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RESISTIVE BALLOONING MODES IN AN AXISYMMETRIC TOROIDAL PLASMA WITH LONG MEAN-FREE PATH

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ABSTRACT

Tokamak devices normally operate at such high temperatures that the resistive fluid description is inappropriate. In particular, the collision frequency may be low enough for trapped particles to exist. However, on account of the high conductivity of such plasmas, one can identify two separate scale lengths when discussing resistive ballooning modes. By describing plasma motion on one of these, the connection length, in terms of kinetic theory the dynamics of trapped particles can be incorporated. On the resistive scale length, this leads to a description in terms of modified fluid equations in which trapped particle effects appear. The resulting equations are analyzed and the presence of trapped particles is found to modify the stability properties qualitatively.

I. INTRODUCTION

The simple resistive magnetohydrodynamic (MHD) model predicts the presence of unstable resistive ballooning modes in toroidal confinement systems with a finite pressure gradient.[1,2] However, this model does not provide an adequate description of the hot plasmas encountered in present experiments. A number of calculations have introduced various aspects of the Braginskii two-fluid equations[3] in order to provide a more realistic model. Thus ion parallel and perpendicular collisional viscosities have been shown to provide a stabilizing influence.[4] Similarly, a fuller treatment of the electron dynamics to include the diamagnetic drift, electron temperature gradient, and the thermal force in Ohm's law, together with parallel thermal conductivity in the electron temperature equation, leads to greater stability at higher temperatures.[5]

However, all these improvements remain within a fluid description of the plasma, valid as long as a particle suffers a collision before completing a transit of the torus - the so-called Pfirsch-Schlüter regime. Unfortunately, typical tokamak devices do not lie in this parameter range, rather they belong to the 'banana' regime where trapped particles can bounce before being scattered by Coulomb collisions. Such a situation requires a full kinetic description of the plasma and that is the subject of this paper.

In the treatment of resistive ballooning modes one can exploit the small parameter ℓ^2/S_R, where ℓ is the mode number and S_R the magnetic Reynolds number, to establish two different scale lengths.[1] In the ballooning representation stability problems reduce, in leading order, to equations defined on a coordinate along the magnetic field line.[6] These two scale lengths then appear as the connection length, reflecting the toroidal periodicity, and a longer length inversely related to the resistive layer. It is then possible to average over the shorter connection length to derive eigenvalue equations defined on the longer resistive scale. This process naturally introduces various averages of toroidally modulated quantities which, when a fluid description is employed, are reminiscent of Pfirsch-Schlüter factors, average curvature, etc.

If one wishes to explore resistive ballooning equations in the banana regimes, a similar averaging process applied to the kinetic equations will generate trapped particle effects in addition to the fluidlike factors mentioned above. Thus neoclassical modifications of the conductivity, perturbed bootstrap currents,[7] etc. can be expected to enter. In this paper we wish to give a systematic treatment of these effects in an arbitrary axisymmetric toroidal geometry based on a gyrokinetic description of the plasma particles. In this way we generalize and place on a firmer basis the ideas of Callen and Shaing.[7]

In order to do this, we introduce an ordering scheme designed to introduce consistently the requisite physical effects and apply this to the solution of the gyrokinetic equations in parallel with Maxwell's equations. This ordering is chosen to introduce diamagnetic effects, perturbed bootstrap currents, and trapped particles, but still corresponds to the collisional fluid limit $\omega\nu_e > k_\parallel^2 v_{Te}^2$ on the long resistive scale. (Here ω and $k_\parallel$ are the mode frequency and wave number parallel to the magnetic field, while ν_e and v_{Te} are the electron collision frequency and thermal velocity.) The resulting eigenvalue equations therefore have a similar form to those from the two fluid models, but with new interpretations of the coefficients which now involve trapped particle effects. (These can greatly exceed the Pfirsch-Schlüter like terms of the fluid model !)

Finally, we discuss the stability properties of the resulting equations.

II. THE GYROKINETIC EQUATIONS AND THE ORDERING SCHEME

The electron and ion distribution functions $f_j (j = i,e)$ can be taken to satisfy the gyrokinetic equations[8]

$$v_\| \underset{\sim}{n}\cdot\nabla g_j - i(\omega - \omega_{Dj})g_j + \oint \frac{d\eta}{2\pi}\, e^{-iL_j} C_j\left(g_j e^{iL_j}\right)$$

$$= \frac{-ie_j}{T_j} F_{Mj}\left(\omega - \omega^{*T}_j\right)\left[J_0(\alpha)\left(\phi - \frac{v_\|}{c}A_\|\right) + J_1(\alpha)\frac{v_\perp}{k_\perp}\frac{\delta B_\|}{c}\right] \quad (1)$$

where

$$f_j = -\frac{e_j\phi}{T_j}F_{Mj} + g_j e^{iL_j} \quad (2)$$

with F_{Mj} a Maxwellian of density n_j and temperature T_j, and ϕ, $A_\|$, and $\delta B_\|$ are the perturbed electrostatic potential and the components of the vector potential and perturbed magnetic field parallel to the equilibrium magnetic field $\underset{\sim}{B}(\underset{\sim}{n} = \underset{\sim}{B}/B)$. High mode-number perturbations have eikonal representations such as $\phi \sim \phi(\underset{\sim}{x})e^{i\ell S(\underset{\sim}{x})-i\omega t}$ where ϕ and S are slowly varying functions of the spatial position $\underset{\sim}{x}$, the mode number is $\ell \gg 1$, and ω is the mode frequency. Equation (1) is expressed in terms of the velocity space variables E, μ, and η where E is the particle energy, μ the magnetic moment per unit mass and η the gyrophase angle. Thus $E = (v_\|^2 + v_\perp^2)/2$ and $\mu = v_\perp^2/2B$ where $v_\|$ and $v_\perp$ are the velocities parallel and perpendicular to $\underset{\sim}{n}$

$$\underset{\sim}{v} = \sigma|v_\||\underset{\sim}{n} + v_\perp(\cos\eta\, \underset{\sim}{e}_\psi + \sin\eta\, \underset{\sim}{e}_b) \quad (3)$$

where σ is the sign of $v_\|$, and we have introduced a set of orthogonal unit vectors $\mathbf{n}$, $\mathbf{e}_\psi$ and $\mathbf{e}_b = \mathbf{n} \times \mathbf{e}_\psi$, with $\mathbf{e}_\psi$ normal to a magnetic surface, ψ = constant. J_0 and J_1 are Bessel functions and the quantities L_j and α_j, representing finite larmor radius (FLR) effects, are given in terms of the gyrofrequency Ω_j and wave vector $\mathbf{k} = \ell \nabla S$ by

$$L_j = \frac{v_\perp}{\Omega_j} (k_\psi \sin\eta - k_b \cos\eta), \quad \alpha_j = \frac{k_\perp v_\perp}{\Omega_j} . \tag{4}$$

The diamagnetic frequency $\omega^*_j = (c\ell T_j/e_j)\ \partial \ln n_j/\partial\psi$ and the relative temperature gradient $\eta^T_j = \partial \ln n_j/\partial \ln T_j$, so that

$$\omega^{*T}_j = \omega^*_j \left[1 + \eta^T_j \left(\frac{m_j E}{T_j} - \frac{3}{2}\right)\right] . \tag{5}$$

The magnetic drift frequency ω_{Dj} takes the form

$$\omega_{Dj} = \frac{1}{\Omega_j} \mathbf{k}_\perp \cdot \mathbf{n} \times \left(\mu \nabla B + v_\parallel^2\ \mathbf{n} \cdot \nabla\ \mathbf{n}\right) \tag{6}$$

and C_j is the Coulomb collision operator for species j. We will characterize it by a collision frequency ν_j.

We use an orthogonal flux coordinate system in configuration space: ψ the poloidal flux acts as a radial coordinate, ζ is the axisymmetric toroidal angle, and χ is a poloidal anglelike variable. In terms of the Jacobian J, major radius R, and poloidal magnetic field B_χ, we have the gradient operator

$$\nabla = \mathbf{e}_\psi\ R\ B_\chi \frac{\partial}{\partial\psi} + \mathbf{e}_\chi \frac{1}{JB_\chi}\frac{\partial}{\partial\chi} + \mathbf{e}_\zeta \frac{1}{R}\frac{\partial}{\partial\zeta} , \tag{7}$$

where $\mathbf{e}_\psi$, $\mathbf{e}_\chi$, and $\mathbf{e}_\zeta$ are unit vectors. In general, the magnetic field can be expressed as

$$\underset{\sim}{B} = -\underset{\sim}{\nabla}\psi \times \underset{\sim}{\nabla}\zeta + I(\psi)\underset{\sim}{\nabla}\zeta \tag{8}$$

with the safety factor

$$q(\psi) = \frac{1}{2\pi} \oint \nu d\chi \;\; ; \;\; \nu \equiv \frac{IJ}{R^2} . \tag{9}$$

In order to discuss the stability of high mode numbers, we introduce the ballooning representation,[6] writing, for example,

$$\phi(\psi,\zeta,\chi) = \sum_{p=-\infty}^{\infty} \bar{\phi}\,(\psi,\zeta,\chi - 2\pi p) \tag{10}$$

where p is an integer. Equation (1) is now to be solved on the infinite domain $-\infty < \chi < \infty$ without the need to consider periodicity constraints - periodicity is guaranteed by the construction Eq. (10), if $\bar{\phi}$ etc. vanish sufficiently fast as $|\chi| \to \infty$ for this sum to exist. Since $\bar{\phi}$ need not be periodic, we can introduce the eikonal representation

$$\bar{\phi} = \phi(\chi)\; e^{i \ell S} \tag{11}$$

with

$$S = \zeta - \int^{\chi} \nu d\chi + \int^{\psi} k(\psi) d\psi \tag{12}$$

so that $\underset{\sim}{n} \cdot \underset{\sim}{\nabla} S = 0$; S is of course a secular function of χ. The boundary conditions for Eq. (1) are that $g \to 0$ as $|\chi| \to \infty$ for circulating particles and the matching of forward and backward streams at the turning points for trapped particles.

To solve Eq. (1) we introduce an ordering scheme constructed to include the desired physical effects. This scheme is based on the small parameter ε, where $\varepsilon^4 = (n^2/S_R)^{1/3}$, which is to be regarded as a technique for bookkeeping physical effects rather than a guarantee of numerical smallness. We introduce the quantities ω_{bj}, the bounce of transit frequency of a particle over a connection length, the FLR parameter $b_j = k_\perp^2 a_j^2$, where a_j is a larmor radius, and θ_0, the characteristic length in ballooning space (in terms of connection lengths) associated with resistive effects,[1] i.e., $\theta_0 \sim \varepsilon^{-4}$. Note that, due to the secularity of k_ψ, $b_j \sim \theta_0^2 b_{j0}$ where $b_{j0} = k_b^2 a_j^2$.

The first condition that we impose is the "banana" regime of collisionality

$$\frac{\nu_j}{\omega_{bj}} < 1 \; . \tag{13}$$

Secondly, we seek a competition between the effects of ohmic currents driven by parallel electric fields on the resistive scale θ_0 and bootstrap currents due to the perturbed pressure gradient. By analogy with the neoclassical transport theory[9] this requires

$$\frac{\omega_{D\psi e}}{\omega_{be}} \sim \frac{\omega_{be}}{\nu_e \theta_0} \; . \tag{14}$$

Since we are considering pressure driven modes, we balance the expansion energy against inertia

$$\omega^2 \theta_0^2 \sim \frac{\omega_* \omega_D}{b_{i0}} , \tag{15}$$

where ω_D does not contain the secularity in $\omega_{D\psi}(\omega_{D\psi} \sim \theta_0\omega_D)$ and $\omega \sim \omega_*$ due to diamagnetic effects. Further we choose an optimal ordering

$$\omega \sim \nu_i b_i \tag{16}$$

to include ion collisional perpendicular transport and viscosity. Now, since $\omega_{D\psi} \sim b^{1/2}\,\omega_{bi}$, condition Eq. (15) requires

$$b_i \sim \left(\frac{\omega_{bi}}{\omega\theta_0}\right)^2 , \tag{17}$$

where as condition Eq. (14) implies

$$b_i \sim \frac{m_i}{m_e}\left(\frac{\omega_{bj}}{\nu_j}\right)^2 \frac{1}{\theta_0^{\;2}} . \tag{18}$$

From conditions Eqs. (16), (17), and (18) we have

$$b_i \sim \left(\frac{\omega_{bj}}{\nu_j}\right)^{2/3} \frac{1}{\theta_0^{\;2/3}} , \frac{\nu_j}{\omega_{bj}} \sim \frac{1}{\theta_o}\left(\frac{m_i}{m_e}\right)^{3/4} . \tag{19}$$

As will be seen, it is convenient to choose $\nu_j \sim \varepsilon\omega_{bj}$ so that $m_e \sim \varepsilon^4 m_i$ and $b_i \sim \varepsilon^2$ (i.e., $b_{i0} \sim \varepsilon^{10}$) from conditions Eq. (19). This implies that the plasma is collisional on the resistive scale length since

$$\frac{\omega\nu_e}{k_\parallel^2\, v_{Te}^2} \sim \frac{\omega\nu_e\theta_0^2}{\omega_{be}^2} \sim \varepsilon^{-2} . \tag{20}$$

In addition, ion sound effects are negligible on the resistive scale

$$\frac{\omega}{k_\parallel v_{Ti}} \sim \frac{\omega}{\omega_{bi}}\,\theta_0 \sim \varepsilon^{-1} . \tag{21}$$

In summary, we solve Eq. (1) with the orderings

$$\frac{\omega_{D\psi i}}{\omega_{bi}} \sim \varepsilon, \quad \frac{\omega}{\omega_{bi}} \sim \varepsilon^3, \quad \frac{\omega_{Di}}{\omega_{bi}} \sim \varepsilon^5, \quad \frac{\nu_i}{\omega_{bi}} \sim \varepsilon, \quad b_i \sim \varepsilon^2 \tag{22}$$

for ions, and

$$\frac{\omega_{D\psi e}}{\omega_{be}} \sim \varepsilon^3, \quad \frac{\omega}{\omega_{be}} \sim \varepsilon^5, \quad \frac{\omega_{De}}{\omega_{be}} \sim \varepsilon^9, \quad \frac{\nu_e}{\omega_{be}} \sim \varepsilon, \quad b_e \sim \varepsilon^6 \tag{23}$$

for electrons. Implied in these orderings is weak curvature $r_n/R \sim \varepsilon^2$, where r_η is the density scale length, which follows from $\omega \sim \omega_*$; thus $\beta < \varepsilon^2$ from the ideal ballooning limit.

In the next section we obtain solutions to the gyrokinetic Eq. (1) in parallel with the quasineutrality condition

$$\sum_j e_j \int d^3v \; f_j = 0 \tag{24}$$

and Maxwell's equations

$$k_\perp^2 A_\parallel = \frac{4\pi}{c} j_\parallel \;, \tag{25}$$

$$i \; k_b \; \delta B_\parallel = \frac{4\pi}{c} j_\psi \;. \tag{26}$$

Although we compute j_ψ directly from f_j, it is more convenient to consider a moment $\sum_j e_j \int d^3v \exp(iL)$ of the gyrokinetic equation Eq. (1) rather than directly evaluate $j_\parallel$

$$\frac{i}{\omega}\,\underset{\sim}{B}\cdot\underset{\sim}{\nabla}\left(\frac{j_\parallel}{B}\right) =$$

$$-\sum_j \frac{e^2}{T}\int d^3v\ F_m\ \left\{\left[1-\left(1-\frac{\omega_*^T}{\omega}\right)J_0^2\right]\phi-\left(1-\frac{\omega_*^T}{\omega}\right)J_0\ J_1\ \frac{v_\perp}{k_\perp}\frac{\delta B_\parallel}{c}\right\}$$

$$+\sum_j\ e\int d^3v\ g\ \frac{\omega_D}{\omega}\ J_0+\frac{i}{\omega}\sum_j\ e\int d^3v\ \sigma\ g\left|v_\parallel\right|\ \underset{\sim}{n}\cdot\underset{\sim}{\nabla}\ J_0 \qquad (27)$$

since this automatically accounts for much cancellation in $j_\parallel$ due to quasineutrality.

We have formulated the problem to account for equilibrium temperature gradients and also perpendicular transport effects. In the present paper we will ignore these complications to emphasize the essentials of banana regime dynamics, but propose to continue the full treatment in a later paper.

III. SOLUTION OF THE GYROKINETIC EQUATIONS

Before solving Eq. (1), it is helpful to introduce an auxiliary function h_j such that

$$g_j = \frac{e_j}{T_j}\left(1-\frac{\omega^*_j}{\omega}\right)\Psi\ F_{Mj} + h_j \qquad (28)$$

where

$$A_\parallel = \frac{c}{i\omega}\,\underset{\sim}{n}\cdot\underset{\sim}{\nabla}\Psi\ . \qquad (29)$$

All quantities are assumed to have a dependence on two scales in the variable χ. Thus we seek solutions for the field quantities of the form Eq. (11) in which

$$\phi(\chi) = \phi^{(0)}(\chi, z) + \frac{1}{z}\phi^{(1)}(\chi, z) + \ \ldots \ , \tag{30}$$

where $\phi^{(0)}$ etc. are regarded as periodic over a connection length χ_0 in χ but have secular dependence on the resistive scale variable $z = \int^{\chi} \nu' d\chi$, $(\nu' \equiv \partial\nu/\partial\psi)$ which has typical scale length $\sim \varepsilon^{-4}\chi_0$.

First we consider the solution of the ion gyrokinetic equation. We expand the function h_i in Eq. (28) in powers of ε, solving for the response of h_i to the field quantities order by order according to Eq. (22). Initially, however, we note that

$$\omega_{Lj} = \frac{m_j c}{e_j} \ell \left[\frac{v_\parallel}{B} \frac{I}{J} \frac{\partial}{\partial\chi}\left(\frac{v_\parallel}{B}\right) z + \mu \frac{\partial B}{\partial\psi} + \frac{v_\parallel^2}{B^2} \frac{\partial}{\partial\psi}\left(4\pi p + \frac{B^2}{2}\right)\right] . \tag{31}$$

Thus

$$\frac{\partial h_i^{(0)}}{\partial\chi} = 0 \tag{32}$$

and

$$\frac{v_\parallel}{JB}\frac{\partial h_i^{(1)}}{\partial\chi} + \frac{i m_i c}{e} \ell z I \frac{v_\parallel}{JB}\frac{\partial}{\partial\chi}\left(\frac{v_\parallel}{B}\right) h_i^{(0)} + C_i\left(h_i^{(0)}\right)$$

$$= - i \ F_{Mi}\Psi^{(0)} \left(1 - \frac{\omega_{*i}}{\omega}\right) \frac{m_i c}{T_i} \ell z I \frac{v_\parallel}{JB}\frac{\partial}{\partial\chi}\left(\frac{v_\parallel}{B}\right) , \tag{33}$$

since $g_i^{(0)}$ and $h_i^{(0)}$ differ only by a perturbed Maxwellian in C_i. Anticipating that we shall be able to demonstrate that $\Psi^{(0)} \equiv \Psi(0)(z)$, the periodicity properties of $h_i^{(1)}$ on the short scale χ produce a solubility condition on h_i (0)

$$\int \frac{JBd\chi}{v_\parallel} C\left(h_i^{(0)}\right) = 0 \tag{34}$$

where the integral $\int$ represents an integration $\oint$ $(JBd\chi/v_\parallel)$ over a connection length for passing particles and $\sum_\sigma \int_{\chi_1}^{\chi_2} (JBd\chi/|v_\parallel|)$, where $\chi_{1,2}$ are the turning point for the trapped particles. Thus

$$h_i^{(0)} = \frac{n_i(z)}{n} F_{Mi} \tag{35}$$

where n is the equilibrium ion and electron density. Integrating Eq. (33)

$$h_i^{(1)} = -\frac{i\, m_i \ell c}{e} z\, I \frac{v_\parallel}{B} \left\{n_i + \frac{e}{T_i}\Psi^{(0)}\left(1 - \frac{\omega_{*i}}{\omega}\right)\right\} F_{Mi} + \bar{h}_i^{(1)}(z) . \tag{36}$$

In next order

$$\frac{v_\parallel}{JB}\frac{\partial}{\partial\chi} h_i^{(2)} + \frac{m_i c}{e} \ell z I \frac{v_\parallel}{JB}\frac{\partial}{\partial\chi}\left(\frac{v_\parallel}{B}\right) h_i^{(1)} + C_i\left(h_i^{(1)}\right) = 0 , \tag{37}$$

and finally

$$\frac{v_\parallel}{JB}\frac{\partial h_i^{(3)}}{\partial\chi} + i\,\frac{m_i c}{e}\,\ell\, z\, I \frac{v_\parallel}{JB}\frac{\partial}{\partial\chi}\left(\frac{v_\parallel}{B}\right) h_i^{(2)} + C_i\left(h_i^{(2)}\right) - i\,\omega\, h_i^{(0)}$$

$$= \frac{-ie}{T_i} F_{Mi}\ (\omega - \omega_{*i}) \left\{\phi^{(0)} - \Psi^{(0)} + \frac{mv_\perp^2}{2e}\frac{\delta B_\parallel^{(0)}}{B}\right\} . \tag{38}$$

An equation for n_i follows from annihilating the terms in Eq. (38) involving $h_i^{(3)}$ and $h_i^{(2)}$ by applying the operator $\oint Jd\chi \int d^3v$. Use is made of integrations by parts in χ, Eq. (37), and conservation of momentum in ion-ion collisions to eliminate the second term on the left-hand side. The result is

$$n_i = \frac{ne}{T_i}\left(1 - \frac{\omega_{*i}}{\omega}\right) \langle\phi^{(0)} - \psi^{(0)} + \frac{T_i}{e}\frac{\delta B_\parallel}{B}\rangle \tag{39}$$

where $\langle A\rangle \equiv \oint JAd\chi/\oint Jd\chi$. Equation (37) leads to the solubility condition

$$\int \frac{JBd\chi}{v_\parallel} C_i\left(h_i^{(1)}\right) = 0 . \tag{40}$$

Since C_i annihilates a Maxwellian, Eq. (36) implies $h_i^{(1)}$ satisfies Eq. (34) and can therefore be absorbed in $h_i^{(0)}$. Consequently, the ions possess a parallel flow velocity

$$u_{\parallel i} = \frac{-i\ell IzT_i c}{eBn}\left\{n_i + \frac{e\psi^{(0)}}{T_i}\left(1 - \frac{\omega_{*i}}{\omega}\right)\right\} \tag{41}$$

which provides a contribution to the fluid inertial energy. Note that this velocity is generated by the trapped ions - a much smaller velocity exists in the Pfirsch-Schlüter regime.

Now we turn to the electron equation to determine a form of Ohm's law valid in the banana regime. Inserting the ordering Eq. (23),

$$\frac{\partial h_e^{(0)}}{\partial\chi} = 0 , \tag{42}$$

$$\frac{v_\parallel}{JB}\frac{\partial h_e^{(1)}}{\partial\chi} + C\left(h_e^{(0)}\right) = 0 ; \tag{43}$$

so that again we have a solubility constraint to determine $h_e^{(0)}$. Thus

$$h_e^{(0)} = \frac{n_e(z)}{n} F_{Me} , \quad h_e^{(1)} = \bar{n}_e^{(1)}(z) . \tag{44}$$

In next order we repeat Eq. (43) and conclude that

$$\bar{h}_e^{(1)} = 0 \ , \quad h_e^{(2)} = \bar{h}_e^{(2)}(z) \ . \tag{45}$$

The following order yields

$$\frac{v_\parallel}{JB}\frac{\partial h_e^{(3)}}{\partial\chi} - \frac{i m_e c}{e}\, \ell\, z\, I\, \frac{v_\parallel}{JB}\frac{\partial}{\partial\chi}\left(\frac{v_\parallel}{B}\right) h_e^{(0)} + C_e\left(\bar{h}_e^{(2)}\right)$$

$$= -i\, F_{Me}\Psi^{(0)}\left(1 - \frac{\omega_{*e}}{\omega}\right)\frac{m_e c}{T_e}\, \ell\, z\, I\, \frac{v_\parallel}{JB}\frac{\partial}{\partial\chi}\left(\frac{v_\parallel}{B}\right) \ . \tag{46}$$

The solubility condition on $h_e^{(3)}$ shows $\bar{h}_e^{(2)}$ can be absorbed in $h_e^{(0)}$, i.e., $\bar{h}_e^{(2)} = 0$, and thus

$$h_e^{(3)} = \frac{i m_e c}{e}\, \ell\, z\, I\, \frac{v_\parallel}{B}\, F_{Me}\left[n_e - \frac{e\Psi^{(0)}}{T_e}\left(1 - \frac{\omega_{*e}}{\omega}\right)\right] + \bar{h}_e^{(3)} \ . \tag{47}$$

In fourth order we can introduce the resistive scale into the operator $\underline{B}\cdot\underline{\nabla}$

$$\frac{v_\parallel}{JB}\left(\frac{\partial}{\partial\chi} h_e^{(4)} + \nu' \frac{\partial h_e^{(0)}}{\partial z}\right) + C\left(h_e^{(3)}\right) = 0 \ . \tag{48}$$

The solubility conditions on $h_e^{(4)}$ are

$$\left\langle\frac{\nu'}{J}\right\rangle \frac{\partial n_e}{\partial z} F_{Me} + \left\langle\frac{B}{v_\parallel} C\left(h_e^{(3)}\right)\right\rangle = 0 \qquad \text{for passing particles;}$$

and (49)

$$\sum_\sigma \int_{\chi 1}^{\chi 2} \left(JB d\chi / |v_\parallel|\right) C\left(h_e^{(3)}\right) = 0 \qquad \text{for trapped particles.}$$

Equations (47) and (49) are analogous to the equations of banana neoclassical theory,[9] with a perturbed pressure gradient across the field playing the role of the equilibrium gradient and a parallel pressure gradient playing the role of the parallel electric field. Although we could invoke the techniques of neoclassical theory to solve Eq. (49) for the full electron collision operator, we restrict ourselves for simplicity to a Lorentz collision operator. Allowing for the nonzero ion velocity given by Eq. (41), we have

$$C_{ei}\left(h_e^{(3)}\right) = \nu_{ei}\left(v_\parallel \frac{\partial}{\partial\mu}\frac{\mu v_\parallel}{B}\frac{\partial}{\partial\mu} h_e^{(3)} + \frac{m_e}{T_e} u_{\parallel i}\, v_\parallel\, F_{Me}\right) . \qquad (50)$$

From Eqs. (47), (49), and (50) we can solve for $\bar{h}_e^{(3)}$ and compute

$$u_{\parallel e} = \frac{ic\ell\, T_e}{e\, Bn} I\, z\left[n_e - \frac{e\Psi^{(0)}}{T_e}\left(1 - \frac{\omega_{*e}}{\omega}\right)\right] - \frac{1}{\nu_{ei}}\left\langle\frac{V'}{J}\right\rangle \frac{\partial n_e}{\partial z}\frac{T_e}{m_e}\frac{B}{\langle B^2\rangle}\frac{K}{n}$$

$$- \frac{ic\ell\, T_e\, Iz}{e\ n}\left[n_e + \frac{e\Psi^{(0)}}{T_e}\left(\frac{\omega_{*e} - \omega_{*i}}{\omega}\right) + \frac{T_i}{T_e} n_i\right] \frac{B}{\langle B^2\rangle} , \qquad (51)$$

where

$$K = \frac{3}{4}\langle B^2\rangle \int_0^{B^{-1}_{max}} \left[\lambda d\lambda/\langle(1-\lambda B)^{1/2}\rangle\right] . \qquad (52)$$

The effects of trapped particles are proportional to 1 - K. Finally, in fifth order

$$\frac{v_\parallel}{JB}\frac{\partial h_e^{(5)}}{\partial\chi} + C\left(h_e^{(4)}\right) - i\,\frac{m_e c\ell}{e} zI\, \frac{v_\parallel}{JB}\frac{\partial}{\partial\chi}\left(\frac{v_\parallel}{B}\right) h_e^{(2)} - i\omega h_e^{(0)}$$

$$= \frac{ie}{T_e} F_{Me}\left(\omega - \omega_{*e}\right)\left[\phi^{(0)} - \psi^{(0)} - \frac{m_e v_\perp^2}{2e}\frac{\delta B_\parallel^{(0)}}{B}\right] \qquad (53)$$

so that the solubility condition on $h_e^{(5)}$ determines n_e as in Eq. (39)

$$n_e = -\frac{ne}{T_e}\left(1 - \frac{\omega_{*e}}{\omega}\right)\langle \phi^{(0)} - \Psi^{(0)} - \frac{T_e}{e}\frac{\delta B_\parallel^{(0)}}{B}\rangle . \quad (54)$$

Implicit in Eqs. (39), (41), (51), and (54) is the form of Ohm's law appropriate to the banana regime; we will present it in the next section where we analyze Maxwell's equations.

IV. MAXWELL'S EQUATIONS AND THE EIGENVALUE EQUATION

With the information to the distribution functions gleaned in the previous section we can compute the results of Maxwell's equations and derive an eigenvalue equation.

From the quasineutrality condition, Eq. (24), we obtain a simple result. Using the definitions, Eqs. (2) and (28) for the distribution functions and the solutions Eq. (35) with Eq. (39) and Eq. (44) with Eq. (54), it is evident that quasineutrality implies $\phi^{(0)}(\chi,z) \equiv \phi^{(0)}(z)$ if our similar assumption about $\Psi^{(0)}$ is correct.

Now considering the perpendicular current equation, we use Eqs. (2), (28), (35), (39), (44), and (54) to calculate j_ψ. Equation (26) then implies

$$\frac{\delta B_\parallel^{(0)}}{B} = \frac{4\pi}{B^2}\sum_j \left[\frac{\omega_{*j}}{\omega} n\, e_j\, \phi^{(0)} - \left(1 - \frac{\omega_{*j}}{\omega}\right) nT_j \left\langle\frac{\delta B_\parallel^{(0)}}{B}\right\rangle\right] . \quad (55)$$

Thus $\delta B_\parallel^{(0)}/B \sim O(\beta) < \varepsilon^2$ and we can simplify the result Eq. (55), using the definition (5) for ω_{*j}:

$$\frac{\delta B_\parallel^{(0)}}{B} = \frac{4\pi \ell p' c}{\omega B^2}\phi^{(0)} \quad (56)$$

where we have defined $p' = (T_i + T_e)\,\partial n/\partial\psi$.

At this point it is convenient to give an explicit form for Ohm's law using Eqs. (39), (41), (51), and (54) now that we can ignore contributions from $\delta B_{\parallel}^{(0)} \sim \varepsilon^{2}$.

$$\frac{j_{\parallel}}{B} = \frac{ic^2\ell^2 I\, z\, p'\,\phi^{(0)}}{\omega}\left(\frac{1}{B^2} - \frac{K}{\langle B^2\rangle}\right)$$

$$+ \frac{\langle V'/J\rangle}{\langle B^2\rangle}\left(1 - \frac{\omega_{*e}}{\omega}\right)\frac{K}{\eta_{\parallel}}\frac{d}{dz}\left(\phi^{(0)} - \Psi^{(0)}\right) \qquad (57)$$

where $\eta_{\parallel} = m\nu_{ei}/ne^2$ is the parallel resistivity in the Lorentz model. The first term in $j_{\parallel}$ is the perturbed bootstrap current proportional to the fraction of trapped particles while the second term illustrates the role of trapped particles in reducing the parallel conductivity.

We can determine the relationship between $\phi^{(0)}$ and $\Psi^{(0)}$ through Maxwell's Eq. (25) which takes the form

$$j_{\parallel} = \frac{c^2\ell^2}{4\pi i\omega}\frac{|\nabla s|^2}{JB}\frac{\partial\Psi}{\partial\chi} \; . \qquad (58)$$

In leading order we verify our assumption that $\Psi^{(0)}(\chi, z) \equiv \Psi^{(0)}(z)$, while in next order we obtain a solubility condition

$$\left\langle\frac{j_{\parallel}B}{|\nabla s|^2}\right\rangle = 0 \; . \qquad (59)$$

Using the form Eq. (57) for $j_{\parallel}$, we obtain

$$\frac{d\Psi^{(0)}}{dz}\left[1 + \frac{\eta_{\parallel}c^2\ell^2}{4\pi i(\omega-\omega_{*e})K}\frac{\langle B^2\rangle}{\langle B^2/|\nabla s|^2\rangle}\right] =$$

$$\frac{d\phi^{(0)}}{dz} + \frac{i\ell^2 Izp'\phi^{(0)}\eta_{\parallel}c^2}{(\omega-\omega_{*e})\;\langle V'/J\rangle K}\left[\frac{\langle B^2\rangle\langle 1/|\nabla s|^2\rangle}{\langle B^2/|\nabla s|^2\rangle} - K\right] \; . \qquad (60)$$

Now we turn to the vorticity Eq. (27), expanding the Bessel functions for $\alpha << 1$ and performing the integrals over Maxwellians. The first nontrivial order yields

$$\frac{1}{4\pi}\frac{\partial}{\partial\chi}\left[\frac{|\nabla s|^2}{JB^2}\left(\frac{\partial\Psi^{(1)}}{\partial\chi} + \nu'\frac{d\Psi^{(0)}}{dz}\right)\right] = -zIp'\phi^{(0)}\frac{\partial}{\partial\chi}\left(\frac{1}{B^2}\right) \tag{61}$$

where we have evaluated the dominant contribution to $\int d^3v\, \omega_D g$ arising from the secular term in ω_D, using the results Eqs. (35), (39), (44), and (54). Integrating with respect to χ yields

$$\frac{1}{4\pi}\frac{|\nabla s|^2}{JB^2}\left(\frac{\partial\Psi^{(1)}}{\partial\chi} + \nu'\frac{d\Psi^{(0)}}{dz}\right) = \frac{-zIp'}{B^2}\phi^{(0)} + \bar{\Psi}(z) \tag{62}$$

where the constant of integration $\bar{\Psi}(z)$ can be determined from the solubility condition for $\Psi^{(1)}$. Equation (62) then yields an equation for $\partial\Psi^{(1)}/\partial\chi$, which is a quantity required in the final order of the vorticity equation

$$\begin{aligned}\frac{\ell^2}{4\pi\omega}\frac{1}{J}\Big[&\frac{\partial}{\partial\chi}\left(\frac{|\nabla s|^2}{JB^2}\frac{\partial\Psi^{(2)}}{\partial\chi}\right) + \frac{\partial}{\partial\chi}\left(\frac{|\nabla s|^2}{JB^2}\nu'\frac{\partial}{\partial z}\Psi^{(1)}\right)\\ &+ \nu'\frac{\partial}{\partial z}\left(\frac{|\nabla s|^2}{B^2}\frac{\partial\Psi^{(1)}}{\partial\chi}\right) + \nu'\frac{\partial}{\partial z}\left(\frac{|\nabla s|^2}{JB^2}\nu'\frac{d\Psi^{(0)}}{dz}\right)\Big]\\ &= -\ell^2\frac{|\nabla s|^2}{B^2}m_i n\,(\omega-\omega_{*i})\,\phi^{(0)} - \frac{2\ell^2}{\omega B^2}p'\frac{\partial}{\partial\psi}\left(4\pi p + \frac{B^2}{2}\right)\phi^{(0)}\\ &+ \frac{\ell z I}{c}\sum_j\int d^3v\,\frac{mv_\parallel}{B}\frac{1}{J}\frac{\partial}{\partial\chi}\left(\frac{v_\parallel}{B}\right)g' \ .\end{aligned} \tag{63}$$

Here g' represents corrections to $g^{(0)}$ which couples to the secular geodesic part of ω_D and competes with the contributions from $g^{(0)}$ and the remainder of

ω_D; i.e., the effects of normal curvature. The eigenvalue equation manifests itself as the solubility condition for $\Psi^{(2)}$ in Eq. (63).

First we note that the definition of h, Eq. (28), implies that the contribution of the last term to the solubility condition is

$$\frac{\ell zI}{c} < \sum_j \int d^3v \frac{m_j v_\parallel}{BJ} \frac{\partial}{\partial\chi} \left(\frac{v_\parallel}{B}\right) g'_j >$$

$$= \frac{I\ell^2 p' z}{c\omega} < \frac{1}{JB^2} \frac{\partial\Psi^{(1)}}{\partial\chi} > - \frac{\ell zI}{c} <\sum_j \int d^3v \frac{m_j v_\parallel}{B} \frac{v_\parallel}{JB} \frac{\partial h_j}{\partial\chi} > \tag{64}$$

where we have integrated by parts in χ. The term in $\partial\Psi^{(1)}/\partial\chi$ can be evaluated from Eq. (62). Using the kinetic equation for h, correct to fourth order in the expansion parameter ε, and taking account of total momentum conservation in collisions between species, we can write the last term as

$$-i < \sum_j \int d^3v \frac{m_i v_\parallel}{B} \left[\frac{v_\parallel}{JB} \nu' \frac{\partial h^{(0)}}{\partial z} - i\omega h_j^{(1)} + i\omega_{Dj} \sum_{k=1}^{3} h_j^{(k)}\right] > . \tag{65}$$

In deriving this result we have used the facts that the ion-ion collision term annihilates a shifted Maxwellian and conserves ion momentum. As discussed earlier, we have also ignored perpendicular transport effects arising from the ion collision operator. The last term in Eq. (65) can be shown to vanish by successive integrations by parts and use of Eqs. (36), (37), and (38), again noting that the ion collision term annihilates a shifted Maxwellian. Consequently, we can express Eq. (65) as

$$-\frac{\ell zI}{c} \left[m_i n\omega i < \frac{u_{\parallel i}}{B} > - < \frac{\nu'}{JB^2} > \sum_j T_j \frac{\partial n_j}{\partial z}\right] \tag{66}$$

or, introducing the results for $u_{\parallel i}$ and n_j given in Eqs. (39), (41), and (54),

$$- \ell^2 z^2 I^2 m_i n(\omega - \omega_{*i}) \left< \frac{1}{B^2} \right> \phi - \frac{\ell^2 I}{\omega} \left< \frac{\nu'}{JB^2} \right> p' z \frac{d}{dz} \left(\phi^{(0)} - \psi^{(0)}\right) . \quad (67)$$

When Eq. (41) with the result Eq. (54) is used in expression Eq. (65), we obtain a modification to the inertial term arising from the polarization drift in Eq. (63), i.e., the first term on the right-hand side.

The solubility condition on Eq. (63) together with Eqs. (60), (62), and (67) can be combined to obtain, after considerable algebra, the desired eigenvalue equation. Introducing the scaled variables

$$X = Z_0\, z \,, \quad Q = -\, i\, Q_0\, \omega \quad (68)$$

where

$$Z_0 = \left(\frac{c^2 \eta_\| \ell^2 \langle B^2 \rangle}{4\pi \langle B^2/R^2 B_\chi{}^2 \rangle}\right)^{1/3} \left[\frac{4\pi m_i n \langle R^2 \rangle \langle B^2/R^2 B_\chi{}^2 \rangle}{\left(4\pi^2 dq/dV\right)^2}\right]^{1/6} ,$$

$$Q_0 = \left(\frac{c^2 \eta_\| \ell^2 \langle B^2 \rangle}{4\pi \langle B^2/R^2 B_\chi{}^2 \rangle}\right)^{-1/3} \left[\frac{4\pi m_i n \langle R^2 \rangle \langle B^2/R^2 B_\chi{}^2 \rangle}{\left(4\pi^2 dq/dV\right)^2}\right]^{1/3} , \quad (69)$$

with V representing the volume within a flux surface so that $\langle \nu'/J \rangle = 4\pi^2\, dq/dV$, we obtain

$$\frac{d}{dX} \frac{X^2}{\Gamma} \frac{d\phi^{(0)}}{dX} + L(1 - T) \left(1 - \frac{1}{\Gamma}\right) X \frac{d\phi^{(0)}}{dX}$$

$$= \phi^{(0)} \left\{X^2 Q Q_i - D_R + \frac{H}{\Gamma}\left(H + 1 - \frac{2}{\Gamma}\right) - L\left(1 - \frac{1}{\Gamma}\right)\left[H + T\left(L - 1 + H - \frac{2}{\Gamma}\right)\right]\right\} . \quad (70)$$

Here Q_j are the scaled versions of $\omega - \omega_{*j}$, $\Gamma = 1 + X^2/Q_e$, and H and D_R have been previously defined by Glasser, Greene, and Johnson in their study of resistive modes in a torus[10]; in particular $D_R > 0$ implies instability for resistive interchange modes in a torus. We have introduced the new quantities

$$L = \frac{Ip'}{\pi(dq/dV)} \frac{\langle B^2/R^2B_X{}^2\rangle}{\langle B^2\rangle} \tag{71}$$

and

$$T = 1 - K \quad , \tag{72}$$

which is proportional to the fraction of trapped particles. It is to be expected that T will dominate D_R, H, and L since, for example, in a large aspect ratio tokamak $T \propto (a/R)^{1/2}$ whereas the other quantities are of order $(a/R)^2$ (a is the minor radius). Trapped particle effects manifest themselves in two ways. First, they modify the conductivity and introduce a perturbed bootstrap current through Eq. (60). Second, they produce anisotropies in the perturbed pressure producing the contributions from Eq. (67), which are absent in a collisional fluid description. Formally, we can recover the resistive MHD limit by letting $L, T \to 0$.

V. RESISTIVE STABILITY

In this section we discuss the stability properties of the modes determined by the eigenvalue Eq. (70). Solutions of this equation must converge as $|X| \to \infty$ and connect onto solutions of the ideal ballooning equation as $|X| \to \infty$.

Let us consider the behavior of Eq. (70) as $X \to 0$. We obtain, reverting to the variable z,

$$\frac{d}{dz} z^2 \frac{d\phi^{(0)}}{dz} + \phi^{(0)} D = 0 \qquad (73)$$

where D = E + F + H is the quantity that characterizes the Mercier stability criterion $D < 1/4$. The solution of Eq. (73) is

$$\phi = C_1 z^{d_+} + C_2 z^{d_-} \qquad (74)$$

where

$$d_{\pm} = -\frac{1}{2} \pm [(\frac{1}{4} - D)^{1/2}] \qquad . \qquad (75)$$

The coefficients C_1 and C_2 must be obtained by numerical solution of the ideal ballooning equation, which symptotically approaches the form Eq. (74) as $z \to \infty$[4].

The solution Eq. (74) provides a boundary condition at X = 0 for Eq. (70) which itself will require numerical solution in general. In this paper we investigate resistive modes which are independent of the matching procedures, i.e., are not driven by the energy available in the ideal ballooning region $X \ll 1$. Rather we focus on the modes dominated by their behavior in the region $X \gtrsim 1$. These are rapidly growing modes with $\gamma \propto (n^2/S_R)^{1/3}$, analogues of the resistive interchange modes of resistive MHD.

By considering the case where D_R and T are small, it is possible to obtain analytic results. Formally ordering $D_R \sim T /\sim \delta \ll 1$, we seek solutions in the region $X^2/Q \sim \delta^{-1} \gg 1$[1], i.e., essentially electrostatic perturbations. Equation (70) then simplifies to

$$\Omega_e \frac{d^2}{dX^2}\phi^{(0)} + L\left(1 - T - \frac{\Omega_e}{X^2}\right) X \frac{d\phi^{(0)}}{dX}$$

$$= \phi^{(0)} \left\{X^2\Omega\Omega_i - D_R - L\left[H + T\left(L - 1 + H\right)\right] + \frac{\Omega_e}{X^2} H\left(H + 1 + L\right)\right\} \tag{76}$$

where it is necessary to expand to $O(\delta)$.

We now demonstrate instability in the limit $H = 0$. [A full solution of Eq. (76) is presented in the Appendix.] In this case, Eq. (76) has a solution

$$\phi^{(0)} = e^{-\hat{\alpha}X^2/2} \tag{77}$$

provided that

$$\hat{\alpha}\Omega_e(L - 1) = - D_R - L(1 - 1)T ,$$

$$\hat{\alpha}^2\Omega_e - \hat{\alpha}L(1 - T) = \Omega\Omega_i . \tag{78}$$

Thus we have an eigenvalue equation

$$\left[D_R + L\left(L - 1\right)T\right] \left[D_R + L\left[L - 1\right)\right] = \Omega\Omega_i\Omega_e\left(L - 1\right)^2 \tag{79}$$

provided

$$\mathrm{Re}\,\hat{\alpha} > 0 . \tag{80}$$

Recalling that $L \sim 1$, $|T| >> |D_R|$, we then have

$$\Omega\Omega_i\Omega_e = L^2 T . \tag{81}$$

Since

$$\hat{\alpha} = \frac{-LT}{|\Omega_e|^2}\,\Omega_e^* \quad , \tag{82}$$

Re $\hat{\alpha} > 0$ for unstable modes (Re $\Omega > 0$) if $LT < 0$, which is the case in a normal tokamak with $p'q' < 0$. Furthermore, $X^2/\Omega_e \sim 1/\hat{\alpha}\Omega_e \sim 1/T \sim \delta^{-1}$ as we assumed in deriving Eq. (76).

In the limit $\omega_* \to 0$ Eq. (81) implies

$$\Omega = \left(L^2 T\right)^{1/3} \; ; \tag{83}$$

whereas if $\omega_* > \Omega\Omega_0$ we find an unstable root with a reduced growth rate

$$\Omega = \left(\frac{\Omega_0}{\omega_*}\right)^2 L^2 T \; . \tag{84}$$

It is interesting to evaluate these results for a large aspect ratio tokamak with circular surfaces and $\beta_p \sim 1$, where $\beta_p \sim = 8\pi p/B_\chi^2$. We find that [11]

$$D_R \sim \frac{\varepsilon\alpha}{s^2} , \quad H \sim \frac{\varepsilon\alpha}{s} \quad , \quad L \sim \frac{\alpha}{s\varepsilon} , \quad T \sim \varepsilon^{1/2} \tag{85}$$

where $\varepsilon = r/R < 1$,

$$\alpha = \frac{-8\pi Rq^2}{B_0{}^2}\frac{dp}{dr} \sim \beta_p\, \varepsilon\, \frac{r}{p}\frac{dp}{dr}$$

and $s = (r/q)\,(dq/dr)$ characterize ideal MHD ballooning instability. Thus we are justified in ignoring D_R and H relative to L and T. In terms of these quantities, Eq. (83) can be written

$$Q \sim \varepsilon^{-1/2} \alpha^{2/3} s^{-2/3} . \tag{86}$$

Recalling the definitions given by Eq. (69)

$$Q_0 \sim \varepsilon^{-2/3} s^{-2/3} \left(\frac{S_R}{\ell^2}\right)^{1/3} \tau_A \tag{87}$$

where $S_R = \tau_R/\tau_A$ with $\tau_R \sim r^2/\eta_\parallel c^2$ and $\tau_A = Rq/V_A$, V_A being the Alfvén speed. Thus the growth rate of this mode is given by

$$\gamma \sim \varepsilon^{1/6} \alpha^{2/3} \left(\frac{\ell^2}{S_R}\right)^{1/3} \frac{1}{\tau_A} . \tag{88}$$

It is amusing to compare this with the resistive ballooning mode discussed previously in the collisional Pfirsch-Schlüter regime[12,13] which has a growth rate

$$\gamma \sim \alpha^{2/3} \left(\frac{\ell^2}{S_R}\right)^{1/3} \frac{1}{\tau_A} . \tag{89}$$

Thus they have essentially the same growth rate for reasonable values of ε!

In the Appendix we discuss the solutions of Eq. (70) more fully. It is shown that only the lowest harmonic, Eq. (77), is consistent with the expansion Eq. (76) for finite values of L. Further, for a normal tokamak with $p'q' < 0$ there are no additional unstable solutions of the more exact Eq. (70) (we do not discuss the possibility of matching to the ideal ballooning region).

VI. CONCLUSIONS

We have shown that, even in the banana regime of collisionality, resistive modes in arbitrary axisymmetric toroidal geometry can be described by a fluidlike equation. However, the effects of collisionless particle motion, in particular the dynamics of trapped particles, are represented in the coefficients of these fluidlike equations. This is a consequence of the fact that in the ballooning space resistive stability is governed by equations defined on a long resistive scale, which are obtained by averaging over the collisionless dynamics on the scale of the connection length. The essential contribution of this paper is, indeed, to obtain this effective resistive fluid equation, Eq. (70).

The present discussion has been limited for simplicity to the inclusion of trapped particle and diamagnetic effects. Extensions to include thermal effects, i.e., temperature gradients and thermal transport,[5] are envisaged; likewise the effects of ion-ion collisions. Furthermore, the intimate relationship of this theory to neoclassical transport theory suggests that formulations for arbitrary collision frequency are possible.[14]

The eigenvalue equation describing resistive stability, Eq. (70), has been discussed. In general, this requires numerical solution but, when curvature and trapped particle effects are weak, analytic dispersion relations can be derived. Examination of this eigenvalue equation has indicated the existence of modes driven by trapped particle effects, Eq. (83). The effects of diamagnetic drifts are included and, as shown in Eq. (84), tend to suppress but not stablize this instability. For the particular case of a large aspect ratio tokamak with circular surfaces, we find an instability with a growth rate very similar to that obtained by Gribkov et al.,[12] and Carreras and

Diamond[13] from the resistive MHD equation.

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APPENDIX

In this Appendix we analyze Eq. (76) more completely. With the substitutions

$$\phi^{(0)} = X^{(L-1)/2} \exp\left(\frac{-L(1-T)X^2}{4Q_e}\right)\psi(X) \tag{A1}$$

and the change of variable

$$u = \frac{X^2}{Q_e}\left[QQ_iQ_e + \frac{L^2(1-T)^2}{4}\right]^{1/2} , \tag{A2}$$

we find ψ satisfies Whittaker's equation

$$\frac{d^2\psi}{du^2} + \psi\left[-\frac{1}{4} + \frac{\kappa}{u} + \frac{1}{u^2}\left(\frac{1}{4} - \mu^2\right)\right] = 0 , \tag{A3}$$

where

$$\mu = -\frac{1}{4}(2H + L + 1) ,$$

$$\kappa = \frac{\left[2D_R + 2LH + + L(L-1) + TL(2H+L-1)\right]}{8\left\{QQ_iQ_e + \left[L^2(1-T)^2/4\right]\right\}^{1/2}} . \tag{A4}$$

The condition that this solution is well-behaved is[15]

$$\frac{1}{2} + \mu - \kappa = -m , \quad m \text{ is a non-negative integer} , \tag{A5}$$

which provides the eigenvalue equation

$$\left[D_R + LH + \frac{L}{2}(L - 1) + TL\left(H + \frac{L}{2} - \frac{1}{2}\right)\right]^2$$

$$= \left[\Omega\Omega_i\Omega_e + \frac{L^2}{4}(1 - T)^2\right]\left(4m + 1 - L - 2H\right)^2 . \tag{A6}$$

Equation (79) will be recognized as the special case $H = 0$ and $m = 0$. We note that if $m \neq 0$ we have (with $H = 0$ for simplicity)

$$\Omega\Omega_i\Omega_e = \frac{-2L^2 m(2m+1-L)}{4m+1-L} \sim 1 , \tag{A7}$$

but these eigenvalues are inconsistent with the expansion $X^2/\Omega_e \ll 1$ used in deriving Eq. (76). If we seek modes with $X^2/\Omega_e \sim 1$, then the eigenvalue is determined by the finite quantities H and L in the limit D_R, $T \to 0$. Ignoring FLR effects, setting $H = 0$ for simplicity and introducing $Y^2 = X^2/\Omega$, Eq. (70) becomes

$$\frac{d}{dy}\frac{Y^2}{1+Y^2}\frac{d\phi^{(0)}}{dY} + \frac{LY^3}{1+Y^2}\frac{d\phi^{(0)}}{dY} = \Omega^3 Y^2 \phi^{(0)} . \tag{A8}$$

Numerical solution shows no unstable modes exist for a normal tokamak with $L < 0$.

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