward method of increasing its time constant.

12. *Circuits which Neutralize Resistance for a Narrow Range of Frequencies near a Desired Frequency and Introduce Resistance and Reactance at other Frequencies.* In the discussion of the triode as a resistance neutralizer, it has been shown that if the circuit is to function as a resistance neutralizer, the ratio E_p/E_g must be less than the ratio G_{cp}/G_p, and the grid voltage E_g must have a component 180 degrees out of phase with the plate voltage E_p. Now, if the grid inductance L_g of Fig. 8 is shunted by a condenser, the magnitude of the grid voltage and its phase position relative to the plate voltage will be a function of the frequency of the current flowing in circuit 1. These considerations at once suggest the possibility of adjusting the circuit and triode constants so as to obtain resistance neutralization for a narrow range of frequencies near a desired frequency and to obtain the effect of an added positive resistance and additional reactance in circuit 1 at frequencies removed from the desired frequency.

Such a circuit is shown in Fig. 10. Let the constants of the circuit be designated as shown on the diagram. The steady-state equations will now be derived by means of the complex algebra. As in the previous case, M_p and M_g represent the algebraic values and not the absolute values of the mutual inductances.

A summation of the voltages around circuit 1 yields the equation,

$$E - R_1 I_1 - j\,\omega L_1 I_1 + \frac{j I_1}{\omega C_1} - j\,\omega M_p I_p - j\,\omega M_g I_g = 0 \qquad (85)$$

A like summation around the grid circuit yields,

$$-j\,\omega M_g I_1 - R_g' I_g - j\,\omega L_g I_g + \frac{j I_g}{\omega C_g} = 0 \qquad (86)$$

The voltage of the grid is,

$$E_g = -\frac{j I_g}{\omega C_g} \qquad (87)$$

The plate voltage is,

$$E_p = -R_p I_p - j\,(\omega M_p I_1 + \omega L_p I_p) \qquad (88)$$

The plate current is given by the equation

$$I_p = G_{cp} E_g + G_p E_p$$

$$= G_{cp} \frac{-j I_g}{\omega C_g} + G_p\,(-R_p I_p - j\,\omega M_p I_1 - j\,\omega L_p I_p)$$

$$I_p = -\frac{j\left[\dfrac{G_{cp} I_g}{\omega C_g} + \omega M_p G_p I_1\right]}{D + j\,\omega L_p G_p} \qquad (89)$$

in which $D = 1 + R_p G_p$ **(29)**

Upon solving (**86**) for I_g, we have

$$I_g = -\frac{j\,\omega M_g I_1}{Z_g} \qquad (90)$$

in which

$$Z_g = R_g + j\left(\omega L_g - \frac{1}{\omega C_g}\right) \qquad (91)$$

Substituting the value of I_g from (**90**) in (**89**),

$$I_p = -\frac{\dfrac{M_g G_{cp}}{C_g Z_g} + j\,\omega M_p G_p}{D + j\,\omega L_p G_p} I_1 \qquad (92)$$

Substituting the values of I_g and I_p from (**90**) and (**92**) in (85), and solving,

$$E = I_1 \Bigg[\Bigg\{ R_1 + \omega^2 \frac{M_p^2 G_p D}{W} + \frac{\omega^2}{Z_g^2} M_g^2 R_g - \frac{\omega X_g}{Z_g^2} \frac{S D}{W} - \frac{\omega}{Z_g^2} \frac{S R_g P}{W} \Bigg\}$$

$$+ j \Bigg\{ X_1 - \omega^2 \frac{M_p^2 G_p P}{W} - \frac{\omega^2 X_g}{Z_g^2} M_g^2$$

$$+ \frac{\omega X_g}{Z_g^2} \frac{S P}{W} - \frac{\omega}{Z_g^2} \frac{S R_g D}{W} \Big\} \Big] \quad (93)$$

in which

$$D = 1 + R_p G_p$$
$$P = \omega L_p G_p$$
$$W = D^2 + P^2$$
$$S = \frac{M_g M_p G_{cp}}{C_g}$$
$$X_1 = \omega L_1 - \frac{1}{\omega C_1}$$
$$X_g = \omega L_g - \frac{1}{\omega C_g}$$
$$Z_g = \sqrt{R_g^2 + X_g^2}$$

In equation (**93**) the expressions for the resistance and the reactance each contain five terms. The first term is the resistance or reactance of circuit 1, the second term represents the effect of the current which flows in the plate circuit by reason of the plate conductance, the third term represents the effect of the alternating current circulating around the divided portion of the grid circuit, and the fourth and fifth terms represent the effect of the current in the plate circuit which is the results of the grid control.

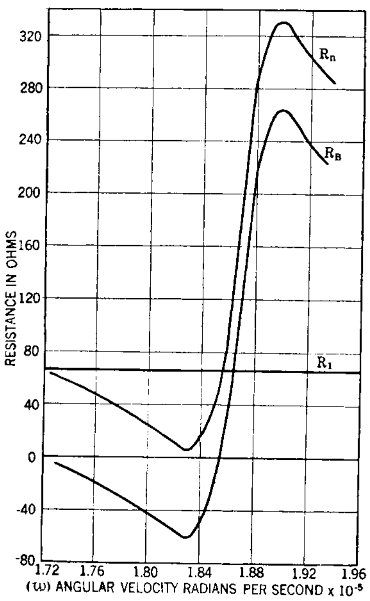

FIG. 11—RELATION BETWEEN RESISTANCE AND IMPRESSED FREQUENCY

R_B—Resistance due to neutralizer.
R_1 —Ohmic resistance of circuit.
R_n —Net resistance.

The fourth term is the important term in the expression for the resistance. It changes sign as the impressed frequency passes through the resonant frequency of the circulatory portion of the grid circuit, contributing a positive resistance on one side and a negative resistance on the other side of this resonant frequency. The first and fifth terms are the important terms of the expression for the reactance. The fifth term becomes large for frequencies near the resonant frequency of the grid circuit and may be used to increase the steepness of the curve between the resultant reactance of circuit 1 and the impressed frequency.

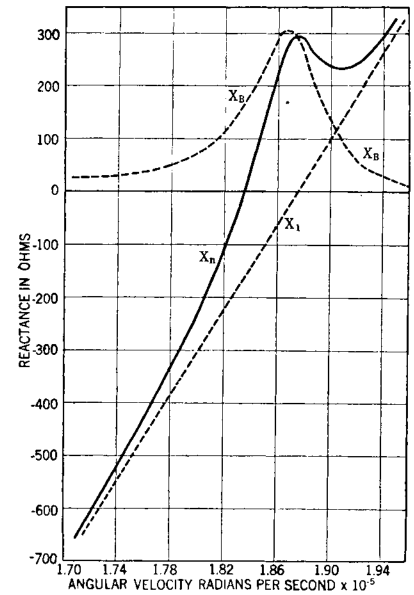

FIG. 12—RELATION BETWEEN REACTANCE AND IMPRESSED FREQUENCY

$X_1 = \omega L_1 - \frac{1}{\omega C_1}$

X_B—Reactance of circuit 1 due to triode.
X_n—Net reactance of circuit 1.

To obtain a better idea of the relations expressed in equation (**93**), let us apply this equation to a particular circuit by plotting the resistance and reactance of the circuit as functions of the frequency of the alternating

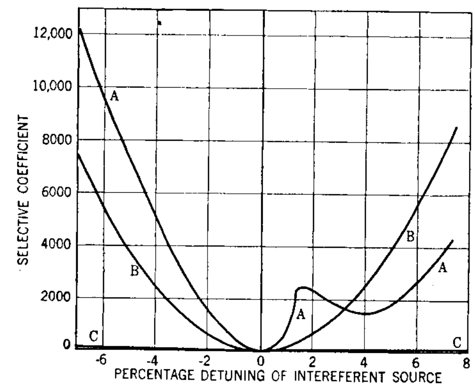

FIG. 13—STEADY STATE SELECTIVE COEFFICIENTS

A—Triode associated with circuit as per Fig. 10.
B—With pure resistance neutralization.
C—Without triode.

electromotive force impressed in circuit 1. Let the coupling be as represented in Fig. 11, that is M_p is positive and M_g is negative. The constants of the circuit with the exceptions noted below are to be the same as given in Section 8 for Fig. 8. The exceptions are:

$R_1 = 66$ ohms
$M_g = -35.7$ microhenries
$C_g = 16.6$ microfarads
$\omega_{rg} = 1.8715 \times 10^5$ rad. per sec.

The curves of Fig. 11 show how the net resistance of circuit 1 varies with the impressed frequency, while those of Fig. 12 show the variation of the reactance with the frequency. An examination of these curves shows that the circuit constants have been proportioned so that the net resistance is near the minimum value at the angular velocity of 1.84×10^5 radians per second, at which the net reactance is zero. It will be seen that the slope of the curve for the net reactance is much greater near this angular velocity than is the slope of the curve X, showing the reactance of circuit 1 without the neutralizer associated with it. This means that the selective coefficient against frequencies detuned by a few per cent is much greater than for a Fig. 8 connection in which pure resistance neutralization is alone utilized.

The selective coefficients, against frequencies detuned by one or more per cent, of a circuit having the constants of circuit 1 of Fig. 10 have been plotted in Fig. 13 for the following conditions. The steady-state selective coefficient is given by the expression,

$$S_c = Z_1^2/Z_c^2 \qquad \textbf{(40)}$$

If the correspondent station has a wave length corresponding to = 183,500 radians per sec., $Z_c = 6$ ohms. Z_1 for any other frequency can be found by combining the resistance values given by curve R_n of Fig. 11 and the reactance values given by curve X_n of Fig. 12. The manner in which the selective coefficient varies with the dissonant frequency in this circuit when the correspondent has a wave length corresponding to an angular velocity of 183,500 radians per sec. is shown by curve A of Fig. 13.

Curve B shows the selective coefficient of the same power circuit for the case in which the neutralizer circuit is adjusted (as in Fig. 8) to give *pure resistance neutralization,* reducing the net resistance of the circuit to 6.0 ohms. For curve B and for curve C the angular velocity of the correspondent station was taken as 187,500 radians per second. Curve C shows the selective coefficient of circuit 1 without resistance neutralization, its net resistance at the resonant frequency being the resistance R_1 of 66 ohms. The increased selectivity against frequencies detuned by 1 or 2 per cent which results from the rapid increase of the reactance and the addition of resistance at frequencies slightly removed from the resonant frequency is illustrated by these curves.

Summary. The paper has been summarized in the review of the subject which is printed at the beginning of the paper.

Appendix A

Compilation of Symbols

A_f represents the abstractive factor of the antenna

A stands for the amplifier or resistance neutralizing device

a represents the area of the antenna network in sq. cm.

B stands for the generator or device supplying the driving force

b represents a factor expressing the ratio of the energy actually delivered to the circuit (expended and stored) during the interval of excitation to the energy which would be expended in the resistance if the current in the circuit jumped immediately to its final value and remained there during the interval of excitation

C represents capacity

D represents the expression $1 + R_p G_p$

F_m represents the peak value of the electric intensity in volts per cm.

f_r represents the resonant frequency of the antenna circuit

G_{cp} represents the controlled plate conductance

$$= \frac{\Delta I_p}{\Delta E_g}\ ;\ (E_p = K)$$

G_p represents the plate conductance

$$= \frac{\Delta I_p}{\Delta E_p}\ ;\ (E_g = K)$$

G_g represents the grid conductance

$$= \frac{\Delta I_g}{\Delta E_g}\ ;\ (E_p = K)$$

G_{cg} represents the controlled grid conductance

$$= \frac{\Delta I_g}{\Delta E_p}\ ;\ (E_g = K)$$

h represents a resistance neutralization factor or the height of the antenna network in cm. according to the text

j represents the rotative operator

k represents the resistance ratio of the antenna

$$= \frac{R_r + R_w}{R_r}$$

L represents self inductance

M represents the mutual inductance

N represents the negative resistance due to the neutralizer

P_T represents the total power delivered to the circuit or device under consideration

P_B represents the power delivered by the generator or driving forces to the circuit or device under discussion

P_A represents the power delivered by the resistance neutralizer to the circuit or device under discussion
P_0 represents the power delivered by the generator with the resistance neutralizer removed from the circuit
p_d represents the decimal part of the resonant frequency by which the interferent source is detuned
p_0 represents the permittivity of air (8.84×10^{-14} farad – cm.)
q represents quantity of electricity
R_w represents the wasteful resistance of the antenna circuit
R_d represents the detector resistance
R_r represents the radiation resistance
R_n represents the net effective resistance of the circuit
R_m represents the minimum allowable circuit resistance
R_t represents the total resistance of the circuit
S_c represents the selective coefficient
s represents the velocity of light
T_c represents the time constant of the circuit $\left(\frac{R}{2L}\right.$ Simple series circuit$\left.\right)$
T_e represents the length of the time interval of excitation
T_i represent the length of the time interval of a voltage impulse
X represents reactance
Z represents impedance
λ represents the wave length
γ represents $\frac{R-N}{R}$, the resistance reduction factor of the neutralizer when associated with a circuit of resistance R.

ELECTRIFICATION OF ITALIAN RAILWAYS

In a report recently received from P. Tuccimei, official correspondent of the National Association of Italian Engineers for the States of North America, considerable progress is indicated in the electrification of the Italian railways and the use of hydroelectric power. It is stated that within the very near future more than one-tenth of the railroads of Italy will have been electrified.

The report begins by giving figures noting that by "electrified miles" is meant the total mileage of electrified road, including double tracks and yard tracks.

At the end of August 1920 the electrified roads aggregated 524 miles.

Between September 1st and June 30th 1921, five lines were electrified with an aggregate length of 289 miles.

Within the first six months of 1922 five more lines will be electrified with a total of 329 miles.

The electrification of the following lines has already been started: Roma-Tivoli; Roma-Nettuno; Sestri Levante-Livorno; Genova-Ovada-Alessandria: Total mileage, 669.

The result of these past and future accomplishments are noteworthy. The saving in fuel alone to June 30th, 1921 has reached the 160,000 tons. It is estimated that from July 1st 1922 the saving in coal will reach the 200,000 lire a day, or over 70 millions lire a year.

For the lines electrified prior to August 1920 the only power house operated by the State Railways was that of Morbegno, with about 5000 h. p. of installed power and about 3800 h. p. actually used.

The rest of the electric power necessary to operate these lines was supplied on contract by the Dinamo, Edison, Maira and Negri companies.

Some time ago the second section of the Superior Council of Waters approved in its general outline an extremely important scheme of contract for the supply of electric power and for the electrification of the Bologna-Milano road, to be awarded to private contractors.

Contractors who intend to compete for the work will be invited to bid; and the one making the bid considered technically and financially most convenient will be called to work out the details of the contract. The plans will then be submitted to the Board of Administration of the Railroads for their final approval.

Projects and schemes of contracts for the electrification of the Firenze-Empoli-Pisa line, and of part of the lines in the Venezia Giulia and Veneto districts, are also almost ready to be submitted for approval to the second section of the Superior Council of Waters.

Furthermore, the electric output of the Bardonecchia plant will be increased to 20,000 kw. with a yearly output of over 50 million kw-hr. This result will be attained by completing before the end of this year the Rochemolles canal in addition to the Melezet Canal already in operation.

The hydrographic basins of the Reno and Limentra will both be more intensely exploited by storing and conveying to the power houses in course of construction the waters from other affluents of the Reno and Limentra, with an installed power of 40,000 kw. and a yearly output of 80 millions kw-hr. The Sagittario plant in Abruzzo is also in course of construction and will have an installed power of 20,000 kw. and an output of about 70 millions kw-hr. a year.

The plans for the utilization of the discharge waters of the several plants have not been neglected: For the Bardonecchia plant, for instance, projects are under way and nearly finished for two successive falls at Ouly and Salbertrand. For those of the Reno and Limentra, plans are being studied for three successive drops along the tract of the river between Castrola and Sasso.