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LABORATORY REPORT

RESONANCE OSCILLATIONS IN A HOT NON-UNIFORM PLASMA

by

Jerald V. Parker

Technical Report No. 20

Nonr 220(13)

May 1963

A REPORT ON RESEARCH CONDUCTED UNDER

CONTRACT WITH THE OFFICE OF NAVAL RESEARCH

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ABSTRACT

The hydrodynamic equations of a hot non-uniform plasma are solved numerically in slab geometry to obtain the resonant frequencies and associated wave functions. The splitting of the various resonances is shown to depend on the parameter: (slab thickness ÷ Debye length).

Resonance Oscillations in a Hot Non-Uniform Plasma

Jerald V. Parker

Until recently the large splittings of the Dattner resonances have remained unexplained, although a small splitting can be shown to result from non-zero temperatures[1].

In a recent letter Weissglas[2] has shown that results in qualitative agreement with observations can be obtained by assuming a non-uniform electron density. In order to facilitate comparison of the theory with experiment it is necessary to calculate the frequencies of the resonances for some approximately correct electron density distributions.

The equations used below to describe the behavior of a non-uniform plasma in the collisionless approximation are the linearized hydrodynamic equations and Maxwell's equations. Since experiments have generally been conducted in plasmas for which one free space wavelength is large compared to the dimensions of the plasma, one can make a quasi-static analysis.

The equations are then

$$\frac{\partial n_1}{\partial t} + \nabla \cdot n_o f(\underset{\sim}{r})\underset{\sim}{V}_1 = 0 \tag{1}$$

$$\frac{\partial \underset{\sim}{V}_1}{\partial t} = \frac{1}{m\, n_o f}\left[- n_1 e\, \underset{\sim}{\mathcal{E}}_o - n_o f\, e\, \underset{\sim}{\mathcal{E}}_1 - \nabla P_1\right] \tag{2}$$

$$\nabla \cdot \underset{\sim}{E}_1 = - \frac{e}{\epsilon_o} n_1 \tag{3}$$

where $n_o f(\underset{\sim}{r})$ is the steady state electron density and where it has been assumed that

$$n = n_o f(\underset{\sim}{r}) + n_1 \qquad f(0) = 1$$

$$\underset{\sim}{V} = \underset{\sim}{V}_1$$

$$\underset{\sim}{\mathcal{E}} = \underset{\sim}{\mathcal{E}}_o + \underset{\sim}{\mathcal{E}}_1$$

$$P = p_o f + P_1$$

Since these waves will propagate adiabatically we can relate p_1 and n_1 as follows:

$$p_1 = \gamma kT\, n_1 \tag{4}$$

Combining equations (1) through (4), assuming an $e^{-i\omega t}$ dependence, and letting $\mathcal{E}_1 = -\nabla\phi_1$

$$\nabla^4\phi_1 - \frac{f'(r)}{\gamma f(r)}\,\nabla^3\phi_1 + \frac{1}{\gamma}\left[\left(\frac{\omega^2}{\omega_{p_o}^2} - f(r)\right)\frac{1}{\lambda_D^2} - \frac{f''}{f} + \left(\frac{f'}{f}\right)^2\right]\nabla^2\phi_1 - \frac{f'}{\gamma\lambda_D^2}\,\nabla\phi_1 = 0 \tag{5}$$

where

$$\omega_{p_o}^2 = \frac{n_o e^2}{m\epsilon_o} \qquad\qquad \lambda_D^2 = \frac{\epsilon_o kT}{n_o e^2}$$

are the plasma frequency and the Debye length at the point $\underset{\sim}{r} = 0$. Despite the complexity of this equation the calculations progress quite easily if one chooses to work in plane geometry.

If one considers the one-dimensional problem illustrated in Figure 1, the following simplifications can be made. First only $\mathcal{E}_z$ exists which

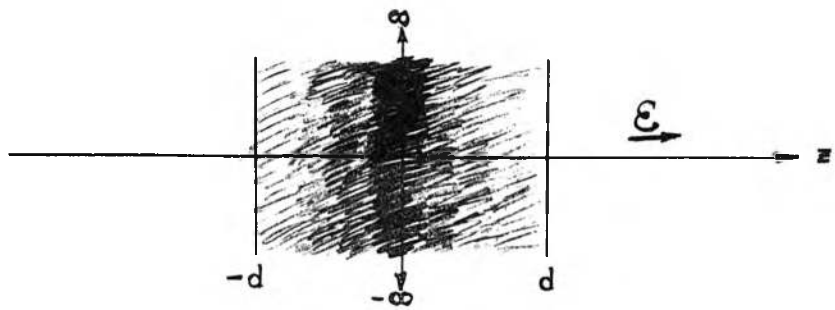

Figure 1.

reduces the order of the equation by one; second, the resulting equation can be integrated once to yield

$$\frac{d^2E_1}{dz^2} - \frac{1}{\gamma}\frac{f'(z)}{f(z)}\frac{dE_1}{dz} + \frac{1}{\gamma\lambda_D^2}\left[\frac{\omega^2}{\omega_{p_o}^2} - f(z)\right] E_1 = K \quad . \tag{6}$$

Using the boundary condition that V_1 normal vanishes at the edge of the plasma, one can evaluate the constant K .

$$K = -\frac{\omega^2}{\omega_{p_o}^2}\frac{1}{\gamma\lambda_D^2} E_1(d) \tag{7}$$

The final simplification results if one changes to the dimensionless variable $s = z/d$ and recognizes that a resonance in this plane geometry is characterized by the vanishing of the external field so that one need solve only the homogeneous equation

$$\frac{d^2E_1}{ds^2} - \frac{f'}{\gamma f}\frac{dE_1}{ds} - A(B - f)E_1 = 0$$

$$A = \frac{1}{\gamma} \left(\frac{d}{\lambda_D}\right)^2 , \qquad B = \left(\frac{\omega^2}{\omega_{p_o}^2}\right) .$$

The density functions $f(z)$ used in these calculations are taken from the work of S. Self[3] and represent theoretical curves for a low density plasma. They are shown in Figure 2. The small slope discontinuities are due to the approximation used in the calculations.

The results of numerical solution of this equation for several values of the parameter $\left(\frac{d}{\lambda_D}\right)^2$ are shown in Figure 3. The circle points are the results of numerical solution of the differential equation. The triangle points were found using the WKB method since the differential equation is difficult to solve for very short wavelengths. The dotted lines connecting the points are included to show which points belong to the same mode and to indicate the general trend of the frequencies but do not represent actual data. The solid line shows the ratio of mean density to the square of the frequency of the lowest mode $\overline{n_o f} / \omega_o^2$.

In general it may be seen that the splitting of the resonant frequencies is dependent on the parameter (d/λ_D) and the splitting is largest for small values of this parameter.

Figures 4 and 5 show how this strong splitting is induced by the non-uniform plasma density. Both figures show the electric field at resonance for various different modes. Figure 4 shows the modes ω_o, ω_1, ω_2, ω_3 for a fixed value of $(d/\lambda_D)^2$. At a given frequency ω that portion of the plasma for which $\omega^2 > \frac{n(z)e^2}{m\epsilon_o}$ $(z > z_c)$ can propagate plasma waves, while the remainder cannot. As ω is increased the propagating region becomes longer and successively more half wavelengths of the wave can be fit into it.

In Figure 5 the mode ω_o is shown for various values of $(\frac{d}{\lambda_D})^2$. One can see that as $(\frac{d}{\lambda_D})^2$ increases the wavelength of the oscillation decreases. Since the thickness of the sheath and the wavelength are proportional to λ_D the resonant frequency of the mode ω_o, which can propagate primarily in the sheath region, tends to remain constant.

It is conceivable that this effect might cause one or more resonances to remain at frequencies below the lowest plasma frequency $\frac{n_o f(d)\, e^2}{m\epsilon_o}$ (see Fig. 2 for $(d/\lambda_D)^2 = \infty$) in the limit $\lambda_D \to 0$. This would mean that the resonances would remain split even in the limit of zero temperature and that the curves in Figure 3 would tend to a limit > 1 as $(\frac{d}{\lambda_D}) \to \infty$. This does not, however, seem to be the case although one cannot be certain of extrapolations based on such limited data.

I wish to express my appreciation to R. W. Gould and to the Office of Naval Research and the National Science Foundation for supporting this research.

References

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3. S. Self, Physics of Fluids (To be published).

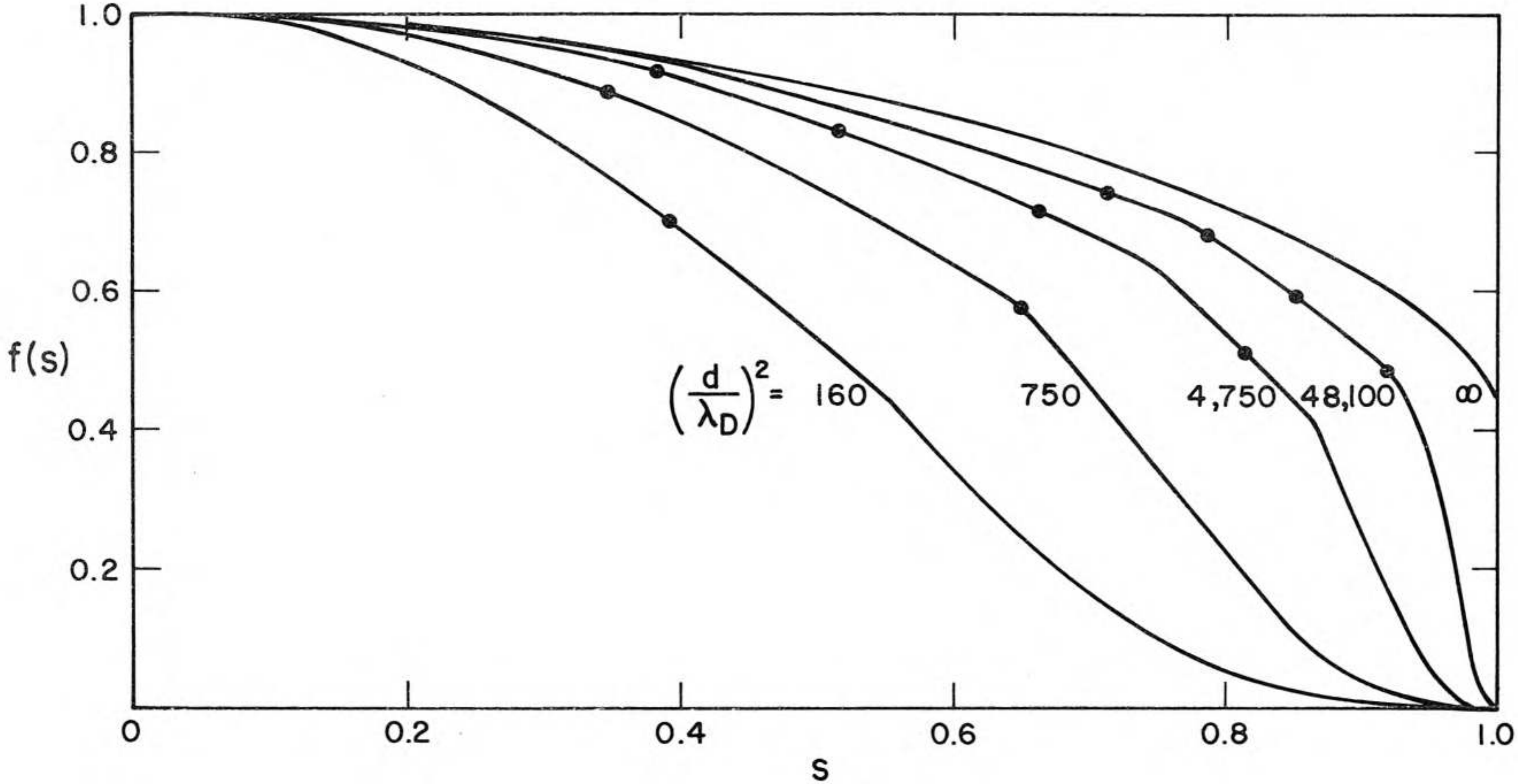

Fig. 2. Electron Density Profiles. The points where the local plasma frequency equals one of the resonant frequencies, i.e., $\frac{n_o e^2}{m\epsilon_o} f(s) = \omega_i^2$ $(i = 0,1,\cdots)$ are indicated.

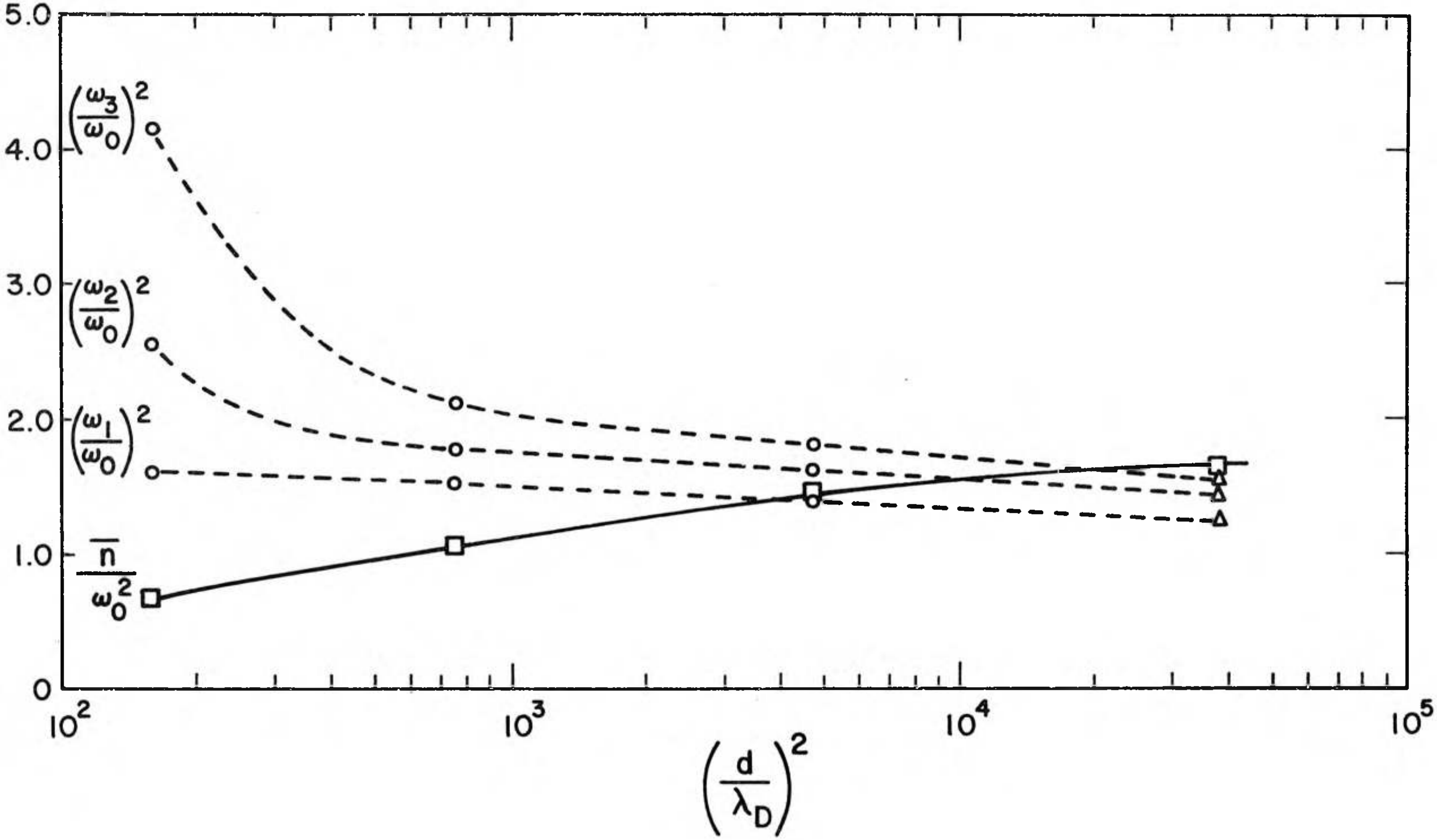

Fig. 3. Resonant Frequencies and Average Density

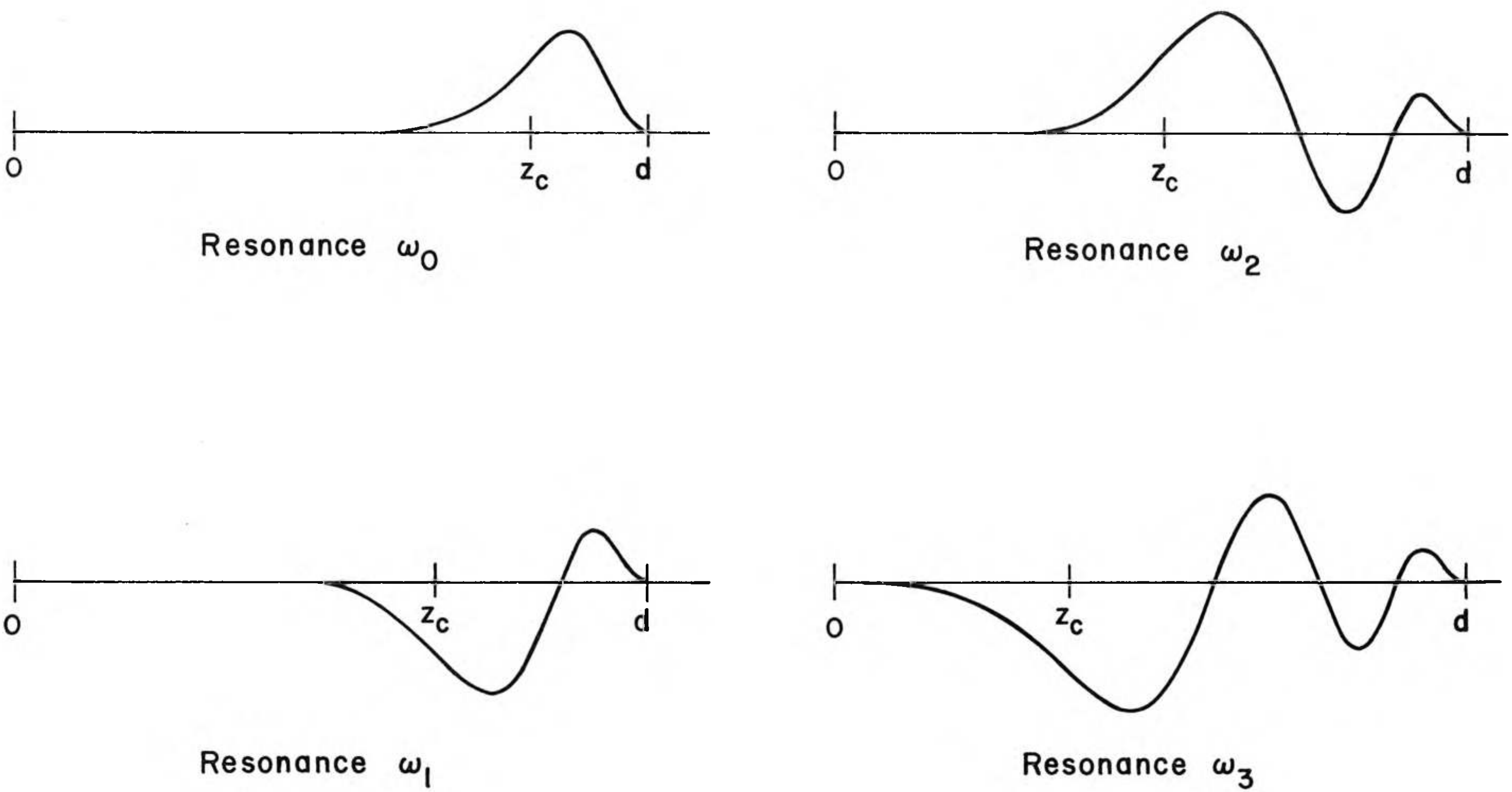

Fig. 4. E_z versus z $(d/\lambda_D)^2 = 4740$

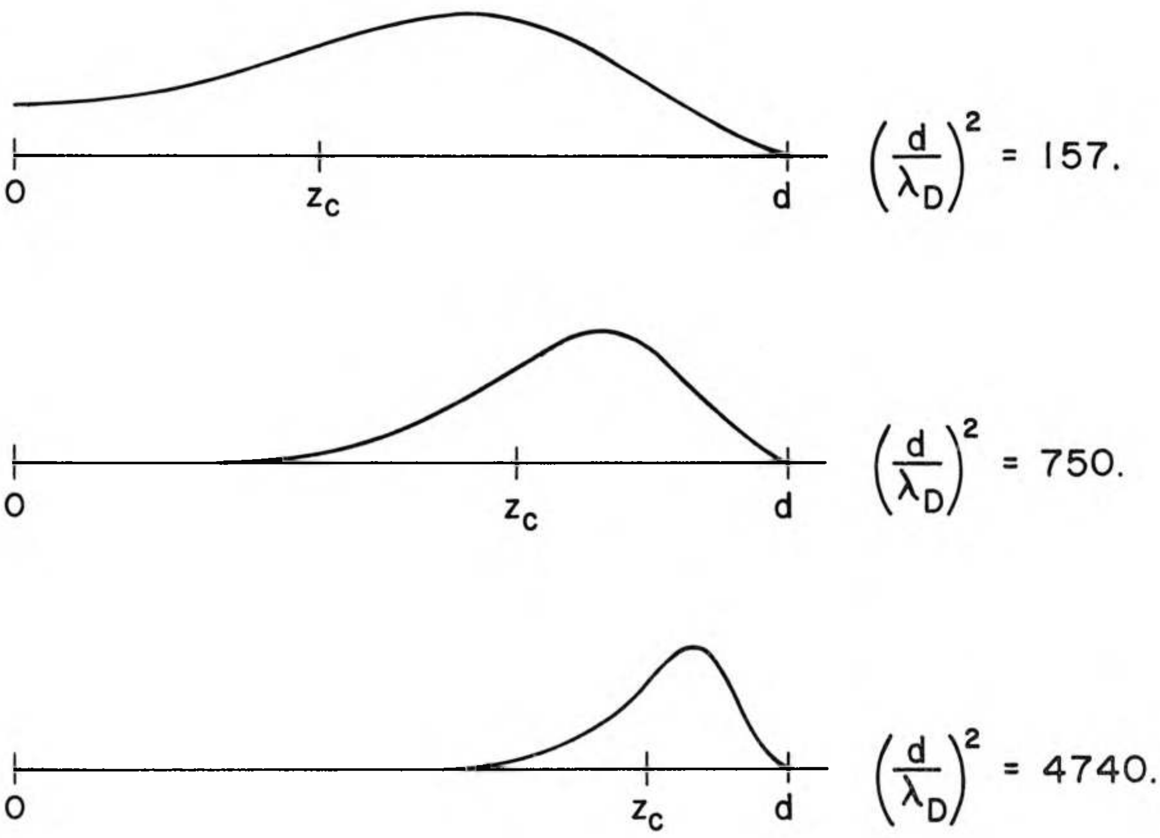

Fig. 5. E_z versus z for the Resonance ω_o