Resonance oscillations of a permanent magnet levitated above granular superconductors

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We have studied the interaction of alternating magnetic fields with granular high temperature superconductors (HTSC) with a magnetization distribution in the HTSC volume. The expressions for resonance frequencies of a permanent magnet levitated above a HTSC sample were found for five oscillation modes. An original method for determination of the superconducting grains' volume fraction is proposed. © *1996 American Institute of Physics.* [S0003-6951(96)02201-4]

There is considerable current interest in systems where a permanent magnet (PM) is levitating above a high temperature superconductor (HTSC) for both practical applications[1,2] as well as for basic research.[3,4] In the latter case the PM is a sensitive detector of structure and macroscopic magnetic properties of granular HTSC.[4,5] In this work we have proposed an original method for determining the volume fraction of superconducting grains by performing the measurement of the resonance frequencies of two modes of PM forced oscillations.

The measurements were performed using the PM-HTSC system that was described in detail in our previous works.[4–7] Figure 1 shows the experimental configuration. Spherical $SmCo_5$ PM of mass $m=0.021$ g, diameter $d=0.16$ cm, and magnetic moment $\mu=1.2$ G cm^3 freely levitated at a distance x_0 above the granular HTSC tablet having a diameter of $D=4$ cm and a thickness of $h=1$ cm. The ratio of sizes ($D>h\gg x_0>d$) allowed the PM to be regarded as a point magnetic dipole resting over the flat semi-infinite granular superconductor.[4] The forced oscillations of the PM were excited by the ac coil located above the magnet so that vectors of the coil magnetic field and its gradient over the PM area did not coincide with x, y, z axes in Fig. 1.

In this work we focused on the granular $YBa_2Cu_3O_{7-x}$ superconductor with $T_c=92$ K which was prepared by a standard sintering method.[8] Typical dimensions of the component grains were 10–20 μm. Two types of samples were used in our experiments. They were the ceramic sample (No. 1) and the composite sample (No. 2). The first one was cut from a sintered superconductor. The composite sample was formed from dispersed grains of superconductors in an insulating paraffin. The dispersed grains were obtained by grinding a piece of Y superconductor. For comparison we have also investigated the ceramic (No. 3) and composite (No. 4) samples that were made of $(Pb_{0.16}Bi_{0.84})_2Sr_2Ca_2Cu_3O_y$ superconductor with $T_c=120$ K.

There are five modes s of PM oscillations in such a PM-HTSC system: three translation modes, $s=x,y,z$ (along the corresponding axes); and two rotation (or torsion) modes, $s=\psi$ and θ (around x and y axes, respectively). Every mode has a resonance frequency ω, damping δ, and a dependency on the PM amplitude A,[5,9] but in this work our attention is focused on extrapolated values of ω when $A\to 0$ (Table I).

Earlier we had studied theoretically the elastic interaction of the PM with granular HTSC[6] and found the expressions for resonance frequencies of PM small oscillations.[7] In those works two assumptions were made: (i) the granular HTSC at 77 K may be considered as a set of isolated superconducting grains with a density α; (ii) the influence of the static magnetization on elastic properties of the PM-HTSC system can be neglected. The first assumption was supported by experimental results for Y samples.[5,10] The same cannot be said of the second assumption. Moreover, the previously derived expressions[7] yield unphysical values of the superconducting volume fraction ($\alpha>1$).[11]

To account for elastic properties of the PM-HTSC system we consider its total free energy in more detail:

$$F(\mathbf{r})=-m\mathbf{gr}-\int_{V_s}d\mathbf{r}'^3\int_0^{H(\mathbf{r}'-\mathbf{r})}\mathbf{M}(\mathbf{H}')d\mathbf{H}', \qquad (1)$$

where $\mathbf{r}$ is the PM position, $\mathbf{H}$ is its magnetic field, $\mathbf{M}(\mathbf{H})$ is the magnetization of HTSC grains, V_s is the superconducting grains' volume. For elasticity k of small PM displacements $\delta\mathbf{q}$ relative to generalized coordinates $q_s=x,y,z,x_0\psi,x_0\theta$, when $\delta\mathbf{B}=\delta\mathbf{H}-4\pi\delta\mathbf{M}=$ const, we may write

$$k_s=G_s\omega_s^2=\frac{\partial^2F}{\partial q_s^2}=\alpha[I_1(s)+I_2(s)], \qquad (2)$$

where

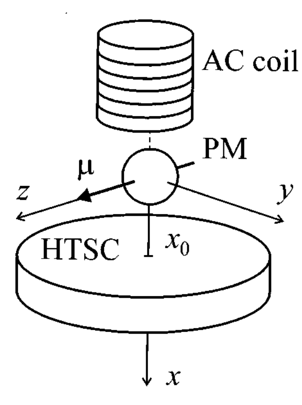

FIG. 1. The experimental configuration.

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TABLE I. The experimental and theoretical parameters of PM-HTSC system for different HTSC samples (No. 1: ceramic Y sample, No. 2: composite Y sample, No. 3: ceramic Bi sample, No. 4: composite Bi sample); x_0: the PM stable equilibrium position; ω: the values of PM resonance frequency for all five modes of its oscillations; α: the superconducting volume fraction; β: the parameter from Eq. (9).

Sample Nos.	x_0, cm	ω_x, Hz (exp.)	ω_y, Hz (exp./theor.)	ω_z, Hz (exp.)	ω_ψ, Hz (exp./theor.)	ω_θ, Hz (exp./theor.)	α	β
1	0.23	24.0	7.0/6.9	12.0	40/41.6	69/72.0	0.76	0.50
2	0.16	34.5	10.7/10.9	18.6	42/44.8	71/74.9	0.27	0.44
3	0.25	21.0	7.2/6.4	11.0	37/41.4	63/70.1	0.90	0.46
4	0.17	29.5	10.6/10.6	18.4	43/47.1	69/73.9	0.29	0.32

$$I_1(s)=\frac{1}{4\pi}\int_V d\mathbf{r}^3\left(\frac{\partial \mathbf{H}}{\partial q_s}\right)^2, \tag{3}$$

$$I_2(s)=-\int_V d\mathbf{r}^3\left(\mathbf{M}\cdot\frac{\partial^2\mathbf{H}}{\partial q_s^2}\right). \tag{4}$$

There $G_x=G_y=G_z=m$, $G_\psi=G_\theta=(1/10)m(d/x_0)^2$, V is the volume of the HTSC sample.

The first integral Eq. (3) is free from any information about physical properties of the specific HTSC sample. Then considering that $\mathbf{H}(\mathbf{r})=-\nabla[(\mu\mathbf{r})/r^3]$, where $\mu=\mu(\sin\theta, \cos\theta\sin\psi, \cos\theta\cos\psi)$, we may evaluate it symbolically

$$I_1(s)=\xi_s\frac{\mu^2}{x_0^5}, \tag{5}$$

where, for $\mu\|z$, $\xi_s=3/16,3/64,9/64,1/16,1/8$ (for $s=x,y,z,\psi,\theta$).

As for the second integral Eq. (4), it depends on the $\mathbf{M}(\mathbf{H})$ function and hence both on equilibrium magnetization $\mathbf{M}_0(\mathbf{H})$ that is a thermodynamic function and on the non-equilibrium part $\Delta\mathbf{M}=\mathbf{M}-\mathbf{M}_0$ that is a consequence of the strong pinning[7] in HTSC.

The experimental parameters presented in Table I were obtained for the PM stable equilibrium position x_0. There is a wide range of quasistable ones in the PM-HTSC systems[6,7] but only one of them is a really stable position from the HTSC thermodynamic standpoint.[5] To carry the PM to the x_0 position we vibrated it with 0.1 cm amplitude for a time. Such drift was investigated early for HTSC samples levitating in a magnetic field.[12]

From the above reasoning for the PM in x_0 position we may believe that

$$\int_V d\mathbf{r}^3\left(\Delta\mathbf{M}\cdot\frac{\partial^2\mathbf{H}}{\partial q_s^2}\right)\approx 0 \tag{6}$$

and magnetization in Eq. (4) is the equilibrium one $\mathbf{M}_0(\mathbf{H})$. Consequently, the second integral in Eq. (2) carries the information mainly about the thermodynamical magnetic properties of the HTSC grains, and we must know the $\mathbf{M}_0(\mathbf{H})$ function for calculating its contribution in Eq. (2).

Just the same, it turned out that the relative values of $I_2(s)$ for the different modes do not depend practically on this function. Figure 2 shows the results of numerically integrating Eq. (4) over the area $(x_0<x<3x_0,\ -3x_0<y<3x_0,\ -3x_0<z<3x_0)$ for $\mathbf{M}(\mathbf{r})=(-1/4\pi)\mathbf{H}(\mathbf{r})\chi[H_c-H(\mathbf{r})]$, where χ is the Heaviside step function. It can be easily seen that the ratios of $I_2(s)$ to $I_2(x)$ do not depend on the H_c parameter over a wide range for the translation modes $(s=y,z)$. We may write

$$I_2(s)=\beta\zeta_s\frac{\mu^2}{x_0^5}, \tag{7}$$

where β is a unitless constant that depends on the HTSC material (and generally, on the PM-HTSC configuration), and ζ_s can be obtained from the following expression

$$\zeta_s\frac{\mu^2}{x_0^5}=\frac{1}{4\pi}\int_V d\mathbf{r}^3\left(\mathbf{H}\cdot\frac{\partial^2\mathbf{H}}{\partial q_s^2}\right), \tag{8}$$

and $\zeta_s=3/16,-3/64,-9/64,-1/16,-1/16$ (for $s=x,y,z,\psi,\theta$).

Then Eq. (2) can be written as follows:

$$G_s\omega_s^2=\alpha(\xi_s+\beta\zeta_s)\frac{\mu^2}{x_0^5}. \tag{9}$$

Thus, only two parameters are necessary to describe an elasticity of the PM translation displacements in the PM-HTSC system. Two resonance frequencies for the x and z modes were chosen. The values of α, β and other frequencies that were obtained from Eq. (9) are presented in Table I for

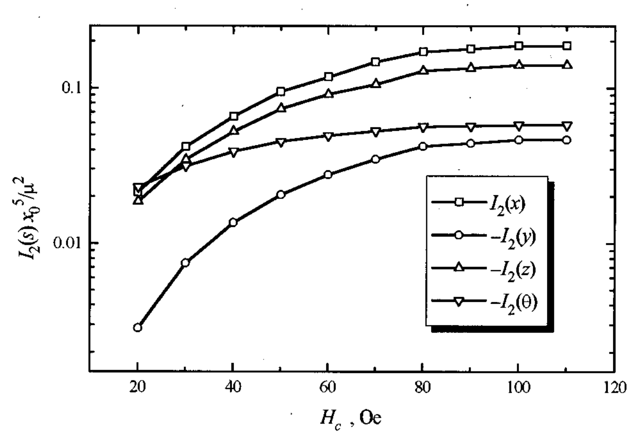

FIG. 2. The evaluation of contribution from intragrain magnetization to elasticity of the PM-HTSC system. The results of numerically integrating Eq. (4) over the area $(x_0<x<3x_0,\ -3x_0<y<3x_0,\ -3x_0<z<3x_0)$ for $\mathbf{M}(\mathbf{r})=(-1/4\pi)\mathbf{H}(\mathbf{r})\chi[H_c-H(\mathbf{r})]$, where χ is the Heaviside step function.

the different HTSC samples. The good agreement between experimental and theoretical values of the resonance frequencies for the *Y* samples (No. 1 and No. 2) and for the composite Bi sample (No. 4) demonstrates the correctness of our assumptions that were made in this work. The small discrepancy between these values for ceramic Bi sample (No. 3) may be explained by the weak intergrain pinning that was observed early for the same one.[13]

In summary, it should be noted that α is the fraction of the magnetic flux that was pinned. Strictly speaking, it is somewhat less than the volume fraction of superconducting grains because nonpinned vortices may exist, but we have every reason to believe that in our experiment (in the limit of small PM amplitudes) these values coincide because of the large intragrain vortex motion viscosity in HTSC.[4,10] In this work our attention was focused on α determination, but the β parameter may be a very useful one when a model of $\mathbf{M}_0(\mathbf{H})$ will be checked.

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